ALMA MEMO # 560

Methods for the Characterization and Measurement of the Gain Fluctuations of Cryogenic Amplifiers

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ABSTRACT

Since ALMA instrument will have low noise receivers with very large instantaneous bandwidth, gain stability should be carefully specified to avoid limiting the sensitivity of the total system. This memorandum reviews the methods for the characterization of gain stability in the time and frequency domain and presents examples of experimental setups used for its measurement in the case of cryogenic amplifiers of the type used in the IF of ALMA as well as its practical limitations. Special attention is paid to some important but not widely known effects, as the dependence of gain fluctuation with bias and its variation across the band of the amplifier. Finally, the compliance of one prototype 4-12 GHz InP cryogenic IF amplifier with the requirements of ALMA project is discussed.

Methods for the Characterization and Measurement of the Gain Fluctuations of Cryogenic Amplifiers

1. ALMA Specification: time and frequency domain

The specification of gain stability for the cryogenic IF amplifier of an ALMA receiver is derived from the total System Gain Stability allowed. With the present system breakdown, the assignment for the gain stability of the cryogenic amplifier was specified in terms of *Allan Variance* in the time interval from 0.1 to 1.0 seconds.

$$s^2(t) \le 2 \cdot 10^{-8} \qquad 0.1 \le t \le 1 \quad \text{(sec)} \qquad [1]$$

A recent proposal from System Engineering (September 2006) sets different limits. The new values would be:

$$\begin{cases} s^2(t) \le 2 \cdot 10^{-8} & 0.05 \le t \le 100 \quad \text{(sec)} \\ s^2(t) \le 2 \cdot 10^{-9} & 100 < t \le 300 \quad \text{(sec)} \end{cases} \qquad [2]$$

So the specification is expanded to higher and lower times. Initially it was only for one decade and now it covers a factor of 6000 in time span (almost four decades).

Instead of Allan Variance, it may be more convenient to measure the Spectrum of Normalized Gain Fluctuations (SNGF) since this allow easy identification and removal of some effects which otherwise will interfere with the measurement and mask the intrinsic fluctuations (i.e. 1 Hz gain oscillation due to refrigerator cycle or 50 Hz power line interference). If the quantity measured is the frequency spectrum and it is then transformed mathematically to Allan Variance[1] one should first determine the frequency range in which the spectrum data needs to be known. We shall assume a SNGF of the form:

$$S(f) = b \cdot f^{-a} \qquad [3]$$

Where S(f) is the unilateral spectral density (SNGF) with units $Hz^{-1/2}$. As a representative example we can set the parameters (ideal 1/f noise) as:

$$\begin{aligned} a &= \tfrac{1}{2} \\ b &= 1 \cdot 10^{-4} \end{aligned} \qquad [4]$$

We shall assume that the spectrum is truncated outside the defined frequency range [f_{min}, f_{max}], let:

$$\begin{aligned} f_{\min} &= 2 \cdot 10^{-4} \cdot Hz \\ f_{\max} &= 20 \cdot Hz \end{aligned} \qquad [5]$$

[1] The definition of the Allan Variance used in this work and the transformation from SNGF to Allan Variance are given in Appendix II.

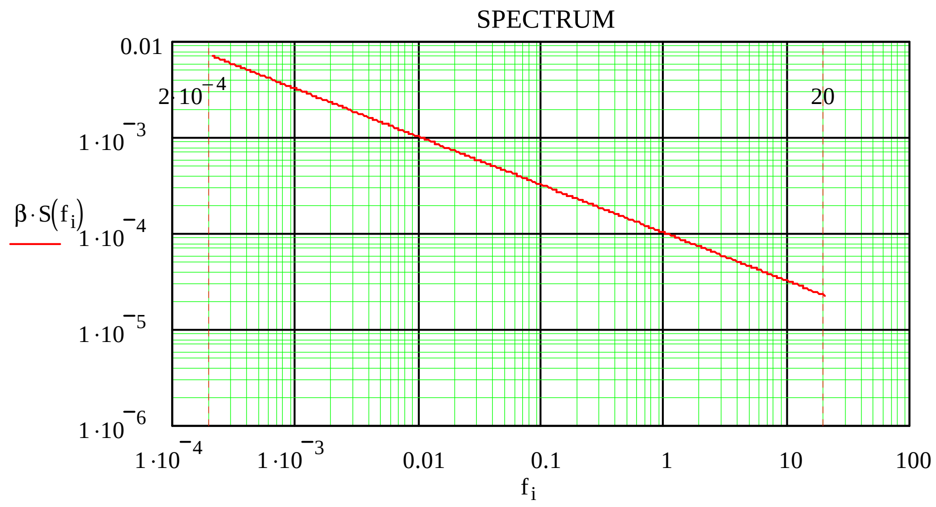
Then, the spectrum and the calculated Allan Variance will appear as in Figure 1. Ideally, for a non-truncated spectrum with α=1/2, the Allan Variance will be constant for any time interval, and its value would be:

$$s^2(t) = b^2 \cdot 2 \cdot \ln(2) \qquad \text{(only for } a = \tfrac{1}{2}\text{)} \qquad [6]$$

However, the truncation ads a “band pass” filter effect in the time domain that is easily recognized in the Allan Variance plot of Figure 1. Note that the values of frequency are chosen to provide a good accuracy (error<2.5%) in the time range required in the new specification. As a rule of thumb, strictly valid only for this type of spectrum, the range of frequencies for which the spectrum needs to be known is given by:

$$f_{\min} \approx \frac{0.06}{t_{\max}} \quad ; \quad f_{\max} \approx \frac{1}{t_{\min}} \qquad [7]$$

Note also that the frequency range required in the measurement expands five decades. This is about one decade more than requested in the time domain specification. The necessity of one decade more in frequency appears due to the fact that the truncation in the frequency domain is transformed into a soft “band-pass” shape in the time domain.



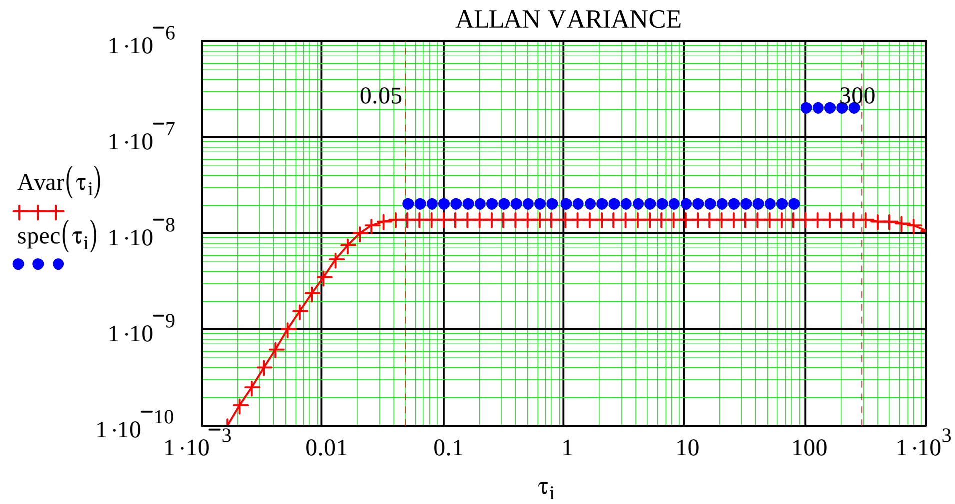


Figure 1: Spectrum truncated at [fmin,fmax] (up) and the calculated Allan Variance (down). The blue dots in the plot represent the proposed new specification.

2. Sensitivity of the measuring system

Intuitively it should be clear that for fluctuation measurements to make sense, the fluctuation of the measurement system should be less than that of the device to characterize. This basic idea will define the range, either in the frequency or in the time domain, in which the measuring system is usable. Two different setups have been used to measure gain stability in the cryogenic amplifiers in our laboratory. Both (see Figure 2) are based in the measurement of a CW signal. In system A the signal generated by a synthesizer is feed into the amplifier and wide band detected by a Schottky diode. The DC output is then sent to a low 1/f noise amplifier and introduced at the input of an Agilent 35670A FFT analyzer. System B uses an HP 8510 C Vector Network Analyzer as generator-detector-sampler at a single CW frequency. While system A is only sensitive to amplitude fluctuations, system B allows characterizing phase fluctuations as well. The power level at the output of the amplifier is set to -20 dBm in both systems to avoid any possibility of compression.

Figure 3 presents the noise floor of the two systems compared with the example non-truncated spectrum of relations [3] and [4]. Noise floor values were obtained by measuring the fluctuation spectrum of the system without DUT at the same power level. It is interesting to note that the HP 8510 C system provides lower fluctuation floor at the low frequency end, but its white noise component makes it less sensitive than the FFT system over ~0.3 Hz.

The HP 8510 C is an excellent system for measurement of stability at low Fourier frequencies. However it is not an instrument designed for this application and it can not be set for measurement of very long times (very small Fourier frequencies). The measurements of Figure 3 were taken initially with an averaging value of 1024 and 401 points, which gives a total time of 85.3 sec. per scan (minimum frequency of $12 \cdot 10^{-3}$ Hz). The maximum values for the averaging and number of points are 4096 and 801 respectively, giving a total time per scan of 665 sec (minimum frequency of $1.5 \cdot 10^{-3}$ Hz). This is the practical limit of this system. Additional measurements were taken with these values to confirm the performance al low frequency.

The HP 8510 C is a venerable system with a very well performing hardware and is no longer manufactured. It has been substituted by other Vector Network Analyzers made by Agilent, former Hewlett Packard. At the time of writing this report the high end of the line is the PNA model, much more flexible than the 8510 C and including some interesting new features, among others an advanced DSP processor for the detected signal. The PNA model has been tested for gain fluctuations but unfortunately is inferior to the 8510 C at low Fourier frequencies due to limitations of its hardware. It does not offer any improvement on the sensitivity shown in Figure 3.

System A in Figure 2 uses the FFT analyzer Agilent model 35670A. This instrument can cover Fourier frequencies from 122 μHz to 102.4 KHz, which is the 9 decade range shown in Figure 3. Not all this rage has been explored in our measurements. Note that 10^{-4} Hz requires time scans of about 2.8 hours. We usually take 50 time scans to average the spectra and reduce the statistical error. The total measurement time required for this will be about 140 hours (~6 days!), not accounting for the time needed for housekeeping. This seems a little cumbersome for a production test.

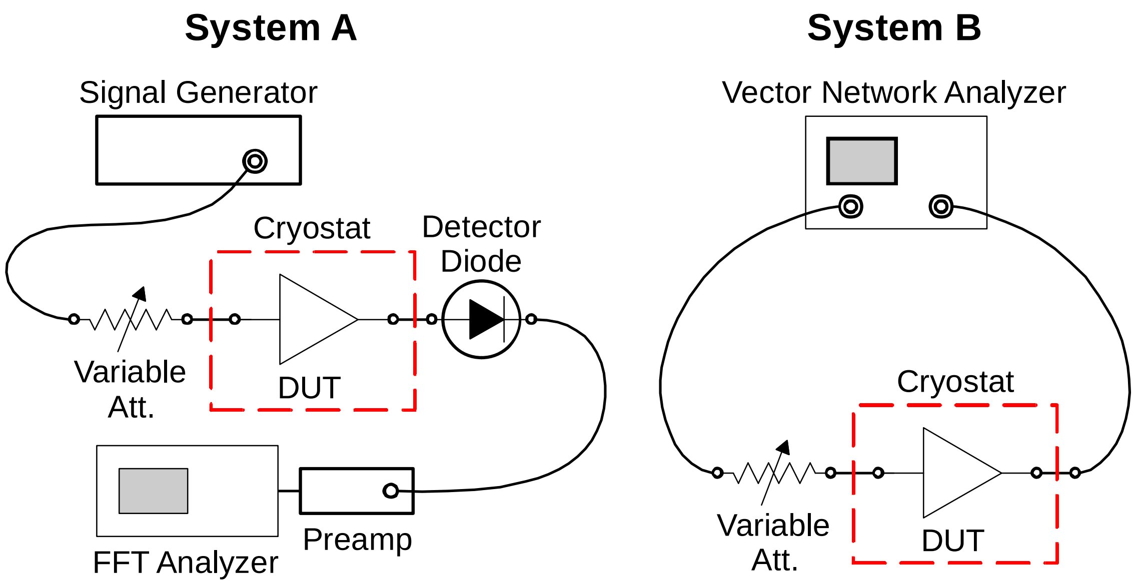


Figure 2: Schematic diagram of the two setups used for gain fluctuation measurement of cryogenic amplifiers. System A is based in a FFT analyzer and system B in a HP 8510 Vector Network Analyzer. Both use a CW signal for the measurement.

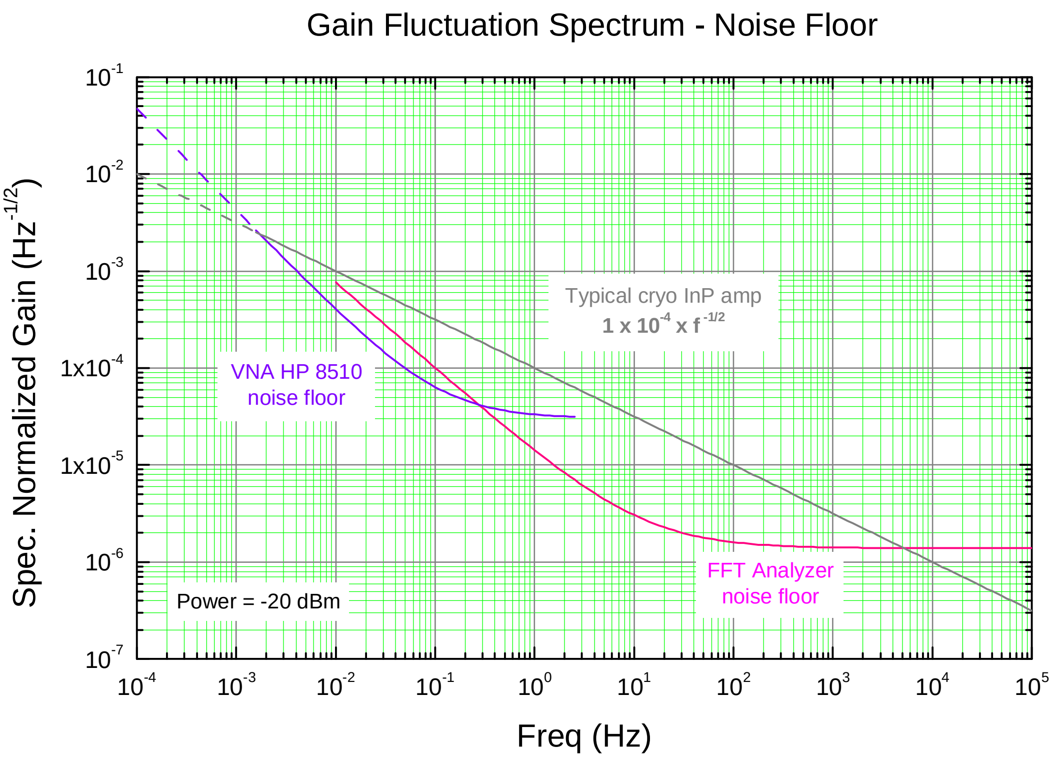


Figure 3: Noise floor of the two experimental setups used for gain fluctuation measurement compared with a typical spectrum. The dashed lines for frequency below $1.5{\cdot}10^{-3}$ Hz represent extrapolation of measured data.

Further analysis of the noise floor of the system A has shown that it is dominated by the noise of the preamplifier (Signal Recovery model 5113), followed very closely by the fluctuations of the synthesizer (HP 83650 B). The effect of the noise of the preamplifier could be reduced by increasing the power level, but then the fluctuations of the synthesizer would start to dominate, and finally very little improvement will be obtained[2].

Taking into account that the typical amplifier in Figure 3 is very close to the maximum allowed by the specification, we can draw the conclusion that the minimum Fourier frequency accessible to our system for obtaining significant results is about $2 \cdot 10^{-3}$ Hz. Above that, the fluctuations of the system will dominate. Incidentally, this is very close to the value of $1.5 \cdot 10^{-3}$ Hz accessible with the HP 8510 C as explained lines above. With these limitations, the conclusion is that the reliable range (error<2.5%) for measurement of Allan Variance (see Figure 4) of our system is from **0.05 to 30 sec**. The maximum extent of the time scans would be 665 seconds (~11 minutes). The total time required for 50 scans would be ~9.2 hours.

3. Expanding Allan Variance measurements to lower frequency

It has been exposed in previous section that the maximum length of time scans which could be obtained with system B of Figure 2 (HP 8510 C) would be 665 seconds. However, due to the time domain filtering of the transformation from spectrum to Allan Variance, this will only provide valid data of Allan Variance up to about 30 seconds. We could, in principle, analyze the time domain scans to calculate directly the Allan Variance. This will allow to fill the gap between 30 seconds and about 333 seconds (half the time of a scan), which will be the absolute maximum accessible with this system. In principle, this time range will be contaminated neither for cryogenic cooler cycle nor for power line related spurious as the effect of those will be filtered out in the averaging previous to Allan Variance calculation. In this way, the value of 300 seconds requested by the specification will be accessible.

The noise floor of systems of Figure 2, assuming the validity of extrapolation lo low frequency and transformed to Allan Variance is shown in Figure 5. The same plot presents the new proposed ALMA specification. It is clear that system A (FFT analyzer) does not have sufficient stability for testing at 100 seconds.

However, the experience has shown that the results obtained from SNGF are more repetitive than those from direct calculation of Allan Variance. For this reason it is convenient to maintain the calculation based in the SNGF as far as possible.

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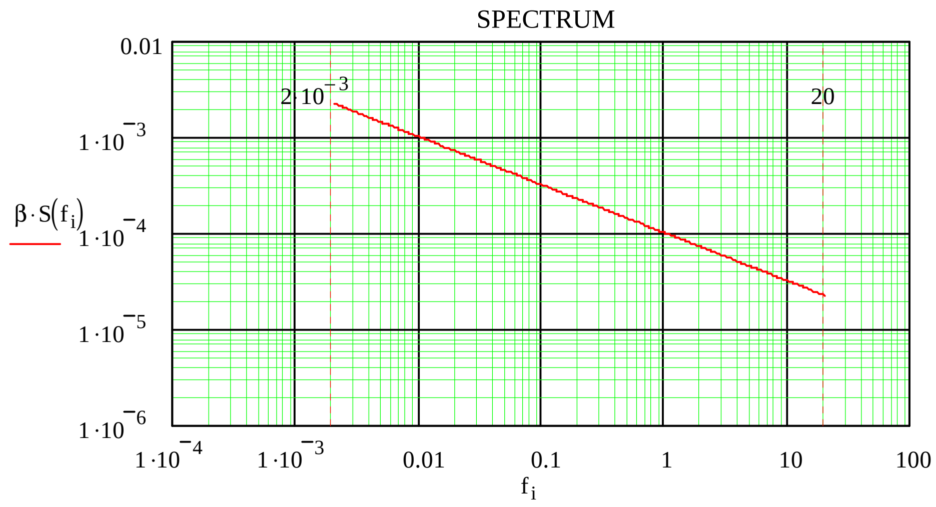
[2] Other signal generators have been tested looking for lower amplitude noise. Only the Low Noise IFR 2042 showed better performance in the range tested (10^{-1}-10^{3} Hz). Unfortunately it did not cover the CW frequency of interest (max. 5.4 GHz). Synthesizers from other manufacturers (Rhode, Racal Dana) performed worse. It is interesting to note that the amplitude noise in signal generators seems to be related with the effect of their power levelling loop. In modern synthesizers there is a 1/f and a white component. Very old oscillators without levelling loop (i.e. HP 860 C VHF vacuum tube generator) show only 1/f noise which drops to very low levels at high Fourier frequencies (above 1 KHz). However, in this case, the noise of the 1/f component is very high at low Fourier frequencies (below 10 Hz).



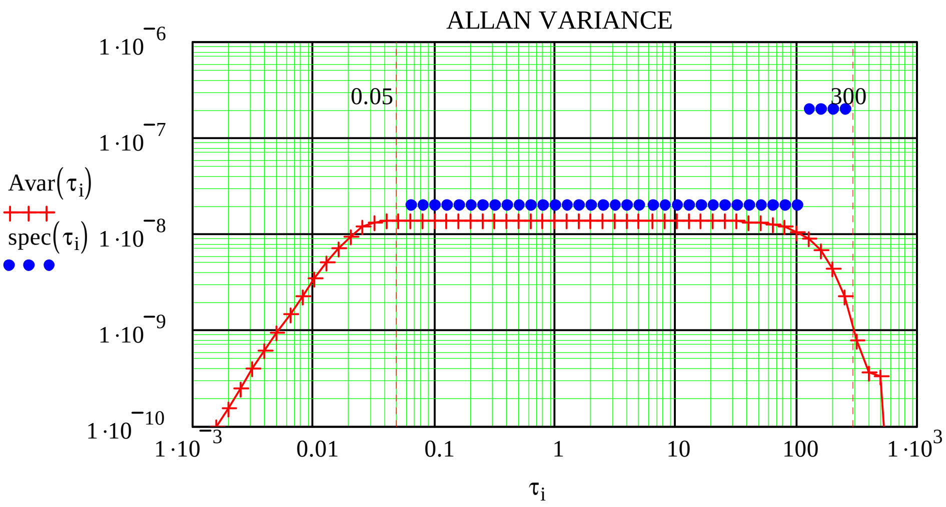


Figure 4: Spectrum truncated at 0.002-20 Hz (up) and the effect on the calculated Allan Variance (down). The value of Allan Variance obtained is **accurate** (error less than -2.5%) from **0.05 to 30 sec**.

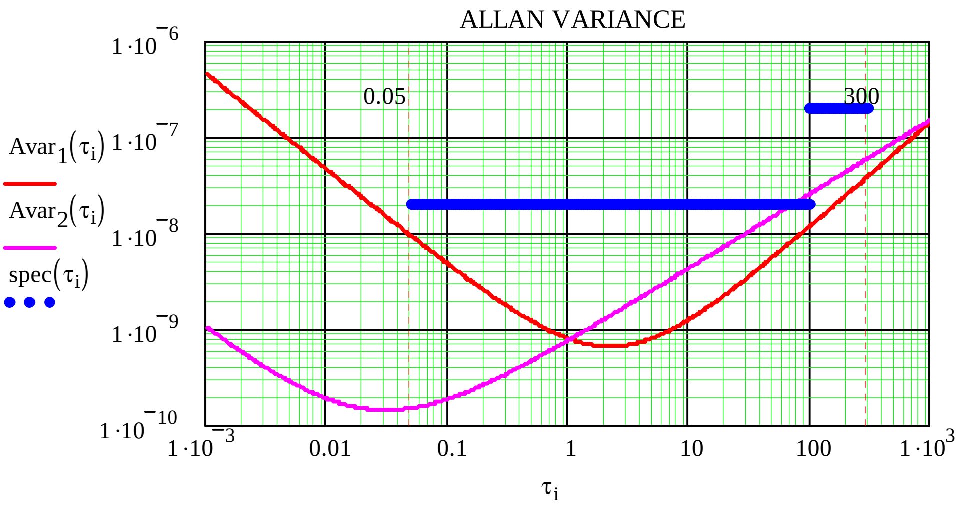
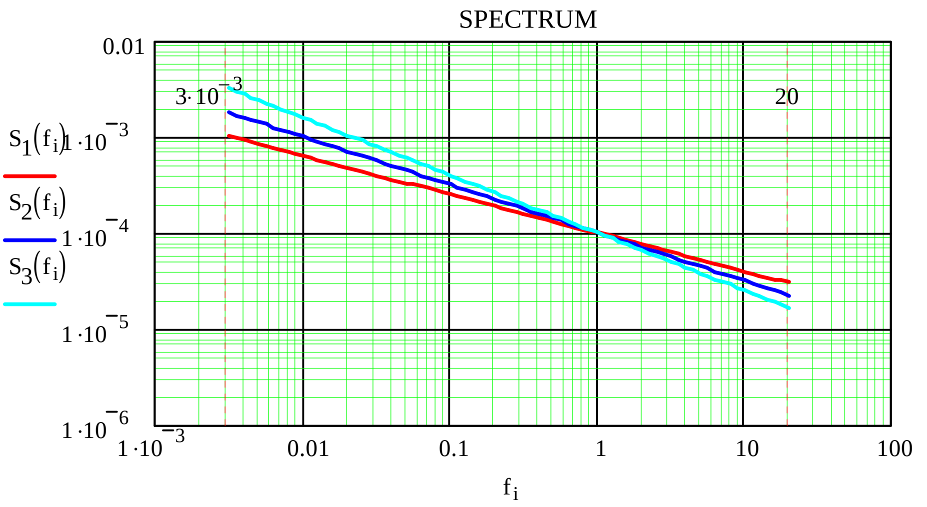


Figure 5: Noise floor of setups from Figure 2 represented in the Allan Variance plot and compared with new proposed ALMA specification. The red curve corresponds to system B (VNA) and the violet to system A (FFT). The measured spectrum has been extrapolated to low frequency to cover the whole range of time presented.

4. Effect of the slope of the SNGF on Allan Variance

In our experience the SNGF of cryogenic amplifiers follows an almost ideal 1/f shape with α=½ in the range accessible to our measurements if measured in a wide frequency band. However, for a restricted frequency range of only one or two decades, the measured spectrum may appear to fit better a spectrum of the form [3] with a value of α slightly different than ½. It is illustrative to show the effects of the difference in the exponent on the shape of the Allan Variance. This is presented in Figure 6 for values of α of 0.4, 0.5 and 0.6. Appendix II presents the relation between the exponents of the plots in the frequency and time domain. For this particular case, the respective exponents in the Allan Variance plot are -0.2, 0, +0.2.



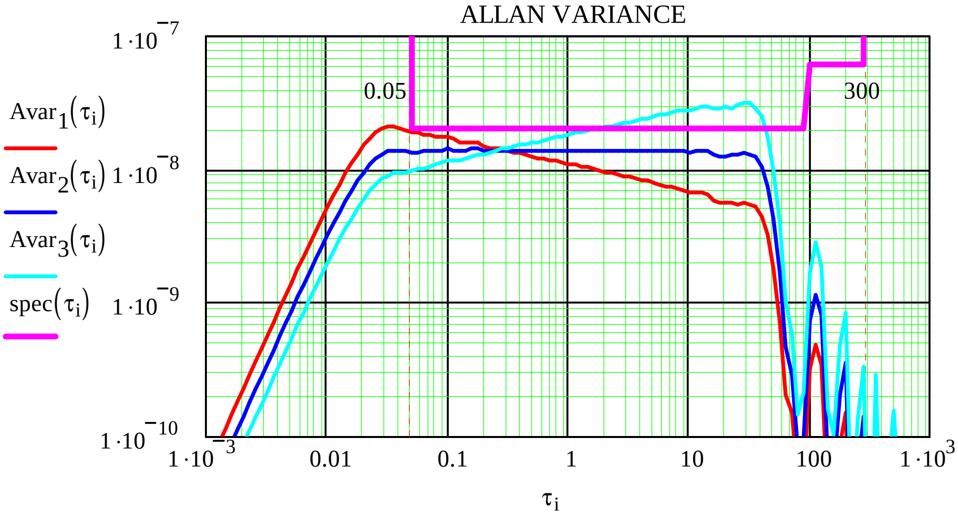


Figure 6: Effect of different exponents of the SNGF on the Allan Variance. RED: α=0.4. BLUE: α=0.5. CYAN: α=0.6. In all cases $b=1\cdot10^{-4}$.

5. Example: prototype ALMA amplifier YXA 001

To illustrate with some real results obtained from a cryogenic amplifier over a broad Fourier frequency range, Figure 7 presents the measurements of the 4-12 GHz ALMA prototype s/n YXA 001. The plot is composed of six spectra covering different frequency ranges superimposed in different colours. Each one is the average of 50 consecutive spectra to reduce the statistical fluctuation. In all cases the results were corrected for the effect of the measuring system fluctuations. The lower Fourier frequencies are covered by two spectra taken with system B (VNA) of Figure 2, while the other four were taken with system A (FFT). Total sampled frequency span covers about six decades. In this wide range, the SNGF follows extremely well the $f^{-½}$ law. Several discrete lines appear clearly in the spectrum. The lines at 1, 2, 3, 4 and 5 Hz correspond to the refrigerator cycle and its harmonics. There is a strong 50 Hz line and many harmonics which are power line related. Note that discrete lines show different amplitude in different spectra. This is due to the different resolution of spectra and to their normalization for noise power density. The lines are effectively "diluted" in the equivalent filter of the FFT channel. Note that the value of b for fitting the spectra is $4.5 \cdot 10^{-5}$, which is about half the value used in the example in [4].

The spectrum fitted in Figure 7, transformed to Allan Variance, is presented in Figure 8 only for the range in which the transformation is valid (up to 30 seconds). The same data used for the computation of the lower end of the spectrum was also used to calculate some points of the Allan Variance which are also presented in Figure 8 as well as the proposed new specification for ALMA. The Allan Variance points were calculated averaging the values obtained from 50 time scans of 665 seconds and 801 points and subtracting the contribution of the measuring system. Note that the value for short time agrees well with the value obtained from transformation of the SNGF. The value of Allan Variance obtained in the proximity of 100 seconds approaches dangerously the maximum allowed by specification. Taking into account that the gain fluctuation of this particular amplifier can be considered low for its type (InP, cryogenic), it can be said that the compliance with the specification at 100 seconds could be problematic for production amplifiers in large quantities.

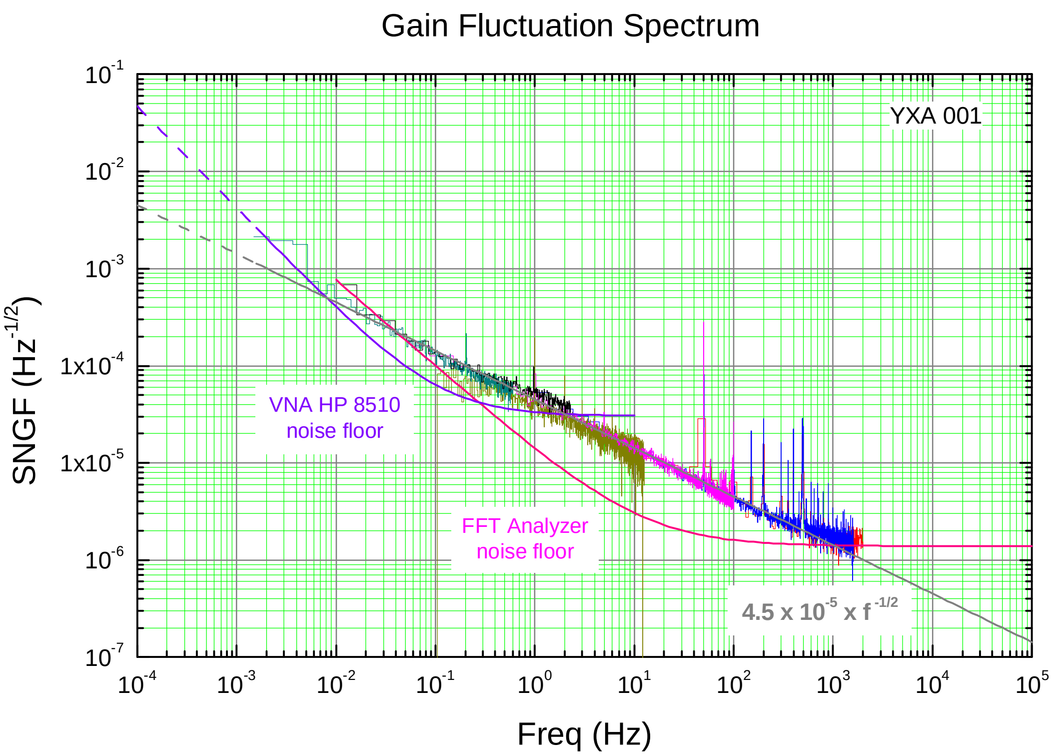


Figure 7: Measurement of SNGF of prototype amplifier YXA 001 at cryogenic temperature. The black and dark cyan lines are the spectra obtained with system B (VNA) of Figure 2. The rest correspond to different sweeps of system A. The fitted SNGF appears in grey.

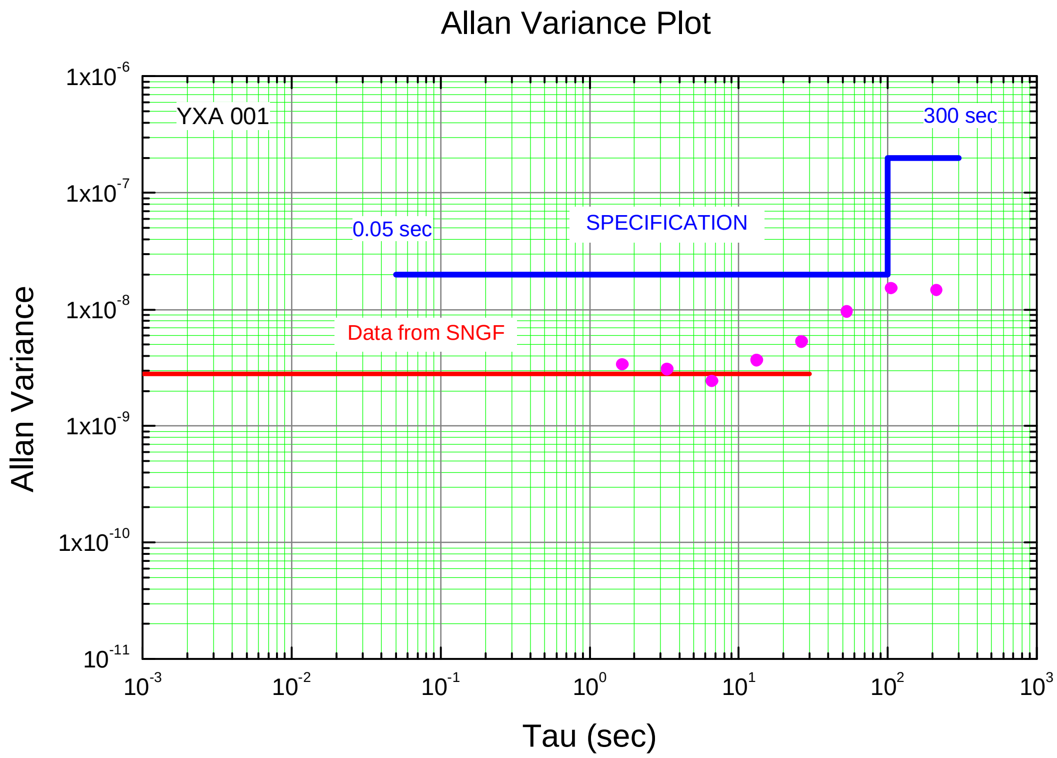


Figure 8: Allan Variance plot of the prototype amplifier YXA 001. The red line represents the transformation of the fitted spectrum of Figure 7 in its range of validity. The discrete Allan variance points (magenta) were calculated directly from the same data used for the low frequency end of the spectrum. Note that although the amplifier can be considered of low gain fluctuations the Allan Variance gets very close to the maximum allowed by the specification in the proximity of 100 seconds.

6. Effect of gain oscillations on Allan Variance

Figure 7 shows that oscillations at discrete frequencies are often found in SNGF. It is instructive to show how they would appear in an Allan Variance plot. We will assume an idealized SNGF of the form presented in [3] and with the values of [4]. Besides, we shall add an idealized oscillation of the normalized gain represented by:

$$\Delta G_n = A \cdot \sin(2 \cdot p \cdot f_0) \qquad [8]$$

This by itself will have a one side spectrum of the form:

$$S(f) = \tfrac{1}{\sqrt{2}} \cdot A \cdot d(f - f_0) \qquad [9]$$

Were δ represents the Dirac's distribution, A is the amplitude of the gain oscillation and f_0 is the frequency of the oscillation. We shall assume the following values for these parameters:

$$f_0 = 1 \cdot Hz \qquad A = 2 \cdot 10^{-4} \qquad [10]$$

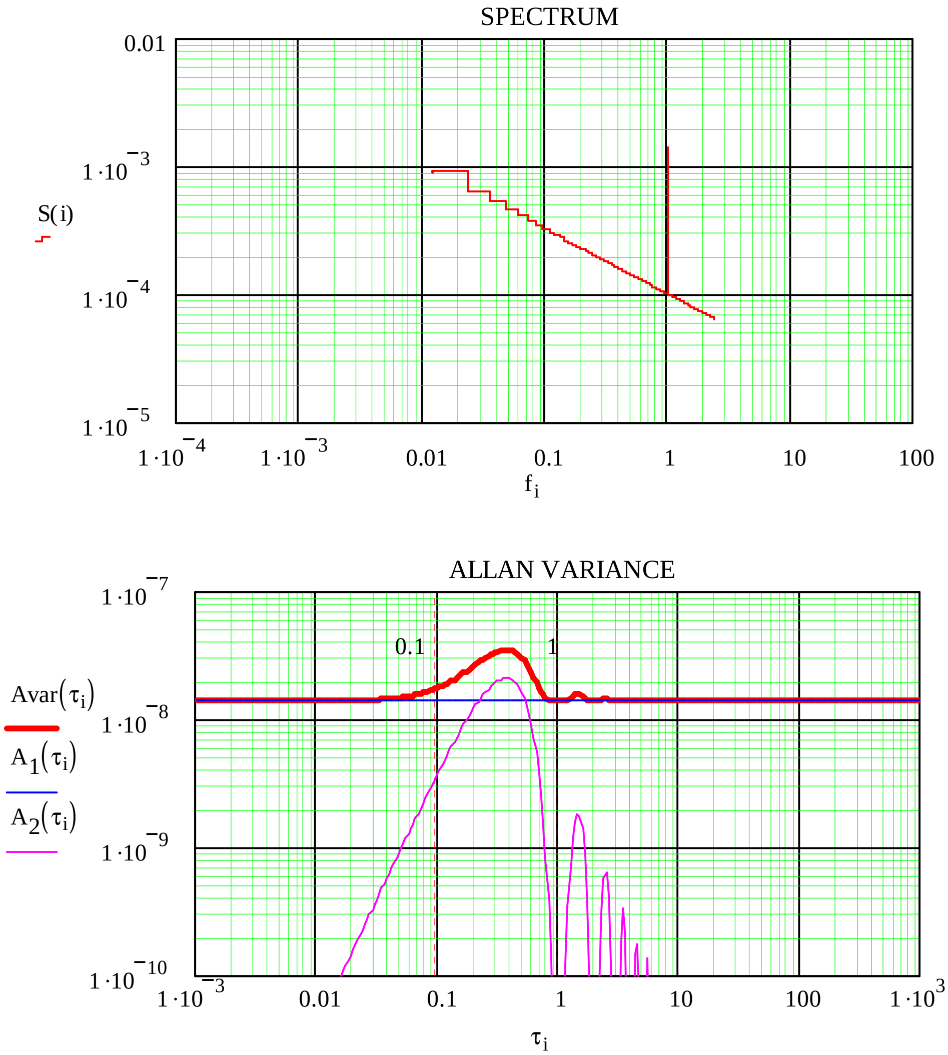


Figure 9: Spectrum and Allan Variance plot (red curve) of the combined *1/f* fluctuation from relations [3] and[4] and 1 Hz oscillation from [9] and [10]. The frequency resolution assumed for the spectrum is 0.012 Hz .The effect observed in the Allan Variance is an additional "bump" spread over the time interval 0.1-1 sec.

Note that this value of amplitude is quite small for practical measurements. It supposes a peak to peak gain variation of 0.0017 dB, difficult to resolve with mainstream microwave power measuring instruments. For example, a 12 bit ADC running at the middle of its range will have a resolution of 0.0021 dB. The amplitude value chosen is within the range observed in practical measurements for the gain oscillation induced by the cryocooler cycle. The spectrum would appear as a discrete line at 1 Hz superimposed over the continuum 1/f noise. The amplitude of the peak will depend on the resolution of the FFT.

From equation [14] we infer that the Allan Variance due to the oscillation would be:

$$A(t) = A^2 \cdot \left(\frac{\sin^2(p \cdot f_0 \cdot t)}{p \cdot f_0 \cdot t} \right)^2 \qquad [11]$$

Figure 9 presents the resultant SNGF and Allan Variance. The height of the peak at f_0 in the spectrum is dependant on the resolution of the FFT. For the plot in Figure 9 the value used is 0.012 Hz, which is the resolution of spectra obtained from the VNA presented in this work.

7. Thermal input termination versus CW signal generator for fluctuation test

Traditionally the tests for gain stability of complete radio astronomy receivers have been performed by measuring total power at the output with a thermal load at the input. The reasons for that are very clear:

- Radio astronomy receivers are designed to amplify noise-like signals from thermal level to power detection. A thermal load is its standard input.
- The output power fluctuation of a thermal load is merely determined by the stability of its physical temperature.
- The statistical fluctuation of a continuum signal is reduced to very low levels if the signal is detected with a large instantaneous bandwidth. For this reason, the tests are usually performed over the whole pass band of the receiver.

However thermal input terminations present some problem for testing gain stability of cryogenic amplifiers. Gain of cryogenic amplifiers is usually not more than 35 dB. Assuming this gain, a noise bandwidth of 8 GHz and an input termination at 290 K, the total noise power at the output of the amplifier would be about -40 dBm. Then, 20 dB more are needed to bring this power to the level of -20 dBm required at the detector of system B in Figure 2. This can be implemented by:

- Using an additional low fluctuation 20 dB amplifier either at the input or at the output of the cryogenic amplifier.
- Using a high ENR (20 dB) noise diode instead of the ambient temperature input termination.

Either solution would have the disadvantage of introducing an additional element in the chain with unknown fluctuations which should be characterized and calibrated.

Using wide band white noise at the input as test signal imposes an additional limitation due to its random nature. It can be shown that the normalized fluctuation of the detected power of a continuum signal shows a white spectrum with a value only dependant on its instantaneous noise bandwidth:

$$S_B(f) = \sqrt{\frac{2}{B}} \qquad [12]$$

This effect will be observed in addition to the fluctuations of Gain. The value of [12] calculated for B=8 GHz is presented in the graphic of Figure 10. The practical effect will be that white noise will dominate over approximately 40 Hz making impossible to appreciate the 1/f shape of amplifier fluctuation for Fourier frequency larger than ~10 Hz. This may not be relevant for the specifications of ALMA, but it is interesting to know that the 1/f shape of gain fluctuation is still valid beyond that and it is not flatten in the range observable with CW measurements.

It should be obvious that measurement of gain fluctuations with wide band noise input will provide a value which is a gain-weighted average over the whole band width of the amplifier. Then, all the details of a possible dependence of the gain fluctuations on the CW frequency are lost. This may be of relevance in some cases, as it will be shown in next section.

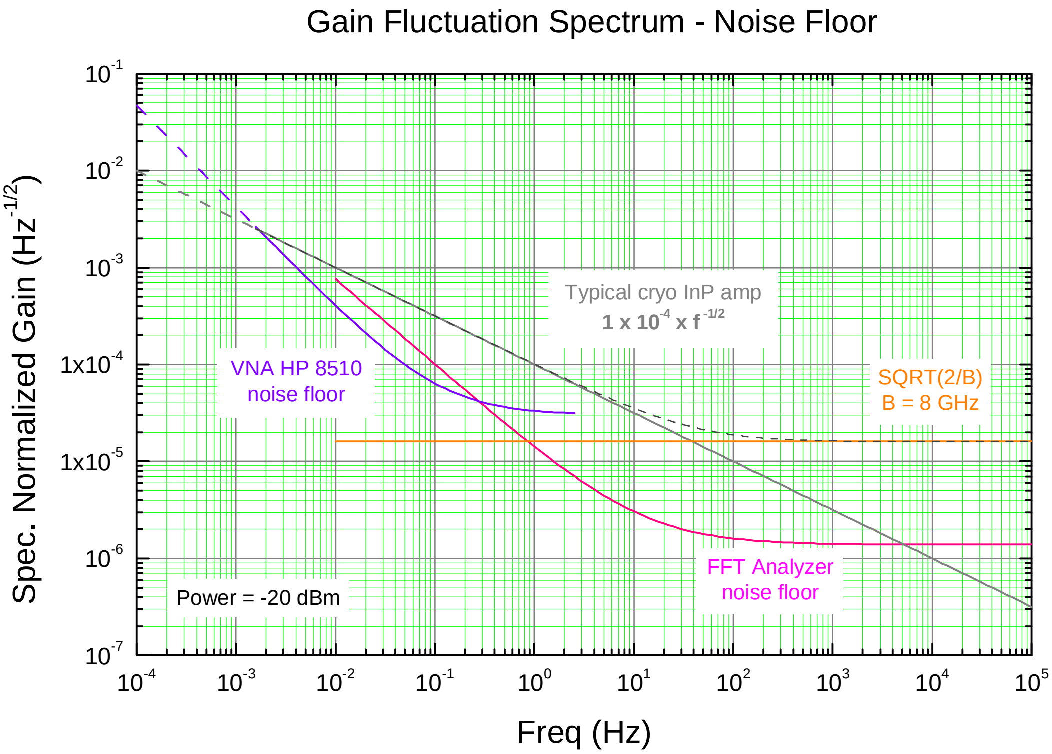


Figure 10: Noise floor of the experimental setups used for gain fluctuation measurement compared with the typical spectrum of a cryogenic amplifier. The orange horizontal line represents the fluctuation appearing if a wide band Gaussian noise signal is used at the input for the test (value given for 8 GHz bandwidth). The rest is the same as in Figure 3.

8. *Variation of gain fluctuations with input frequency and transistor bias.*

In previous sections the SNGF has been assumed a fixed feature of the cryogenic amplifier. Measurements presented have been taken with a CW frequency in the middle of the pass band and at a fixed bias point for the transistors. However, our measurements show a clear evidence of a quite noticeable dependence of the fluctuations on both, input frequency and bias.

To our knowledge, dependence with input frequency has not been previously reported, and it is particularly interesting because it will remain undetectable for the tests usually performed on complete radio astronomy receivers. Still, **it could have an important effect on the quality of the data obtained in spectral line observations**. Figure 11 presents the results obtained in an ALMA pre-production amplifier at cryogenic temperature with nominal bias. There is a factor of three of variation across the band and almost inverse linear dependence with frequency was observed.

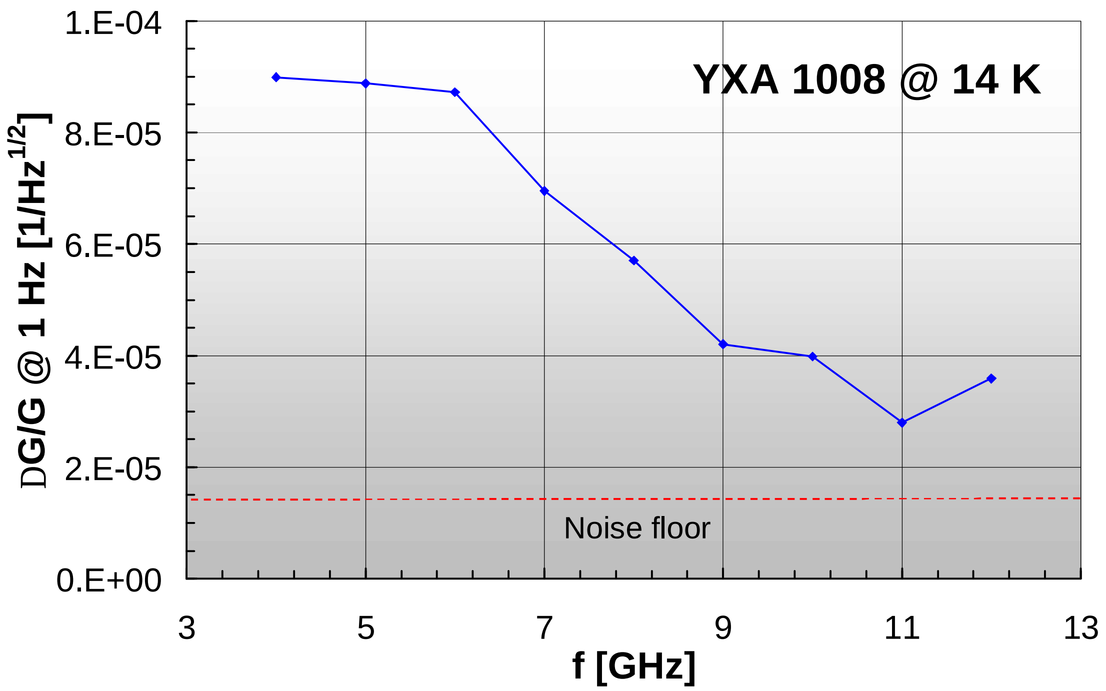


Figure 11: Measured dependence of SNGF of a cryogenic amplifier on input frequency. The plot presents *b*, the value of the spectrum at 1 Hz Fourier frequency.

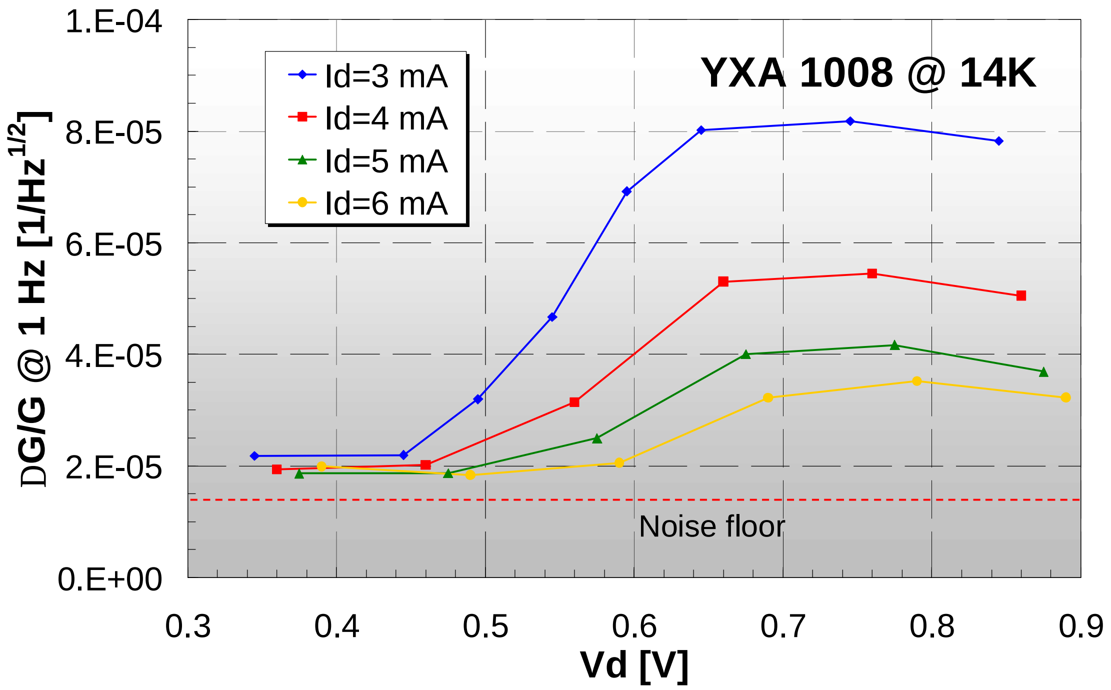


Figure 12: Measured dependence of SNGF of a cryogenic amplifier on bias point of the first stage transistor. As in previous plot, *b* is the quantity shown.

We have tried to explain frequency dependence observed by simulating fluctuations in the values of some of the key elements of the equivalent circuit used to represent the transistor in the computer model of the amplifier. The negative slope observed in Figure 11 could only be explained by fluctuations of G_m (transconductance). Fluctuations simulated in C_{gs} (gate-source capacitance) led to an opposite frequency dependence than observed. If the G_m hypothesis were true, the fluctuations at different frequencies in the band pass, although of different value, will be completely coherent.

Figure 12 shows the dependence of the gain fluctuation with bias measured in the same amplifier. The measurements were taken at cryogenic temperature and at the central frequency. The characteristics of Figure 12 were obtained for InP HEMTs made by HRL, but the same pattern has been observed in InP devices from other sources (NGST, ETH). The main characteristic found is a fast increment of the fluctuations produced when the drain voltage (V_d) is raised from 0.5 to 0.7 Volts. The increment is more pronounced for low values of drain current (I_d). Note that the fluctuations may change by a factor of 8. It is important to remark that the change in gain fluctuation shows a totally different dependence with bias than the noise temperature. If the fluctuation dependence is not taken into account, it is very easy to end up into a region of high fluctuation when trying to optimize other parameters of the cryogenic amplifier by adjusting the bias.

9. Conclusions

- Testing the gain stability of the cryogenic amplifiers in the range required by ALMA specifications is possible using a combination of measurements made with two different systems, one based on a Vector Network Analyzer and the other on a FFT analyzer.
- In the range were interferences from cryogenic refrigerator cycle or power line exist, the best approach would be to measure the SNGF in the frequency domain, fit a simple spectrum excluding spurious lines and transform it to Allan Variance.
- As a rule of thumb (strictly valid for SNGF with of the form $b \cdot f^{-1/2}$ with error less than 2.5%), for the calculation of Allan Variance, the range of frequencies for which the spectrum needs to be known is given by:

$$f_{\min} \approx \frac{0.06}{t_{\max}} \quad ; \quad f_{\max} \approx \frac{1}{t_{\min}}$$

- Covering the largest times required by the ALMA specification can be accomplished by calculating Allan Variance directly from slow time scans.
- Gain stability has been fully characterized in a prototype ALMA cryogenic IF amplifier (YXA 001, BW=4-12 GHz, Tn=5.4 K, G=32 dB).
- The gain stability of the prototype amplifier fits very well a SNGF of the form $b \cdot f^{-1/2}$ (flat Allan Variance) in a large frequency range. However, for measurement times greater than 10 seconds, the direct calculation of Allan Variance show an increase over the value for obtained for short times.
- From the measurements of the prototype amplifier it was found that there is very little margin to achieve the proposed specification (Allan Variance) in the proximity of 100 seconds.
- An important dependence of gain stability of the amplifier on input frequency and bias has been identified in the measurement of a pre-production amplifier. The dependence on input frequency was not previously reported and may have an important impact in total system performance when used for spectral line observation in wide band modes. Bias dependence should be carefully considered in the development of amplifiers.

APENDIX I

Practical configuration of measurement systems

<u>*System A:*</u> Fast Fourier Transform Analyzer

- Agilent 35670A Dynamic Signal Analyzer (FFT)
- Agilent 8474C Detector (10 MHz-33 GHz)
- Continuously Variable Attenuator (ARRA)
- HP34401A Multimeter (for DC value measurement)
- Signal Recovery 5113 Low Noise Pre-amplifier (AC coupled, 10 sec.)
- HP83650B Signal Generator (10 MHz-50 GHz)
- Frequency: 8.45 GHz
- Power level: set to -20 dBm
- Gain of the DC amplifier set to 900 (output of -5 V with -20 dBm input)

<u>*System B:*</u> Vector Network Analyzer

- HP8510C + HP8517A + HP83651A (45 MHz-50 GHz)
- Continuously Variable Attenuator (ARRA)
- Power output at the end of the flexible cable: set to -20 dBm
- Frequency: 8.45 GHz
- Number of averages: 1024 or 4096
- Number of points set to 401 or 801

In the case of the measurements with the FFT analyzer is very important to reduce the 50 Hz noise interference (and harmonics) to the lowest possible level. For a DC voltage of 5 V at the input of the FFT Analyzer, the level of the interference peaks should be below 500µV rms. To avoid higher values of interference is important to:

- Operate the preamplifier with batteries. Do not insert the connector from the power supply in the socket of the preamplifier. Keep the ground of the preamplifier isolated from the earth!
- Disconnect the display of the preamplifier when taking measurement (push the "recover" button).
- Set the input front-end of the FFT analyzer with floating ground (Isolated from earth).
- Do not connect the signal ground of the FET power supply to earth while measuring.
- Use short coaxial cables for signal routing.
- Keep cellular phones strictly off.
- Keep people away from the set up.
- Avoid vibration of coaxial cables transmitted from the refrigerator.

APENDIX II

Definitions used in this document

Let G(t) be a random function of time. It could be, for example, the normalized insertion gain of an amplifier. *Allan Variance* σ^2 is defined as:

$$\sigma^2(\tau) = \frac{1}{2}\left\langle \left(\overline{G}(t+\tau) - \overline{G}(t)\right)^2 \right\rangle \qquad [13]$$

Were $\overline{G}(t)$ and $\overline{G}(t+\tau)$ represent the average of the function G at times t and t + τ respectively, and the brackets represent the expected value.

It can be demonstrated that, with the definition of σ given above, the transformation between the unilateral spectral density and the Allan Variance is given by:

$$\sigma^2(\tau) = 2 \cdot \int_0^{\infty} (S(f))^2 \cdot \frac{\sin^4(\pi \cdot \tau \cdot f)}{(\pi \cdot \tau \cdot f)^2} \cdot df \qquad [14]$$

Being S(f) spectral density of G (SNGF) with dimensions $1/\sqrt{Hz}$.

The SNGF can be calculated numerically by Fourier transformation of the gain. There are numerous programs and subroutines available to perform the calculation. As the normalization criteria is not always known and may vary between different programs is convenient to check the values obtained. This can be easily done using these simple tests:

$$\begin{aligned} & S(0) = \mathrm{mean}(G(t)) = 1 \\ & \int_{f_{\min}}^{f_{\max}} \left|S(f)\right|^2 \cdot df = \mathrm{var}(G(t)) \end{aligned} \qquad [15]$$

That is, for cero frequency the value of the spectrum should be equal to the DC (average) value of the gain (1 for normalized gain) and the integration of the power spectral density should give its variance (classical, not Allan).

Ambiguity of inverse transformation

As it is shown in relation [14], there exists a well known transformation from the SNGF to Allan Variance. However, it has been demonstrated that the unique inversion of [14] is not possible in general[3] since the mapping from SNGF to Allan Variance is not one to one. Then, from the point of view of the analysis of the fluctuations, it can be argued

[3] C. A. Greenhall, "Does Allan Variance determine the Spectrum?", 1977 Proceedings of International Frequency Control Symposium, pp. 358-365.

that the SNGF is more descriptive because the same Allan Variance could be obtained for more than one spectrum. This does not mean that Allan Variance is an inappropriate way to describe the statistical fluctuations of random signals. In many practical applications it may be very useful. In the case of Radio Astronomy it is used because it provides a direct measurement of the fluctuations in the receiver as a function of the integration time.

However, in practice, for most physical situations, random gain fluctuations[4] can be described with a SNGF represented by a linear combination of functions of the form $f^{-\alpha}$. In this particular case, the mapping to Allan Variance is one to one. The corresponding slopes in the time and frequency domain are related as:

$$S(f) = \beta \cdot f^{-\alpha} \quad \Leftrightarrow \quad \sigma^2(\tau) = K_\alpha \cdot \beta^2 \cdot \tau^{2\alpha-1} \qquad \text{(valid for } -\tfrac{1}{2} < \alpha < \tfrac{3}{2}) \qquad [16]$$

Some values of K_α where a simple analytical solution exists are:

$$\begin{aligned}
K_1 &= \frac{2}{3}\cdot\pi^2 \\
K_{3/4} &= \frac{32}{15}\cdot\pi\cdot\left(\sqrt{2}-1\right) \\
K_{1/2} &= 2\cdot\ln(2) \\
K_{1/4} &= \frac{4}{3}\cdot\left(2-\sqrt{2}\right) \\
K_0 &= \frac{1}{2} \\
K_{-1/4} &= \frac{1}{2\pi}\cdot\left(4-\sqrt{2}\right)
\end{aligned} \qquad [17]$$

[4] Excluding some possible deterministic effects like gain oscillations induced by temperature or vibration of refrigerators or related with power supply ripple.

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