# ALMA Memo No. 545
# Design of the Cone at the Centre of the Subreflector

28th Dec 2005

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## Abstract

The distribution of the power reflected from the subreflector is calculated for a variety of cases. It is found that good suppression of the reflections can be obtained by placing a cone in the centre, but that this does need to be a good deal larger than is called for in the present specification. The size needed depends on whether or not the subreflector is tilted to optimise the gain when we use off-axis feeds. The question of whether the cone should be a separate component or made as an integral part of the subreflector is also discussed.

The recommendation is for a cone of 60mm diameter and for this to be built into the surface of the subreflector. This does however require that we use a slightly different alignment of the subreflector for each receiver. If this cannot be done then the diameter of the cone needs to increase to about 72mm. This larger size will produce a loss in gain which is of order 0.5%. Perhaps more importantly, however, this analysis has revealed that using off-axis feed positions (e.g. bands 5 and 6) without tilting the subreflector is already producing a significant loss of gain, as well as an increase in the spill-over onto the ground, which together will produce a loss in the sensitivity of ALMA of several percent. This means that having the ability to adjust the tilt of the subreflector is highly desirable anyway.

Finally it is noted that there is a significant reflection back to into the focal plane due to diffraction from the outer edge of the subreflector. This can be suppressed by giving this edge a slightly non-circular form and a variety of ways of doing this are discussed.

## Introduction

It has long been planned to modify the centre of the subreflector so that it takes the form of a (very) blunt cone. The purpose of this cone is to suppress the reflection of signals from the receivers back into the focal plane. The most important such signals are: 1) the local oscillator (LO) which, if it is reflected back into the same receiver, will interfere with itself and is likely to produce a loss of stability because of the long path involved; 2) the harmonics of the LO from one receiver getting into another receiver and causing interference, which might be coherent between antennas in the array; and 3) signals from the source being observed, for which the reflections lead to gain modulation – i.e. a ripple with a characteristic period of ~25MHz for the ALMA case. Reflection of receiver noise, which is well known to cause baseline ripple on single dishes, is less of a problem on an interferometer but may be significant when the antennas are used to obtain "zero-spacing" data.

In this context, the water vapour radiometers are also important since they use Schottky mixers with a much higher level of LO than in the astronomical receivers. The WVR's are designed to make the LO's on different antennas deliberately incoherent (they are offset by typically a few hundred kHz) but it will nevertheless be a good idea to minimize the reflections at the relevant frequencies – 183.3, 275, 366.6GHz, etc.

The reflections were analyzed by Aurore Bacmann and Stephane Guilloteau in ALMA Memo 457. They recommended using a cone which is 1.1 to 1.2 times larger than the geometrically blocked area, but they did not calculate the effect on the overall antenna gain. In general the larger the cone the better the cancellation of the reflections, but a very large cone will produce a reduction in antenna gain, so this needs to be calculated to decide how large a cone can be used. Bacmann and Guilloteau also point out that, for a given size, somewhat lower reflections can be obtained by using a cone that is slightly curved – in the sense that it becomes steeper towards the centre – instead of one with a fixed slope. Memo 457 does not deal with the non-axially-symmetric case, which is relevant if one is thinking about signals reflected from one receiver to another or wanting to take account of the fact that the receiver is off-axis, which is important for us.

The current scheme is defined in the Antenna Specification. This gives a diameter of 48mm for the cone, which is only very slightly larger than the geometrical blocking (~46mm). The angle of the cone is given as 88 degrees, but it seems clear that the intention is that the slope should match that of the subreflector at the point where they join, which means that the correct angle is 87.78 degrees. As we shall see, this rather small cone does not do a good job of cancelling the reflections, especially for the receivers which are significantly off-axis, e.g. bands 1 to 6.

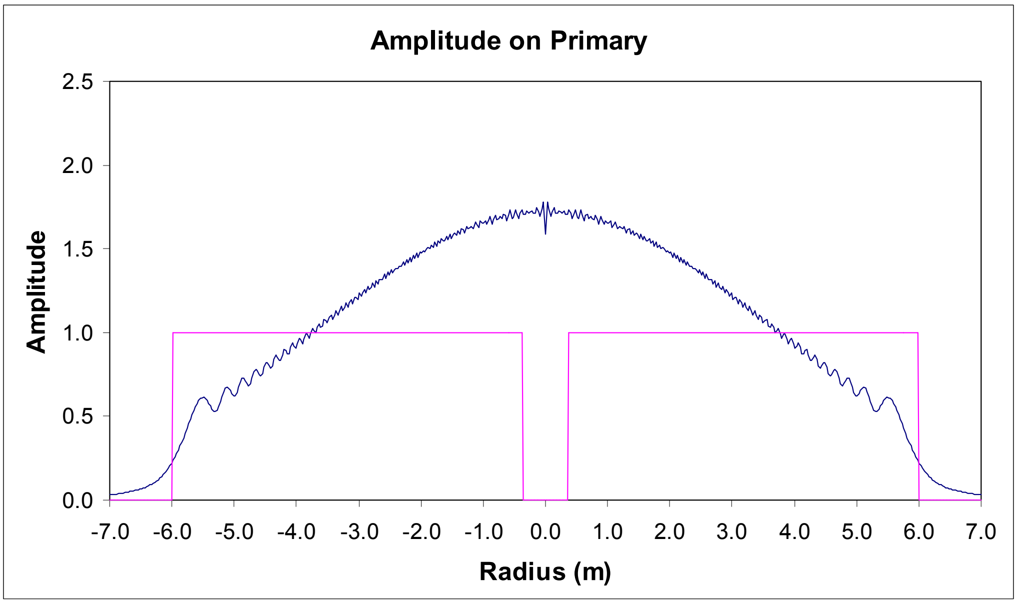
This note presents calculations of the effects of various cones, using physical optics. It should be appreciated that the actual level of the interference or standing waves that are created by the reflections from the subreflector depends on the details of what happens in the receiver system – e.g. the level of the LO signals radiated, the amount of mismatch at the feed, etc. We cannot quantify these things at present, so the considerations here are based entirely on comparisons of the amplitude of the reflection with the cone in place to the amplitude that would be seen with no cone at all – i.e. with a smooth hyperbolic contour. The magnitude of effects like gain ripple due to standing waves will generally be proportional to this amplitude, whereas for interference the power returned will be proportional to the square of the amplitude. It is also worth noting that, all other things being equal, the amplitude of the standing wave will be proportional to the wavelength, so the worst problem is likely be at the lower frequencies. (One way to see this is to note that the effective area of the feed system is proportional to wavelength squared and that the fraction of the power returned by the subreflector that is accepted by the horn is proportional to this area.)

# Calculations

At the relevant wavelengths a diffraction calculation is essential – using Geometrical Optics (GO) only shows what behaviour can be expected in the high frequency limit, which in this case turns out to be above 300GHz. In Memo 457 an analytical diffraction calculation was used, which is accurate on axis but cannot be used to calculate where the scattered energy ends up, which we do need to do. The calculations here employ Physical Optics (PO). In most cases it is sufficient to use the scalar approximation, which means that one evaluates Kirchhoff's integral to find the field at points in the focal plane (or on the primary) by adding up the contributions from a grid of points on the secondary. With this approach it is relatively easy to put in different geometries and it is reasonably fast. With help from Ross Williamson I have, however, set the problem up in GRASP as well and used that to check the results and find the antenna gain and the spill-over. GRASP does a proper vector integration taking account of the currents in the reflectors. This is necessary to model the polarization and edge effects properly. The following description is rather detailed so that people who are interested in such things can understand what is going on. Others can safely skip to the conclusions.

## Smooth Subreflector

We start by looking at the case of a complete subreflector with a hyperbolic profile. We assume a 100GHz feed at the centre of the focal plane (i.e. on axis) with a Gaussian pattern and 12dB edge taper. The amplitude of the illumination on the aperture of the primary then looks like this.

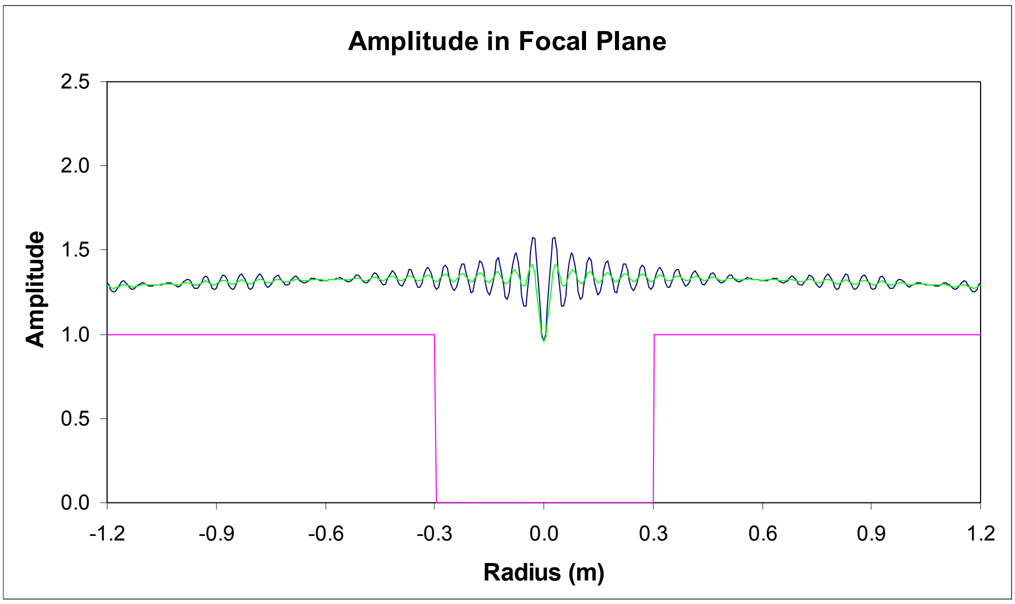



Here the pink line shows both the outer edge of the primary and the central hole in it. The diffraction due to the <u>outer</u> edge of the subreflector shows up in two places:

1) the ripple towards the outside of the primary, which resembles the Fresnel diffraction pattern of a straight edge, and

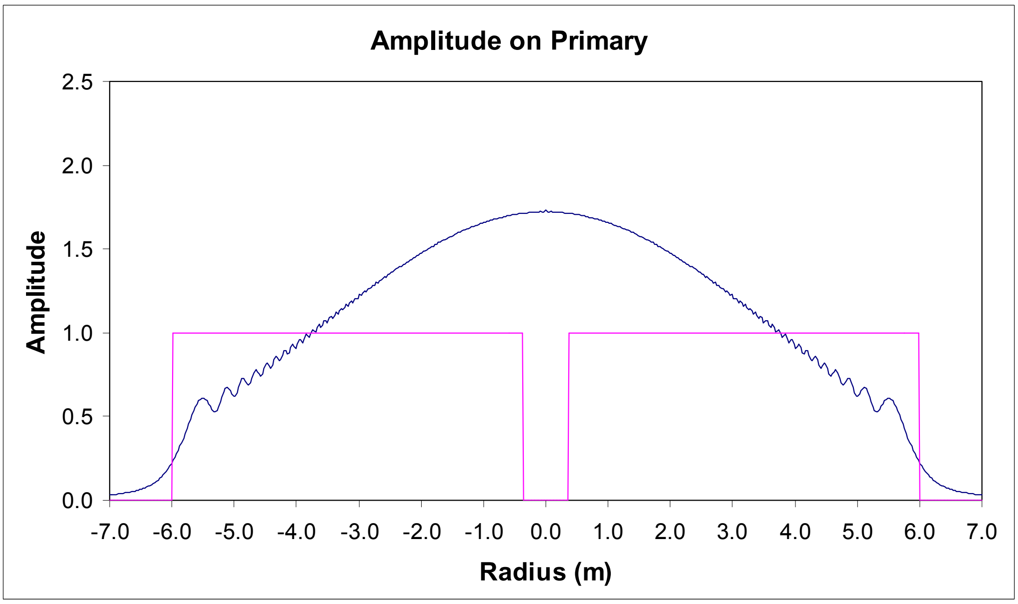
2) the structure in the centre, which is the "Poisson's spot" effect due to the fact that the subreflector is circular so that the diffracted energy all adds up in phase here.

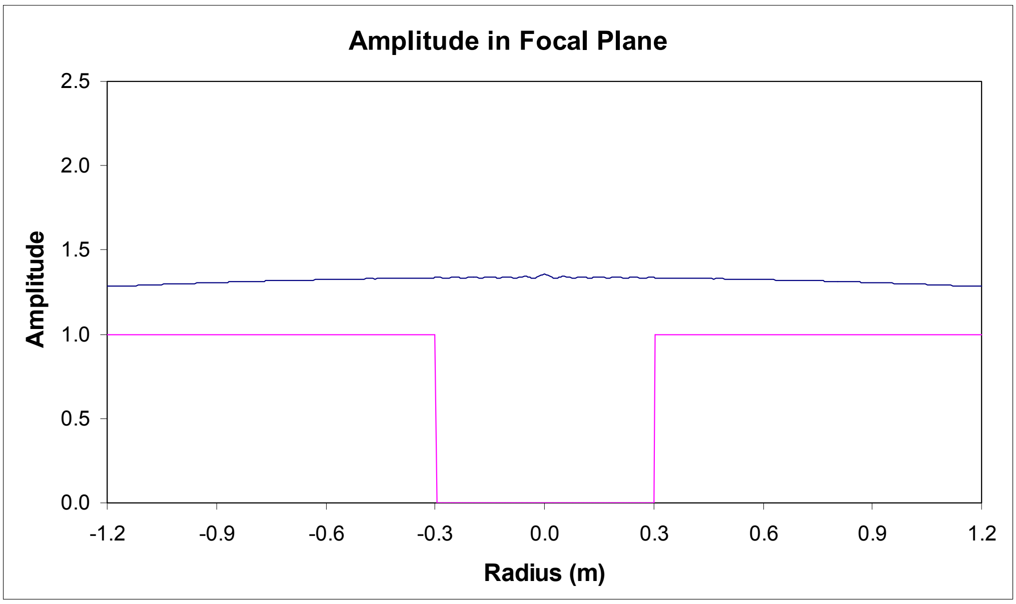
To see the central region in more detail we can examine the signal arriving back at the focal plane (which is 1377mm below the vertex of the primary).




Here the pink line shows the 600mm diameter clear aperture at the focal plane. The blue and green lines show cuts in two directions – blue is the cut perpendicular to the E-plane of the feed and green is parallel to it. The difference between them is due to the way the currents flow perpendicular and parallel to the edges of the subreflector and to the angular dependence of the radiation from those currents. (Here GRASP has been used, which employs PTD – the Physical Theory of Diffraction – to model the edge currents.) The form of the pattern seen is due to the beating of the diffraction from the edge with the signal returned directly from the centre of the subreflector. The path difference between these two signals produces a modulation as a function of frequency in the on-axis reflection coefficient with the period of about 1.2GHz. This can be seen in the plots shown in Memo 457, e.g. figure 6.

We will return to the reflection from the outer edge of the subreflector at the end of this memo, but for the time being we will suppress it, so that the effects of the treatment of the centre of the mirror can be seen clearly. We do this by making the outer edge "fuzzy" – instead of being truncated sharply at a radius of 375mm – the fields at the edge are made to go smoothly from their nominal value to zero as the radius goes from 373 to 377mm. The results for the two polarizations are then essentially identical, and look like this:

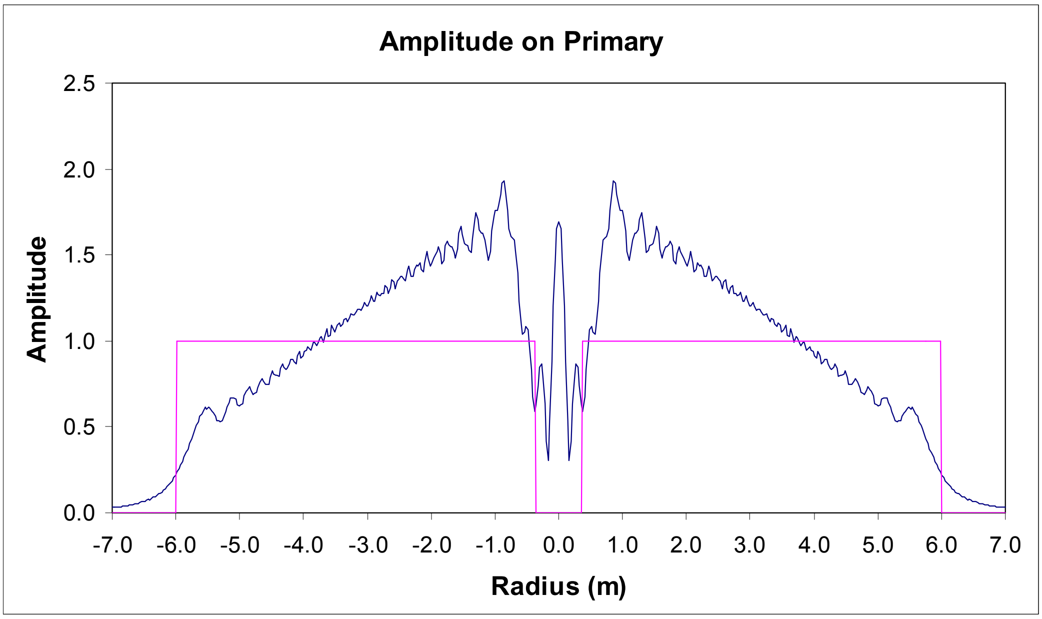


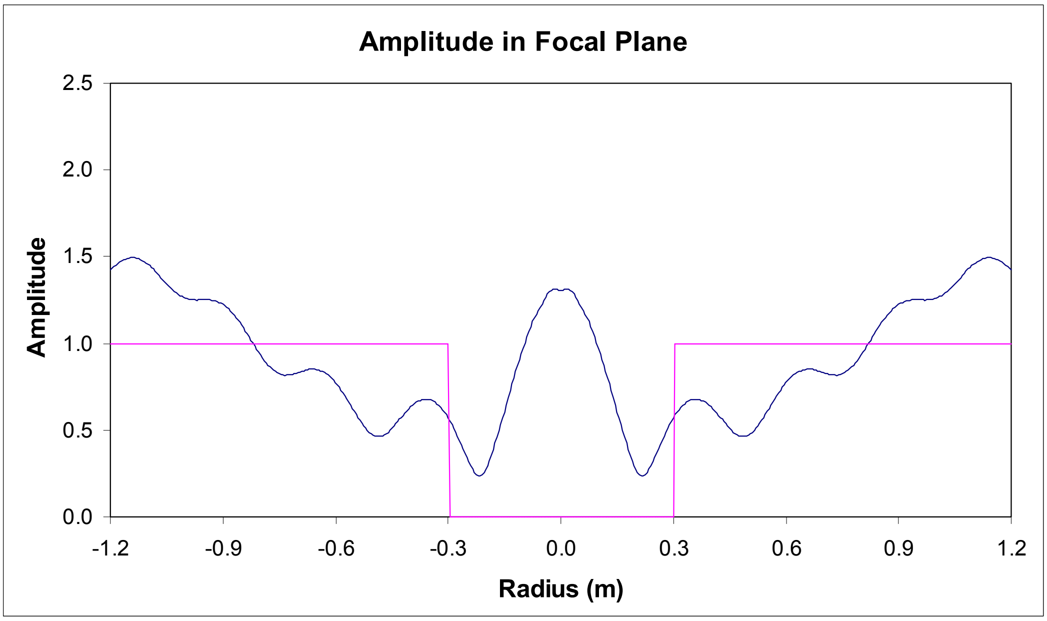




It can be seen that the diffraction effects at the outer edge of the primary are unchanged, but that the central spot is largely eliminated.

**Hole in the centre of the Subreflector**

On the basis of ray optics one would expect that making a hole in the middle of the subreflector would remove the reflections. Here are the plots, still for 100GHz, of the reflected amplitude with a 60mm diameter hole in the centre. (Note that this is already larger than the ~46mm diameter that is blocked in the geometrical case.)



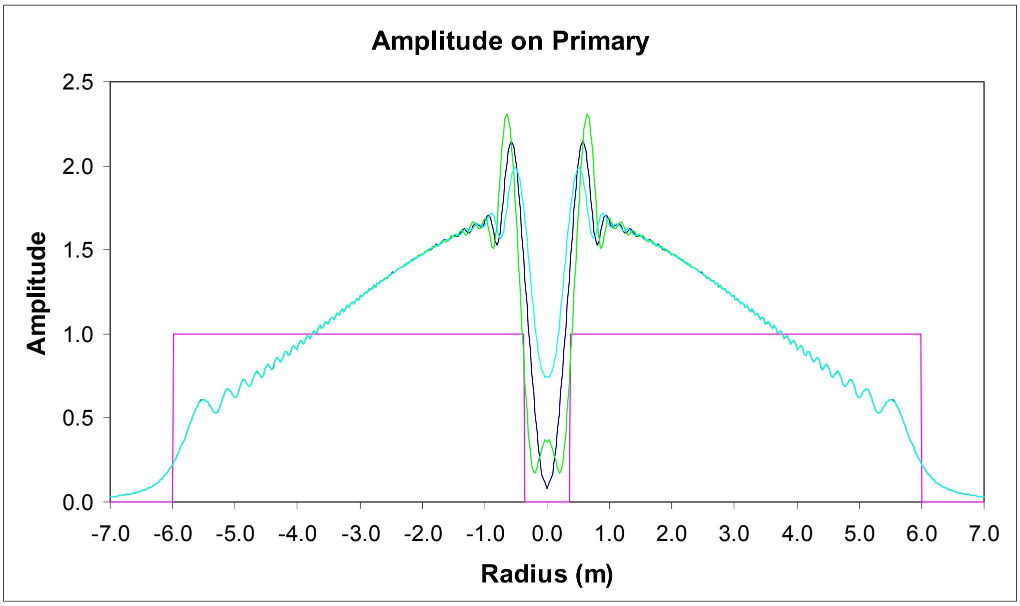



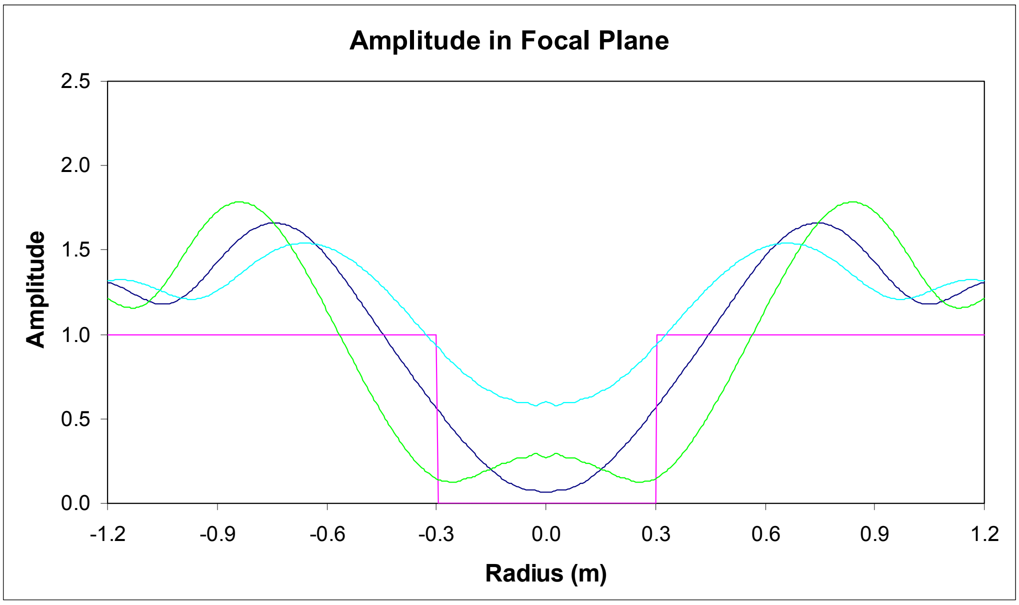

These demonstrate that the "Poission spot" phenomenon applies in this case as well, and that in fact on-axis the amplitude of the reflected wave has essentially the same amplitude as it would if there were no hole. Having a round hole in the centre is clearly not the thing to do!

**Straight Cones**

Previous experience has shown that to suppress the reflections over a wide frequency range one should make the surface continuous and also avoid sudden changes in slope. The simplest circularly symmetric shape is a cone with a slope that matches that of the hyperbolic surface at the point where they join. (We do not consider non-circularly symmetric shapes because of the difficulty of manufacturing them. This could be investigated if we get stuck!)

These plots show the reflected amplitudes for three cases – cones of diameter 48, 60 and 72mm. The corresponding half-angles are 87.781, 87.227 and 86.674 degrees.



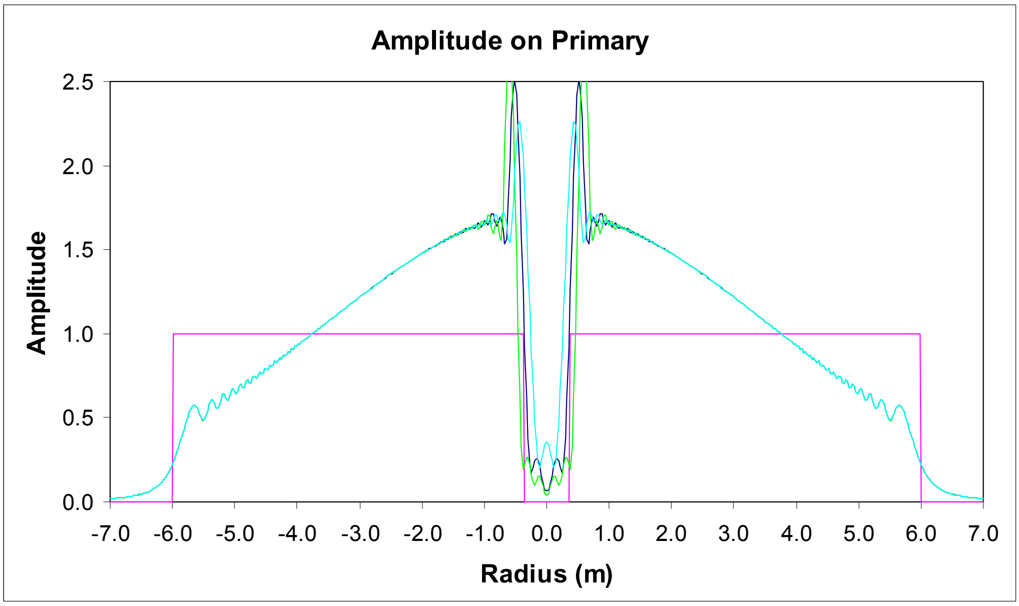



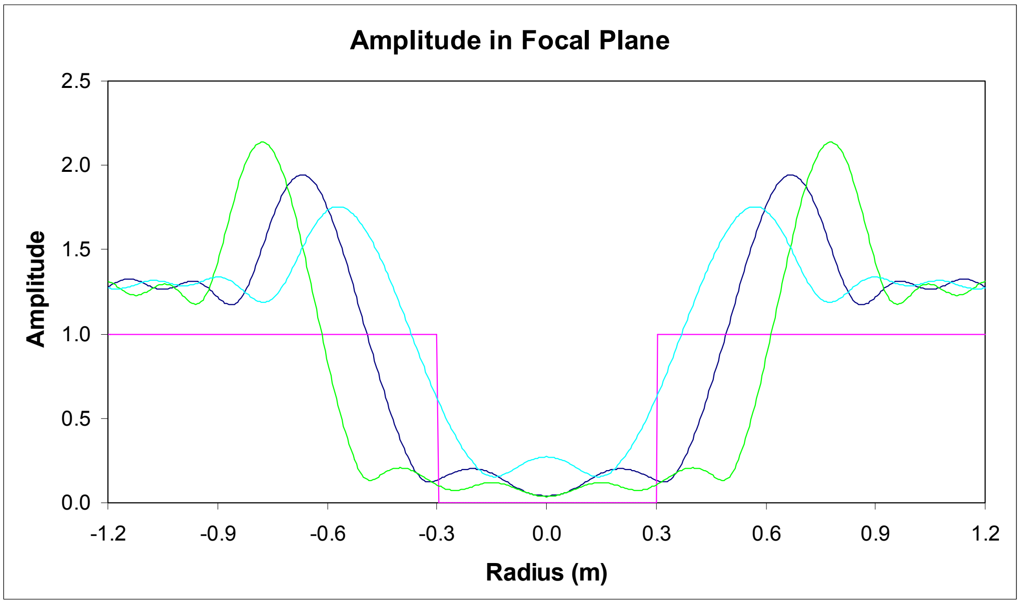

It can seen that the 48mm diameter cone (light blue) does not do a good job of suppressing the reflections – it is simply too small – the 60mm one (dark blue) does reasonably well on-axis, and the 72mm one (green) provides some suppression over most of the focal plane, but at this frequency 72mm is not the optimum size for suppressing the on-axis reflections.

In addition to looking at these patterns we have calculated two quantitative measures of performance. These are peak forward gain of the antenna and an estimate of that part the spill-over which is terminated at ambient temperature. The gain is found by just generating the far-field pattern of the primary and finding the peak value. The latter is taken to be the difference between the amount of energy falling on the secondary and the amount falling on the primary. With a Gaussian pattern and 12dB edge taper, the fraction of the power radiated by the feed that falls on the secondary is always close to 93.7%, while the fraction reaching the primary is typically ~1% less than this. This difference is due to the spill-over past the edge of the primary, which will mostly fall on the ground, <u>and</u> the power that goes back through the central hole and into the volume in front of the receiver. The values of these quantities have been gathered together in a table at the end of this section for convenience, but the significant results will be given as we go along.

For the cases above, which were at 100GHz, it was found that with a cone in place both the gain and the forward efficiency improve compared to the case with an unmodified secondary. The reason for the increase in forward efficiency is that more of the power is being reflected onto the primary instead of going back to the focal plane. This power falls on the primary and, at this relatively low frequency, it is close enough to being in phase for the gain to increases as well (but only by ~0.3%) for the smaller sizes of cone. For a 72mm diameter this is no longer true and the gain is essentially back to where it was for the case with no cone.

The same models were then run for a frequency of 200GHz. It should be appreciated that the compute times are getting quite long here. (The time required goes as the square of frequency for the basic patterns and roughly as the cube if the gain is required.)



It is seen that the behaviour is starting to approach that expected for geometrical optics – the energy that would have been returned into the focal plane is being placed in ring near the inner edge of the primary. The ring moves out further and contains more energy for the larger diameter cones. For an on-axis feed (which only applies exactly to the water vapour radiometer) it can be seen that a 60mm diameter cone (dark blue in the plot above) is large enough to suppress the reflection back to all parts of the focal plane.

Because the patterns are approaching the geometric form at this frequency, the spill-over past the primary is now quite small (see table below). The antenna gain is also greater than for the case with no cone for diameters of 48 and 60mm, but for a cone diameter of 72mm it is falling off quite significantly from this optimum (by ~0.8%).

Here are the values for the spill-over and gain. Also shown is the effect on "G/T", the gain over system temperature ratio. This is calculated by assuming that a spill-over of 1% terminated at ambient adds 5% to the system temperature, which is about right for a system temperature of 50K. The gain and G/T values are given as the changes with respect to the case for an on-axis feed and a smooth hyperboloid. (This table includes values for off-axis cases which are described below.)

| Frequency | Offset in focal plane | Tilt of subreflector | Cone Diameter | Spill % | Gain % | G/T % |
|---|---|---|---|---|---|---|
| 100 GHz | On axis | None | None | 1.71 | 0.00 | 0.00 |
| | | | 48mm | 1.27 | 0.25 | 2.48 |
| | | | 60mm | 0.99 | 0.30 | 3.92 |
| | | | 72mm | 0.79 | 0.02 | 4.65 |
| 200 GHz | On axis | None | None | 1.27 | 0.00 | 0.00 |
| | | | 48mm | 0.74 | 0.28 | 2.95 |
| | | | 60mm | 0.32 | 0.23 | 5.02 |
| | | | 72mm | 0.21 | -0.57 | 4.76 |
| 200 GHz | 245mm | None | None | 1.58 | -1.37 | -2.88 |
| | | | 48mm | 1.32 | -1.28 | -1.49 |
| | | | 60mm | 1.05 | -1.49 | -0.35 |
| | | | 72mm | 0.67 | -1.96 | 1.05 |
| 200 GHz | 245mm | 1.146 deg | None | 1.32 | -0.44 | -0.64 |
| | | | 60mm | 0.36 | -0.23 | 4.34 |

Because of the long compute times, higher frequencies have not been investigated, but it is reasonable to assume that the effects on gain and efficiency will not change a great deal more as the frequency increases beyond 200GHz.

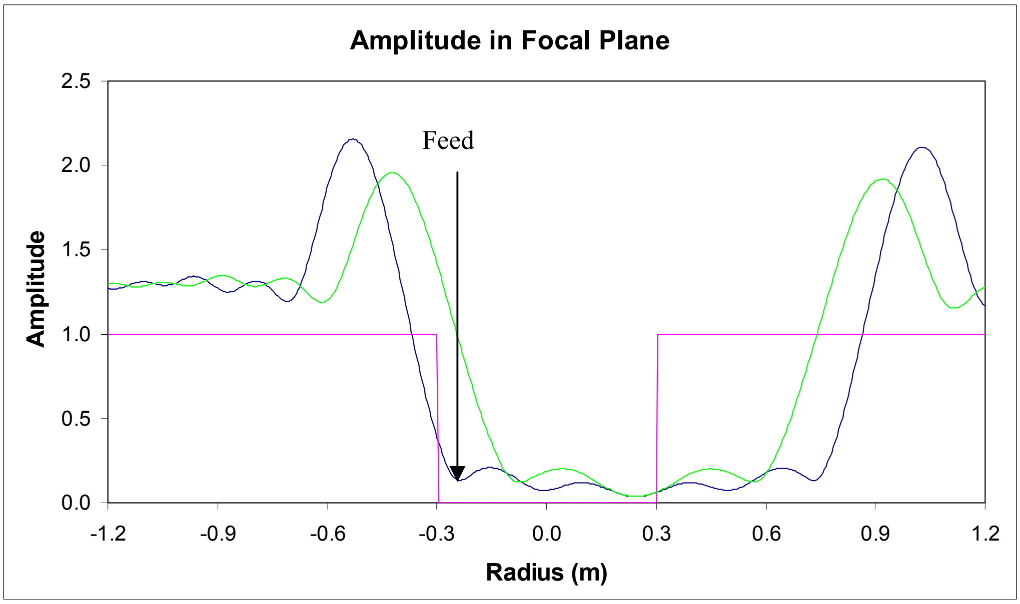
**Off-axis Feeds**

We now need to take account of the fact that on ALMA the receiver feeds are off axis. The radial distances of the feeds[1] from the axis, as given in a note from Matt Carter, are the following:

| Band | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Radius (mm) | 255 | 255 | 188 | 194 | 245 | 245 | 100 | 103.3 | 100 | 100 |
| Angle (deg) | 2.43 | 2.43 | 1.79 | 1.85 | 2.34 | 2.34 | 0.95 | 0.99 | 0.95 | 0.95 |

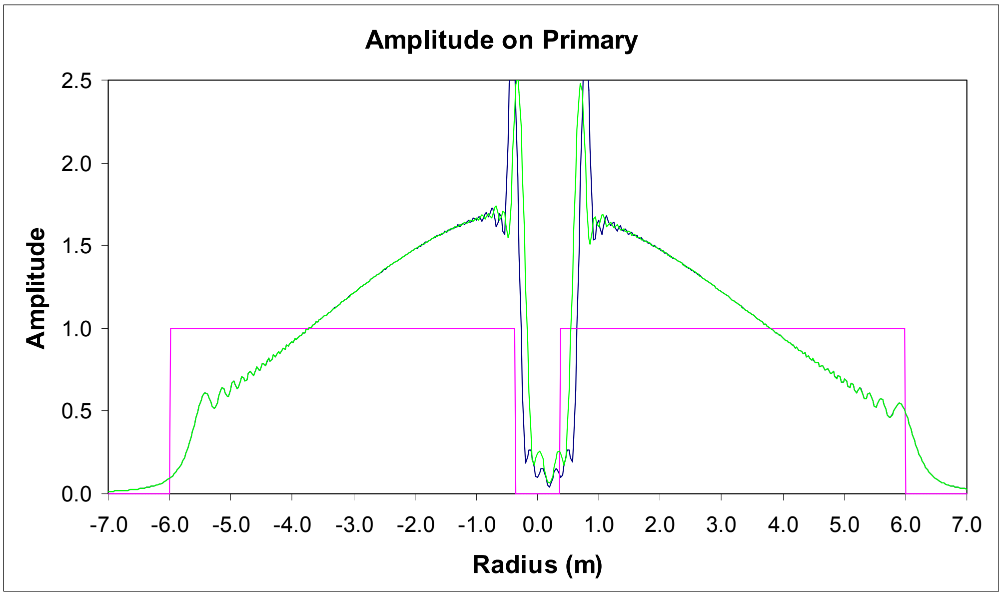
[1] The feeds are assumed to lie in the (flat) focal plane. Because we have additional optics associated with the receivers, these are not actual the positions of the real horns. Instead these represent the positions of the secondary focus for the various bands, but we can model all the relevant effects by placing a virtual feed there.

The angles given here are the amounts by which the feeds are tilted inwards to point at the subreflector (which has here been assumed to be at a distance of 6m from the focal plane). As was pointed out in an e-mail from James Lamb, this is obviously going to cause a problem, because, for bands 1, 2, 5 and 6, the angles of tilt are such that the signals arrive more or less perpendicular to the surface of the cone, so there is going to be a strong reflection. When we examine the illumination for these off-axis cases we can see this happening. Here are plots for a 200GHz feed 245mm off-axis and straight cones with diameters of 60mm and 72mm.




The cut is in the plane containing axis of symmetry and the feed, which is marked with the arrow. It can be seen that the effect of offsetting the feed is to shift the illumination pattern by the same amount, i.e. 245mm, but in the opposite direction. Clearly the 60mm diameter cone (green line) is not satisfactory, since, even at this relatively high frequency, the reflection back into the feed is only slightly reduced relative to that with no cone at all. The diameter of 72mm (dark blue line) is about the right size to ensure that the reflections back into the feed are suppressed for bands 5 and 6.

It is also true that the overall illumination of the primary has shifted, as shown here.



This means that there is an increase in the spill-over past the edge of the dish (on the side opposite that where the feed is) and a loss of gain. We can estimate this amount by looking at the cases for a smooth secondary with no cone. These are given in the table above and show that at 200GHz the additional spill-over past the primary is about 0.3%. With the same assumptions as in the table ($T_{sys}$ ~ 50K) this produces an increase of about 1.5% in the system temperature. The loss in gain is about 1.4%, so the total reduction in sensitivity is nearly 3%.

In the high frequency limit the loss of gain can be calculated by ray optics. In optical terminology the loss is due to "vignetting" – the fact that, for an off-axis source, some of the rays that are gathered by the primary miss the secondary. The reason for this is that the diameter of the secondary (750mm) has been chosen to be only just large enough to collect all the rays in the on-axis case. We could of course avoid this loss by using a larger subreflector, but the consequence of that would be a larger spill-over onto the ground at all frequencies, which would be worse. Doing a Geometric Optics calculation shows that, for a feed 245mm off-axis and with uniform illumination, about 3% of the rays are vignetted. Allowing for the tapering of the illumination reduces this to only ~0.7%, but one has to take account of the fact that the gain of the antenna depends on the square of the amplitude illumination, so this suggests a value of about 1.4% for the loss of gain at high frequencies, which is in good agreement with that found from the diffraction calculation.

The point of the above discussion was to highlight the fact that using an off-axis feed without tilting the subreflector to re-centre the illumination causes a significant loss, independent of the question of the central cone. To use the obvious currency, a three percent loss is about one-and-a-half antennas-worth of collecting area in some of the key observing bands.

The numbers come out slightly differently when we include the cone. With the smooth secondary quite a large fraction of the spill-over was going back through the hole in the primary and (it was assumed here) terminated at ambient somewhere in the cavity in front of the receiver.[2] The cone puts this onto the primary and then onto the sky, so there is an improvement in the forward efficiency and gain. For an on-axis receiver we would be able to use a 60mm diameter cone and the overall G/T at 200GHz improves by ~5% with respect to the smooth case. With this cone and a feed that is 245mm off axis all this is improvement is lost – the calculated G/T is actually down by more than 5%. Using a larger cone helps here and we have already seen that we need this to suppress the standing waves adequately. With a 72mm diameter cone the loss in G/T is closer to 4% (two antennas-worth!). I have not done the detailed calculations for bands 3 and 4 but I would expect the results to be quite similar – the feed offset is rather smaller, but the system temperatures are likely to be even lower so spill-over has a larger effect.

**Tilting the Subreflector**

In the plots and calculations above it has been assumed that the subreflector has <u>not</u> been tilted – i.e. it remains aligned on the optical axis of the telescope. This is what has apparently been assumed in writing the antenna specifications, which refer only to movements along three linear axes. It is clear however that if we can tilt the subreflector we can both recover the lost gain and do a good job of suppressing the reflections without using a large cone.

The tilt required is almost exactly half that of the feed tilt given in the table above. Note that this tilt should be implemented as a rotation of the mirror about an axis passing through the position of the prime focus and perpendicular to the plane containing the feed and the antenna

[2] Note that some of this improvement is probably not real because in reality a good deal of the signal reaching the focal plane would bounce back into the sky anyway. The figures here are a limiting case which would apply if we filled up most of the clear area with calibration loads, polarization widgets, etc.

axis. Rotation of the subreflector about this point produces almost no coma, although it does introduce a small amount of astigmatism. This does not imply that the subreflector mechanism has to provide a mechanical axis of rotation about this point – the required motion can be made up of rotation about some other axis plus a lateral movement. In fact, I believe that the plan is to use a hexapod to adjust the position of the secondary, so the necessary degrees of freedom are available. It will however be necessary to include this control in the software associated with selecting a particular receiver for operation[3]. We will return to the requirements on this tilting below.

When the tilt is implemented (1.15 degrees for a feed at 245mm from the axis), the illumination patterns in the focal plane and on the primary are almost exactly restored to those for an on-axis feed and an un-tilted subreflector. There is no point in showing the plots because they are indistinguishable from those on pages 6 and 7 above. As can be seen from the table of losses and gains (bottom line) the spill-over figure with a 60mm cone is now back down to the very low figure of ~0.3% found for the on-axis case. The gain is however lower than in the on-axis case by nearly 0.5%. This is due to the astigmatism which is introduced by the tilt of the subreflector. This is confirmed by a standard GO analysis. The bottom line here is that applying the correct tilt and using a 60mm diameter cone gains us about 3.3% in sensitivity compared to not tilting and using a 72mm cone.

This loss of gain due to astigmatism will be proportional to frequency squared and will be approaching 1% at the top end of band 6. Fortunately all the higher frequency receivers (bands 7 to 10) have small radial offsets (~100mm) for which any such effects will be small. It fact it will probably not be worth tilting the subreflector for those feeds since neither the spill-over or the standing waves will be greatly changed by making such a small tilt.

**Curved cones**

Apart from being simple to describe, the "straight" cones analysed above – cones with a constant slope – do not have any special RF properties. Bacmann and Guilloteau already pointed out that a lower on-axis reflection at ~100GHz can be obtained with a slightly curved cone. I have therefore examined a variety of shapes to see if an optimum can be found.

Ideally we would like to find a shape that gives low reflections over a wide range of frequencies and over the whole focal plane. An exhaustive study of this would be take a lot of effort, but one take account of the fact that even at high frequencies the cone only fills a few Fresnel zones for waves transmitted from the feed and back into the focal plane, so there is no point in considering shapes which vary rapidly as a function of radius. By the same token, such relatively small and smooth structures cannot give rise to behaviour which varies rapidly as a function of frequency. I have therefore modelled the cone as a surface that deviates from the nominal hyperboloid by a distance *dz* in the axial direction whose variation as a function of radius, *r*, is give by a polynomial with 4 terms, i.e.

$$dz = \mathrm{A} + \mathrm{B}\, q + \mathrm{C}\, q^2 + \mathrm{D}\, q^3 , \text{ where } q = (r_c - r) / r_c$$

and $r_c$ is the outer edge of the cone, taken to be 30mm in all the cases considered from now on.

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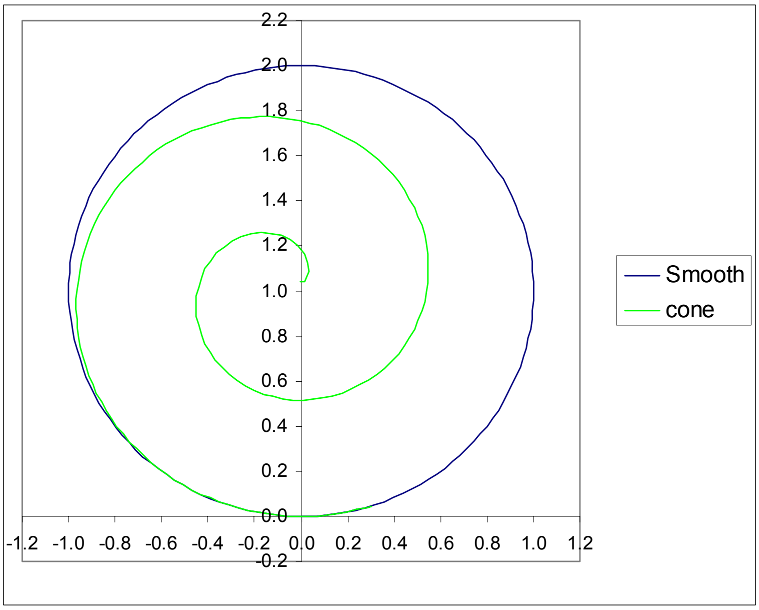
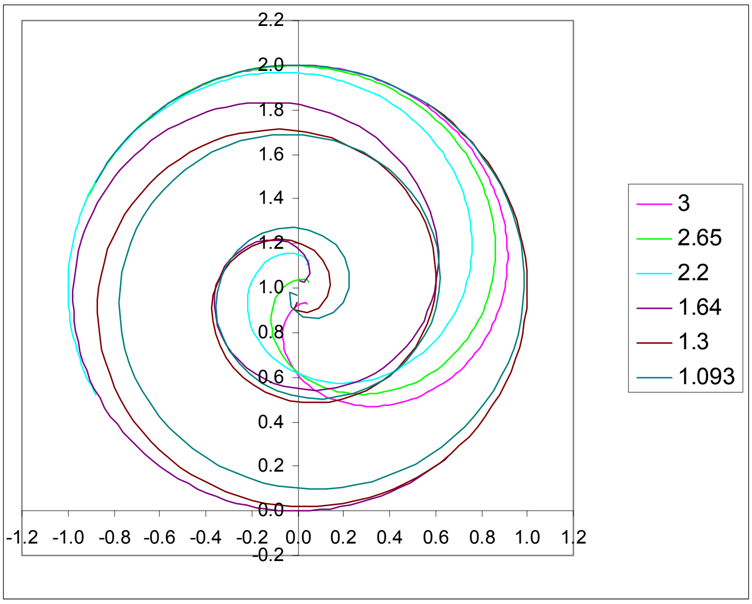
[3] Note that with the offset feed position it was found necessary to move the subreflector downwards by 0.35mm in the axial direction to achieve the maximum gain. This is a result of the fact that the Cassegrain focal plane is curved, with a radius of curvature of ~0.3m. This implies that there is already a software requirement to adjust the subreflector position when changing to a different receiver. When the subreflector is tilted, the amount of axial refocusing required is reduced to 0.13mm.

Although the modelling of gain, spill-over, etc., takes quite a lot of computer time, these quantities will not be significantly affected by the detailed shape of the cone. The on-axis reflection can however be calculated easily by evaluating the Fresnel integral. This can be represented in graphical terms as the real and imaginary terms contributed by a series of rings:

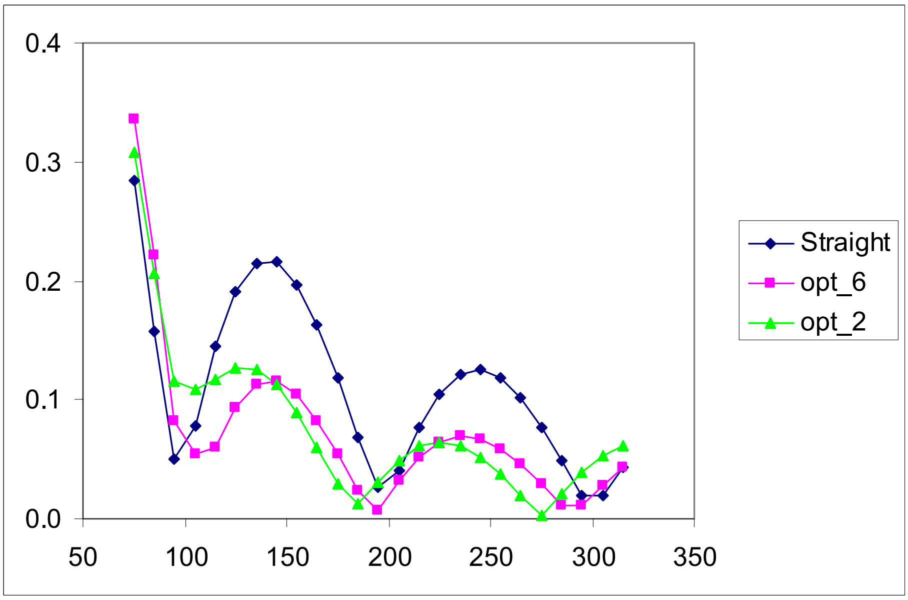



Here the blue curve shows the function for the hyperbolic surface, which is almost exactly a circle (the "Fresnel circle"), and the green curve shows the reflection from a straight cone in the form of a spiral. The goal is to have the spiral converge to the point (0.0, 1.0) which means that the contribution from the cone cancels that from the rest of the subreflector. It can be seen that it does so rather well in this case, which is for 200GHz. The goal now is to find a shape that does this over a range of wavelengths. The next plot illustrates the results one such solution, with the wavelengths in millimetres given in the box.



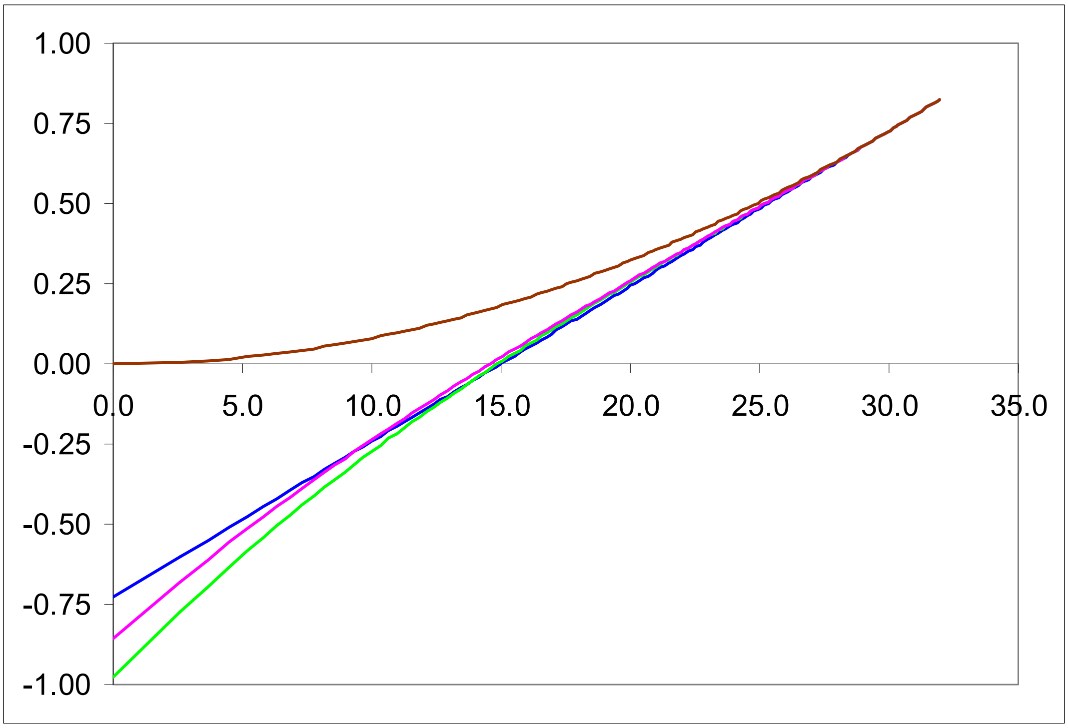

It was found that wide-band solutions always have the offset and slope terms, A and B in the equation above, very close to zero. This is as expected – discontinuities in the surface, or in its slope, will always produce additional reflections which cannot cancel over a range of frequencies. Those terms were therefore set to zero and only C and D were allowed to vary.

Plotted as a function of frequency in GHz, the amplitude of the on-axis reflection, relative to that for a smooth hyperboloid, then looks like this.

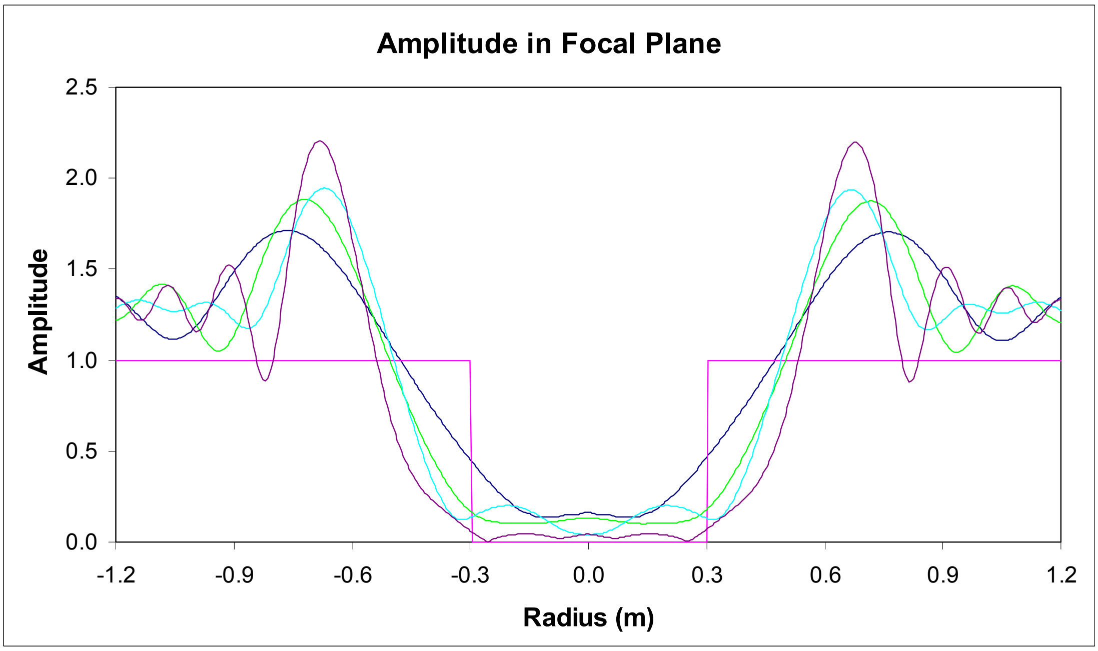



Here the line marked "straight" is the original cone, tangential to the hyperboloid at outer edge of the cone. We see that in terms of the on-axis reflection, the frequencies we chose for the calculations, 100 and 200 GHz were fortuitously good. The pink line, "opt_6", is the case where the choices of C and D minimize the rms of the returns for all 6 of the wavelengths given in the plot at the bottom of page 12, while the green line, "opt_2", is minimized for just the two wavelengths which are most critical for reducing the effects of the WVR local oscillator, 1.64 and 1.09mm, corresponding to 183 GHz and the next harmonic at 275GHz.

The shapes of these cones are illustrated in this plot showing height against radius (same colours, scales in mm – vertical scale highly exaggerated). The differences are quite small.

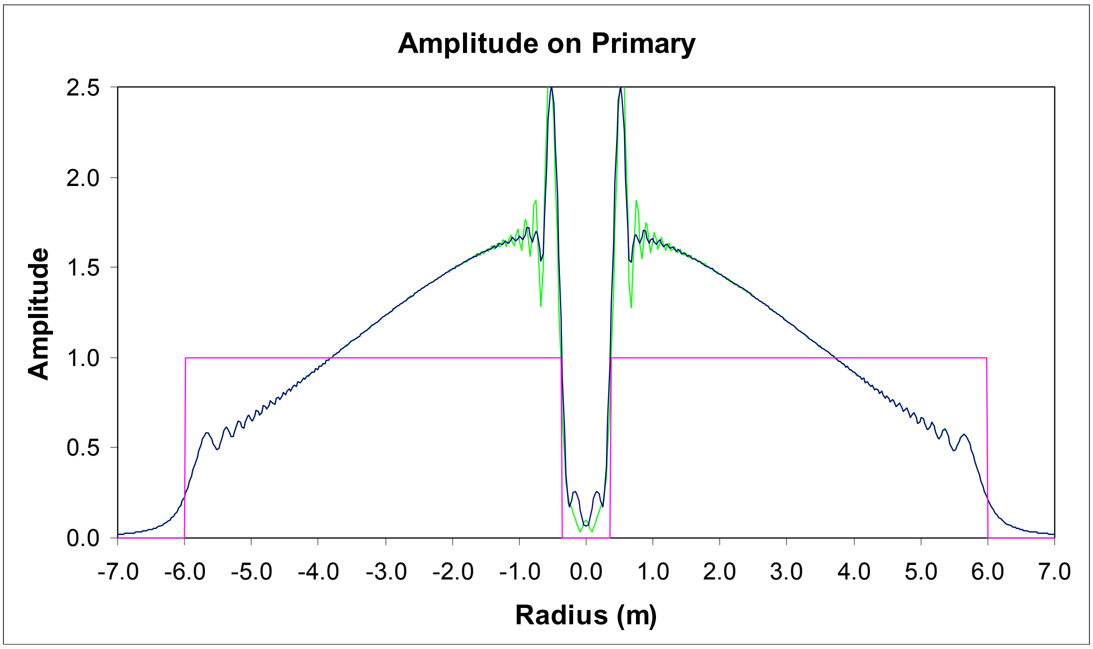


We need to check that this optimisation process of driving down the on-axis reflection has not ended up making the reflections larger in other parts of the focal plane. Here are plots of the reflection for the case "opt_2" at 100, 150, 200 and 275 GHz.



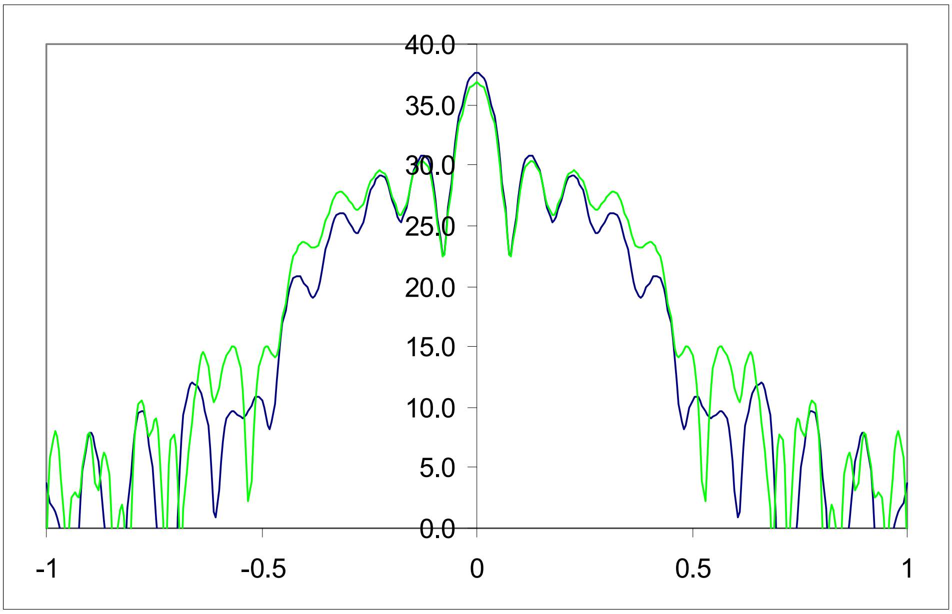

It is apparent that this design is doing about as good a job as can reasonably be expected. Note that these plots are for the case of an offset feed with the subreflector tilted to re-centre the illumination. The fact that this works well confirms that using the curved cone has the expected results for the non-axially symmetric case as well as the symmetric one.

One consequence of the curved design is that the energy that falls on the cone is distributed further out onto the primary than with the straight cone. This is illustrated, for 200GHz, here:




As previously, the blue is the straight cone and the green is the curved design optimized for two wavelengths. One sees a wavy structure in the amplitude extending away from the central hole. At first sight it might be thought that this would have a bad effect on the primary beam, but in fact this is just the beat between the desired (smooth) illumination pattern and the wave from the cone, most of which will propagate away at relatively large angles.[4]

To clarify this, here are 200GHz far-field patterns of the energy scattered by the cones (blue for the straight cone, green for the case "opt_2"). To be precise, this is the power in dB in the vector difference between the beam patterns for the smooth hyperboloid and the cones, plotted against angle in degrees. The feed has been moved back on-axis here for simplicity.

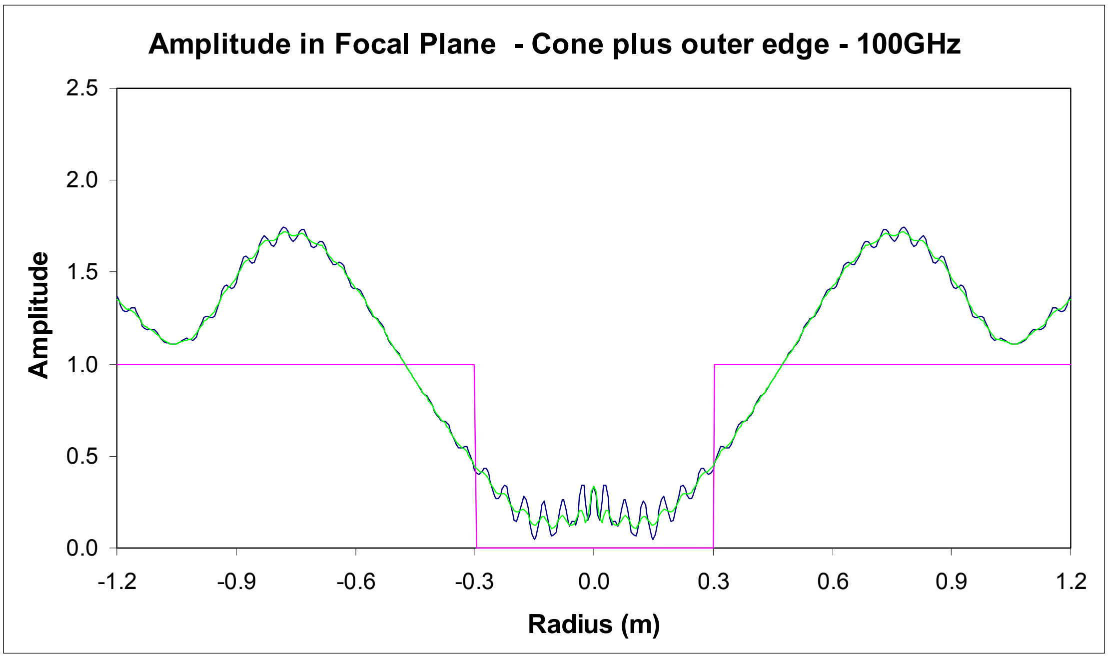


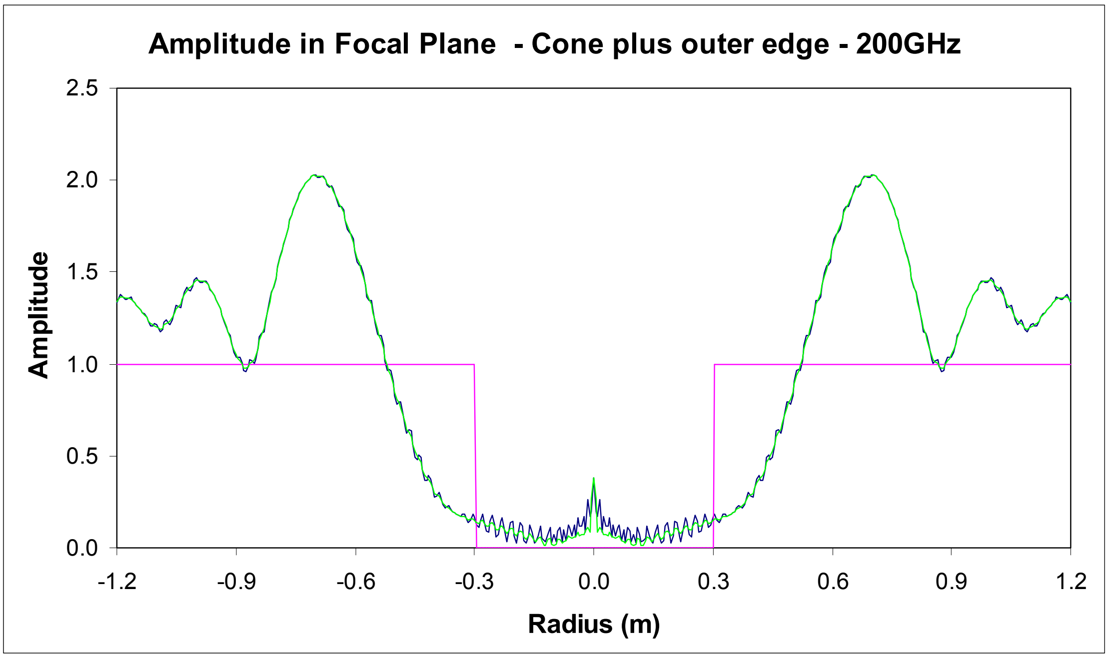

It is seen that the energy is spread over a range of angles, going up to about half a degree from the main beam, and that the pattern for the curved cone is slightly wider than for the straight cone. The main beam has a width of ~0.01 degrees FWHM but does not appear here because we only show the differences die to the cones. The main beam gain is about 87 dB at this frequency, so most of this structure is of order 60dB below the peak. It is hard to see that there will be any undesirable consequences as a result of this. The signals due to the cone are in any case at a rather lower level than the sidelobe pattern due to the edge of the dish.

**Effects of the Outer Edge of the Subreflector**

With such a cone in place at the centre of the subreflector, the principle cause of reflections back into the focal plane will be the diffraction from its outer edge, which was artificially suppressed in all the models above. If we now go back to a circular outer edge and again use GRASP, with PTD turned on, to model the edge currents, we find that the real amplitude of the reflections are as show in the plots below. These are for frequencies of 100 and 200GHz and the cone is the 60mm diameter case "opt_2" as before. The blue lines are for the cuts perpendicular to the E-plane, which show a diffraction pattern over an extended region because of the currents flowing allow the outer edges, and the green lines are for the cuts in the parallel direction, where the effect is only strong near the centre of the focal plane.

[4] These disturbances to the illumination pattern would be a problem if we were doing high resolution holography from the Cassegrain focus, but for ALMA we are not. On JCMT we do the holography that way, so this was a consideration and that was one reason for just using a straight cone.




To repeat the explanation from page 3, the presence of the central peak is the Poisson's spot effect resulting from the circular shape of the subreflector. It follows from the standard analysis of this phenomenon that the amplitude of the central peak relative to that which would be present from the direct signal from the "smooth" subreflector (i.e. an amplitude ~1.4 on these plots, which have arbitrary units on the vertical axis) is given by the amplitude edge taper of the feed. It can only be removed by breaking the circular symmetry in some way. Note that having the feed off-axis by one or two degrees is not sufficient to do this. We need to introduce irregularities that cover at least one Fresnel zone, which in this case means a variation in the radius of at least one wavelength at the lowest frequency where we are trying to suppress the effect, i.e. of order 1% variation in the radius to be effective at 90GHz.

This problem has of course been thought about previously. In particular, Jaap Baars has reminded me that Dave Morris[5] analysed it in the 1970's and that as a result it was decided to

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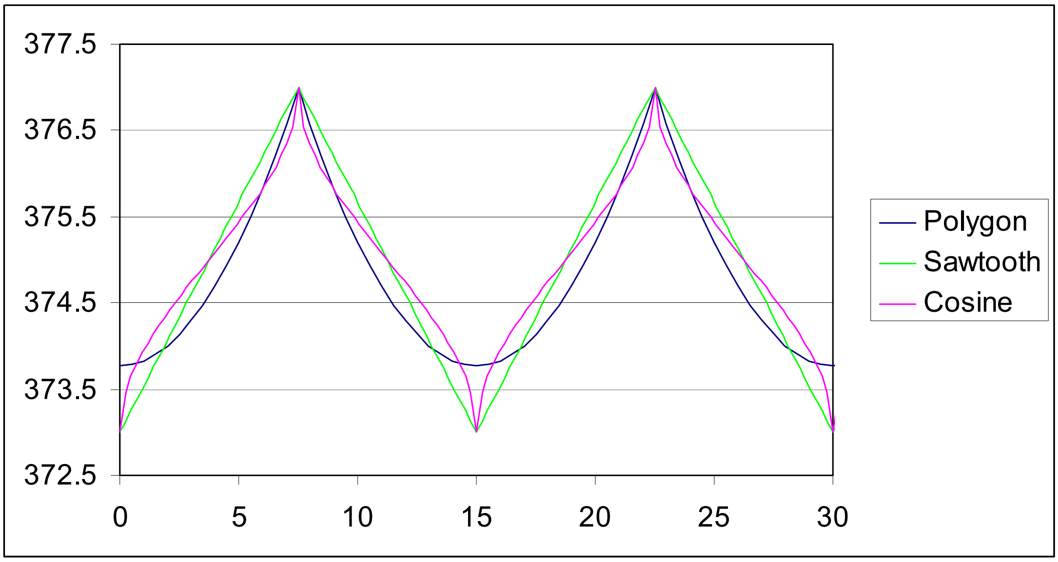
[5] Dave's paper on "Chromatism in Radio Telescopes due to Blocking and Feed Scattering", A&A, 67, 221-228, 1978, contains a wealth of detailed analysis of many aspects of the problems of reflections and baseline ripple.

make the outer edge of the subreflector on the 30m telescope Pico Veleta slightly elliptical. I have looked briefly at the case of an elliptical subreflector. It does suppress the central peak, but much of the structure in the H-plane cut persists. One would also be worried that an elliptical subreflector would introduce subtle polarization effects into the performance of the antenna. I have therefore concentrated on cases with a higher degree of symmetry.

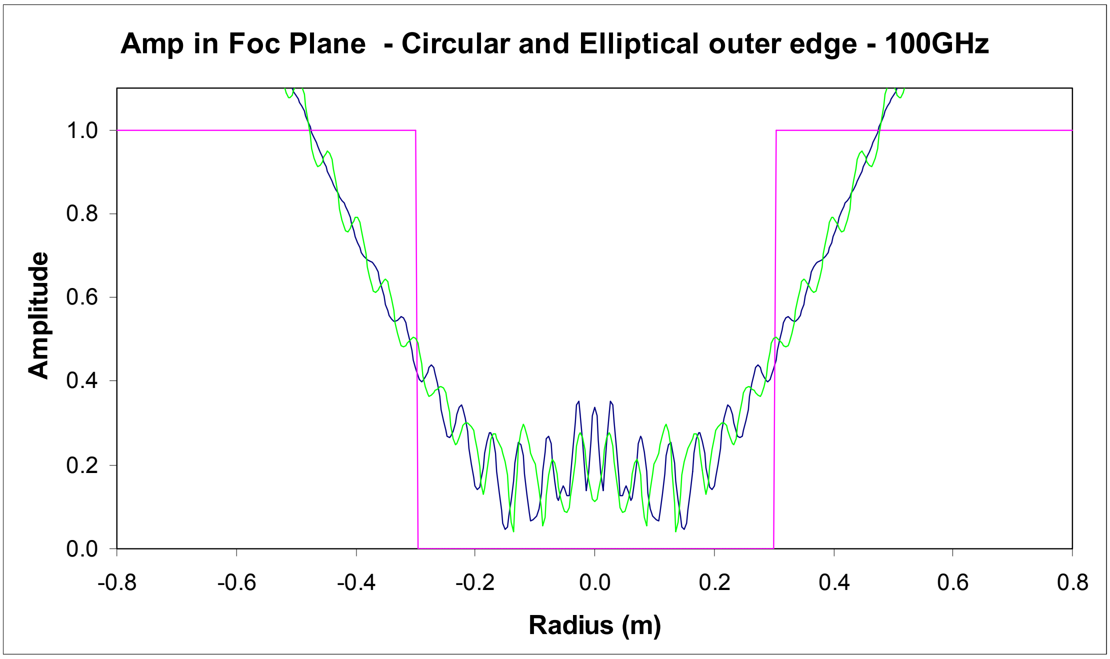
Naturally one is thinking about shapes that are easy to machine, so the most obvious is a regular polygon. If one uses a 24-sided polygon with the points at a radius of 377mm, the minimum radius (at the centre of the flats) is 373.77mm, i.e. a little over 3mm smaller, so one would expect this to provide sufficient variation in radius. One in fact finds that this works well at 200GHz, but is not very effective at 100GHz – plots on following pages. The reason for this is can be seen by considering the radius as a function of angle. This is plotted here as the blue line (radius of outer edge in mm versus angle round the rim in degrees).



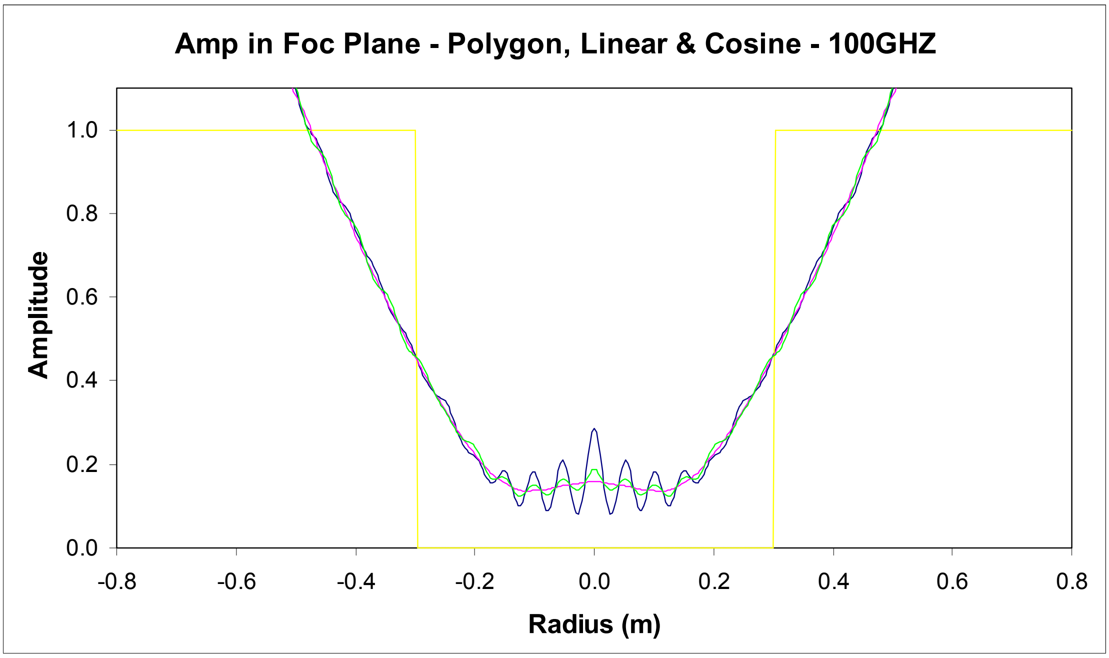

This shows that for the polygon the distribution of radii is very non-symmetric around the mean. The stretches of edge where the radius is only changing slowly still produce quite a strong coherent reflection. A more symmetric distribution would be expected to do better. This could just be a saw-toothed variation of the radius (green line) which gives a nearly linear tapering off of the area as a function of radius. For the similar problem of removing edge diffraction in Compact Antenna Test Ranges, it is however normal practice to use a taper with cosine weighting (pink line) to give even better suppression. Although this looks to be a nasty thing to machine, I do not think that it would really be difficult under numerical control. It should be realised that the vertical scale in this plot is enormously exaggerated in this plot: the peak-to-valley height assumed here is only +/– 2mm while the distance between peaks would be around 100mm for this 24 sided case.

These cases have been calculated and the results are shown in the diagrams on the following two pages, which give the amplitude of the reflected signal in the focal plane on an expanded scale compared to the previous plots. For completeness the circular and elliptical cases are given first followed, by the results for a regular polygon and then the saw-toothed and "cosine-weighted" cases. Here only the cases with the cut perpendicular to the E-plane of the feed are shown since that is the direction where it is most difficult to suppress the reflections.

This is the on-axis case using the “opt_2” cone, with a circular outer edge (blue) and with an elliptical one (green) – semi-major and minor axes 377 and 373mm.

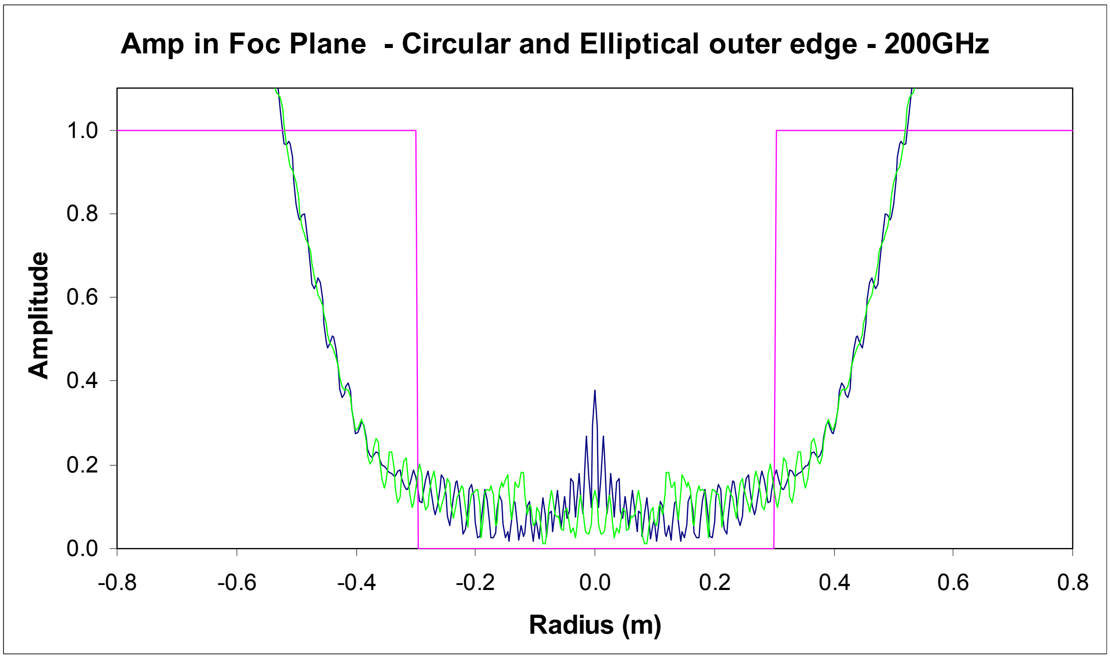


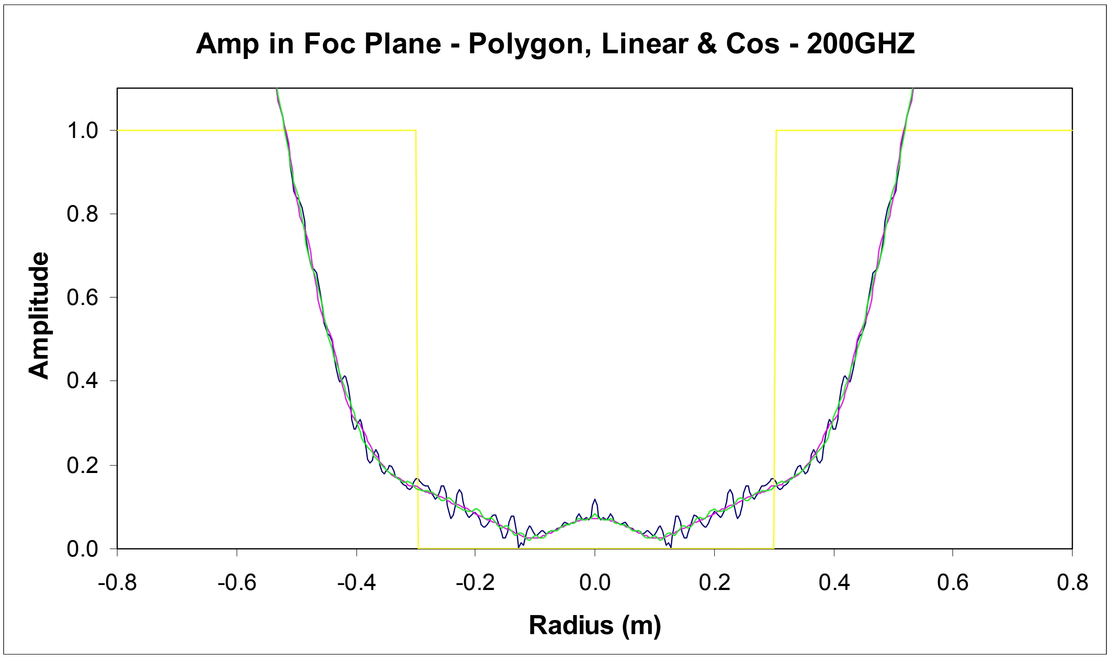

Plot for regular polygon (blue), saw-tooth with radius going from 373 to 377mm (green) and “cosine weighted” shape with same radii (pink) – all with 24-fold symmetry.




It can be seen that polygon is not very effective, but the linear saw-tooth removes most of the reflections from the outer edge, while the cosine-weighted shape apparently eliminates it completely.

Same plots as previous page but for 200GHz.




At this higher frequency the polygon and saw-tooth do rather better than at 100GHz, but the best result is again with the cosine weighting.

As far as I can tell these changes in the outer edge have essentially no effect on the gain and spill-over, although there is a slight doubt here because I had some problems in getting convergence to precise numerical results with these shapes. I also checked that there is no significant change in the patterns when the off-axis feed and tilted subreflector is used.

Finally, it should be pointed out that 24-fold symmetry was chosen in the case of the regular polygon because this is the number of sides that gives roughly the desired 1% variation in radius. There is however no need to have so many sides for the saw-tooth or cosine-weighted shapes. A quick checked showed that 16-fold symmetry works essentially as well as 24, but that with only 8-fold symmetry the results were clearly less good.

## Implementation

### The Cone

The present antenna specification calls for the cone to be a separate part which can be removed from the subreflector. It also assumes that standard manufacturing tolerances can be used, which means (see appendix 1) that a step of up to 50 microns could occur in the surface where the cone meets the subreflector and that there could be a radial gap of up to 25 microns. It is easy to see that neither of these is acceptable. A 50-micron step is clearly not consistent with the surface accuracy required of the subreflector, which is of order 7 microns rms. Similarly a radial gap of 25microns will act as a reduced height waveguide interrupting the surface and disrupting currents flowing in the radial direction at that point. I have not worked out the details, but it is clear that the effective impedance of such a gap will be far greater than that of the conductive surface, especially at the shorter wavelengths at which ALMA is required to have good performance. If it were decided that the cone must be separable then we would have to do more work on these tolerances. I suspect that the result would be that we really need the limits on the allowable step height and the size of any radial gaps to be in the region of 10 microns. This could of course be achieved, but it would require special fitting, which would be expensive.

The first question to ask, however, is whether it is in fact necessary to have the cone detachable. As far as I can ascertain the main reason for doing this was to make it possible to fit an alignment target in its place. Although this is clearly useful, it seems to me that it is by no means essential. In fact the final lateral positioning of the subreflector will be done by making astronomical observations to find the location which gives minimum coma. It is true that these measurements cannot give the tilt of the subreflector around axes passing through the prime focus, but by the same token the requirement here is very loose. We have seen that the original plan called for what was essentially a tilt of more than 1 degree from the optimum position. We have argued that this is excessive, but it is clear that a tilt of 0.1 or even 0.2 degrees is of no real significance. It should be easy to achieve such accuracy in setting the tilt of the subreflector without using a target. For example, on could simply set the dish to point at the zenith and use a spirit level to set the subreflector horizontal.

On this basis then, I believe that we do not need to make the cone removable and the best solution is therefore to manufacture the whole subreflector surface in one piece, e.g. on a numerically controlled lathe. If there are other reasons for having the hole in the middle of the subreflector that I have missed then this will need to be reconsidered

### Tilting of subreflector

We have shown that it is highly desirable to be able to tilt the subreflector around axes passing through the prime focus. To accommodate the planned arrangement of receivers we should allow tilts of up to +/–1.25 degrees in any direction, i.e. two additional degrees of freedom are required.

We have just seen that the requirements for knowing the absolute tilts of the subreflector are quite loose, but it is clear that the stability and repeatability of the positioning must be good, since any errors will need to be taken into account in the pointing error budgets. Short term instability would affect the tracking performance and longer term instabilities and non-repeatability would affect the absolute pointing error. From a simple ray optics model I find that the multiplication factor is 15.5, i.e., if the subreflector is tilted by 15.5 arc seconds around an axis passing through the prime focus, the beam moves by 1 second on the sky.

Note that the intention is that the position of the subreflector will be set to the optimum for the receiver being used for the main astronomical observations. Conceptually this is straight-forward – one imagines that there will be table containing the correct settings for all five degrees of freedom (x, y and z positions as well as the two tilts) for each antenna for each receiver band. If one is doing fast switching to a reference source using a different receiver band, one would not re-position the subreflector. The loss of sensitivity on the calibrator is too small to make this worthwhile. This means that a re-positioning time of say 10 seconds would be adequate, which is compatible with the present specification on axial refocusing. It also means that one would not make it a requirement to be able to shift the position of the subreflector to that for another receiver and bring it back again with sufficient repeatability that one was still within the pointing error required for tracking, as opposed to absolute pointing.

One final point to note is that there is a complication in keeping track of the pointing offsets. It is of course intrinsic to the ALMA optics scheme that each receiver will have a separate set of pointing offsets (collimation terms). The movements of the subreflector, both lateral displacement and tilts, will also need to be taken into account. This means that if we have the subreflector set for say the band 6 receiver and we switch to a reference source using the band 3 receiver, the pointing offsets for band 3 will not be the same as they would have been if we had set up the subreflector for that receiver. In fact the change in the pointing due to the subreflector motions can be calculated quite accurately and it may be adequate to include these directly in the pointing model using a suitable formula. If not then a more elaborate look-up table will need to be derived.

### Outer Edge

We have seen that it is desirable to make the outer edge of the subreflector non-circular. This means of course that it cannot simply be turned on a lathe. I imagine however that modern equipment can in fact produce the shapes discussed here rather easily. The desired profile can easily be provided in the form of a table of numbers (either x and y or radius and angle). No great accuracy is need here since what we are trying to do is create irregularity anyway. It will presumably be necessary to cut the original piece of metal to the right overall shape before the final machining of the reflecting surface is undertaken so the edge could be cut then.

It is worth noting that it is assumed that the edge should be cut parallel to the telescope axis. One could consider alternatives, such as undercutting to make a sharper edge, but I do not believe there are any advantages. (This was looked into for Planck and found to have little effect.)

## Conclusions

The present 48mm-diameter straight cone is too small to give good cancellation of reflections. The diameter should be increased to 60mm and, preferably, a slightly curved shape should be used. It would be best to machine the cone as an integral part of the subreflector surface.

It is highly desirable that the control of the subreflector positioning is extended to five degrees of freedom. This will make it easier to keep down the reflections but, more importantly, it will improve the overall sensitivity significantly.

If possible the outer edge of the subreflector should be made non-circular. A shape with 16-fold symmetry that produces a smooth tapering off of the effective aperture with radius is recommended.

## Appendix 1. Present Design – as in the Antenna Specification