# Relativistic filamentation instability in an arbitrarily oriented magnetic field

E. Perez Alvaro[1] , A. Bret[1]

[1]ETSI. Industriales, Universidad Castilla-La Mancha, Spain
And Instituto de Investigaciones energéticas y Aplicaciones Industriales, Ciudad Real, Spain

## Abstract

Gamma Ray Burst (GRB's) and High Energy Cosmic Rays (HECR's) are important enigmas in the field of Astrophysics. Today the most studied scenario is known as "Fireball", in which particles are accelerated by an ultra-relativistic shock from the explosion of supernovae (SN).

The shock is a direct result of the "collision" between different shells of plasma ejected by the SN during the explosion. In this non-collisional environment, counter streaming plasma shells generate a shock through beam-plasma instabilities.

In fact, the filamentation instability is believed to be the shock mediator [1].

For two counter-streaming plasmas immersed in a magnetic field parallel to the flow, this instability can be canceled for a critical field value.

We investigate the development of the filamentation instability for an arbitrarily oriented magnetic field, in order to know whether such suppression of this instability persists in this new field.

## Relation between HECR's, GRB's and instability

The origin of High Energy Cosmic Rays (HECR's) and Gamma Ray Burst (GRB's) are two mysteries in Astrophysics today.

The most energetic particle ever detected had an energy of 50 Joules.

**Fig.1** *Cosmic rays reaching Earth's surface*

The probably of finding a particle decays with its energy as shown in figure 2.

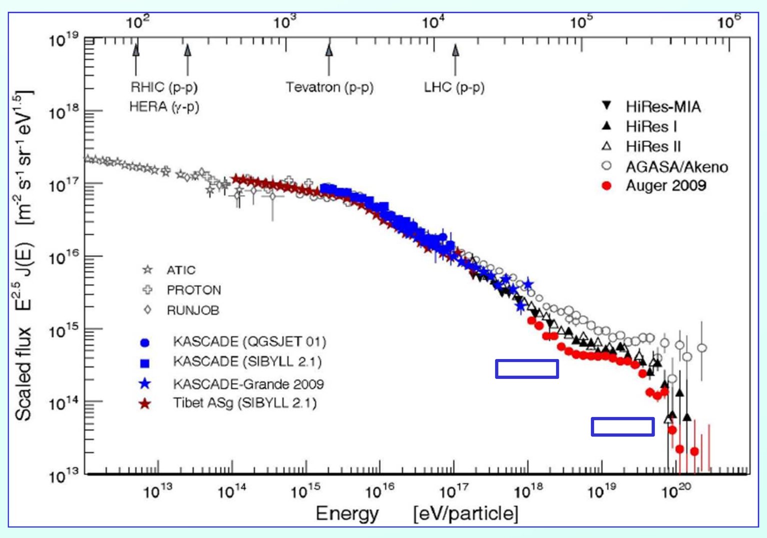



**Fig.2** *Cosmic rays spectrum [2]*

The energy of a GRB's is between $10^{51}$ y $10^{54}$ erg, with an average duration of 2 seconds.

In the so-called "Fireball" scenario (Fig. 3), particles are accelerated by an ultra-relativistic shock from the explosion of a supernova (Figure 3).

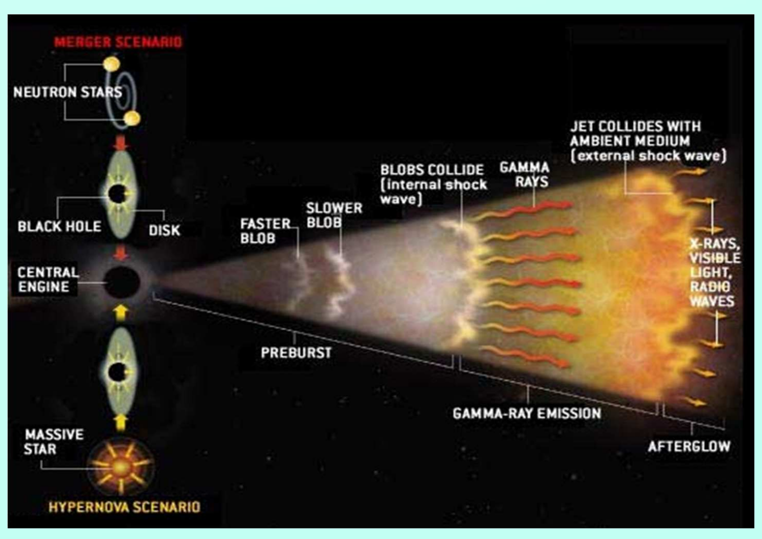



**Fig.3** *Fireball scenario*

This acceleration is based on the idea of Enrico Fermi, in which particles are accelerated by moving magnetic walls, as if they were hit by tennis rackets. These idea is reflected in the following figure:

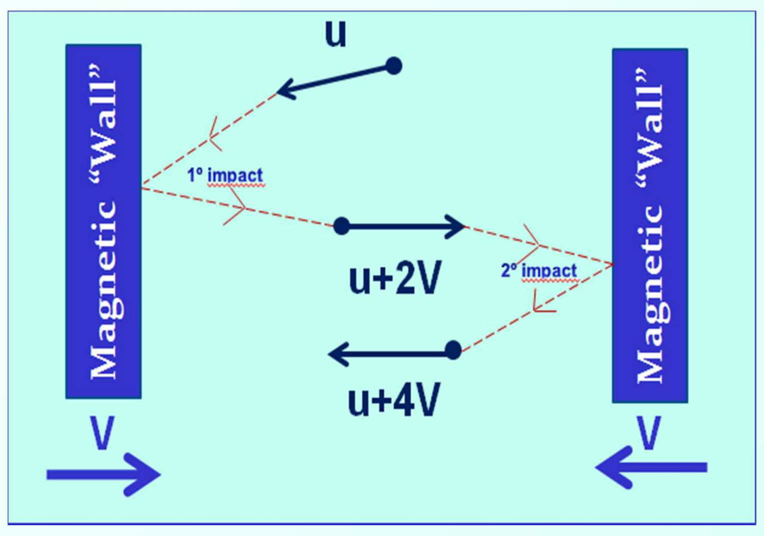



**Fig.4** *Fermi acceleration.* ***u*** *is the initial velocity,* ***u+2V*** *is the velocity after the firs impact and* ***u+4V*** *is the velocity after the second impact.*

Fermi's idea works well but… Where are there MAGNETICS "WALLS" in nature?

A shock in a plasma can act like a magnetic "wall". In Fluids Mechanics a Shock is a jump in the density, like a step function. In reality, the jump is not abrupt. In a plasma, magnetic turbulence takes place in the transition region and in front of the shock, acting like (partial) magnetic walls.

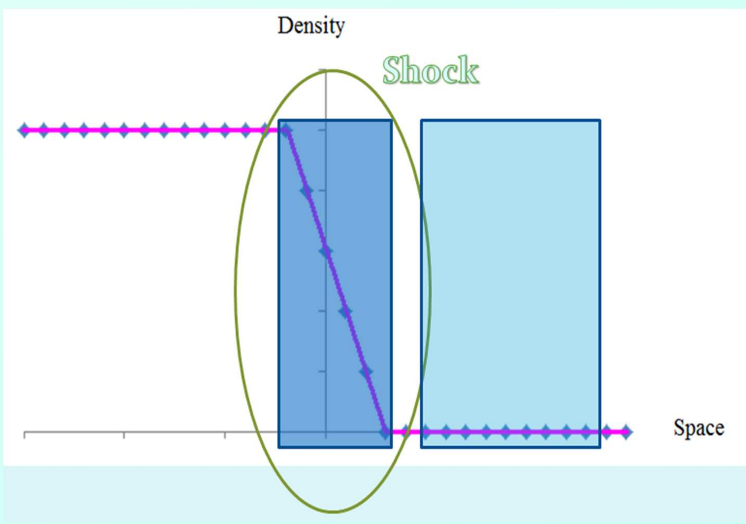
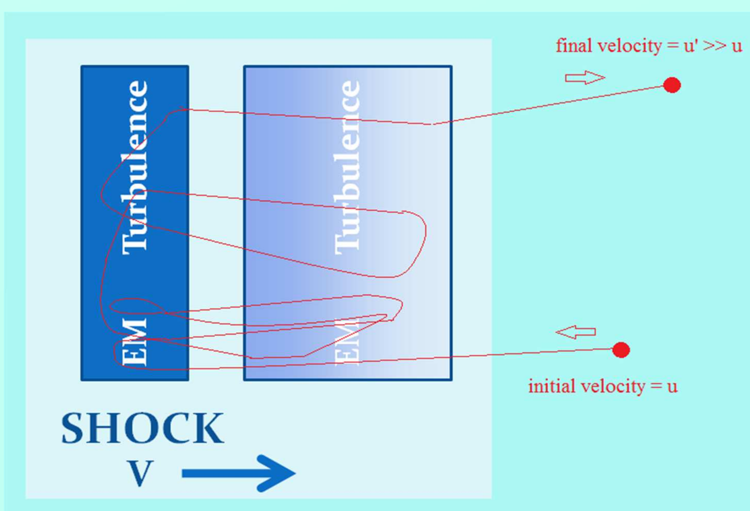



**Fig.5** *Representation of a shock in a plasma with zones of electromagnetic turbulence*

WHERE DOES THE SHOCK COME FROM? Two counter-streaming plasmas shells form an unstable system. The shock is the result of the instabilities triggered.




## Our interest

Particle in Cell (PIC) simulations show the filamentation instability is mediating the shock. This instability develops in the direction perpendicular to the propagation of plasma.

It is well-known that a flow aligned magnetic field can cancel this instability. For symmetric counter-streaming shells, the critical value of the magnetic field is [3]:

$$\Omega_B{}^* > \beta\sqrt{2\gamma}$$

See definition of the parameters bellow.

We investigate the case of an arbitrarily oriented magnetic field.

## Physical Model

We studied the growth rate of the filamentation instability in a system of two infinite identical counter-streaming electron/proton plasma shells, exposed to a magnetic field oriented arbitrarily. The angle $\theta$ is defined as the angle between the magnetic field and the direction of propagation of the plasma:

$$B = B_0 sin(\theta)\overrightarrow{u_x} + B_0 cos(\theta)\overrightarrow{u_z}$$

We assumed both plasmas have the same densities ($n_1 = n_2$), and the same opposite velocities ($V_1 = V_2 \equiv V$) with the relativistic factor:

$$\gamma = \left(1 - \left(\frac{V}{c}\right)^2\right)^{-1/2}$$

The current and the charge are initially neutralized.

The dielectric tensor of the system was obtained from the fluid equations for each plasma and linearizing these expressions in the cold approach. The dispersion equation reads $det\, T = 0$, where $T$ is the dielectric tensor obtained combining Maxwell-Faraday and Maxwell-Ampere equations.

We obtain the dispersion equation from the linearized Maxwell equations:

$$\frac{c^2}{\omega^2} k \times (k \times E_1) + E_1 + \frac{4\pi i}{\omega} J = 0$$

with $J = q\sum_{j=1,2} n_{j0}V_{j1} + n_{j1}V_{j0}$.

We used the reduced variables:

$$x = \frac{\omega}{\omega_p} \quad , \quad \Omega_B = \frac{\omega_b}{\omega_p} \quad , Z_k = \frac{k \cdot V}{\omega_p}$$

Where $\omega_p$ is the electronic plasma frequency and $\omega_b$ is the electronic cyclotron frequency.

## Results

Representing the growth rate ($\delta$) for $\theta$=0, the possible suppression of the instability is retrieved (Fig. 7)

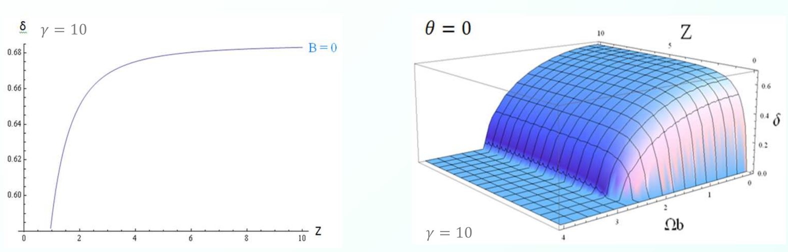
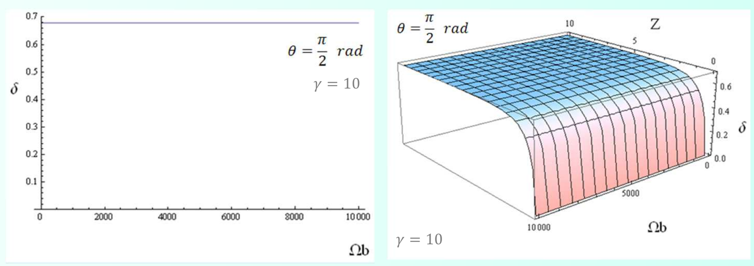
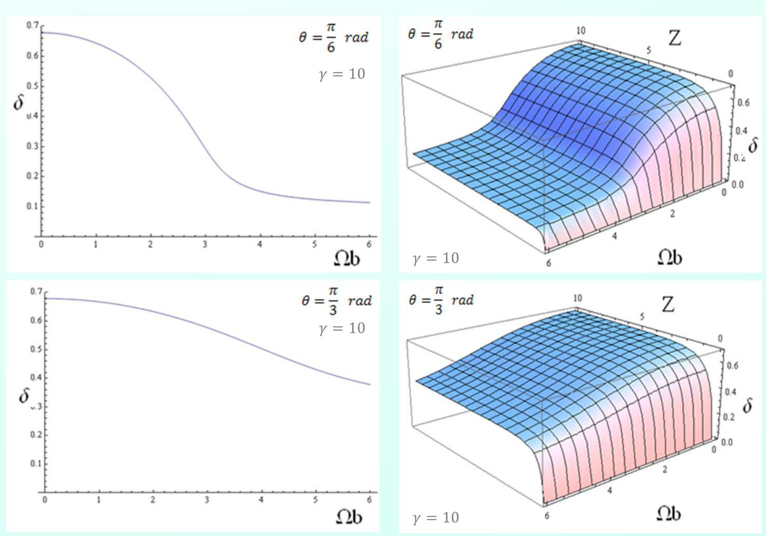


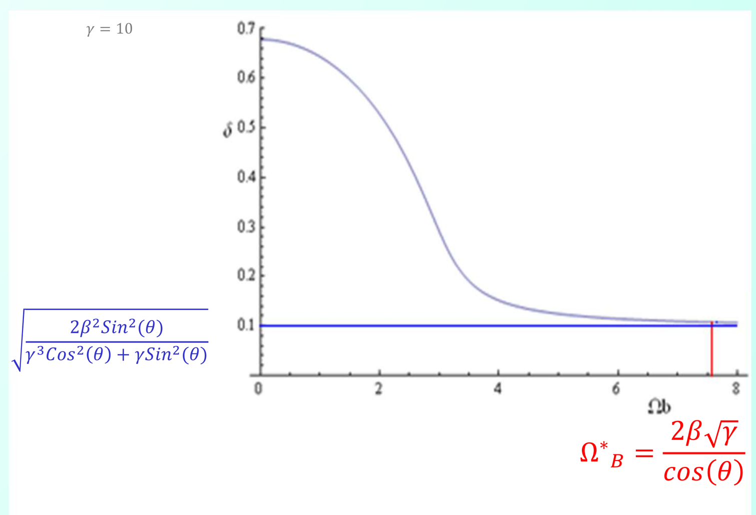

**Fig.7** *Representation of the filamentation growth rate VS the wave vector for θ=0. Representation in 3D is the filamentation growth rate VS the wave vector and VS the external magnetic field*

We repeat this calculation for $\theta \in [0, \pi/2]$ and as examples of the results we present the next figures:




Analytical results:




Instead of falling to zero, the growth rate saturates to a finite value for field strengths larger than [4]:

$$\Omega^*{}_B = \frac{2\beta\sqrt{\gamma}}{cos(\theta)}$$

**In nature it is impossible to find an external magnetic field perfectly aligned with the flow… therefore filamentation instability should always exist.**

**References:**

[1] Medvedev and Loeb, Astrophysycal Journal 526, 698 (1999)

[2] Matthiae et al., New Journal of Physics 12, 075009 (2010)

[3] Godfrey, Physics of fluids 18, 346 (1975)

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