Introduction to Radio Interferometry

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Associated Universities, Inc.

Atacama Large Millimeter/submillimeter Array
Karl G. Jansky Very Large Array
Very Long Baseline Array

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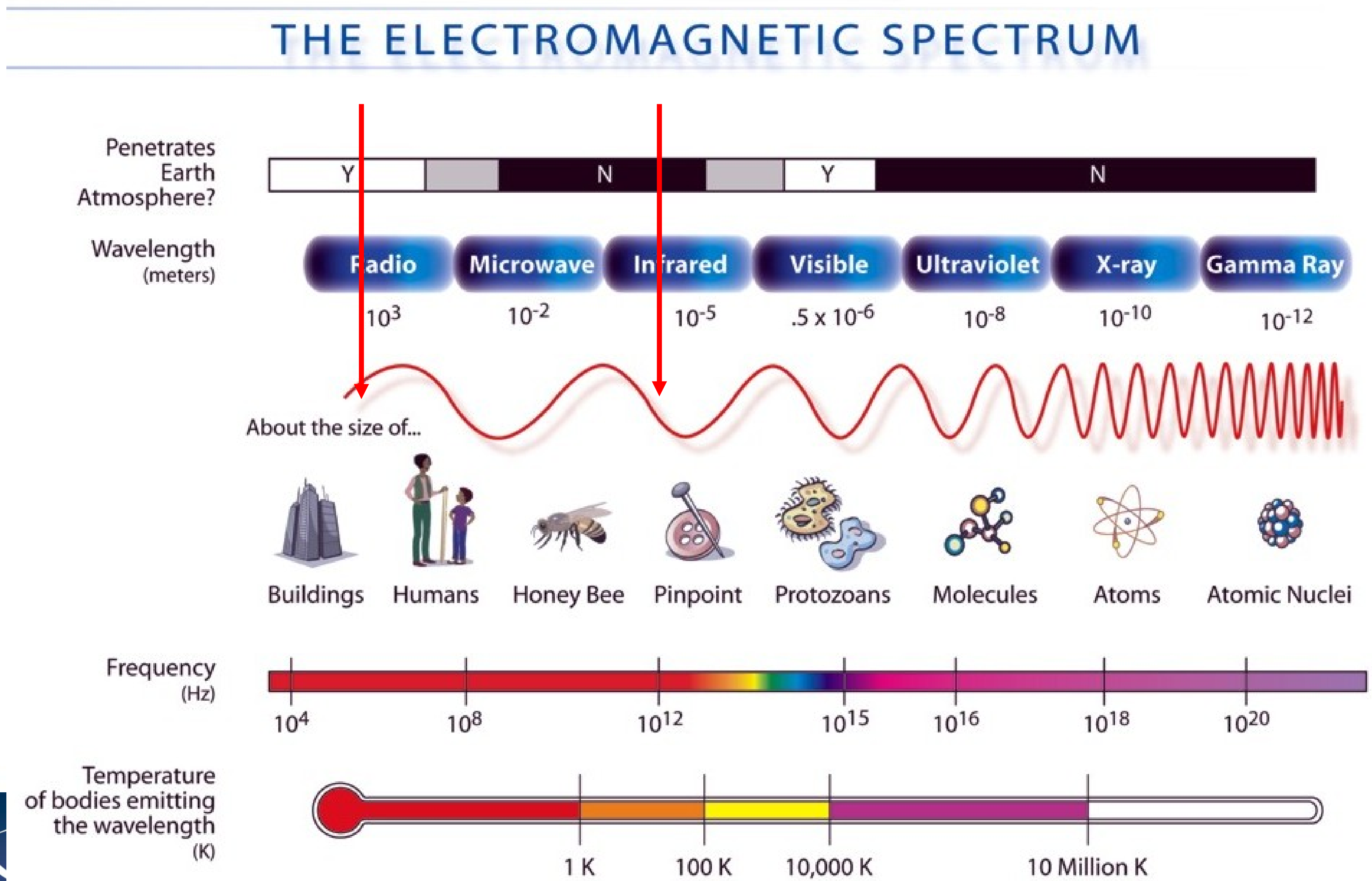
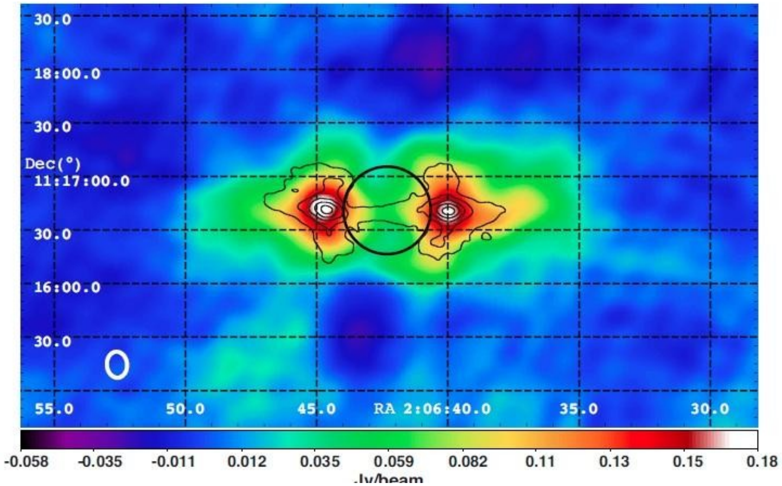
Radio Astronomy

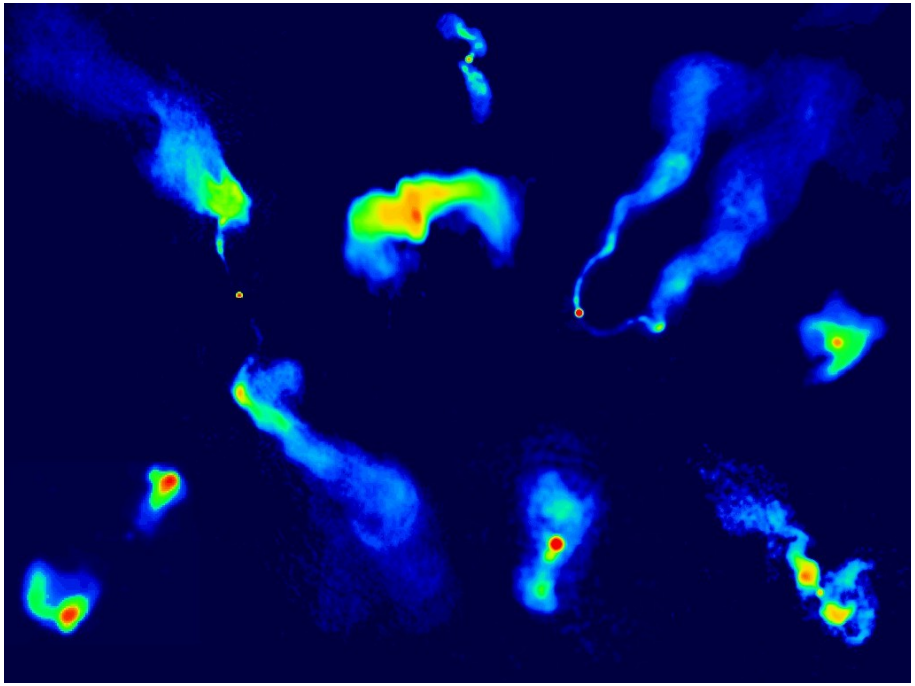
Now used to refer to most telescopes using heterodyne technology



What can we observe? (MHz-GHz range)

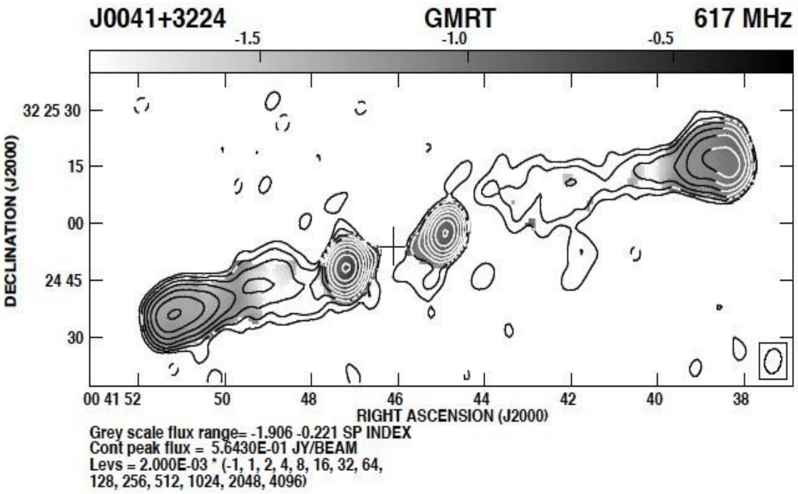
Jupiter's radiation belt at 100MHz





Synchrotron emission from extended radio galaxies (5 GHz)

Relic emission from old radio galaxies

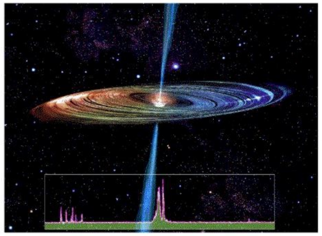


Images from NRAO Image Gallery: http://images.nrao.edu/

Images from NRAO Image Gallery
http://images.nrao.edu/

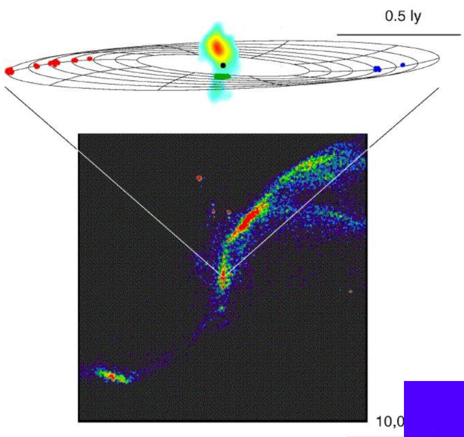
What can we observe?

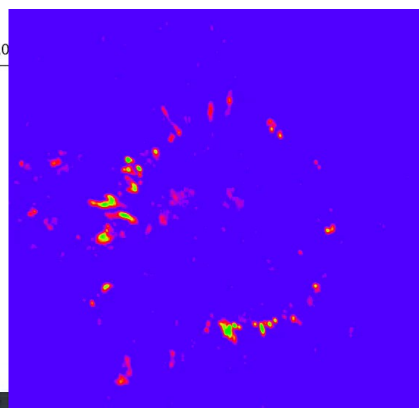
At low frequencies (MHz-GHz):

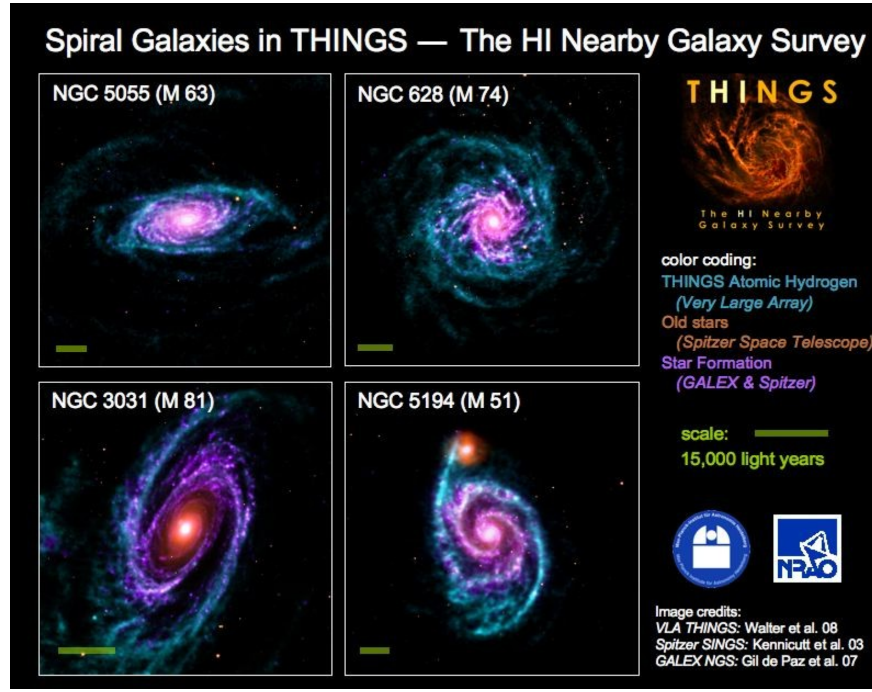


NGC 4258

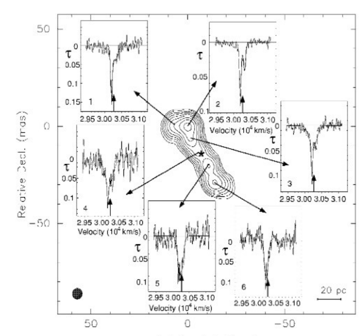
H_2O, OH or SiO masers in galaxies and stars

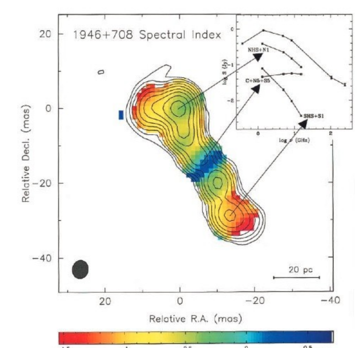






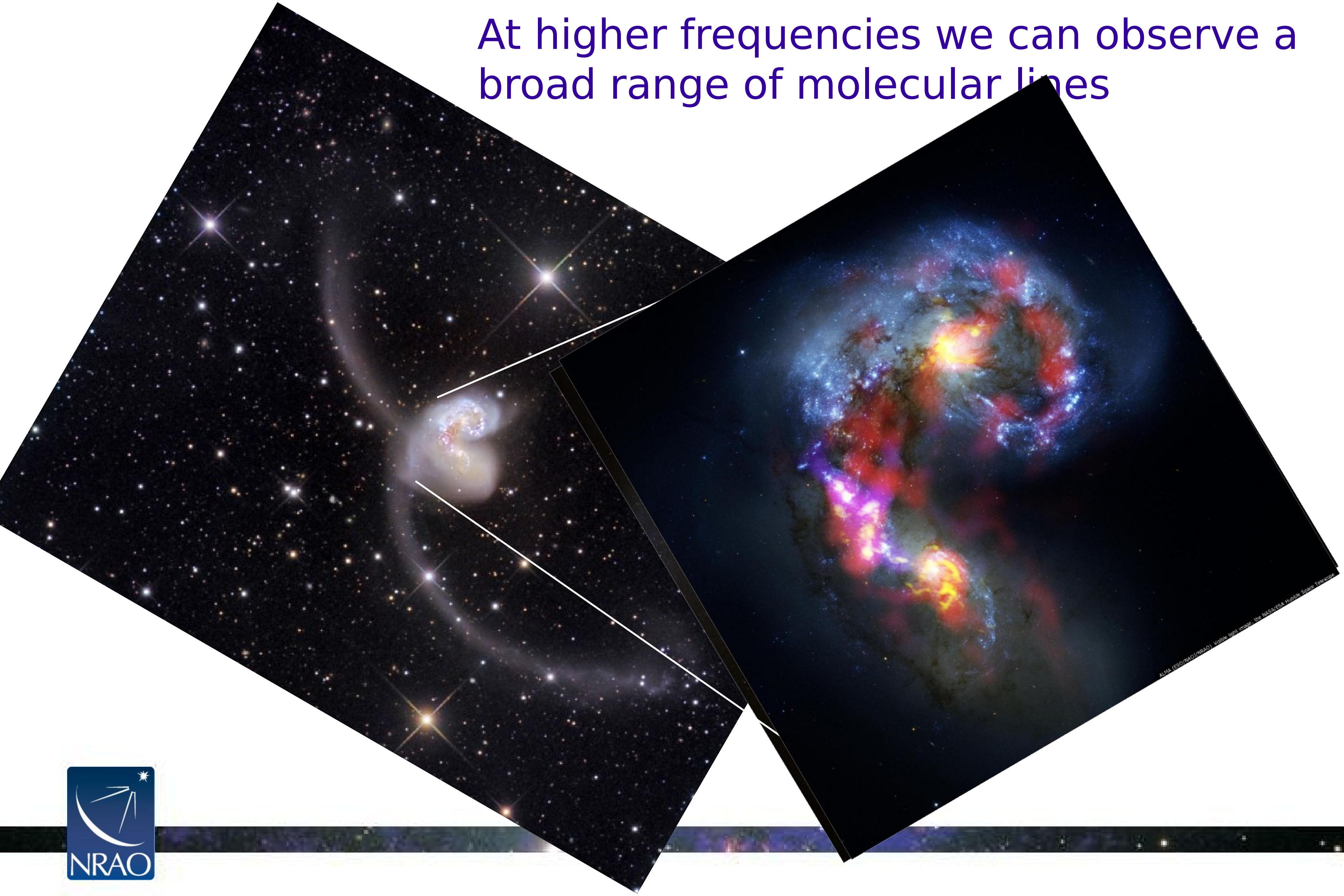
HI emission and absorption, free-free absorption in galaxies





What can we observe?

At higher frequencies we can observe a broad range of molecular lines



Resolution of Observations

Angular resolution for most telescopes is ~ λ/D

D is the diameter of the telescope and λ is the wavelength of observation

For the Hubble Space Telescope:

λ ~ 1um / D of 2.4m = resolution ~ 0.13”

To reach that resolution at λ ~1mm, we would need a 2 km-diameter dish!

Instead, we use arrays of smaller dishes to achieve the same high angular resolution at radio frequencies

This is interferometry!

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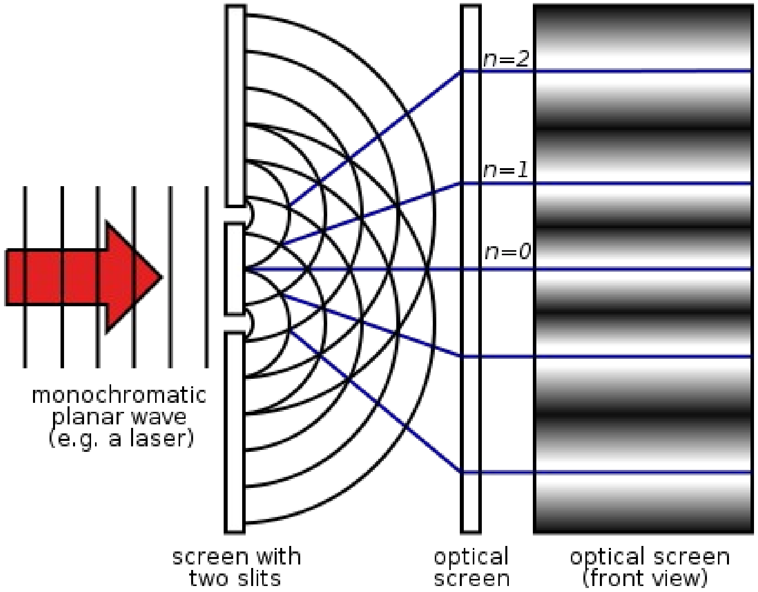
What is an interferometer?

An *interferometer* measures the interference pattern produced by multiple apertures, much like a 2-slit experiment



Note: *However, the interference patterns measured by radio telescopes are produced by **multiplying** - not adding - the wave signals measured at the different telescopes (i.e. apertures)

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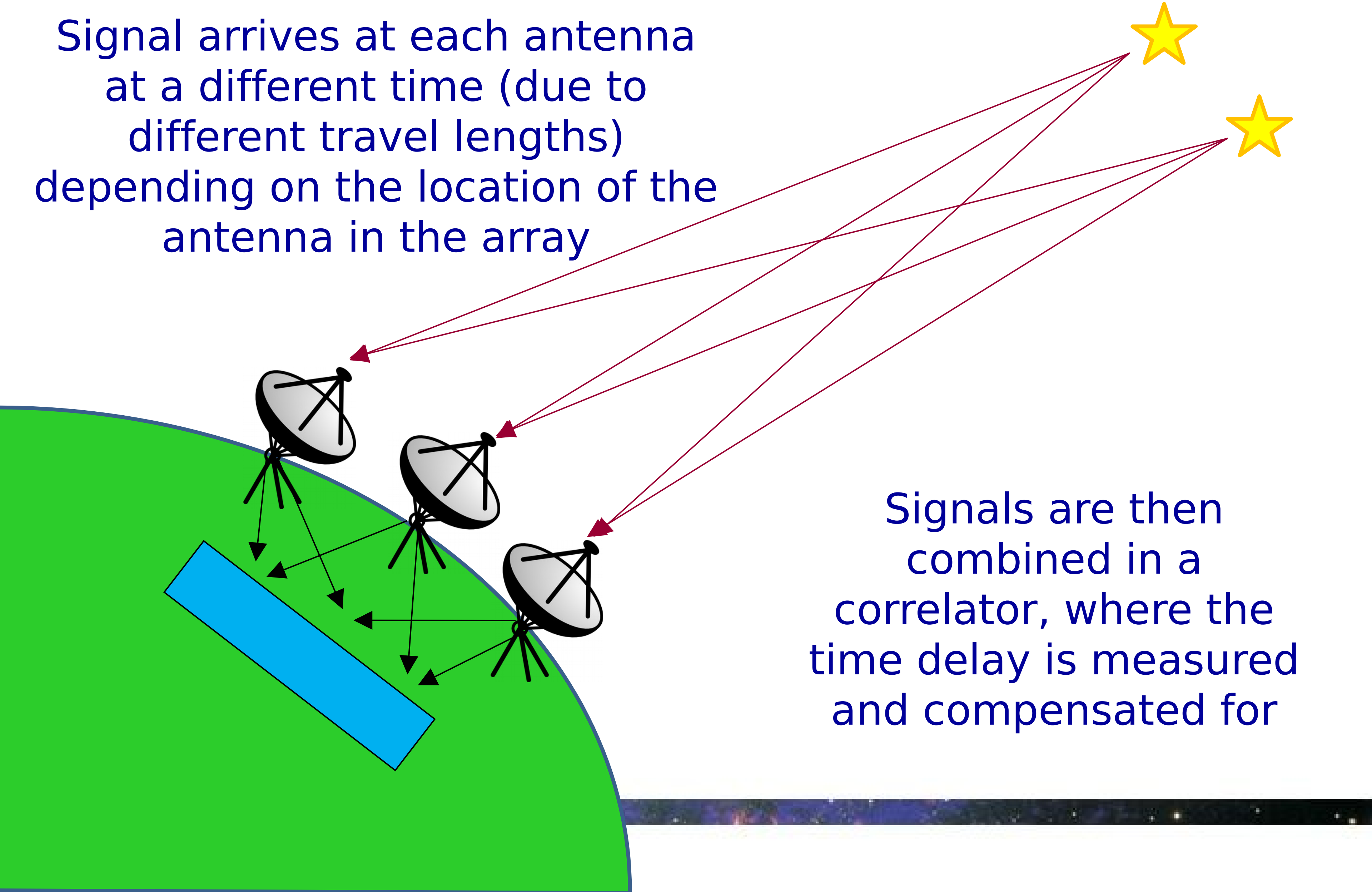
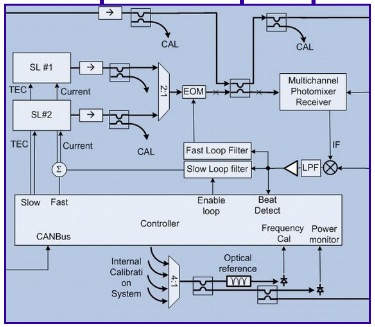
How Do We Use Interferometry?

Signal arrives at each antenna at a different time (due to different travel lengths) depending on the location of the antenna in the array



Signals are then combined in a correlator, where the time delay is measured and compensated for

Some Instrument Details



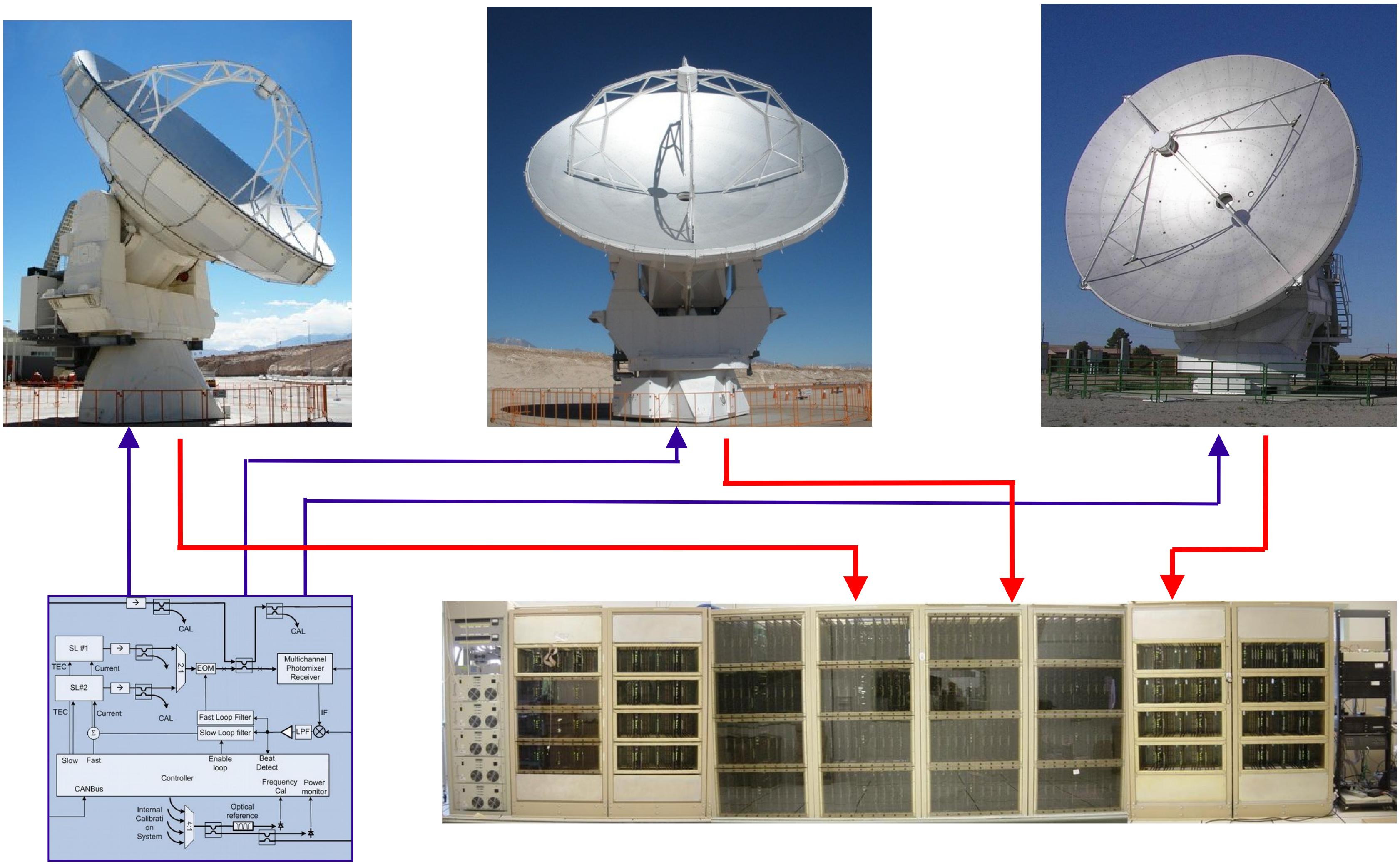
To precisely measure arrival times we need very accurate clocks

At Band 10 one wavelength error = 1 picosecond = 10^{-12} s (!!)

Need << 1 wavelength timing precision, so each antenna has an on-board clock with high sampling rates

Once determined, the reference time is distributed to all antennas

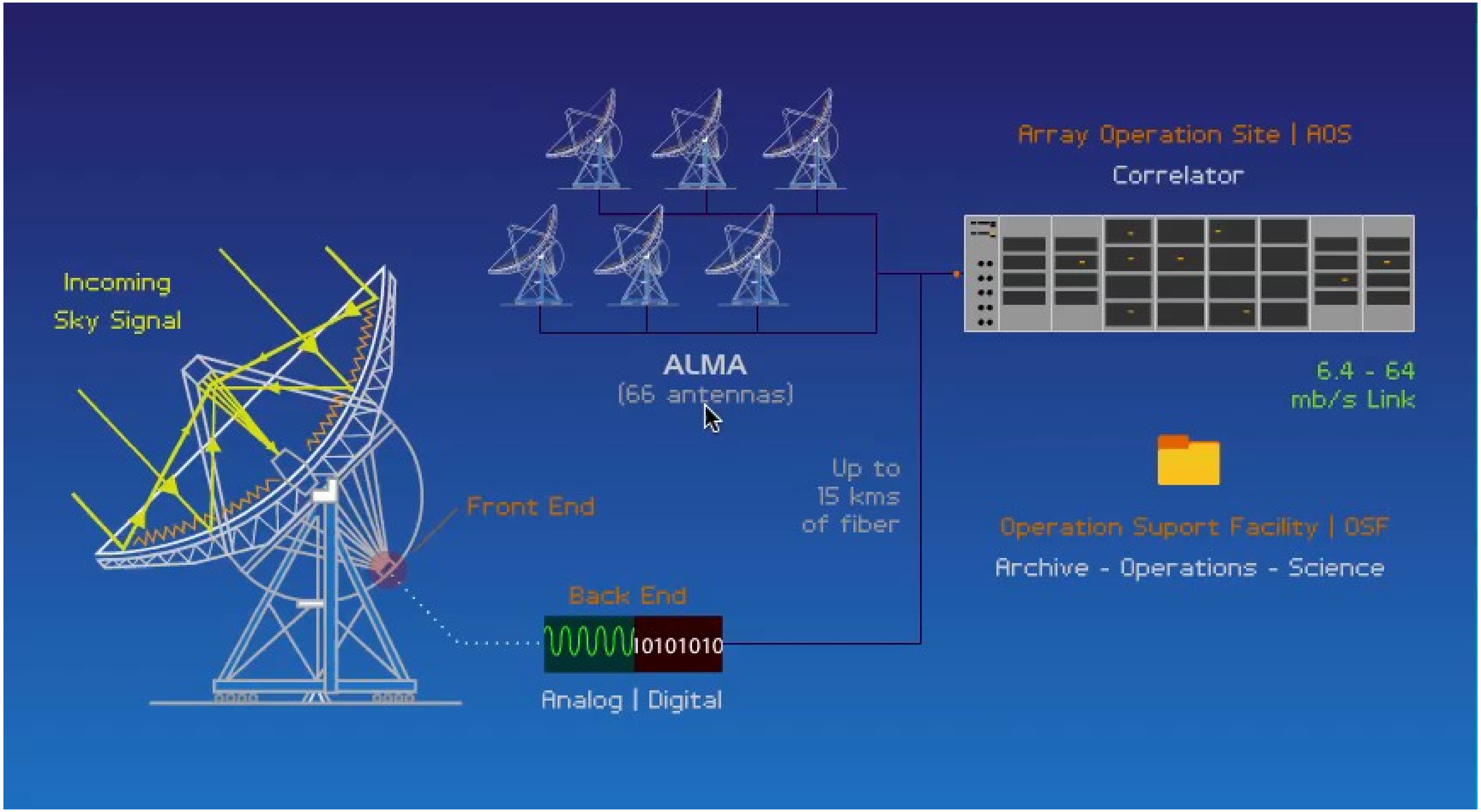
Some Instrument Details



Signals from each antenna are digitized and sent to the correlator for multiplication & averaging

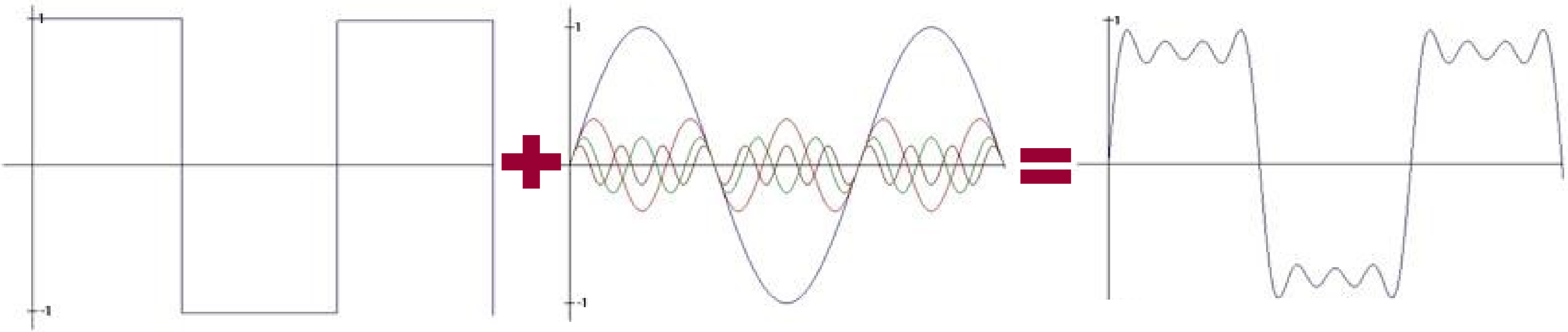
For ~50 antennas, the data rate is 600 GB/sec for the correlator to process

An Interferometer In Action



Introducing the Fourier Transform

Fourier theory states that any well behaved signal (including images) can be expressed as the sum of sinusoids



Reference signal

4 sinusoids

Sum of sinusoids & signal

The Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components

The Fourier transform contains *all* of the information of the original signal

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Visibility and Sky Brightness

(The van Cittert-Zernike theorem)

- **Visibility as a function of baseline coordinates (u,v) is the Fourier transform of the sky brightness distribution as a function of the sky coordinates (x,y)**

$$V(u,v) \underset{\textbf{FT}}{\rightleftarrows} T(x,y)$$

V(u,v) = the complex visibility function $= \iint T(x,y)e^{2\pi i(ux+vy)}dxdy$

T(x,y) = the sky brightness distribution $= \iint V(u,v)e^{-2\pi i(ux+vy)}dudv$

What Are Visibilities?

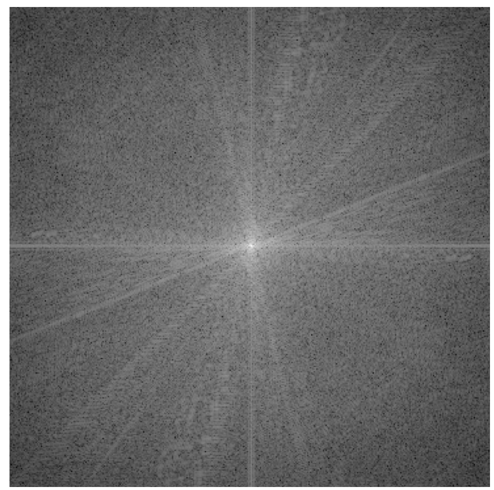
Each V(u,v) contains information on T(x,y) everywhere

Each V(u,v) is a complex quantity

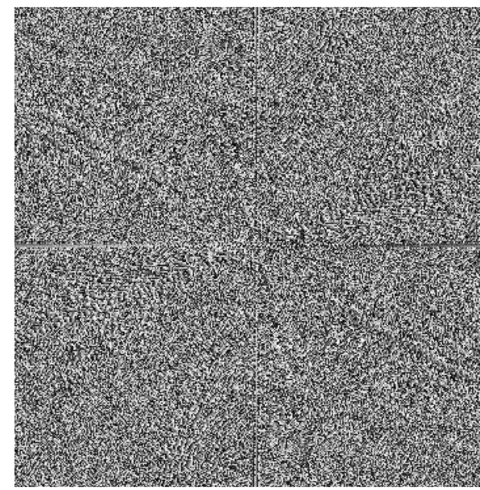
Expressed as (real, imaginary) or (amplitude, phase)

T(x,y)

FT →



V(u,v) amplitude



V(u,v) phase

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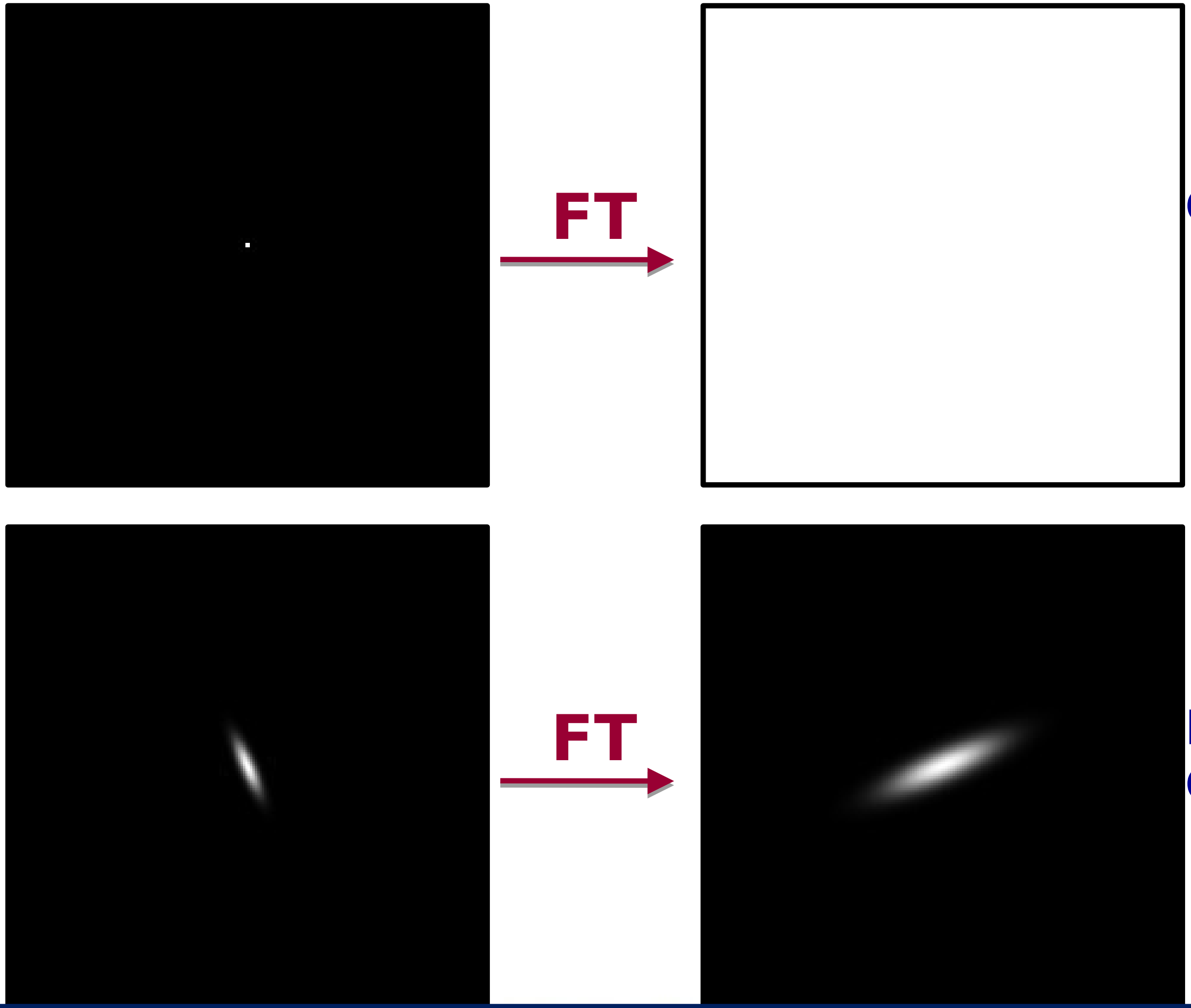
Examples of 2D Fourier Transforms

T(x,y) **V(u,v) amplitude**

δ Function → **Constant**

Elliptical Gaussian → **Elliptical Gaussian**

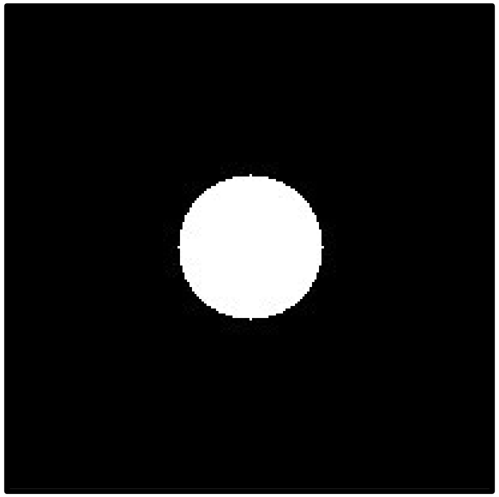


Rules of the Fourier Transform:
Narrow features transform to wide features (and vice versa)

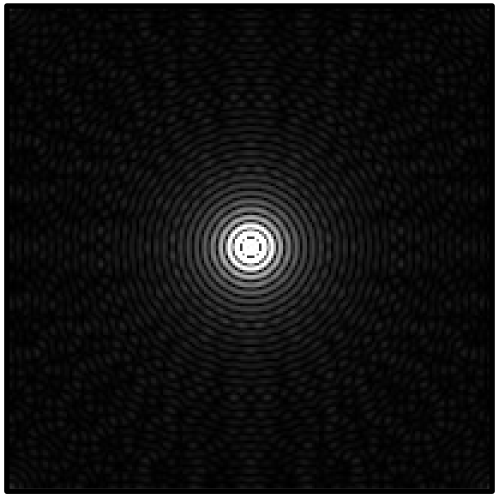
Examples of 2D Fourier Transforms

T(x,y) | **V(u,v) amplitude**

Uniform Disk

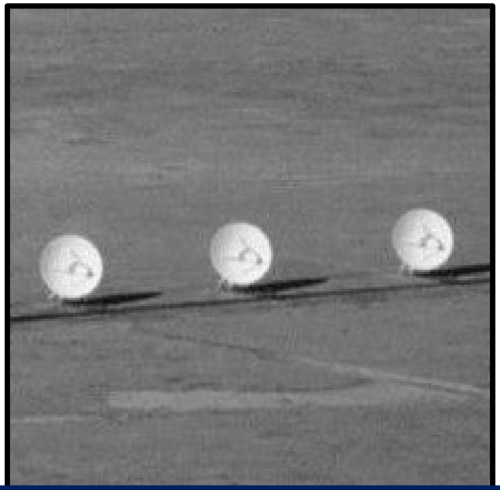


FT →

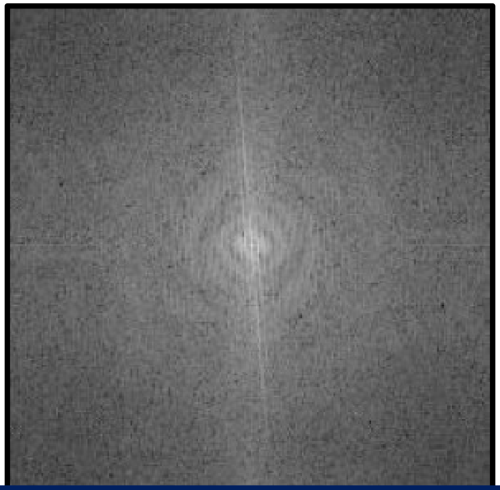


Bessel Function

VLA Antennas



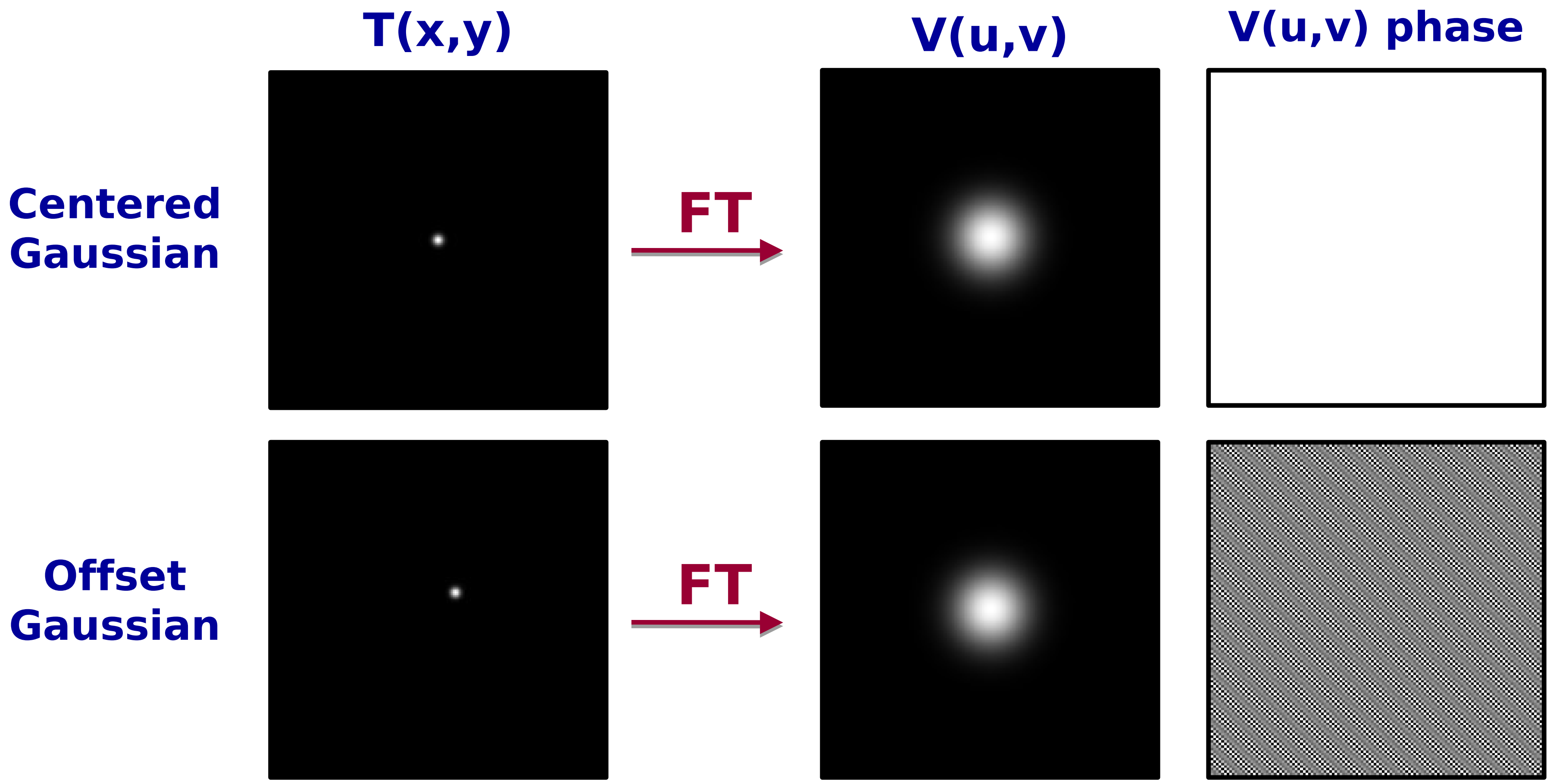
FT →



Bessel Function!

Rules of the Fourier Transform:
Sharp features (edges) result in many high spatial features

Examples of 2D Fourier Transforms



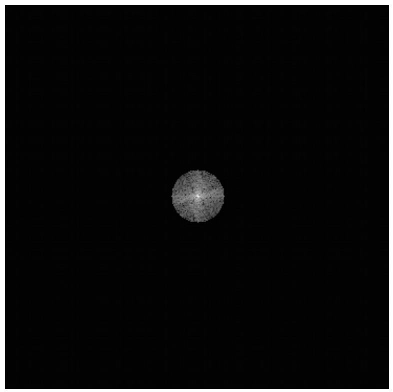
Rules of the Fourier Transform:
Amplitude tells you ‘how much’ of a spatial frequency
Phase tells you ‘where’ the spatial frequency is

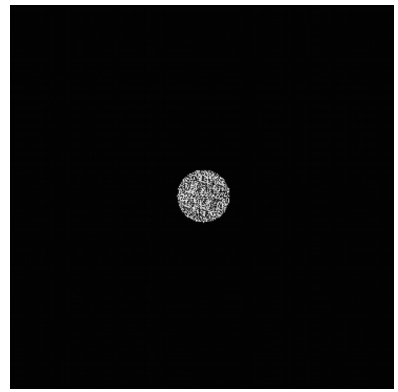
Implications of (u,v) Coverage

What does it mean if our (u,v) coverage is not complete?

V(u,v) amplitude | **V(u,v) phase** | **T(x,y)**

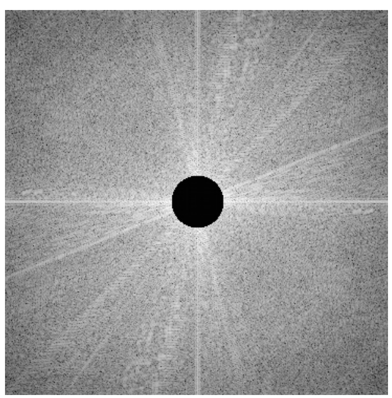
Missing High Spatial Frequencies

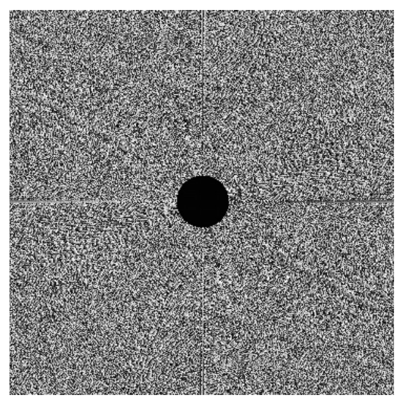




FT →

Missing Low Spatial Frequencies

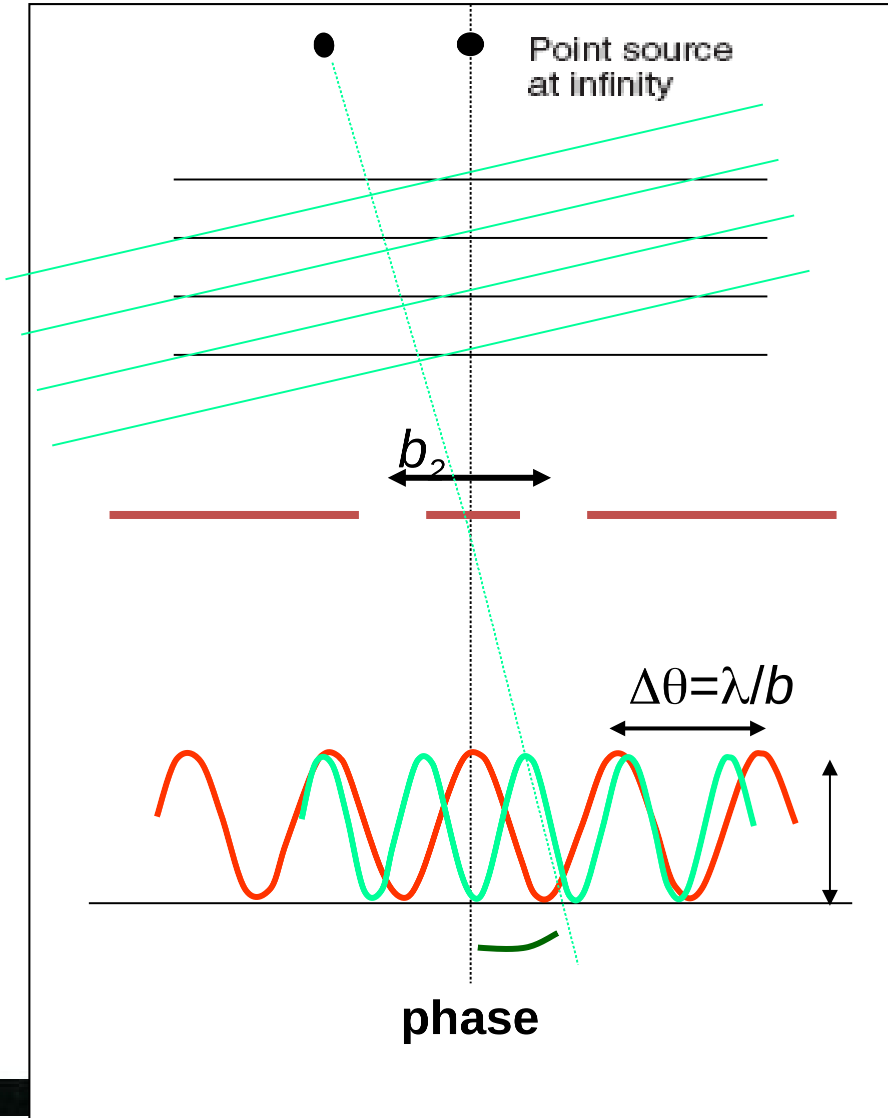


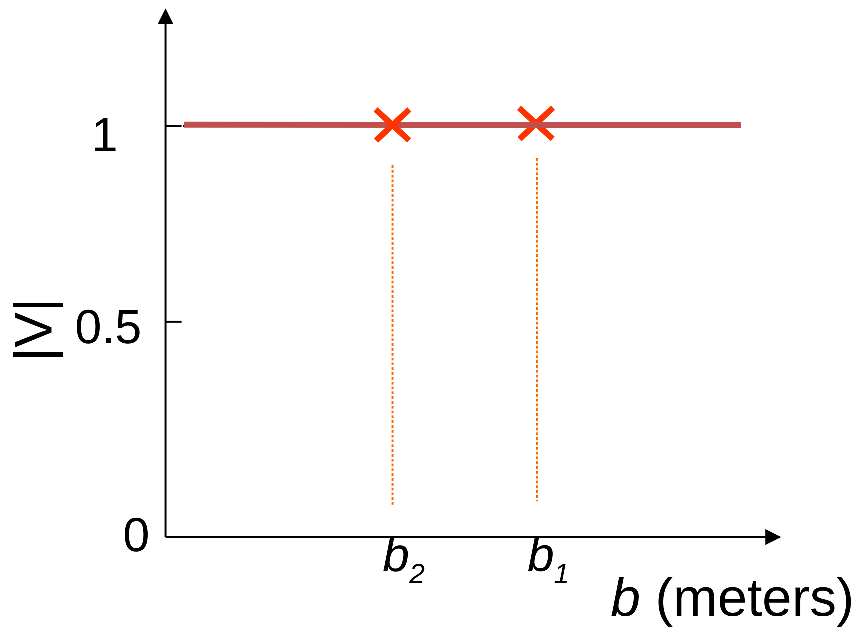


FT →

Visibility and Sky Brightness

Graphic courtesy Andrea Isella





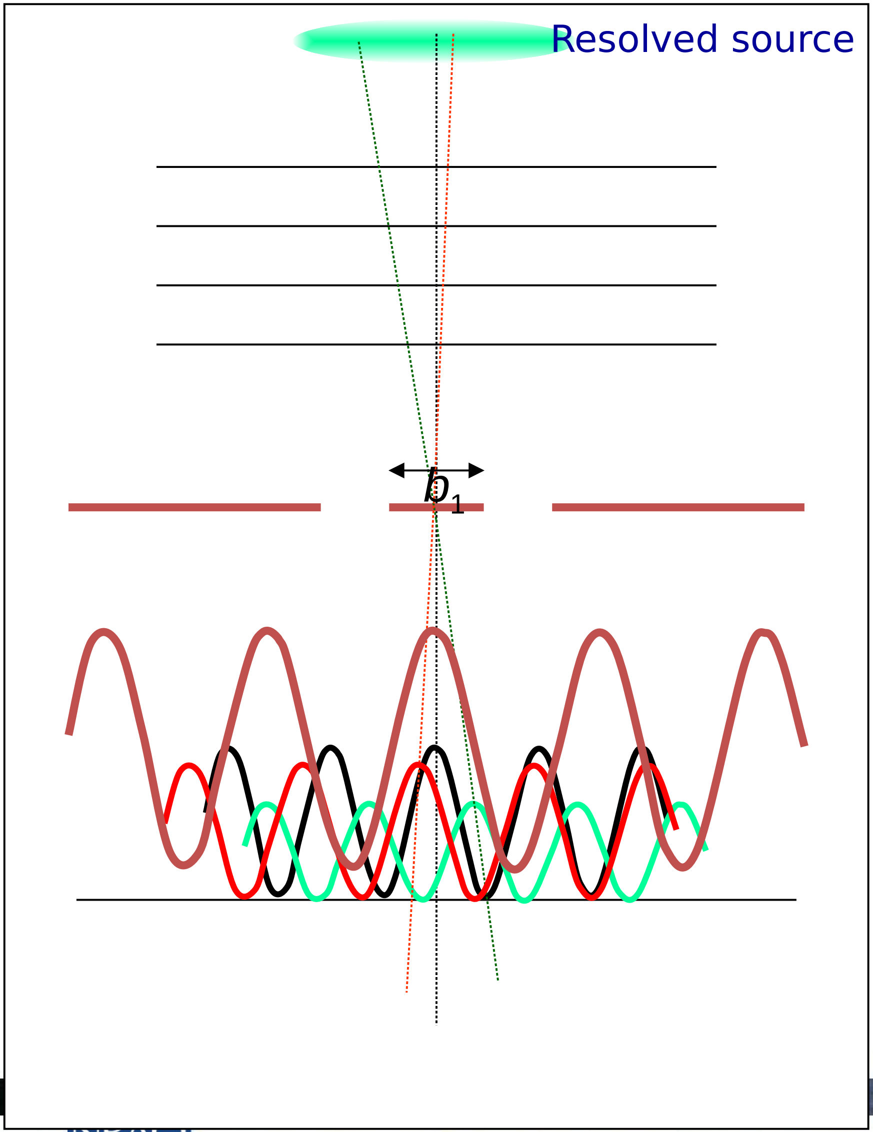
$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

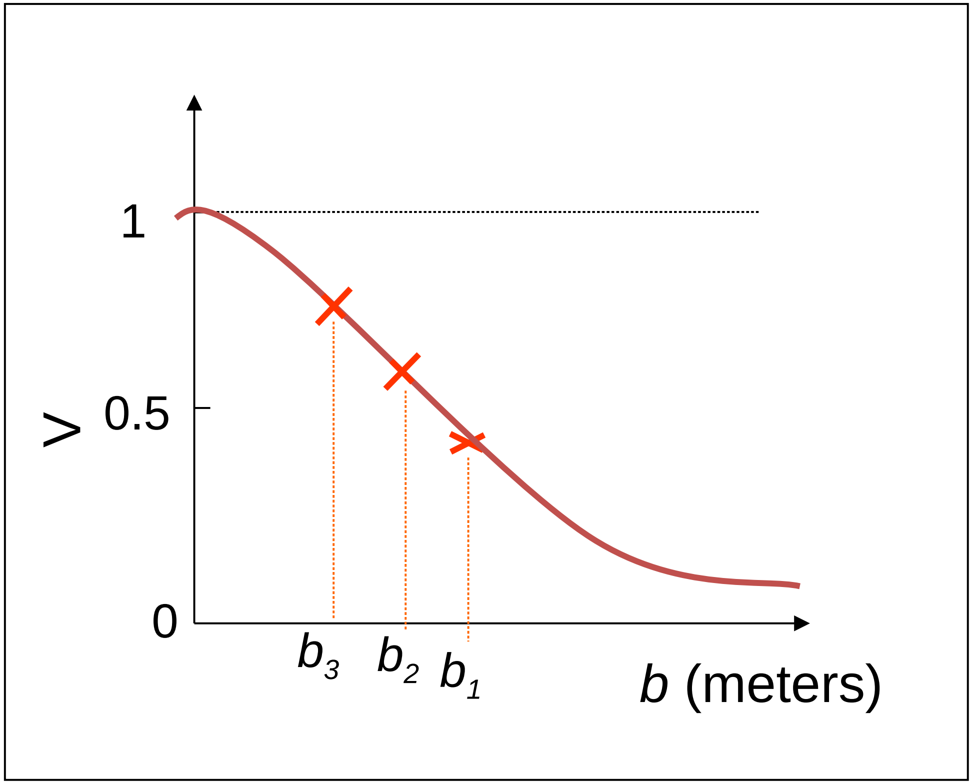
The visibility is a **complex** quantity:

- **amplitude** tells "how much" of a certain frequency component
- **phase** tells "where" this component is located

Visibility and Sky Brightness

Graphic courtesy Andrea Isella





$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

Basics of Aperture Synthesis

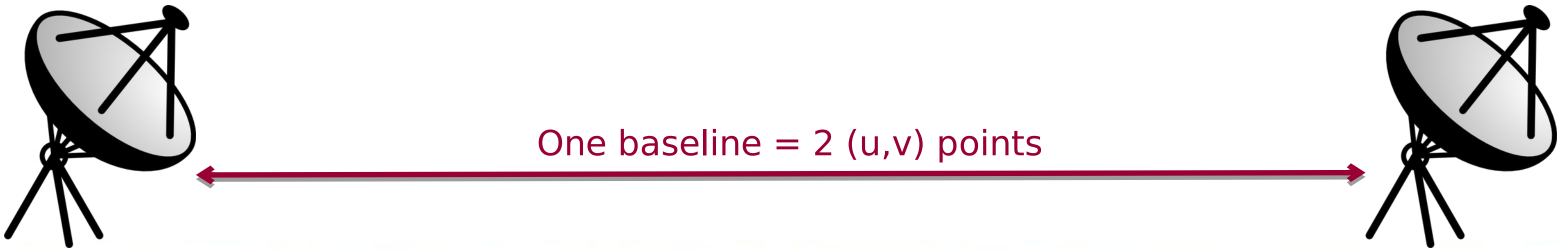
Idea: Sample V(u,v) at a enough (u,v) points using distributed small aperture antennas to synthesize a large aperture antenna of size (u_{max},v_{max})

One pair of antennas = one baseline

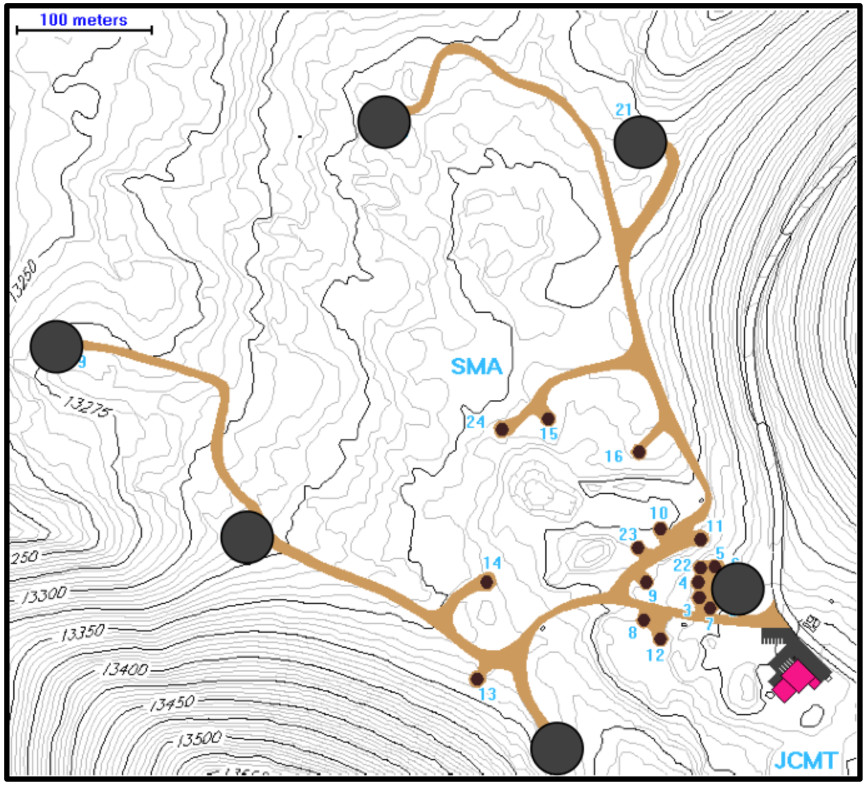
For **N antennas**, we get **N(N-1) samples** at a time

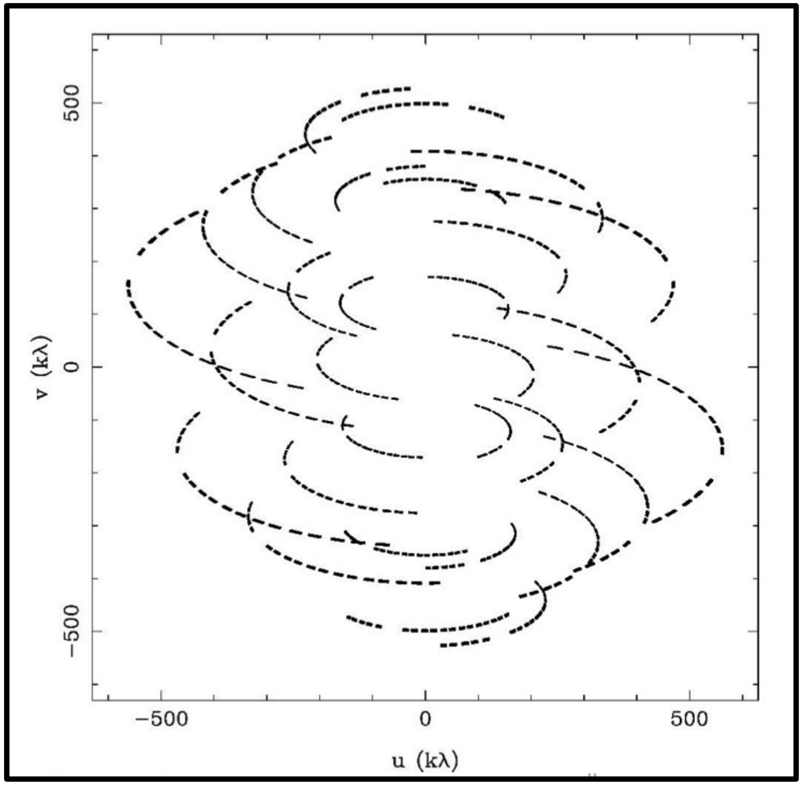
How do we fill in the rest of the (u,v) plane?

1. Earth's rotation
2. Reconfigure physical layout of N antennas



(u,v) Plane Sampling



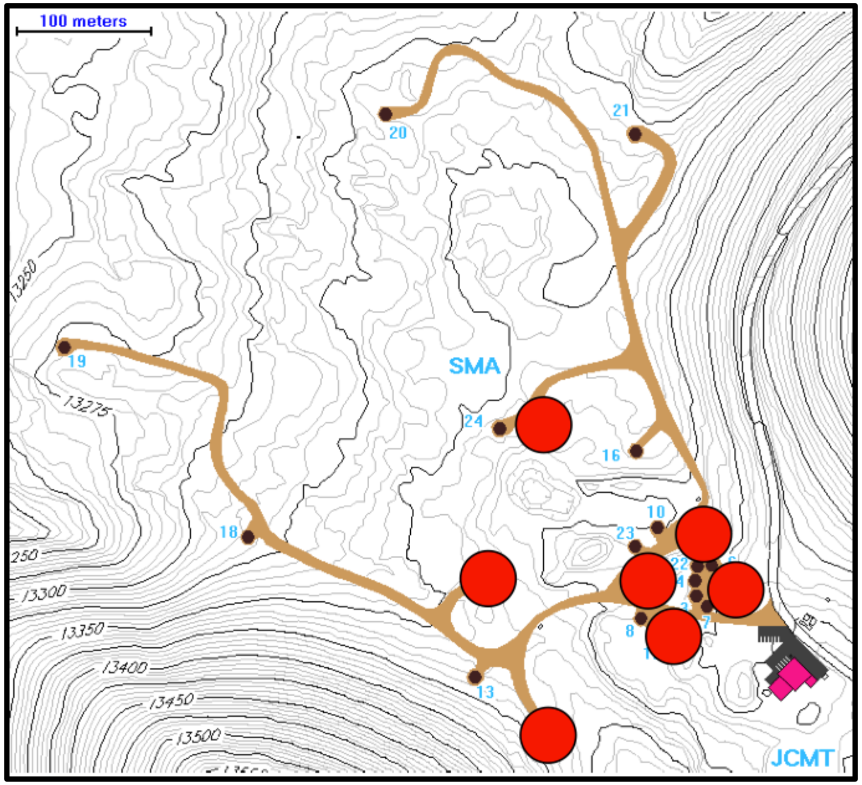


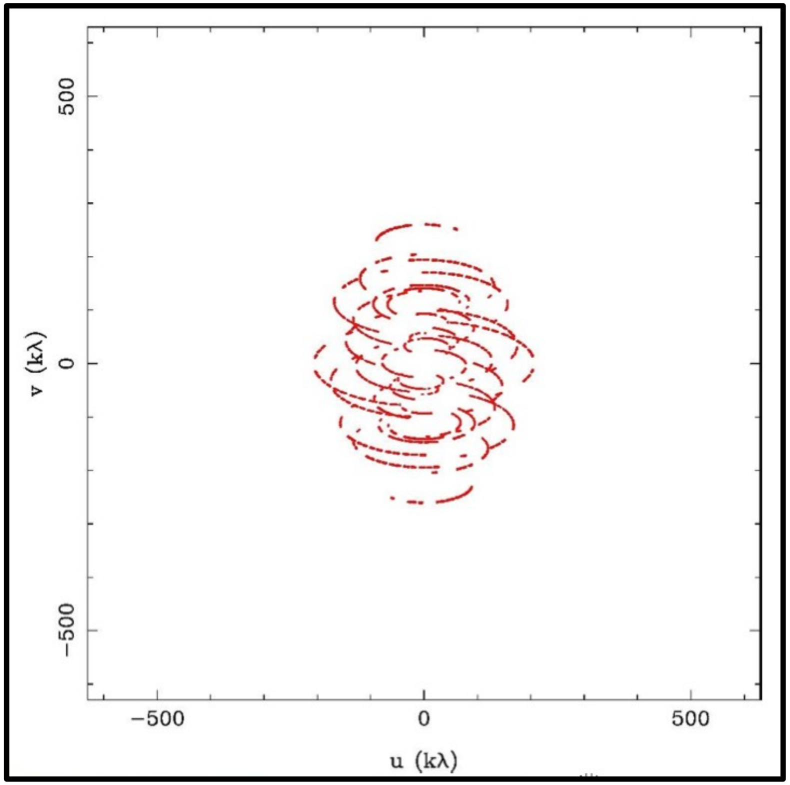
Very Extended SMA configuration

(most extended baselines)

345 GHz, DEC = +22

(u,v) Plane Sampling





Extended SMA configuration

(extended baselines)

345 GHz, DEC = +22

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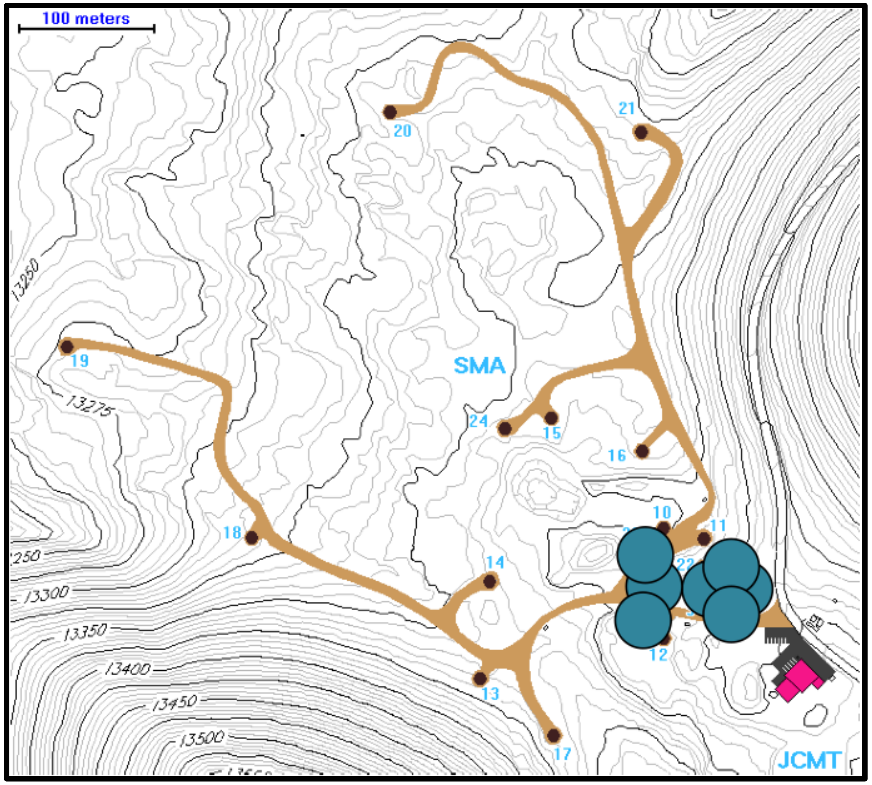
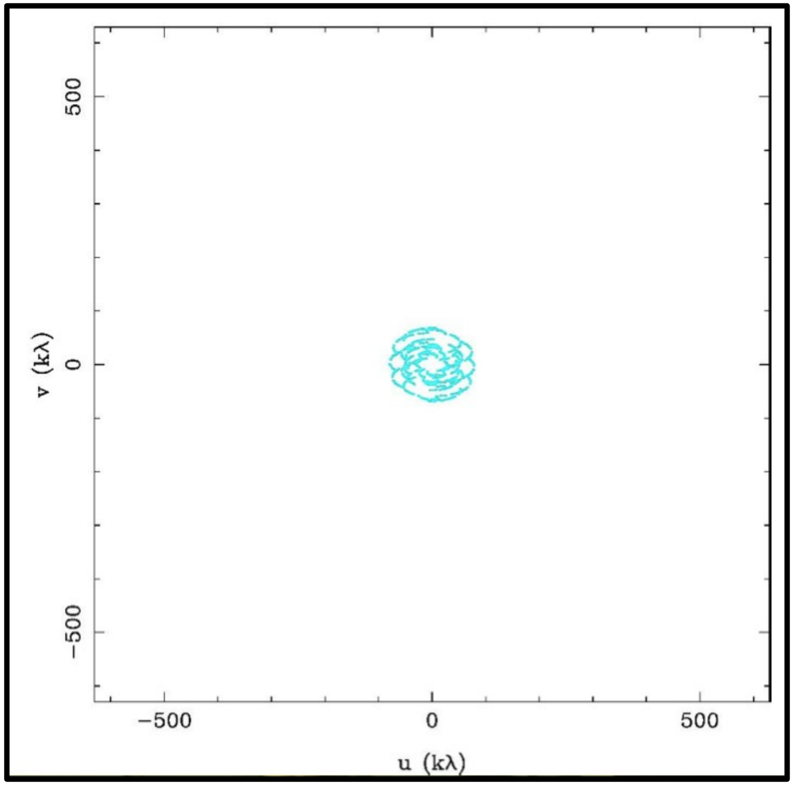
(u,v) Plane Sampling



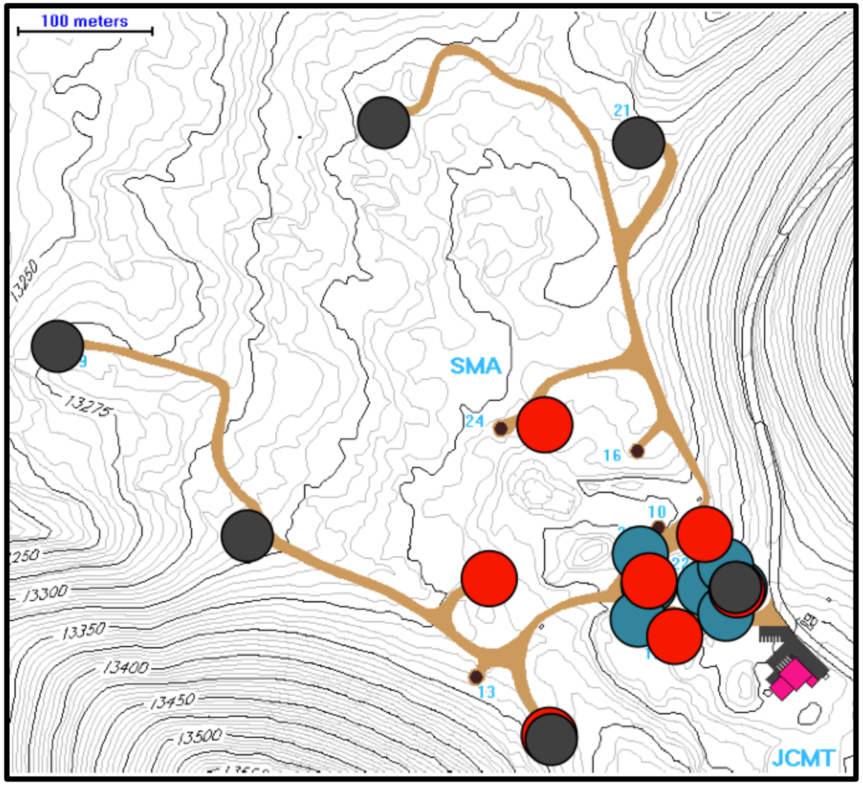


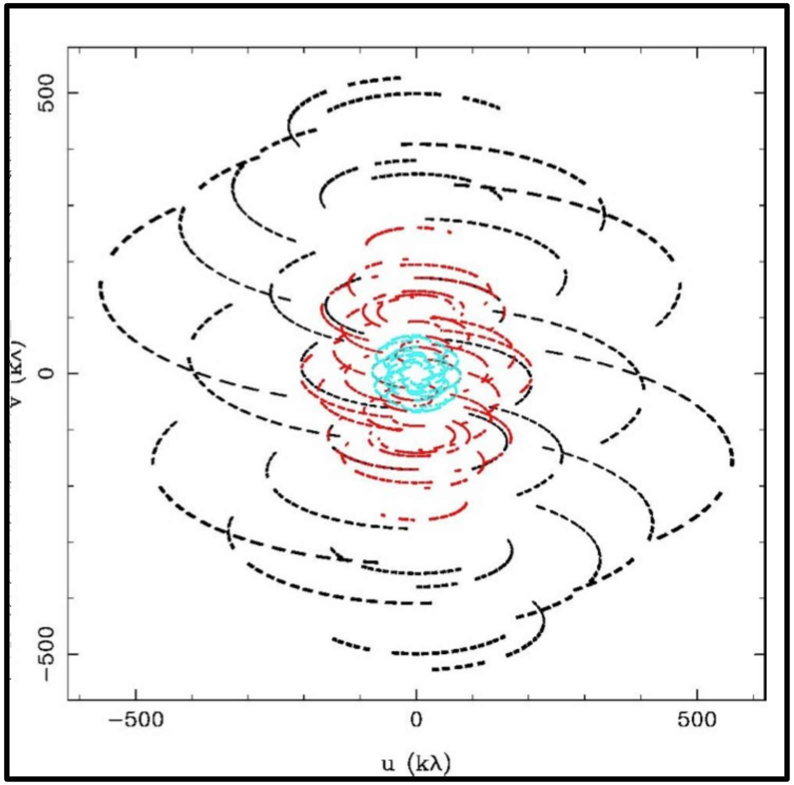
Compact SMA configuration

(compact baselines)

345 GHz, DEC = +22

(u,v) Plane Sampling





Combine multiple configurations to get the most complete coverage of the (u,v) plane

Characteristic Angular Scales

Angular resolution of telescope array:

$\sim \lambda/B_{max}$ (B_{max} = longest baseline)

Maximum angular scale:

$\sim \lambda/B_{min}$ (B_{min} = shortest distance between antennas)

Field of view (FOV):

$\sim \lambda/D$ (D = antenna diameter)

*Sources more extended than the FOV can be observed using multiple pointing centers in a mosaic

An interferometer is sensitive to a range of angular sizes: $\lambda/B_{max} < \theta < \lambda/B_{min}$

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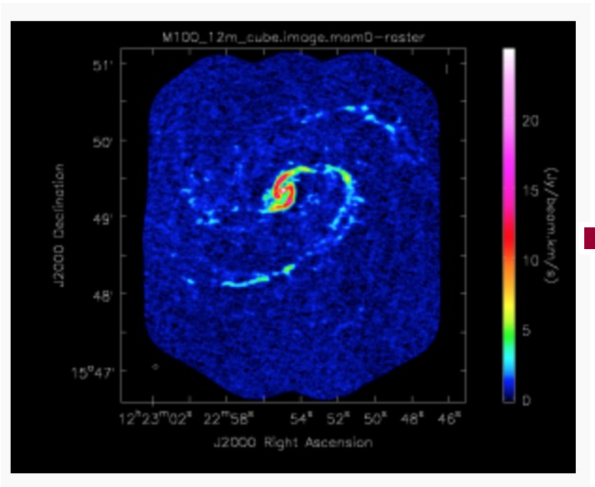
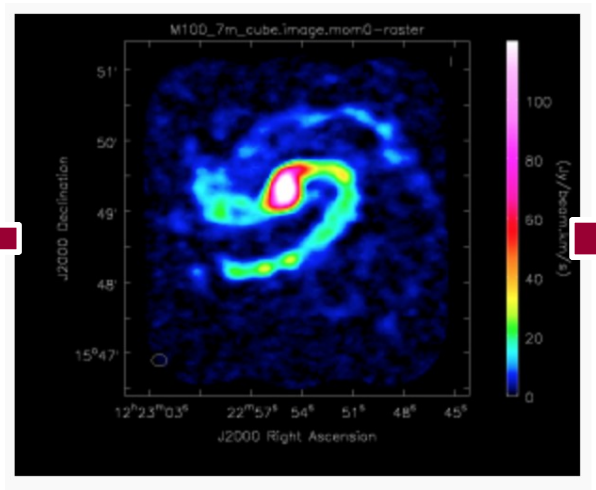
Characteristic Angular Scales: M100

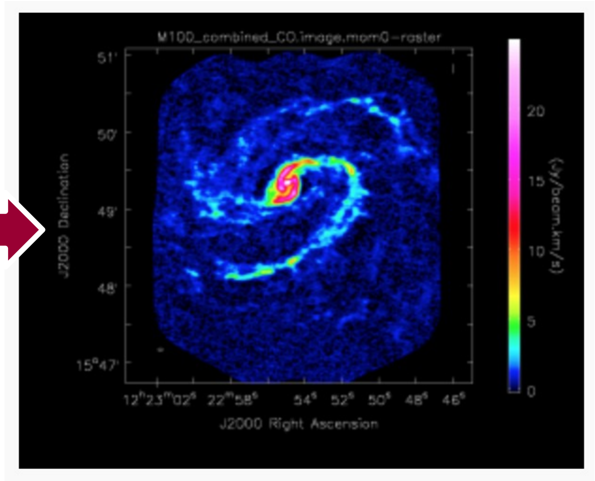
ALMA 12m



ACA 7m



Combined



ALMA 12m shows smaller spatial scales (denser, clumpier emission)

ACA 7m data shows larger spatial scales (diffuse, extended emission)

To get both — you need a combined image!

Interferometry: Spatial Scales

- The **sensitivity** is determined by the number of antennas times their area
- The instantaneous **field of view** ("primary beam") is determined by the beam of a single antenna (corresponding to the resolution for a single dish telescope)
- The **resolution** ("synthesized beam") is determined by the largest distance between antennas
- The **largest angular scale** that can be imaged is determined by the shortest distance between antennas

Angular Scales — A Proposal Tip!

Interferometers act as spatial filters - shorter baselines are sensitive to larger targets, so remember ...

Spatial scales larger than the smallest baseline cannot be imaged

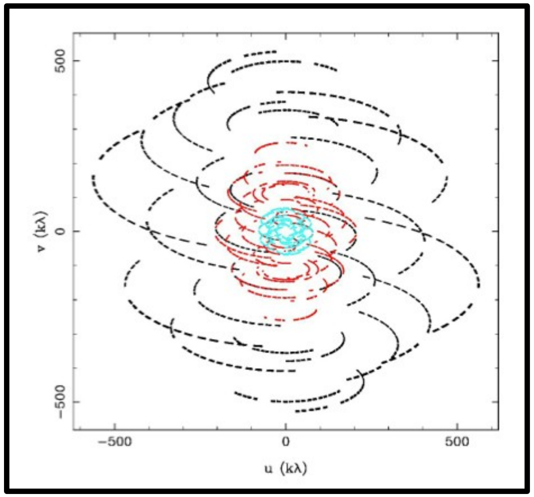
Spatial scales smaller than the largest baseline cannot be resolved

	Band	3	4	5	6	7	8	9	10
	Frequency (GHz)	100	150	185	230	345	460	650	870
Configuration									
7-m	θ_{res} (arcsec)	12.5	8.35	6.77	5.45	3.63	2.72	1.93	1.44
	θ_{MRS} (arcsec)	66.7	44.5	36.1	29.0	19.3	14.5	10.3	7.67
C43-1	θ_{res} (arcsec)	3.38	2.25	1.83	1.47	0.98	0.735	0.52	0.389
	θ_{MRS} (arcsec)	28.5	19.0	15.4	12.4	8.25	6.19	4.38	3.27
C43-2	θ_{res} (arcsec)	2.3	1.53	1.24	0.999	0.666	0.499	0.353	0.264
	θ_{MRS} (arcsec)	22.6	15.0	12.2	9.81	6.54	4.9	3.47	2.59
C43-3	θ_{res} (arcsec)	1.42	0.943	0.765	0.615	0.41	0.308	0.218	0.163
	θ_{MRS} (arcsec)	16.2	10.8	8.73	7.02	4.68	3.51	2.48	1.86
C43-4	θ_{res} (arcsec)	0.918	0.612	0.496	0.399	0.266	0.2	0.141	0.106
	θ_{MRS} (arcsec)	11.2	7.5	6.08	4.89	3.26	2.44	1.73	1.29
C43-5	θ_{res} (arcsec)	0.545	0.363	0.295	0.237	0.158	0.118	0.0838	0.0626
	θ_{MRS} (arcsec)	6.7	4.47	3.62	2.91	1.94	1.46	1.03	0.77
C43-6	θ_{res} (arcsec)	0.306	0.204	0.165	0.133	0.0887	0.0665	0.0471	0.0352
	θ_{MRS} (arcsec)	4.11	2.74	2.22	1.78	1.19	0.892	0.632	0.472
C43-7	θ_{res} (arcsec)	0.211	0.141	0.114	0.0917	0.0612	0.0459	0.0325	0.0243
	θ_{MRS} (arcsec)	2.58	1.72	1.4	1.12	0.749	0.562	0.398	0.297
C43-8	θ_{res} (arcsec)	0.096	0.064	0.0519	0.0417	0.0278	-	-	-
	θ_{MRS} (arcsec)	1.42	0.947	0.768	0.618	0.412	-	-	-
C43-9	θ_{res} (arcsec)	0.057	0.038	0.0308	0.0248	0.0165	-	-	-
	θ_{MRS} (arcsec)	0.814	0.543	0.44	0.354	0.236	-	-	-
C43-10	θ_{res} (arcsec)	0.042	0.028	0.0227	0.0183	0.0122	-	-	-
	θ_{MRS} (arcsec)	0.496	0.331	0.268	0.216	0.144	-	-	-

From the ALMA Cycle 7 Technical Handbook

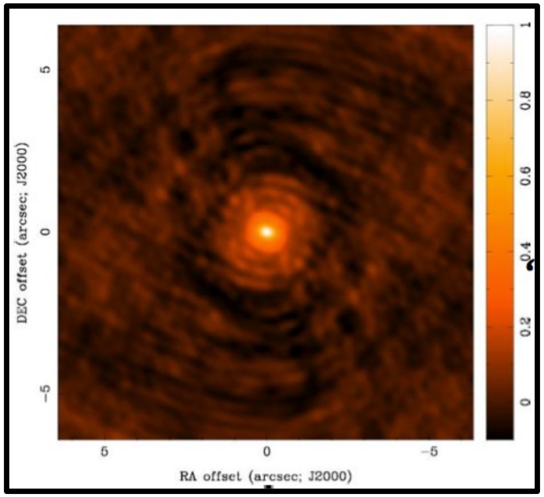
The Dirty Beam

S(u,v)

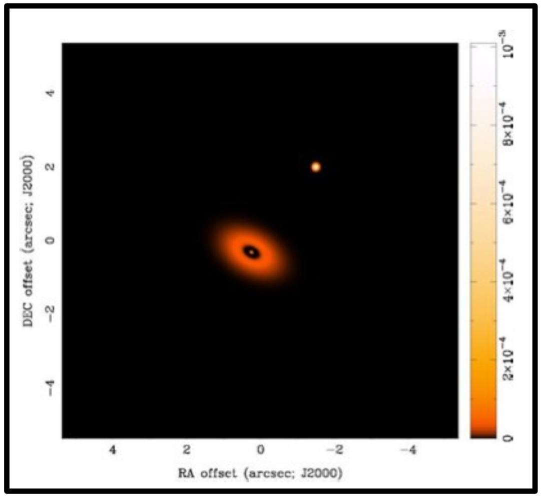


FT

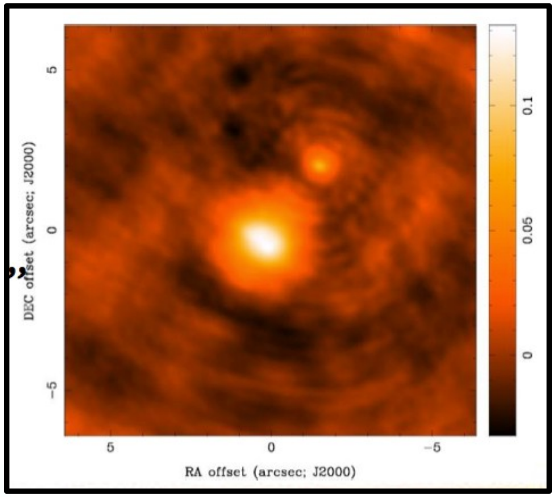
s(x,y)
"Dirty Beam"



* (Convolution)

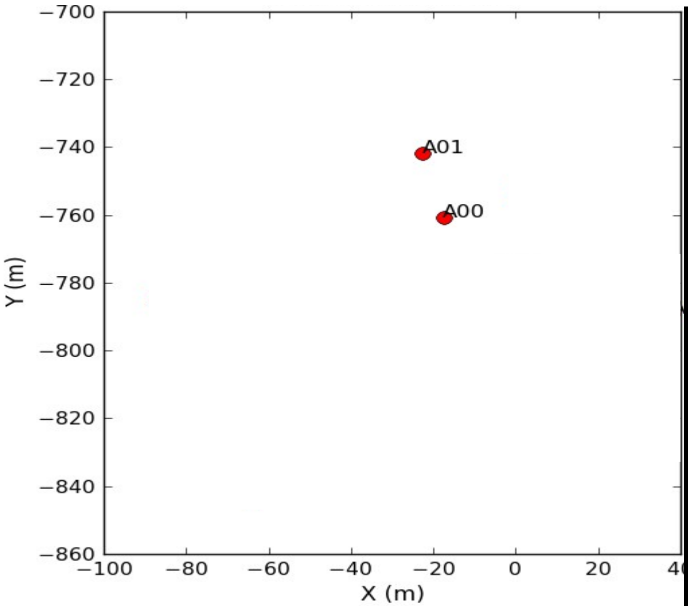


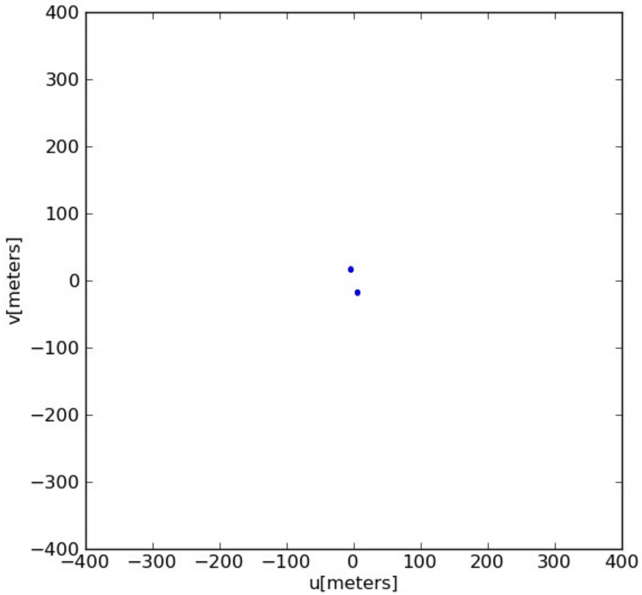
T(x,y)

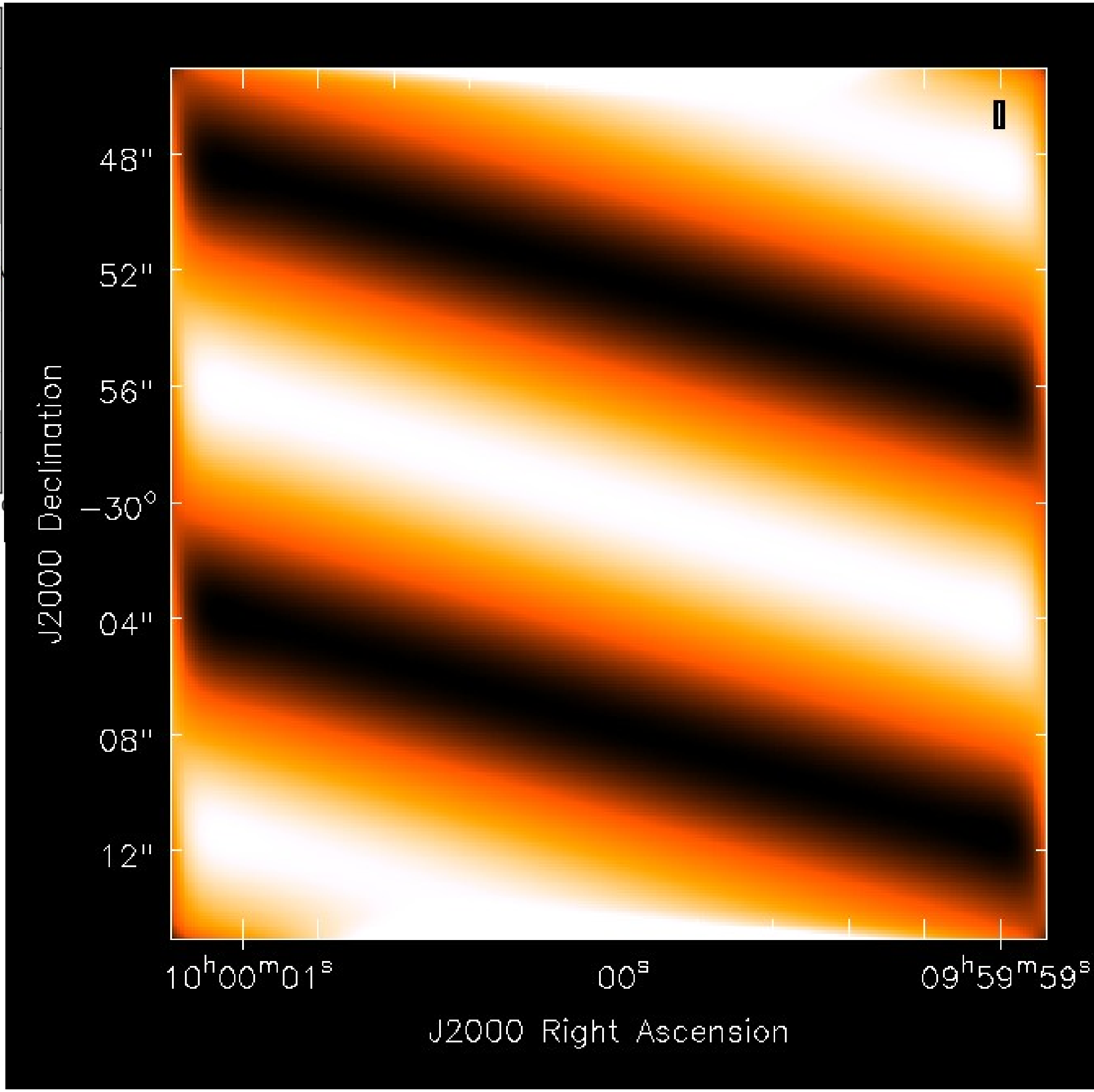


$T_D(x,y)$
"Dirty Image"

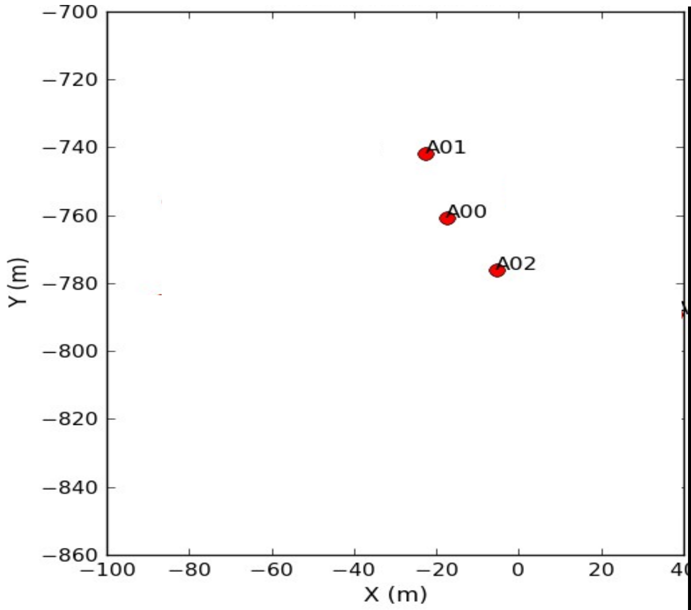
Example: Fringe pattern with 2 Antennas (one baseline)

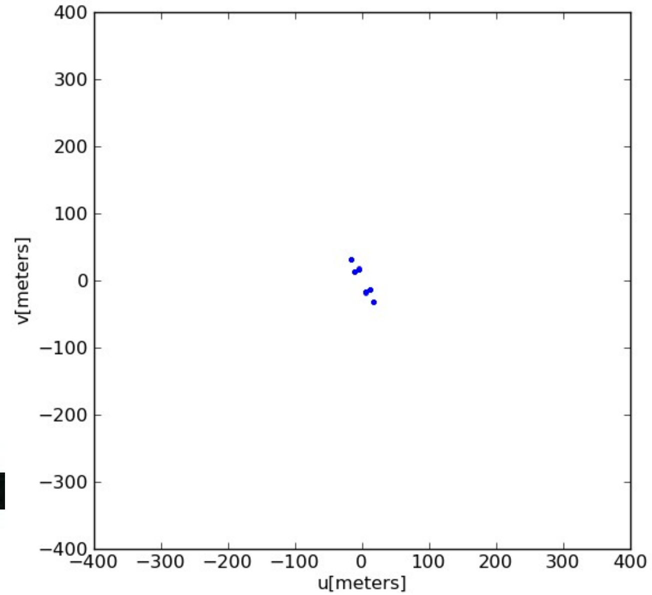


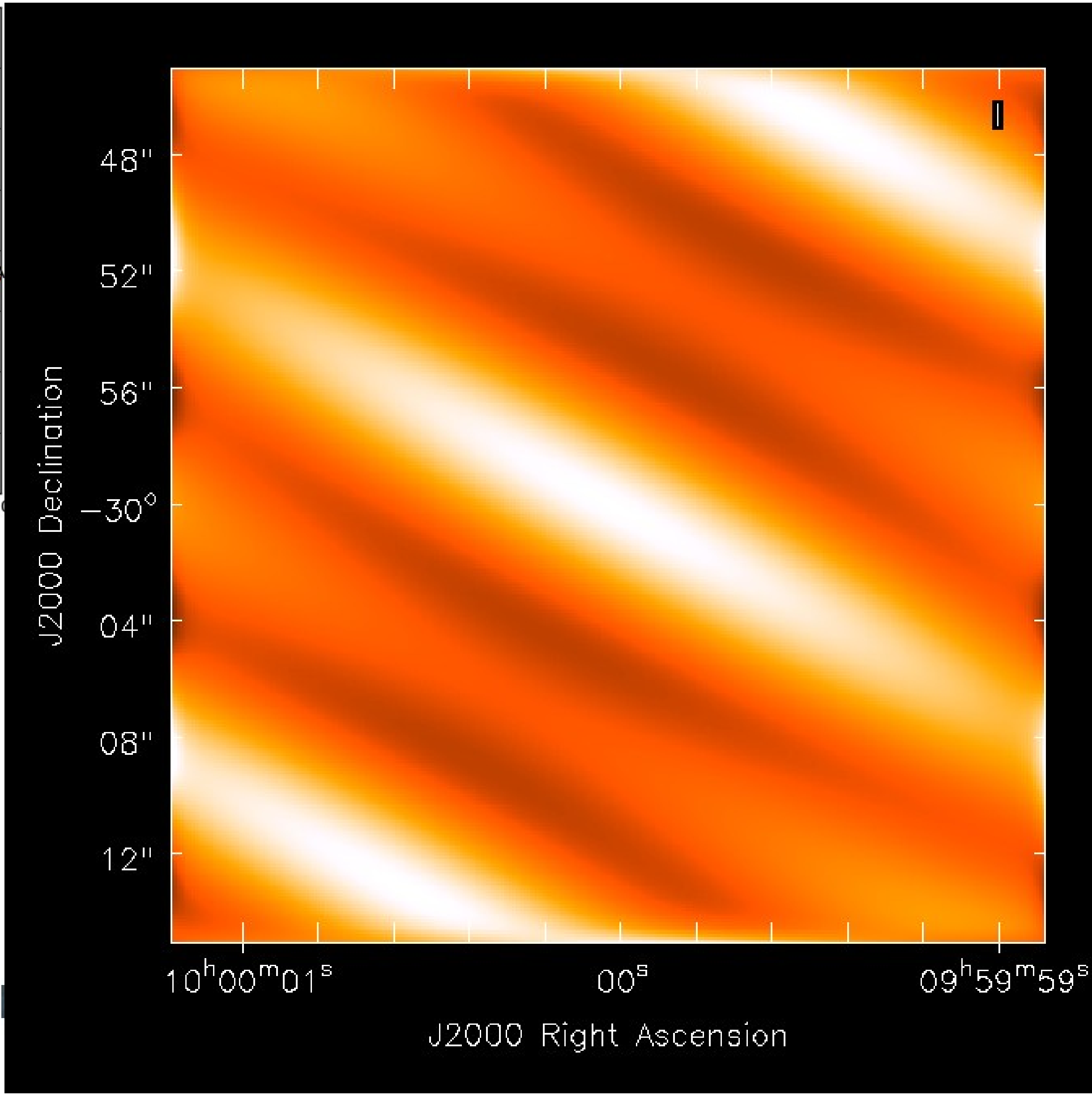




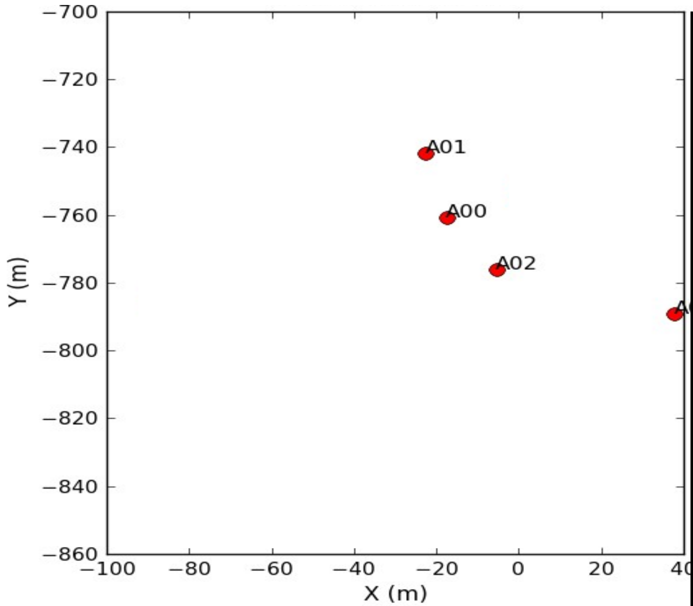
Example: Fringe pattern with 3 Antennas (3 baselines)

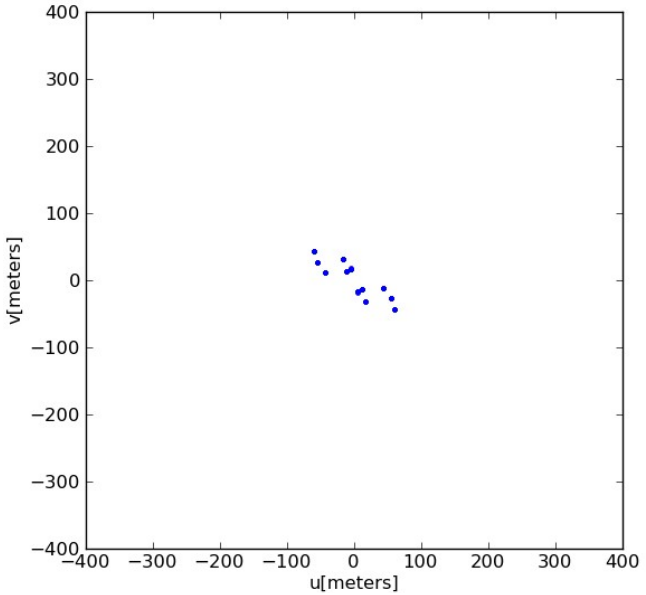


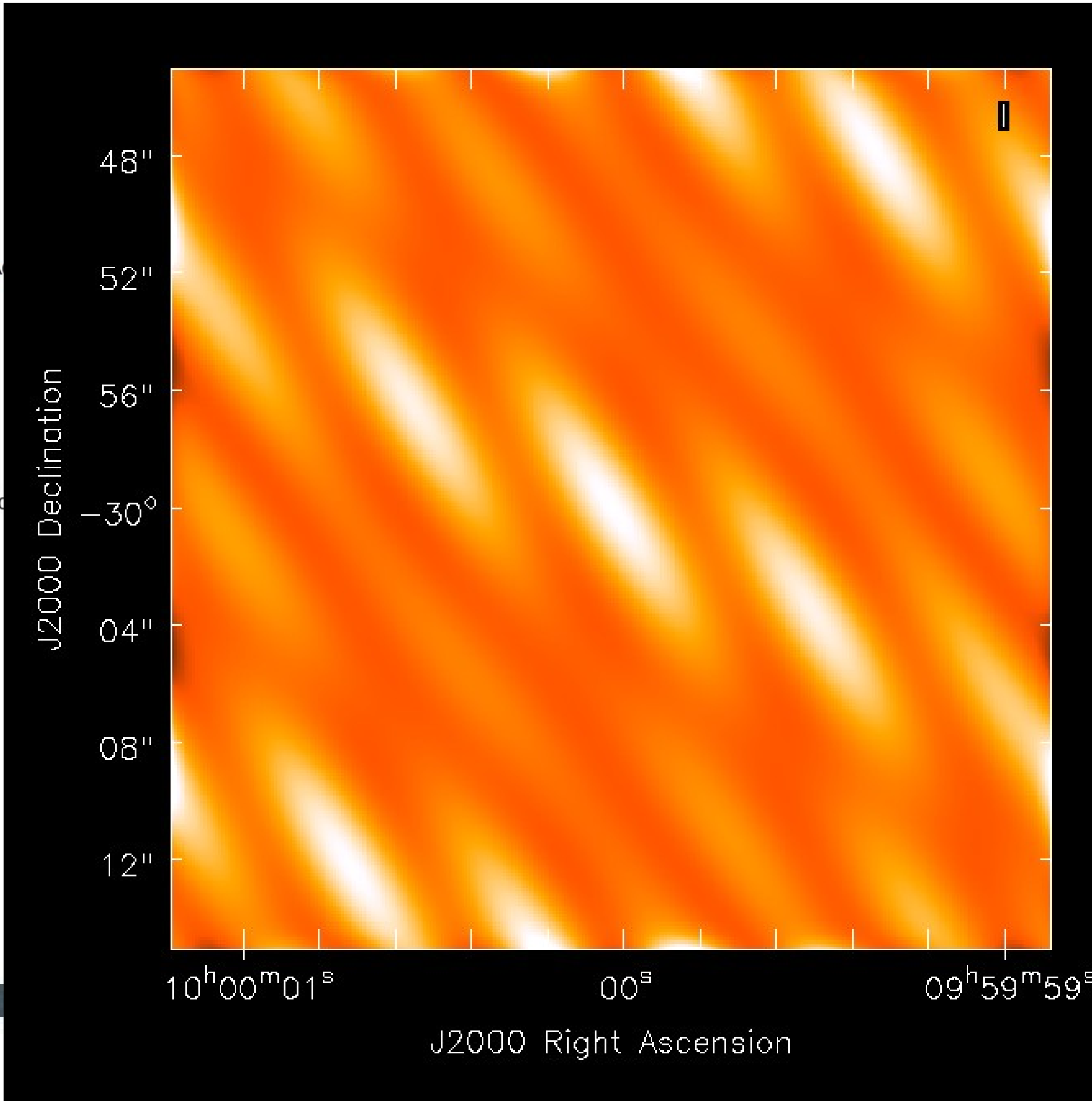




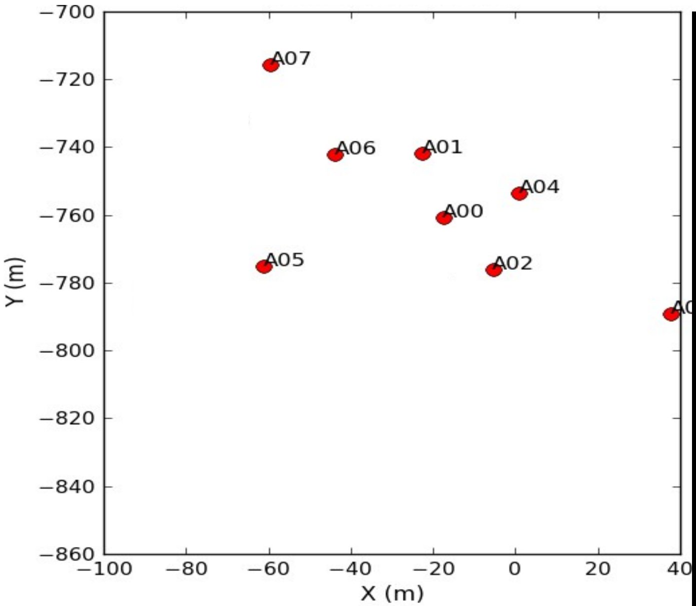
Example: Fringe pattern with 4 Antennas (6 baselines)

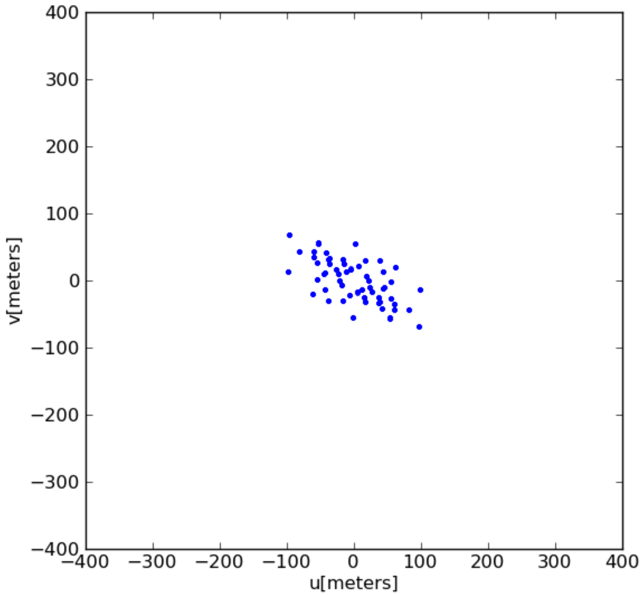


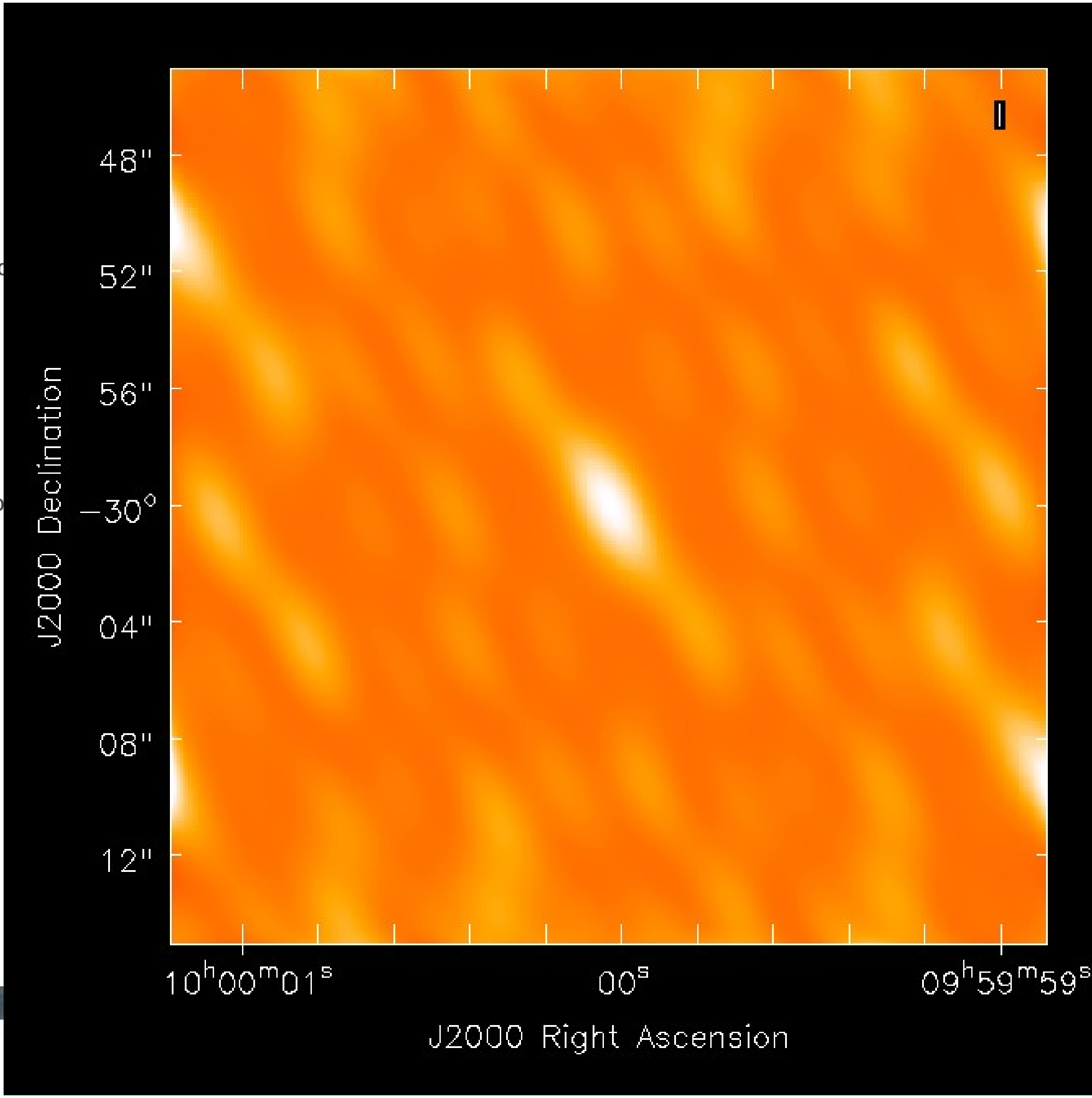




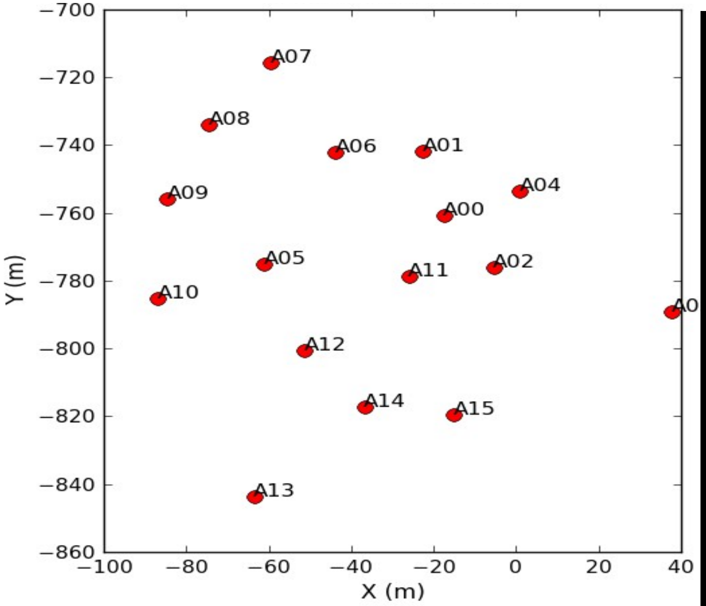
Example: Fringe pattern with 8 Antennas (28 baselines)

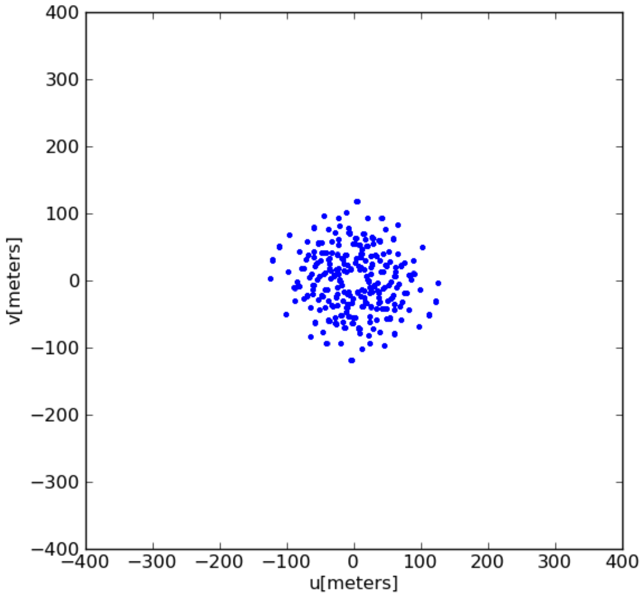


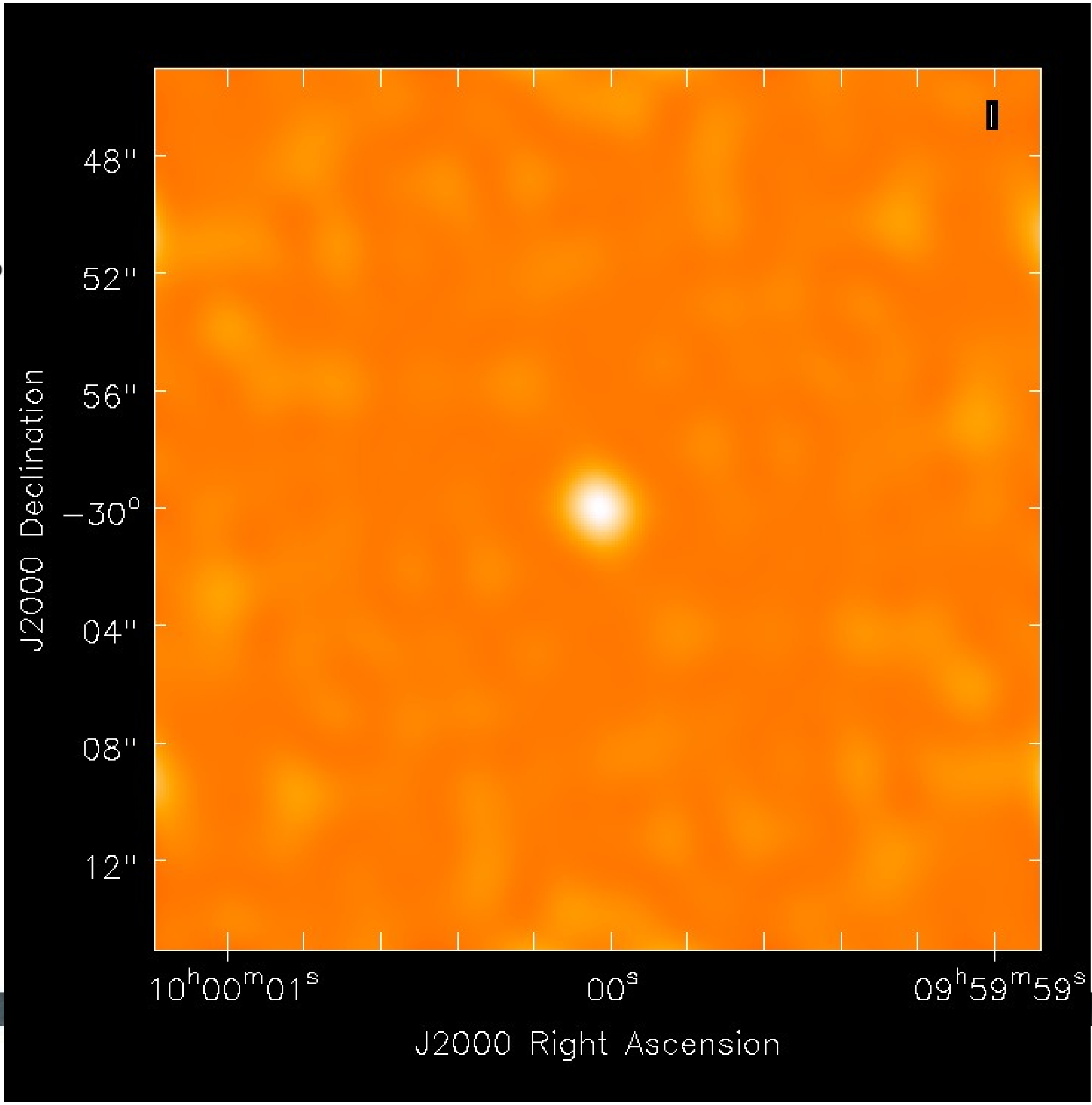




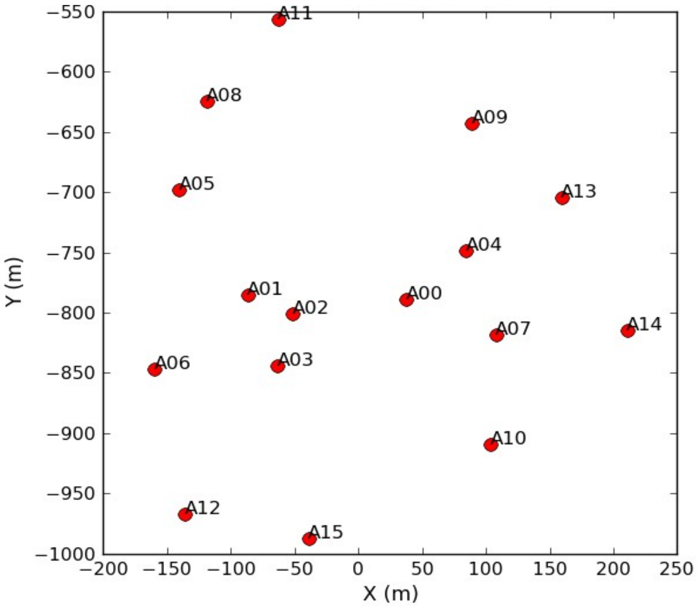
16 Antennas – Compact Configuration

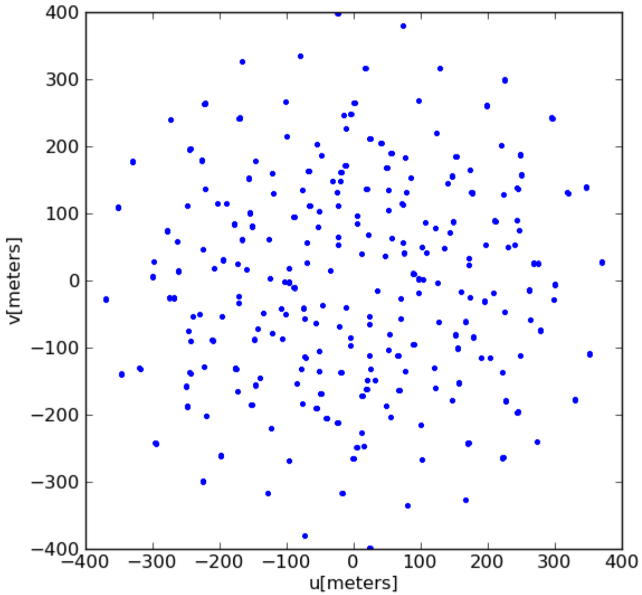


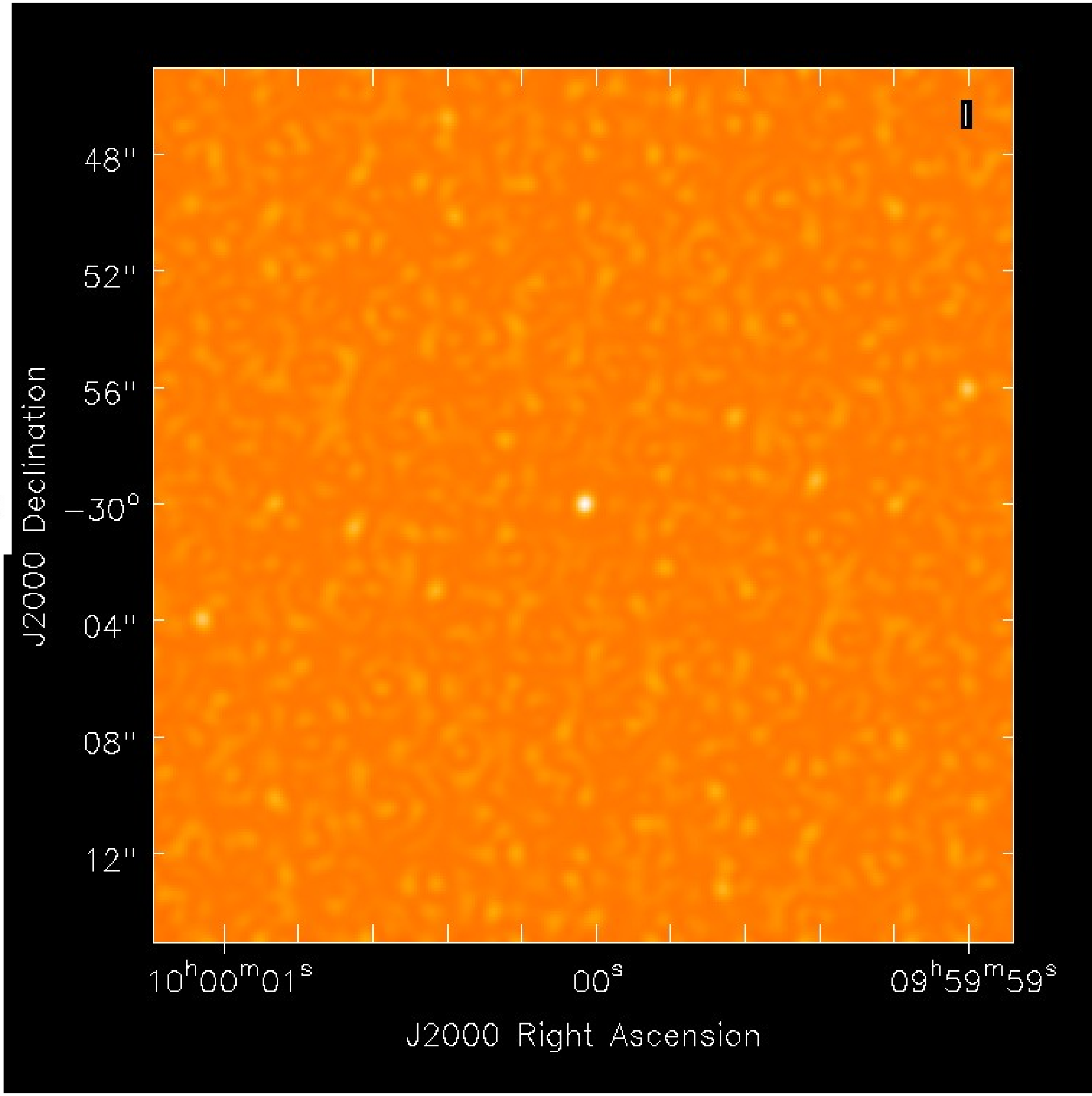




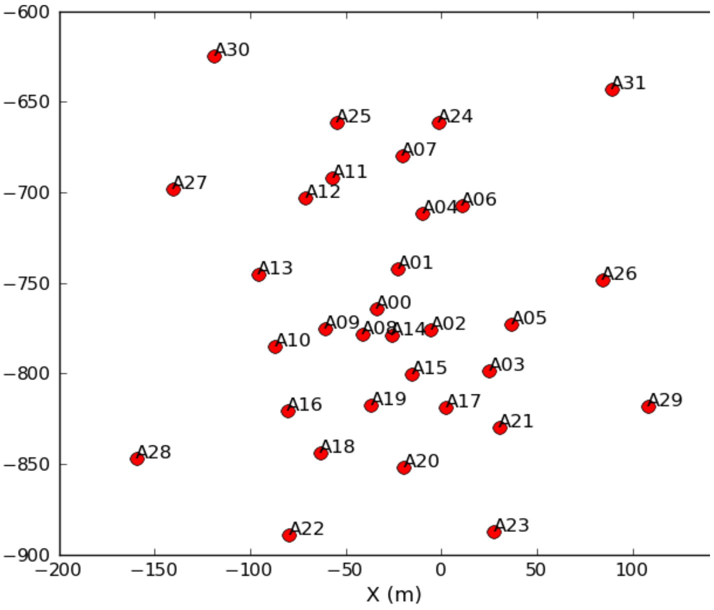
16 Antennas – Extended Configuration

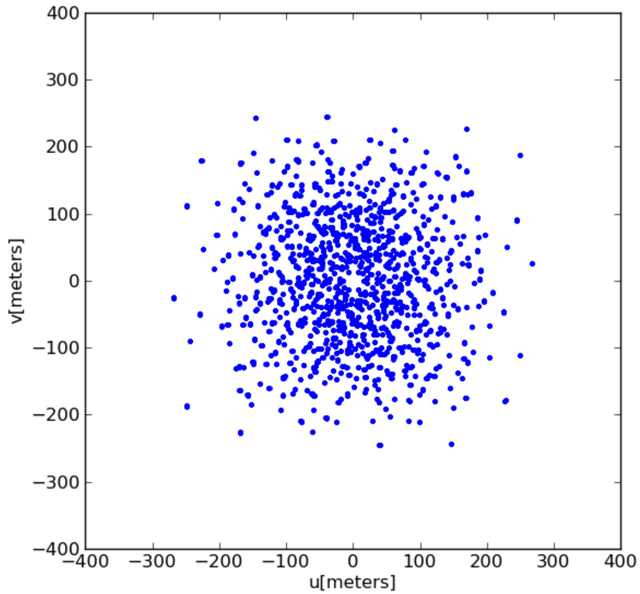


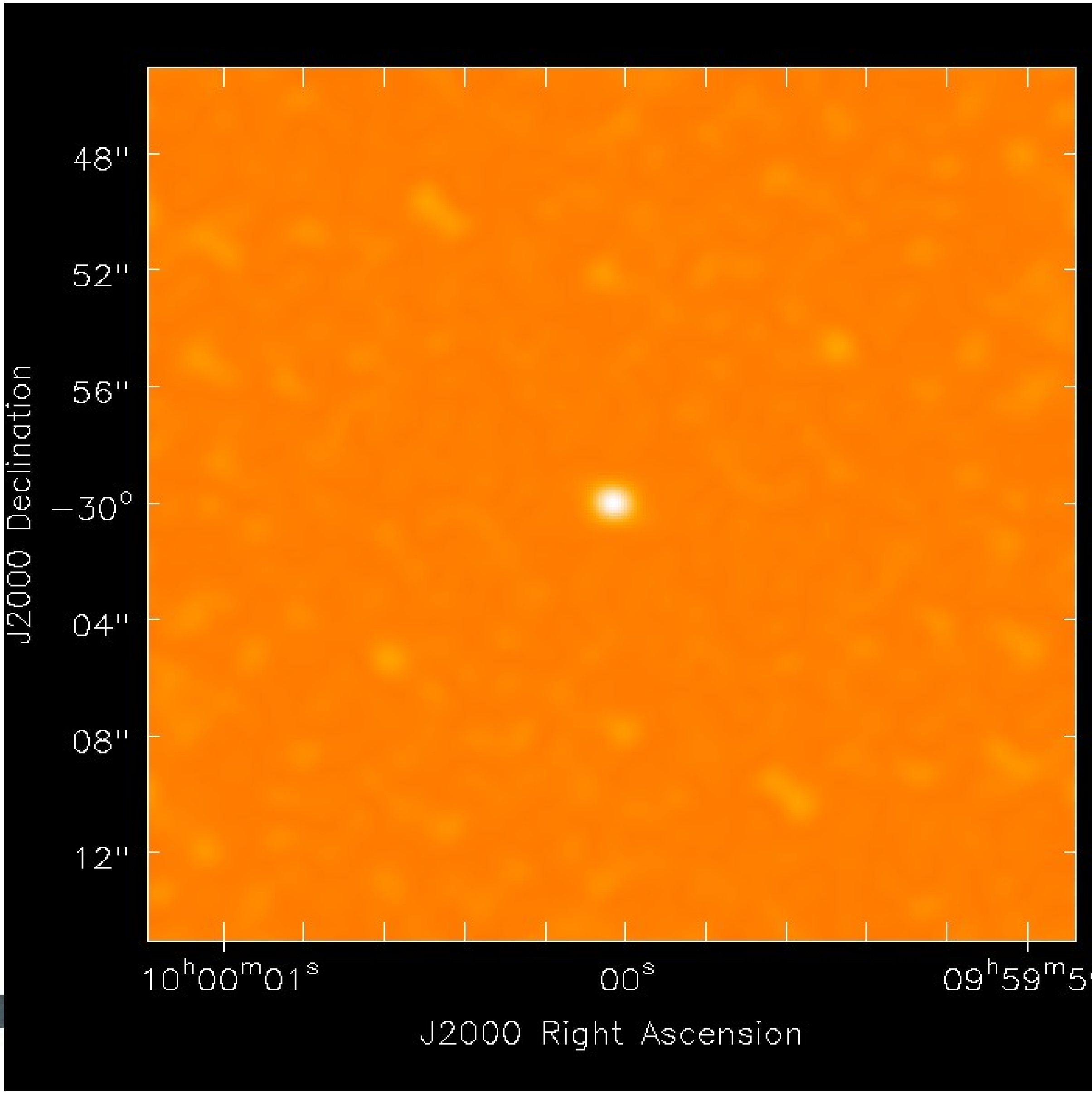




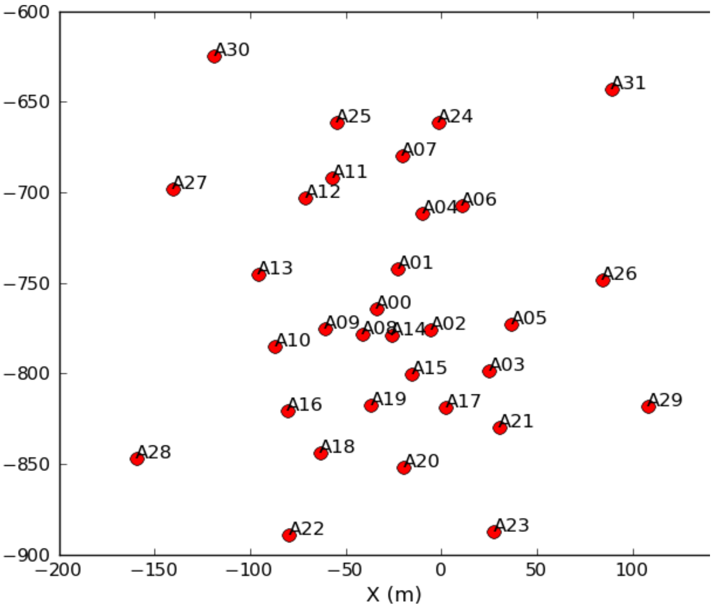
32 Antennas – Instantaneous

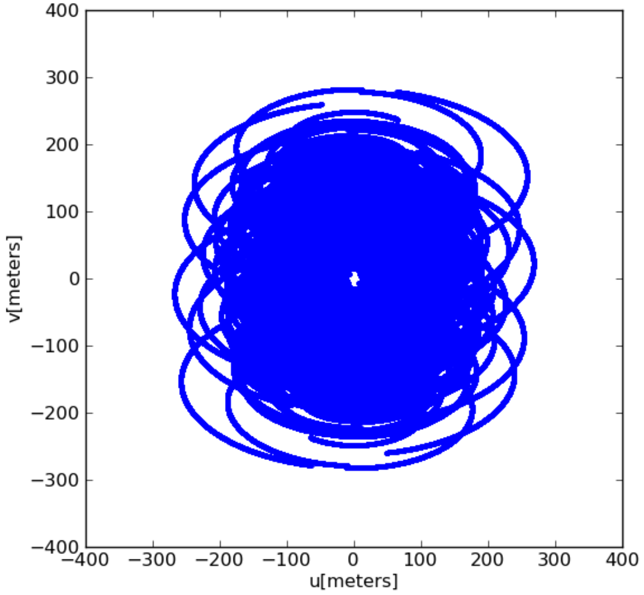


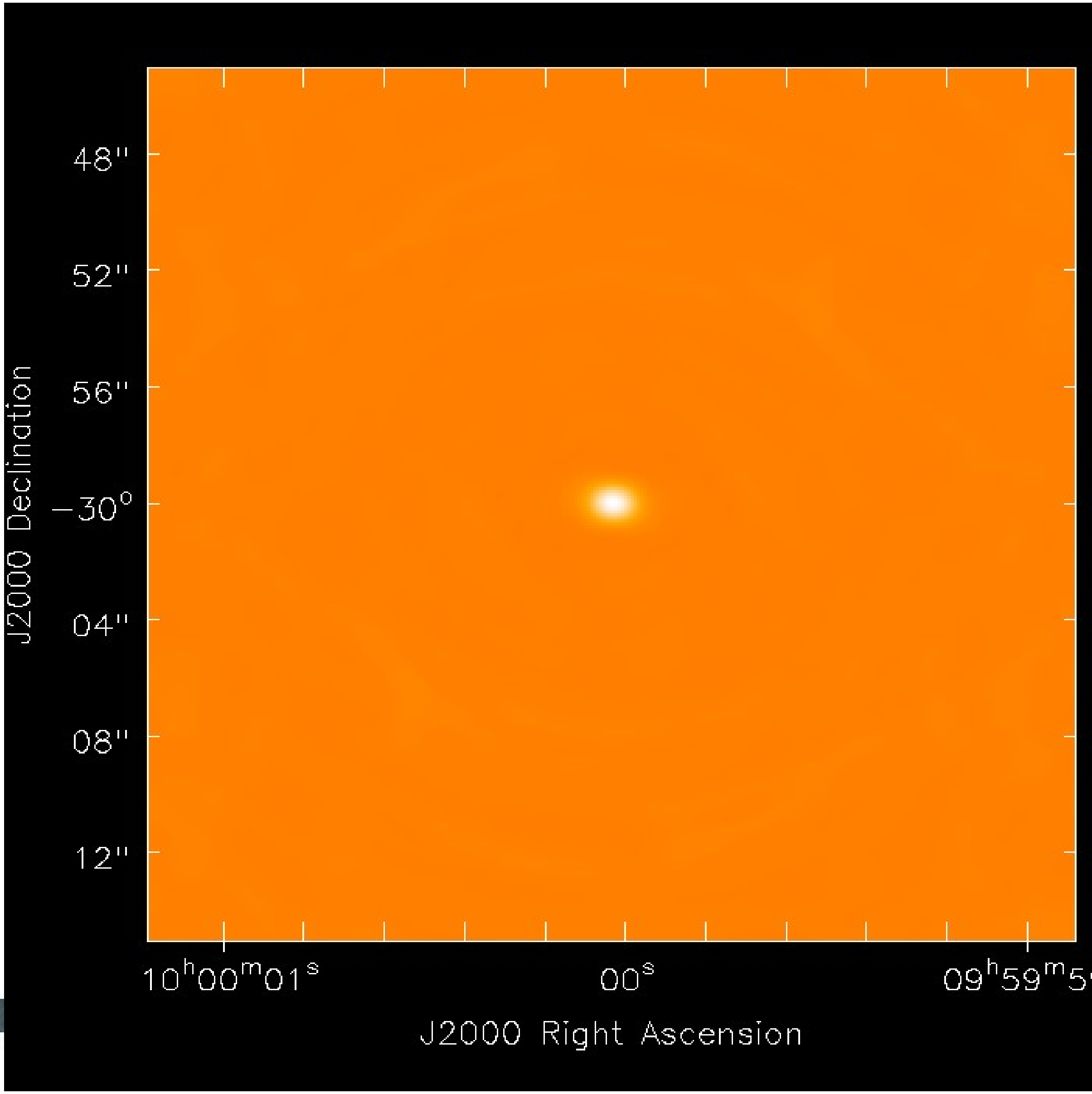




32 Antennas – 8 hours

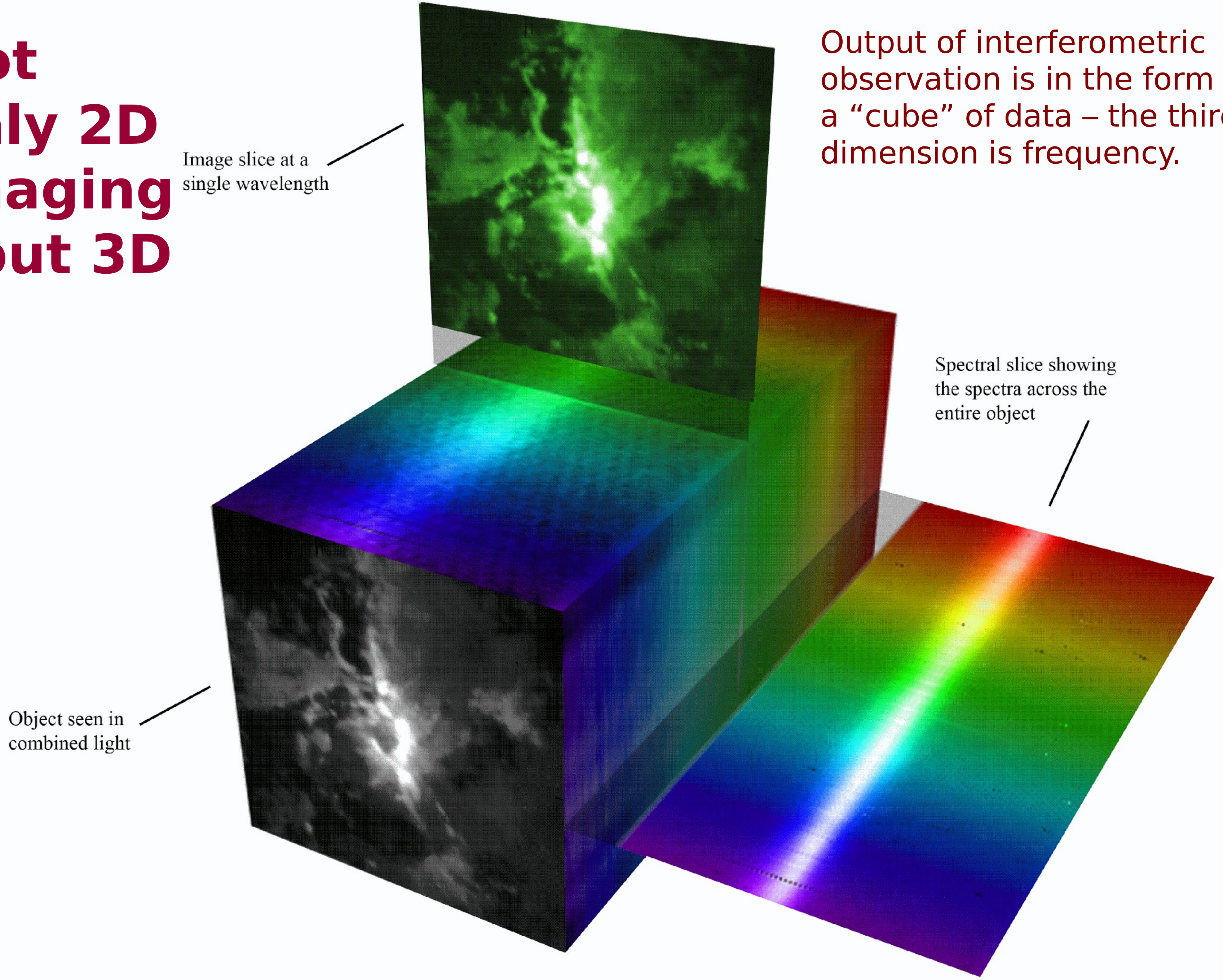






Not only 2D imaging, but 3D

Output of interferometric observation is in the form of a “cube” of data – the third dimension is frequency.



A Brief Word on Calibration

- Interferometers measure visibilities, i.e., the amplitude and phase of the cross-correlated signals between pairs of antennas, as a function of time and frequency.
- We calibrate these data by determining the complex gains (amplitude and phase), the frequency response (bandpass) and flux scale for each antenna.

Calibration Process

Calibration is the effort to measure and remove the time-dependent and frequency-dependent atmospheric and instrumental variations.

Steps in calibrating interferometric data:
(Note: You don't have to worry about these in your observational set up!)

- Bandpass calibration (correct frequency-dependent telescope response)
- Phase and amplitude gain calibration (remove effects of atmospheric water vapor and correct time-varying phases/amplitudes)
- Set absolute flux scale

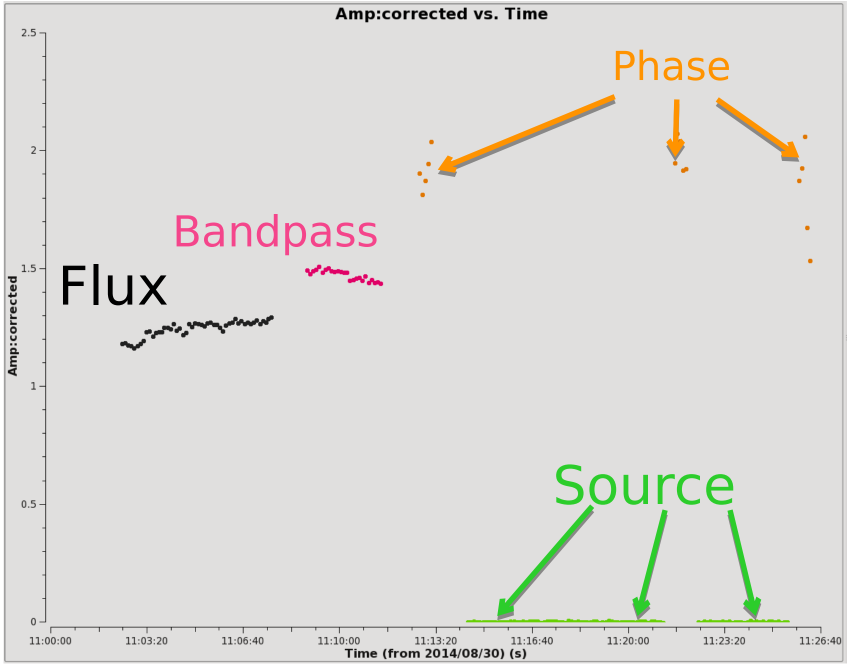
Calibration Requirements

(Details handled by ALMA):

Absolute flux calibrator
(Solar system object or quasar)
Used to scale relative amplitudes to absolute value

Bandpass calibrator
(Bright quasar)
Fixes instrumental effects and variations vs frequency

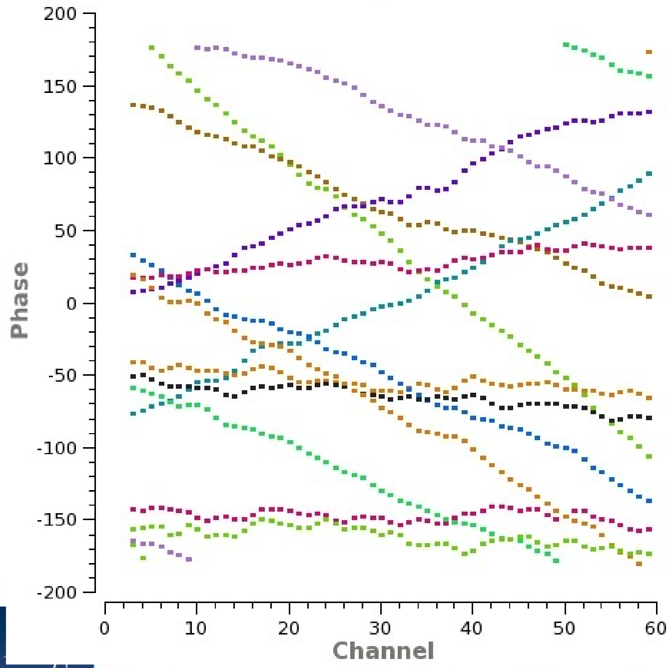
Gain calibrator
(Bright quasar near science target)
Solves for atmospheric and instrumental variations with time



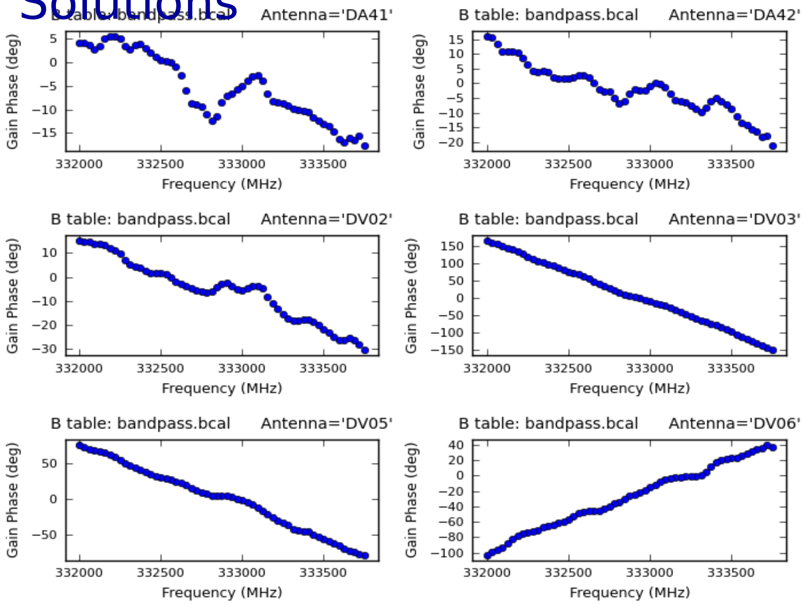
Bandpass Calibration: Phase

* Analogous to optical "flat fielding" + bias subtraction for each antenna.
* Primarily correcting for frequency dependent telescope response (i.e. in the correlator/spectral windows)
* Done once in an SB, uses bright point sources like quasars
* Typically, baseline responses are inverted to antenna-based correction

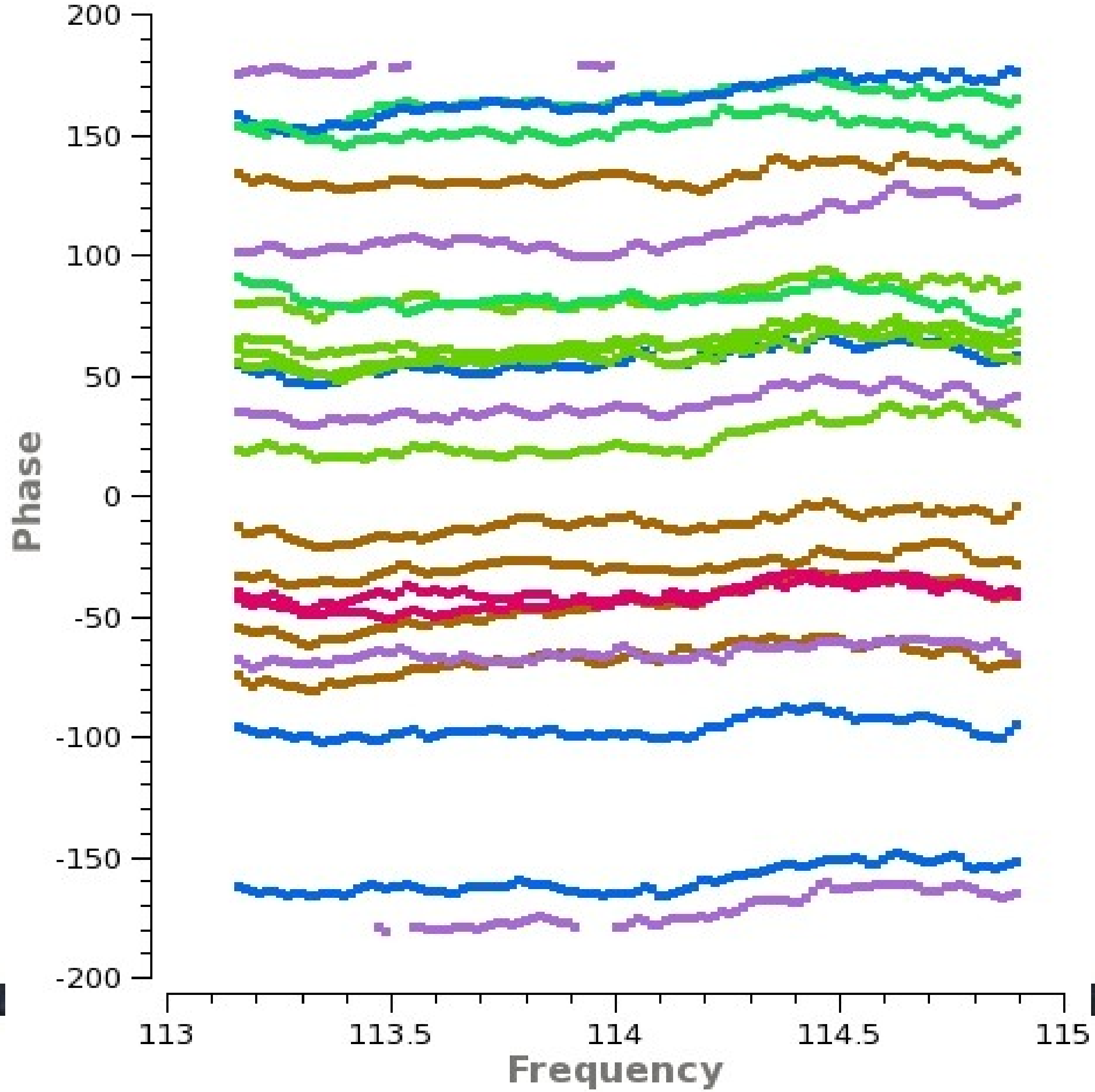
Baselines to one antenna



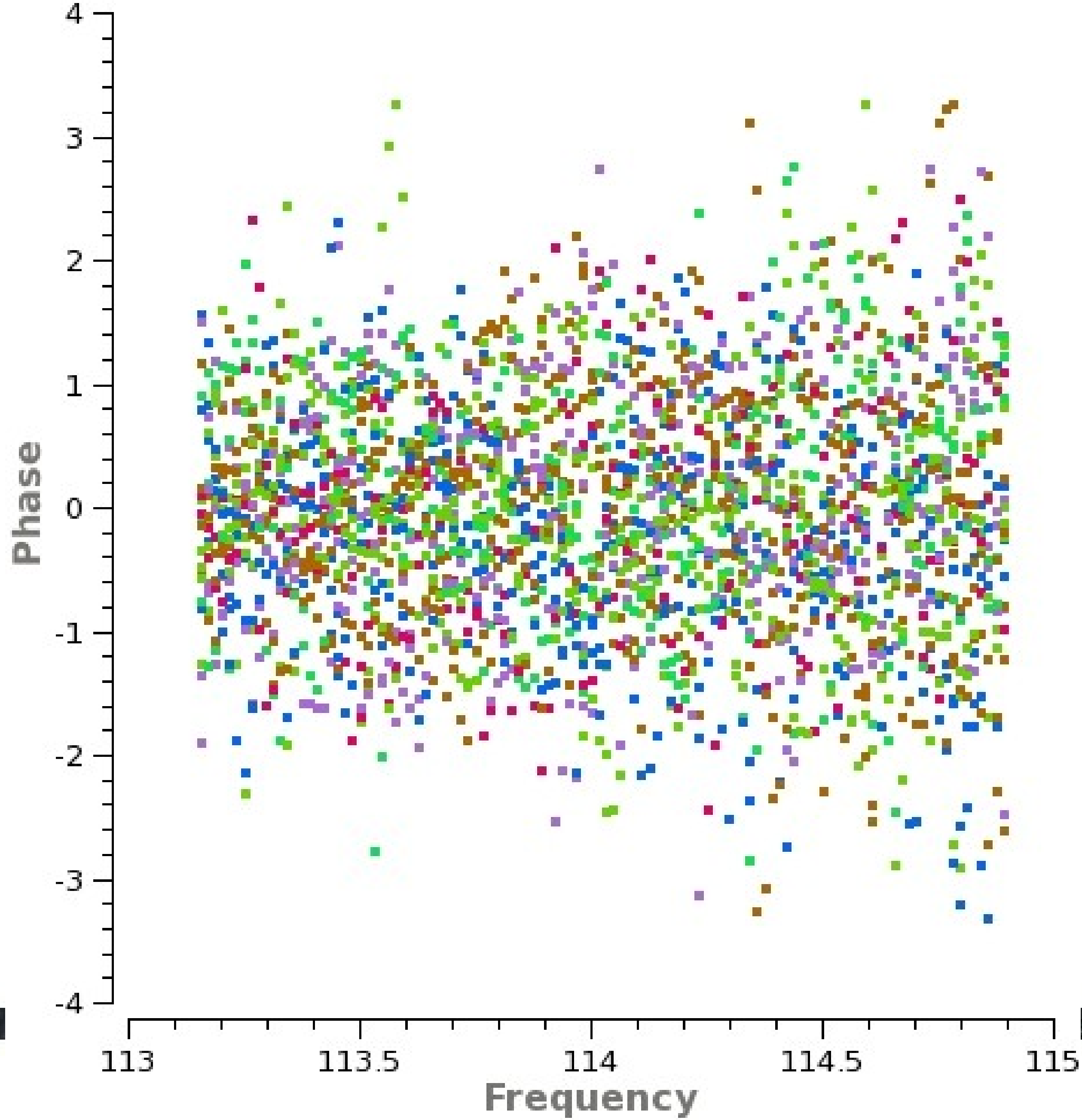
Antenna-based Bandpass Solutions



Bandpass Phase vs. Frequency (Before)

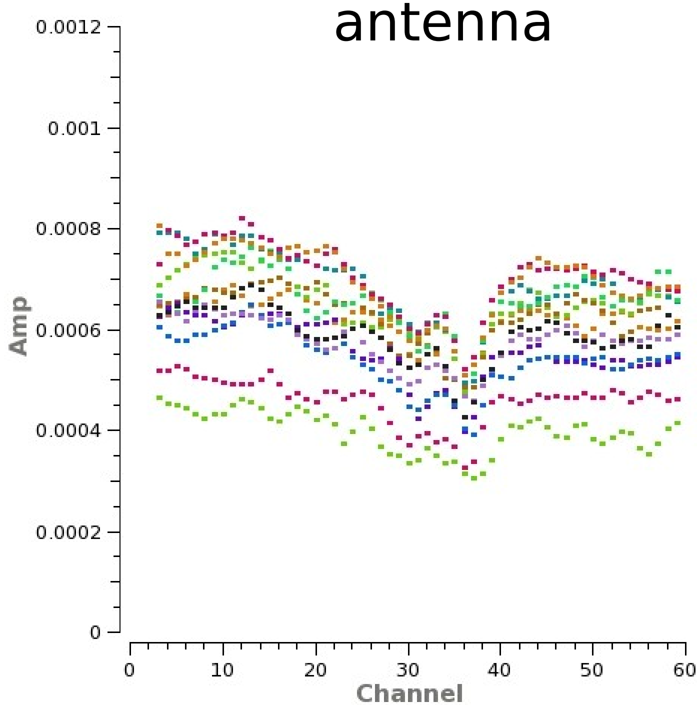


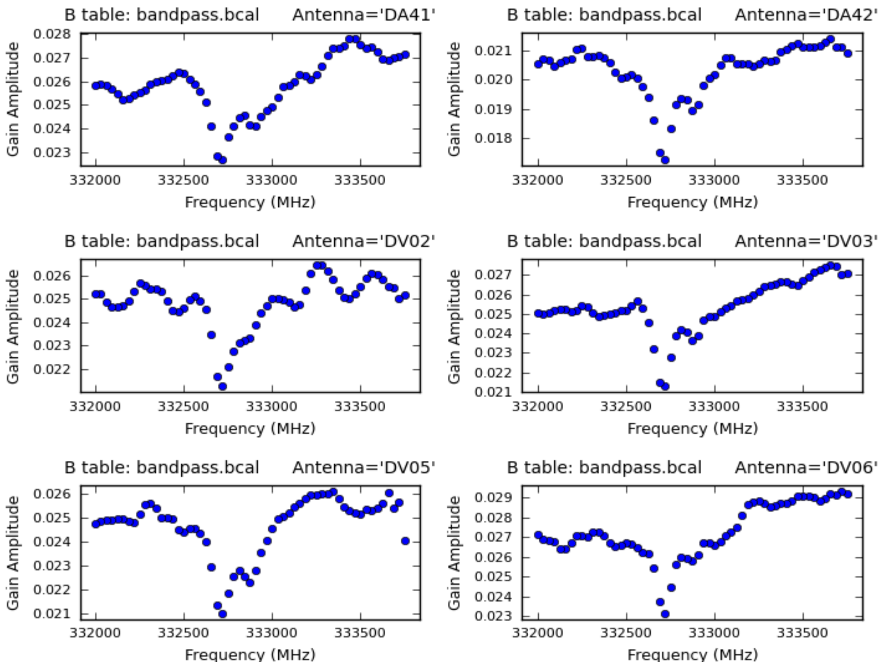
Bandpass Phase vs. Frequency (After)



Bandpass Calibration: Amplitude

Baselines to one antenna





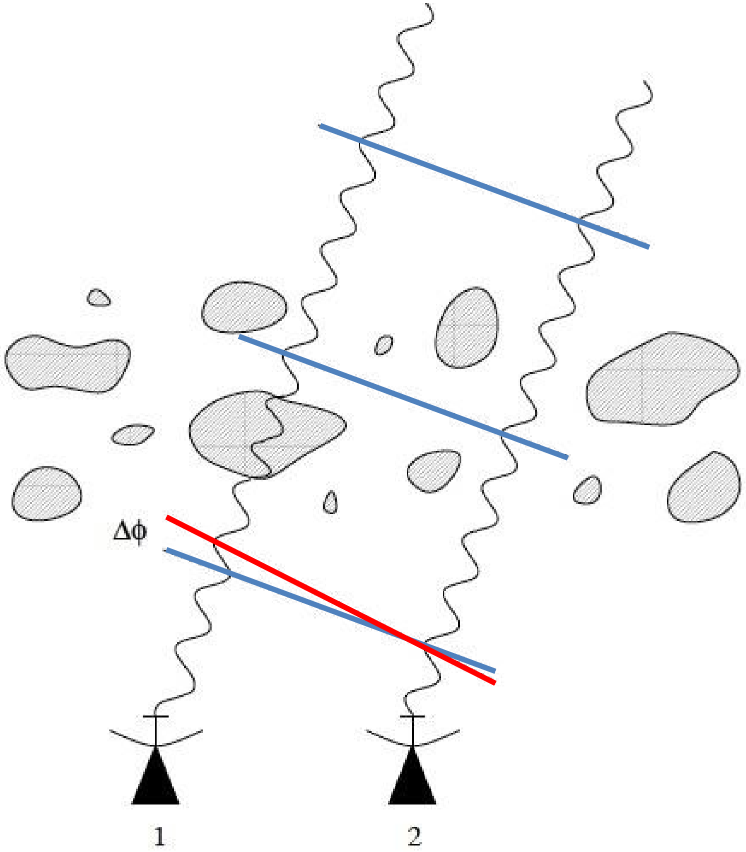
Amplitude Before Bandpass Calibration

Bandpass solutions for individual antennas

Atmospheric Phase Correction

- Variations in the amount of precipitable water vapor cause phase fluctuations that result in:
  - Low coherence (loss of sensit
  - Radio “seeing” of 1arcsec at :
  - Anomalous pointing offsets
  - Anomalous delay offsets

Patches of air with different water vapor content (and hence index of refraction) affect the incoming wave front differently.



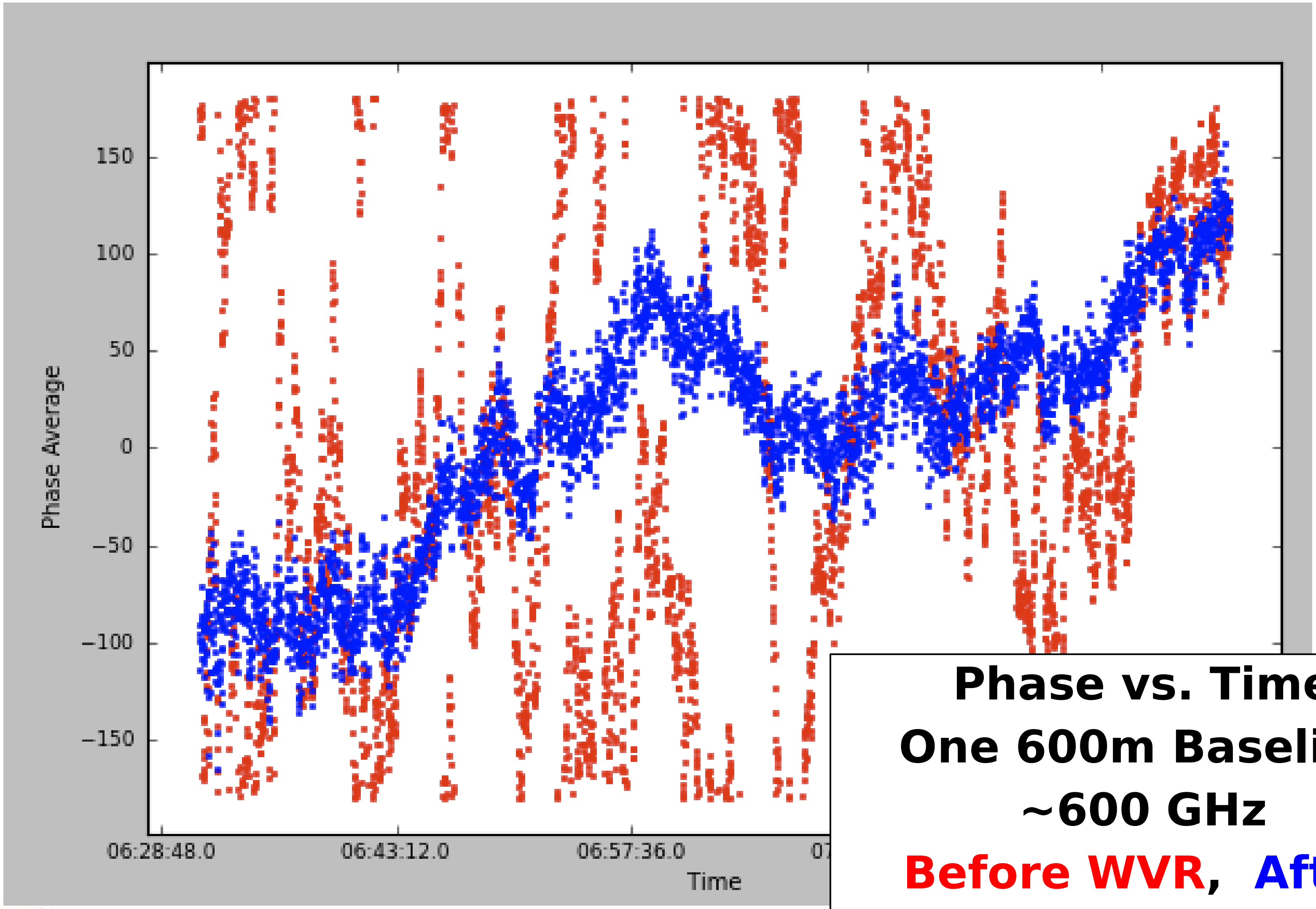
Phase & Amplitude Gain Calibration

Determines the variations of phase and amplitude over time

- First pass is atmospheric correction from Water Vapor Radiometers readings

- Final correction from gain calibrator (point source near to target) that is observed every few minutes throughout the observation (analogous to repeat trips to a standard star)

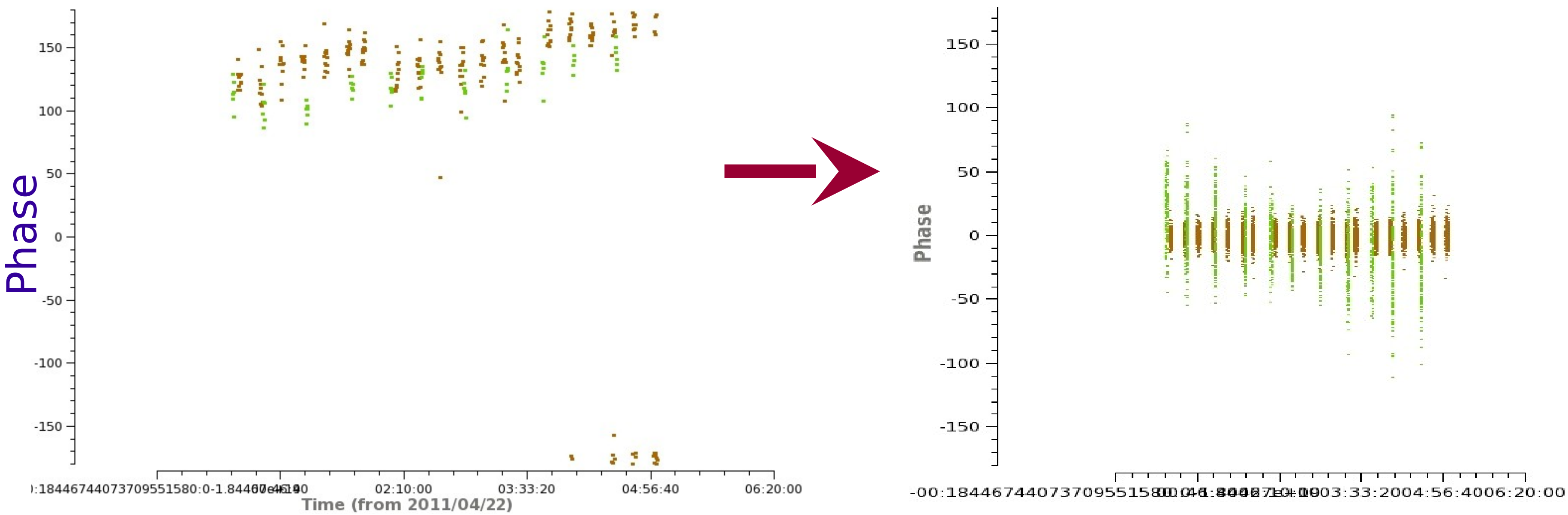
Water Vapor Correction on ALMA



Phase vs. Time
One 600m Baseline
~600 GHz
Before WVR, After WVR

Phase Calibration

The phase calibrator must be a point source close to the science target and must be observed frequently. This provides a model of atmospheric phase change along the line of sight to the science target that can be compensated for in the data.



Time

Corrected using point source model

Flux (or Amplitude) Calibration

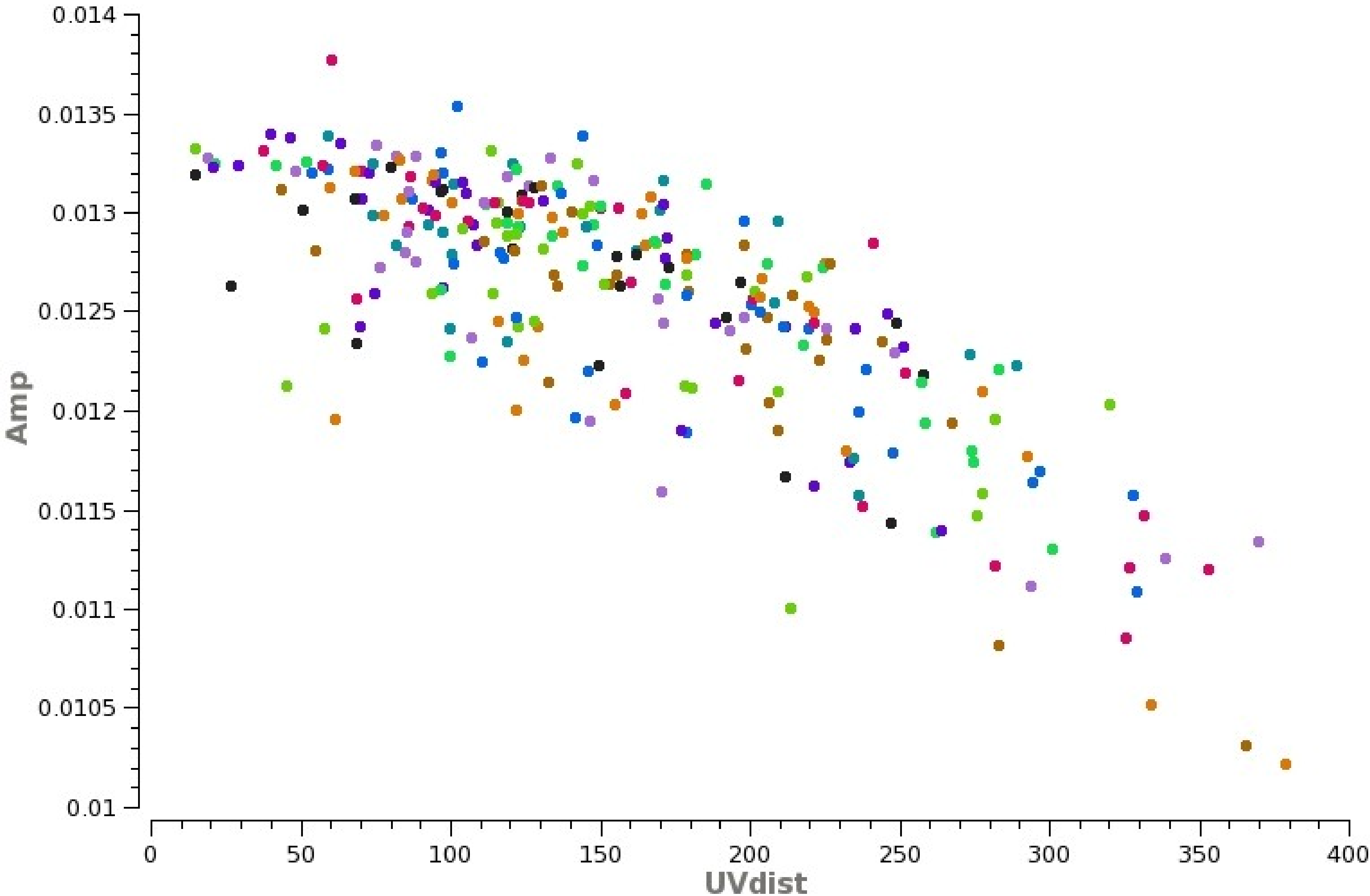
Two Steps:

1. Use calibration devices with known temperatures (hotload and ambient load) to measure System Temperature frequently.

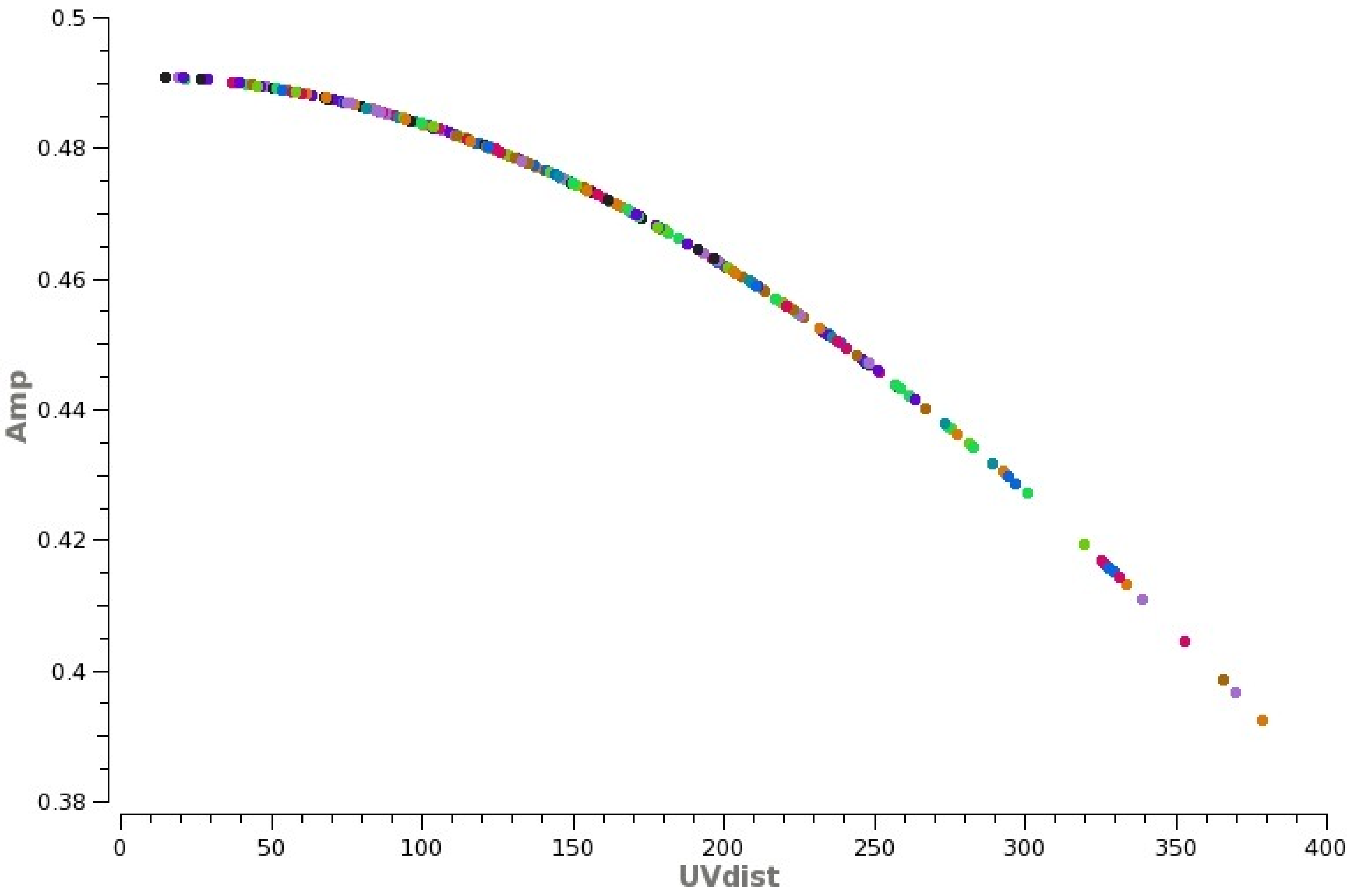
2. Use a source of known flux to convert the signal measured at the antenna to common unit (Janskys). If the source is resolved, or has spectral lines, it must be modeled very well.

The derived amplitude vs. time corrections for the flux calibrator are then applied to the science target.

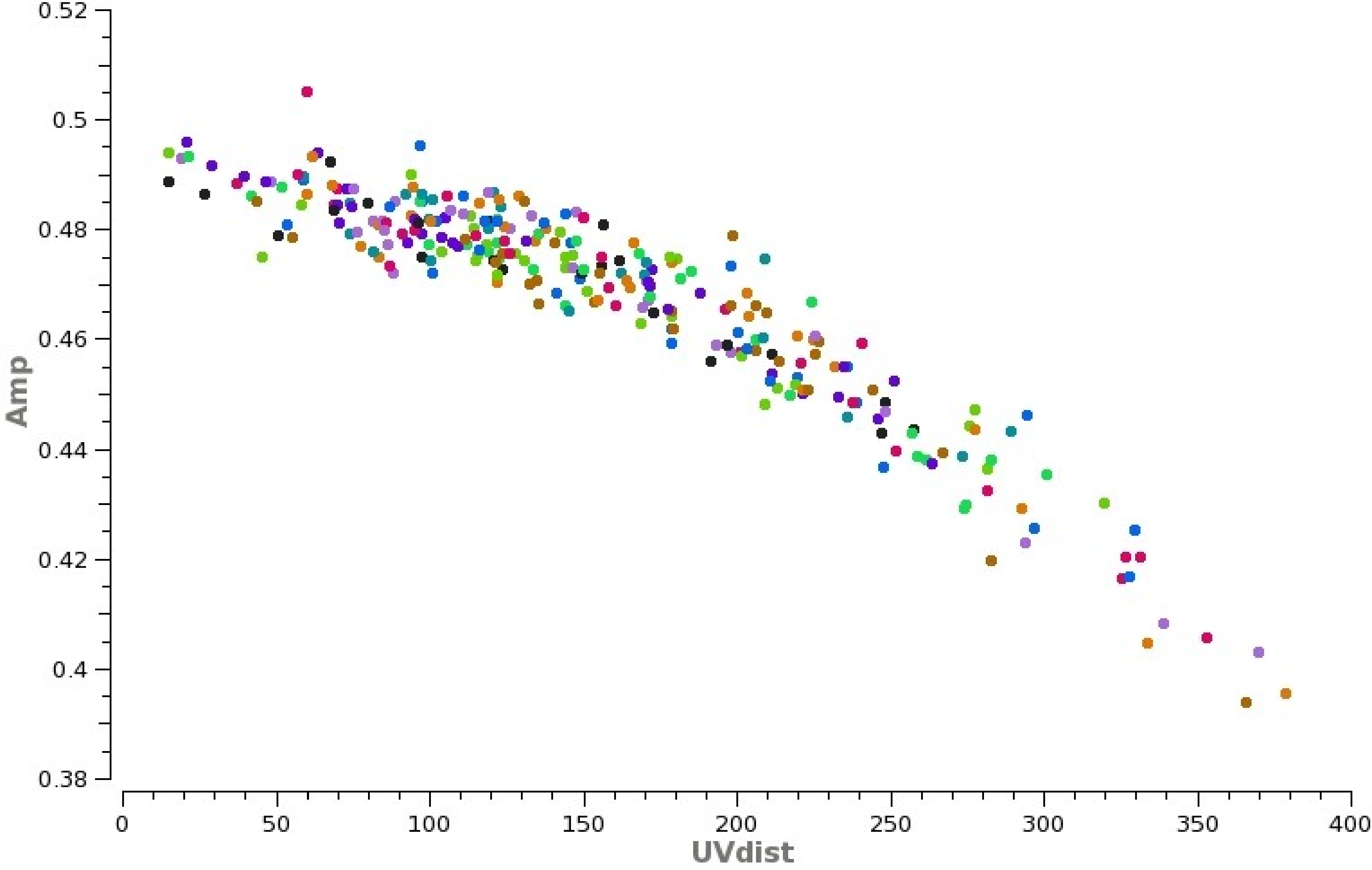
Amp-Calibrators Amp vs. uv-distance (Before)



Amp-Calibrators Amp vs. uv-distance (Model)



Amp-Calibrators Amp vs. uv-distance (After)



Good Future References

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Perley, R.A., Schwab, F.R., Bridle, A.H. eds. 1989 ASP Conf. Series 6 “Synthesis Imaging in Radio Astronomy” (San Francisco: ASP)

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www.iram.fr/IRAMFR/IS/IS2008/archive.html

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www.nrao.edu
science.nrao.edu

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