# Introduction to Radio Interferometry


## Veronica Allen

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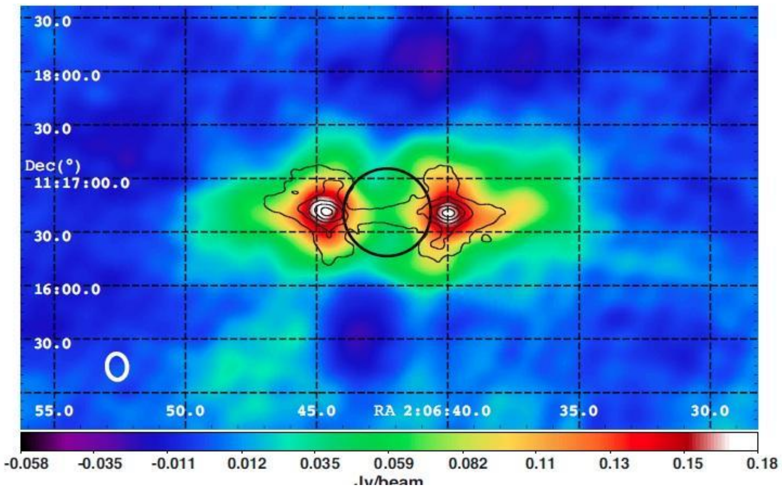
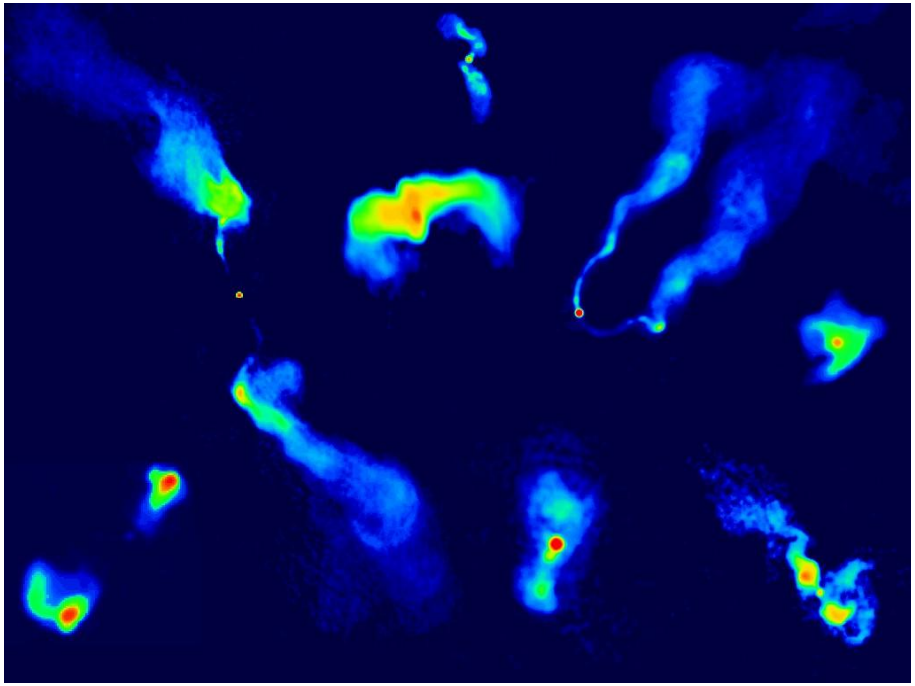
Atacama Large Millimeter/submillimeter Array
Karl G. Jansky Very Large Array
Very Long Baseline Array

# Long wavelength means no glass mirrors

# What can we observe? (MHz-GHz range)

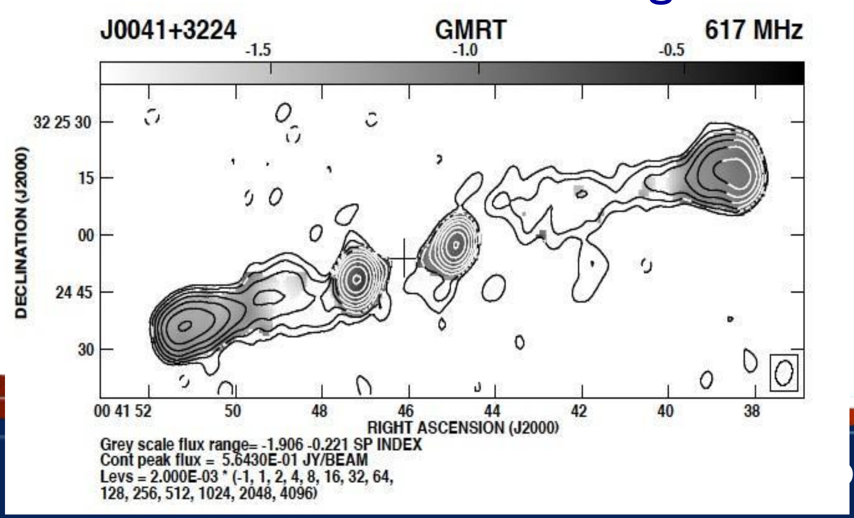
Jupiter's radiation belt at 100MHz



Synchrotron emission from extended radio galaxies (5 GHz)

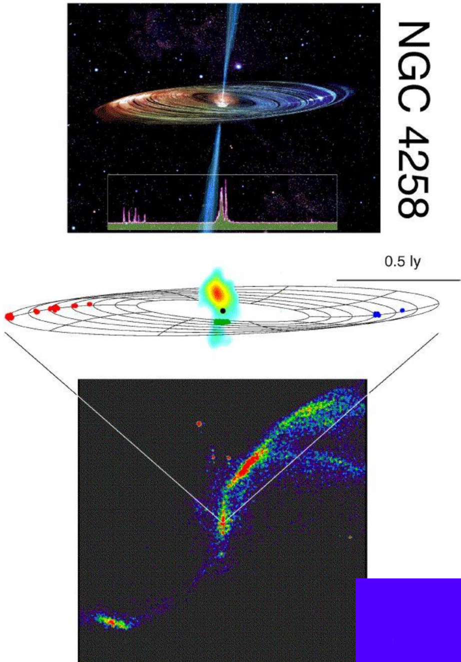
Relic emission from old radio galaxies



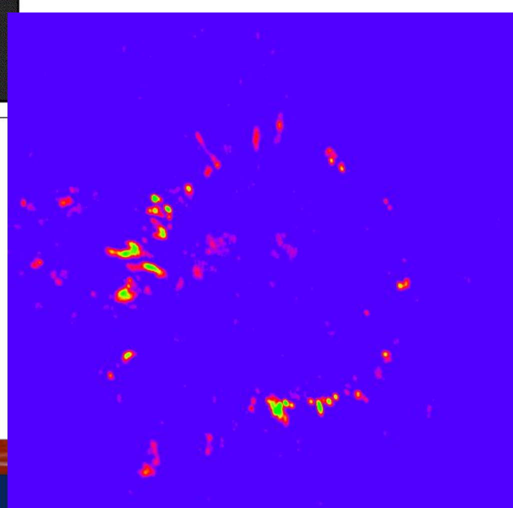
Images from NRAO Image Gallery
http://images.nrao.edu/

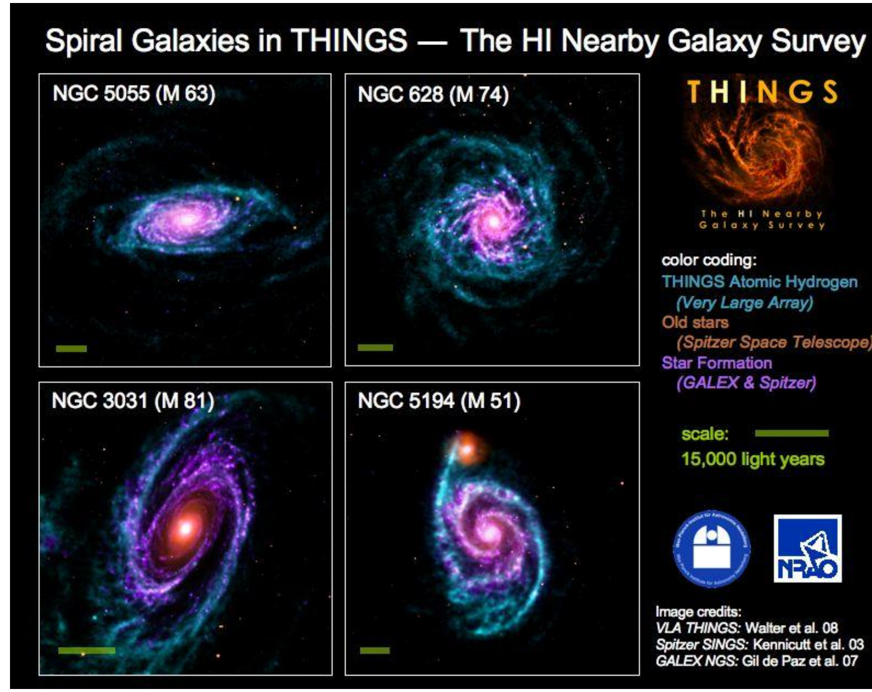
# What can we observe?

At low frequencies (MHz-GHz):

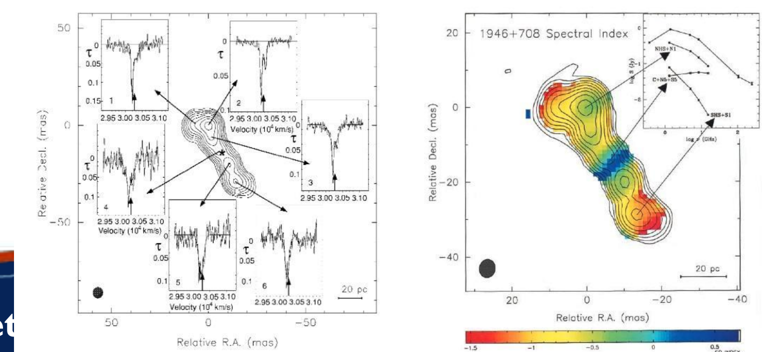



$H_2O$, OH or SiO masers in galaxies and stars



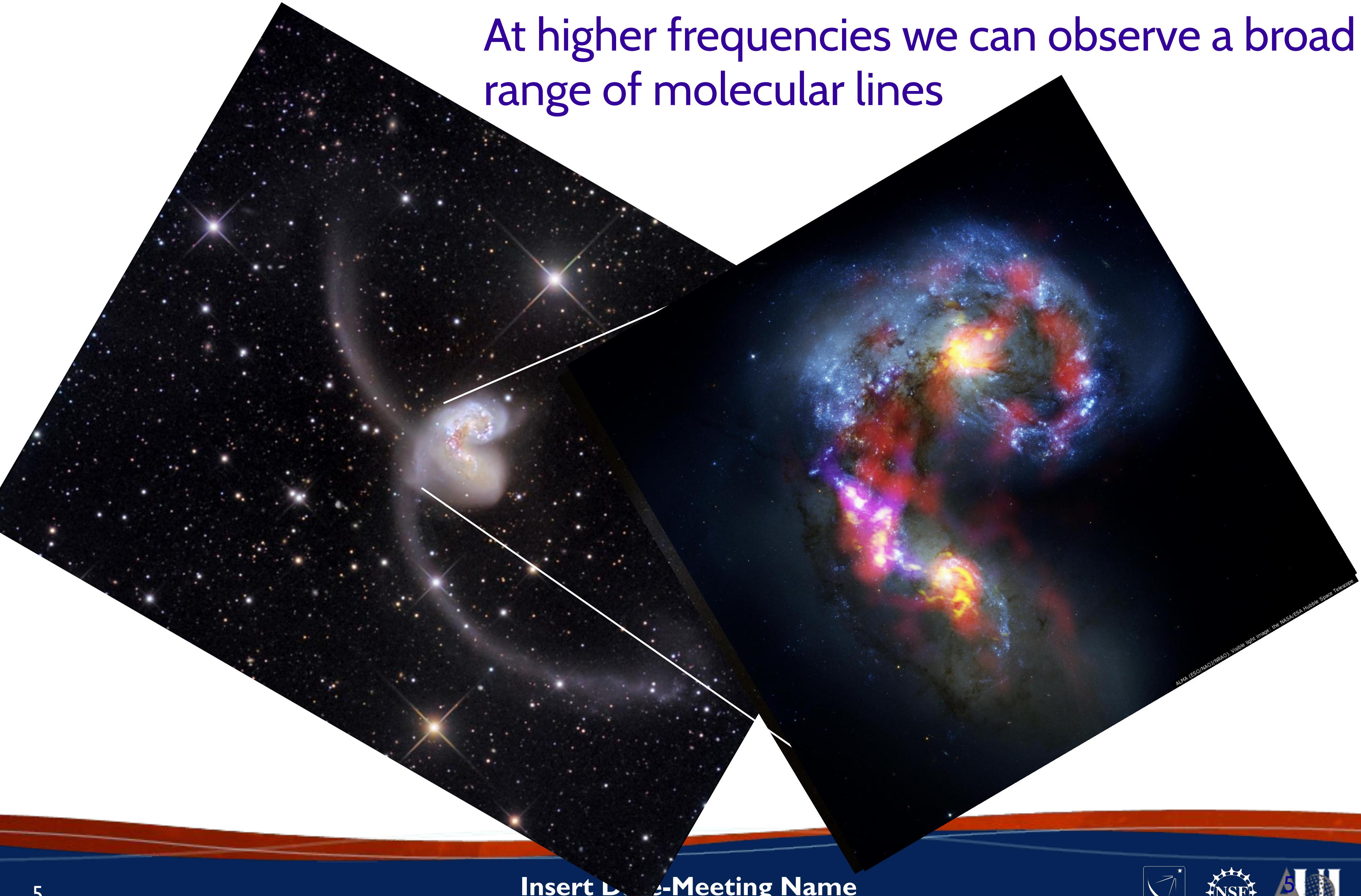



HI emission and absorption, free-free absorption in galaxies



# What can we observe?

At higher frequencies we can observe a broad range of molecular lines



# Resolution of Observations

**Angular resolution for most telescopes is ~ λ/D**

D is the diameter of the telescope and λ is the wavelength of observation

**For the Hubble Space Telescope:**

*λ ~ 1um / D of 2.4m = resolution ~ 0.13"*

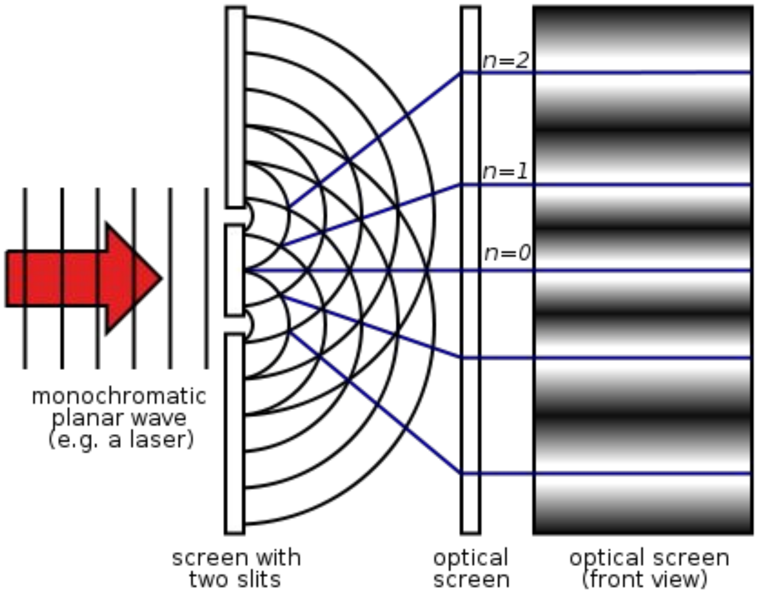
**To reach that resolution at λ ~1mm, we would need a 2 km-diameter dish!**

Instead, we use arrays of smaller dishes to achieve the same high angular resolution at radio frequencies

**This is interferometry!**

# What is an interferometer?

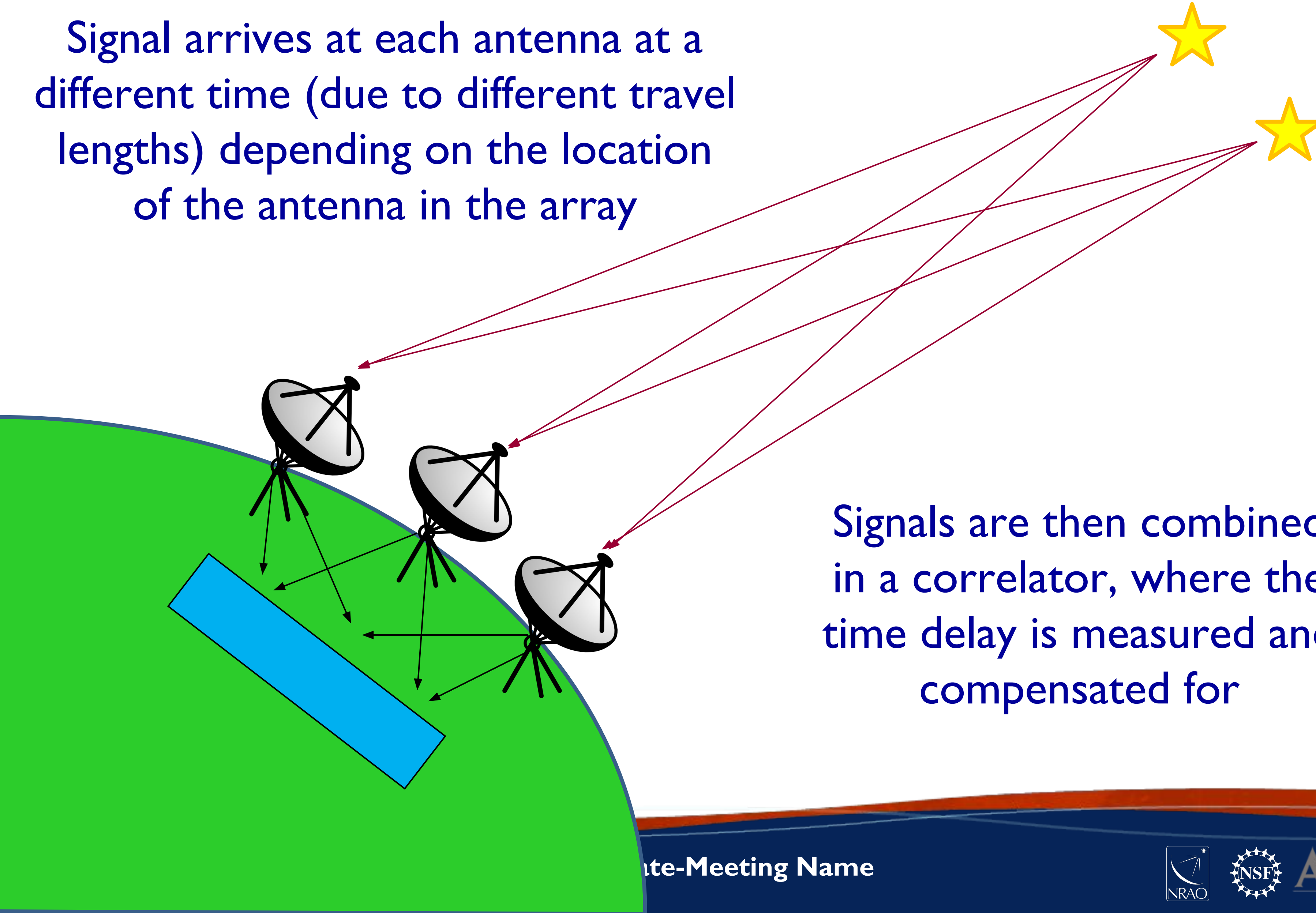
An *interferometer* measures the interference pattern produced by multiple apertures, much like a 2-slit experiment




**However, the interference patterns measured by radio telescopes are produced by* ***multiplying*** *- not adding - the wave signals measured at the different telescopes (i.e. apertures)*

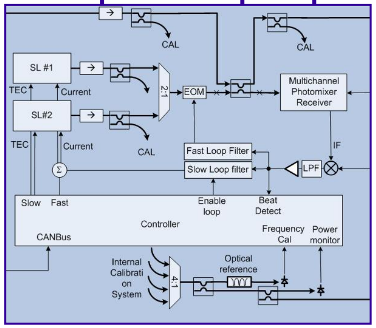
# How Do We Use Interferometry?

Signal arrives at each antenna at a different time (due to different travel lengths) depending on the location of the antenna in the array



Signals are then combined in a correlator, where the time delay is measured and compensated for

# Some Instrument Details



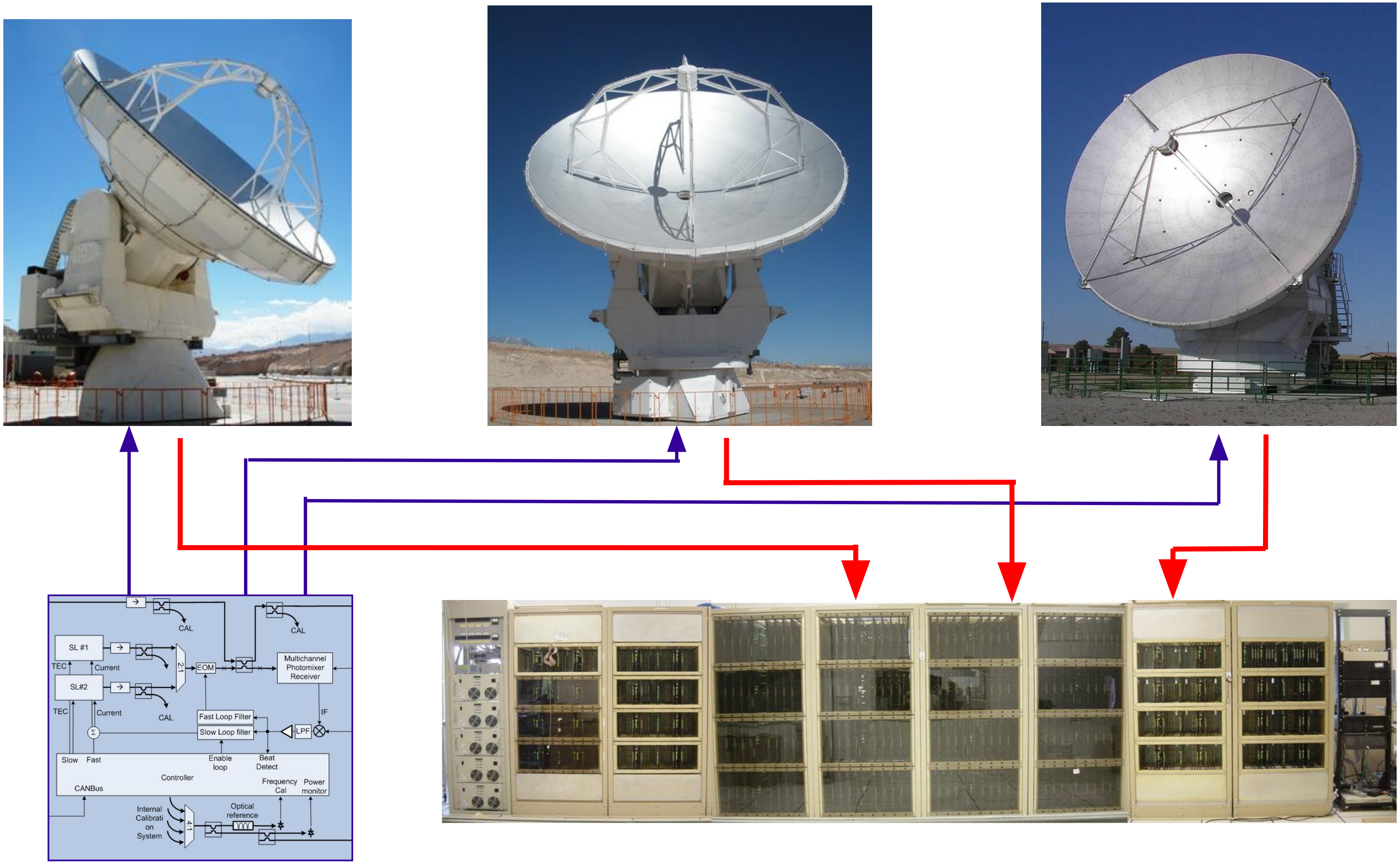

## To precisely measure arrival times we need very accurate clocks

At Band 10 one wavelength error = 1 picosecond = $10^{-12}$ s (!!)

Need << 1 wavelength timing precision, so each antenna has an on-board clock with high sampling rates

Once determined, the reference time is distributed to all antennas

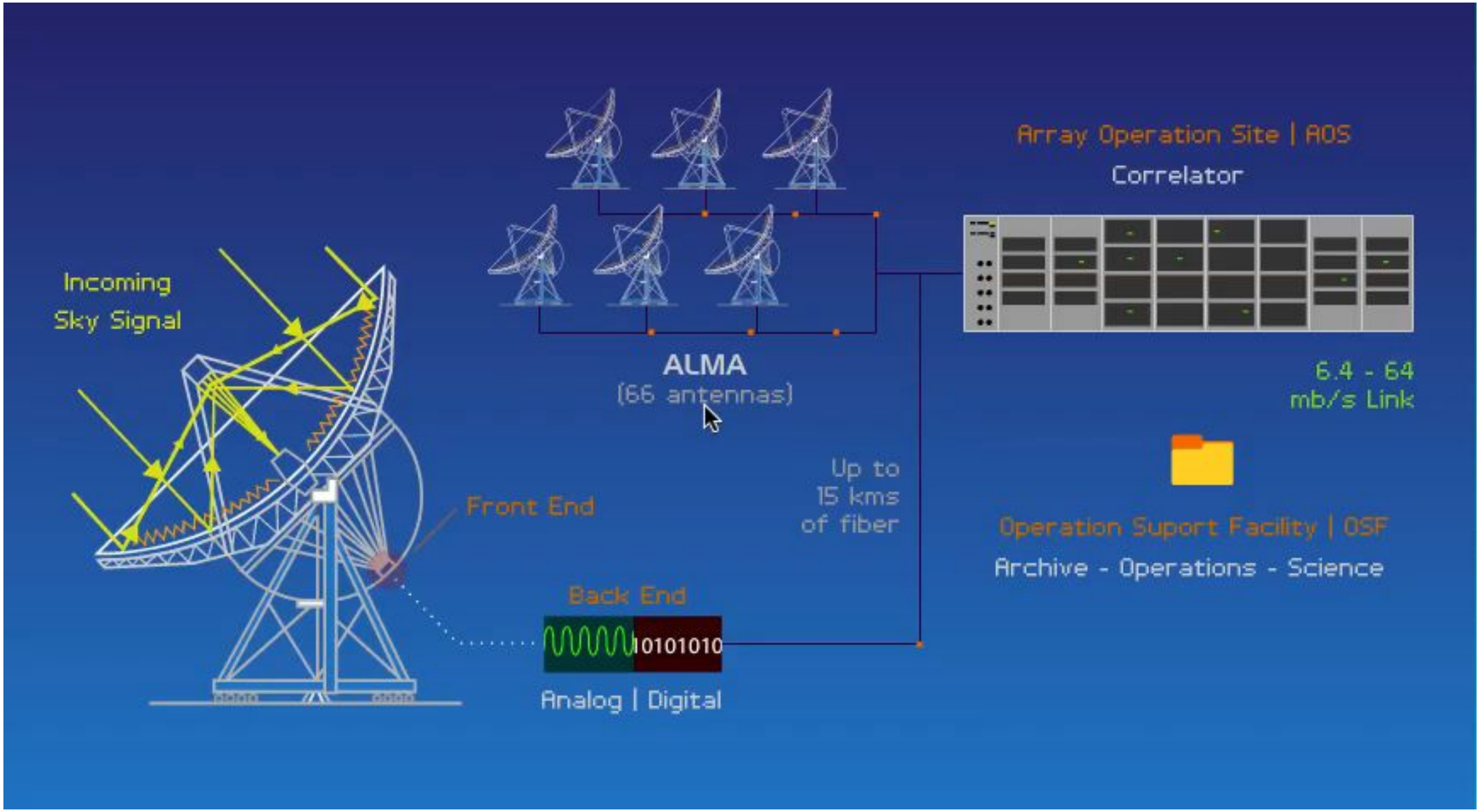
# Some Instrument Details




Signals from each antenna are digitized and sent to the correlator for multiplication & averaging

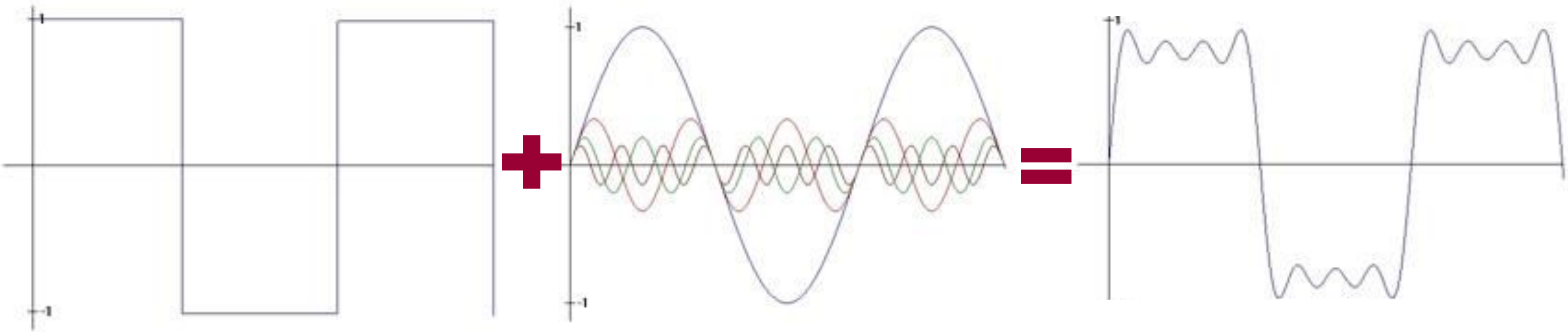
For ~50 antennas, the data rate is 600 GB/sec for the correlator to process

# An Interferometer In Action



# Introducing the Fourier Transform

**Fourier theory states that any well behaved signal (including images) can be expressed as the sum of sinusoids**



Reference signal　　4 sinusoids　　Sum of sinusoids & signal

The Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components

The Fourier transform contains *all* of the information of the original signal

# Visibility and Sky Brightness

*The van Cittert-Zernike theorem*

- **Visibility as a function of baseline coordinates (u,v) is the Fourier transform of the sky brightness distribution as a function of the sky coordinates (x,y)**

$$V(u,v) \xrightarrow{\text{FT}} T(x,y)$$

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V(u,v) = the complex visibility function $= \iint T(x,y)e^{2\pi i(ux+vy)}dxdy$

T(x,y) = the sky brightness distribution $= \iint V(u,v)e^{-2\pi i(ux+vy)}dudv$

# What Are Visibilities?

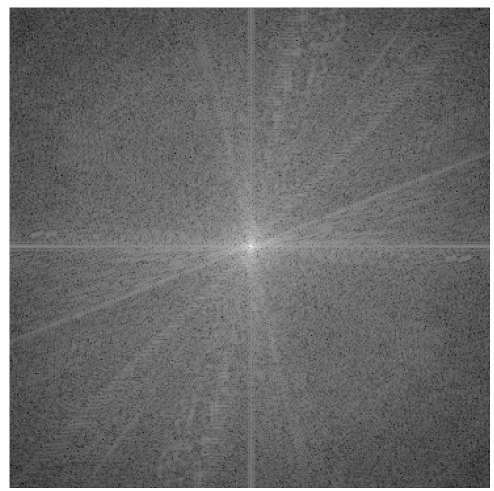
Each V(u,v) contains information on T(x,y) everywhere

Each V(u,v) is a complex quantity

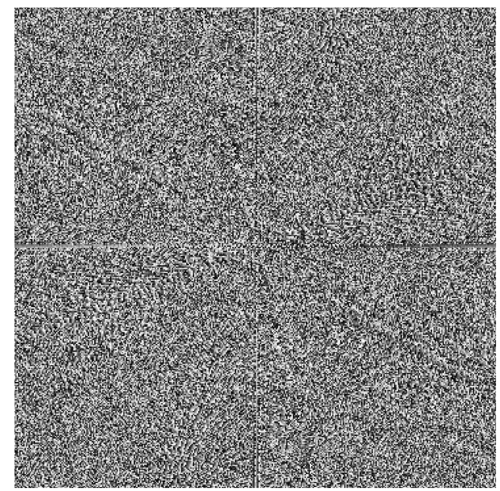
Expressed as (real, imaginary) or (amplitude, phase)

T(x,y)

FT →

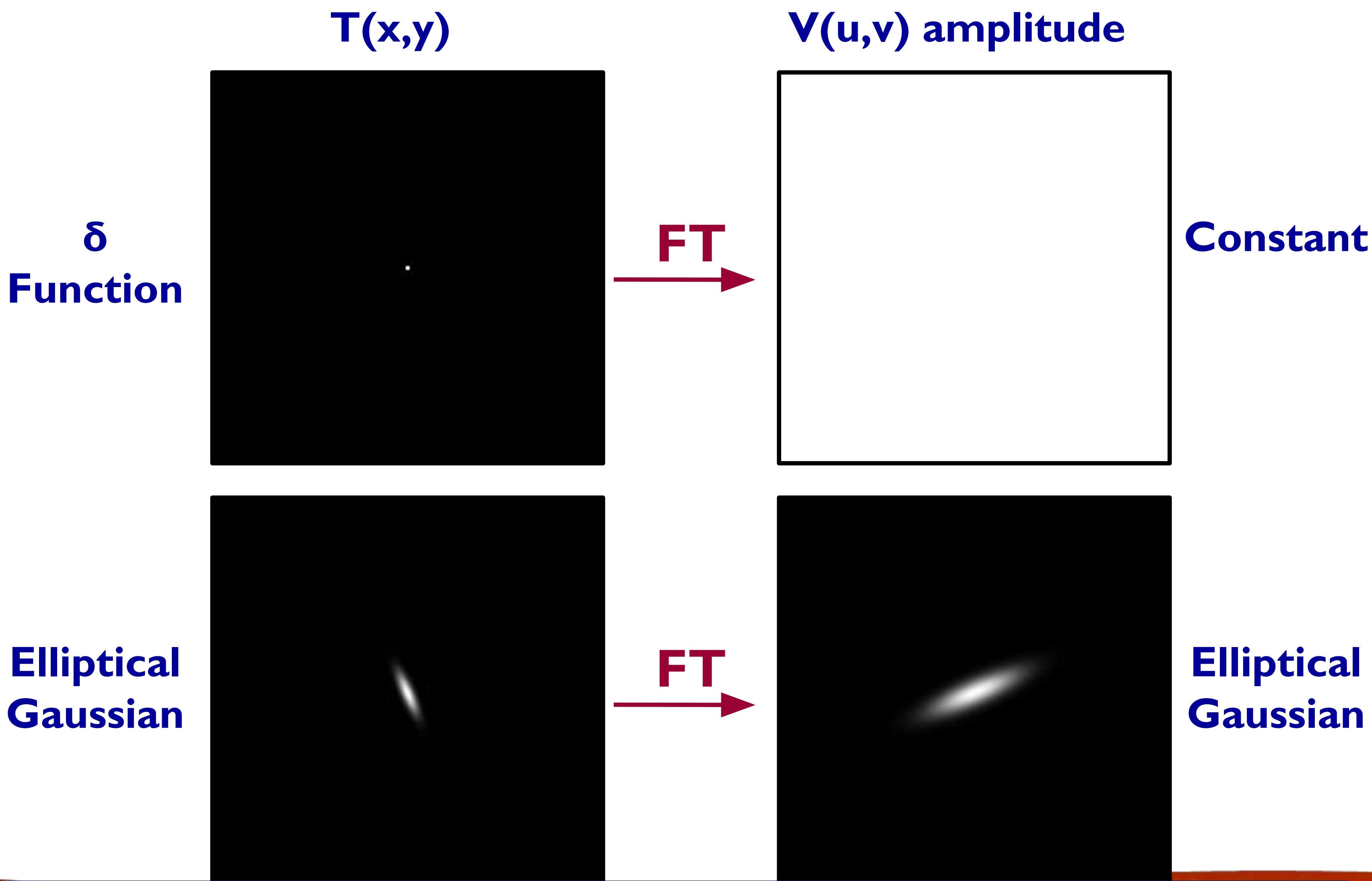


V(u,v) amplitude



V(u,v) phase

# Examples of 2D Fourier Transforms



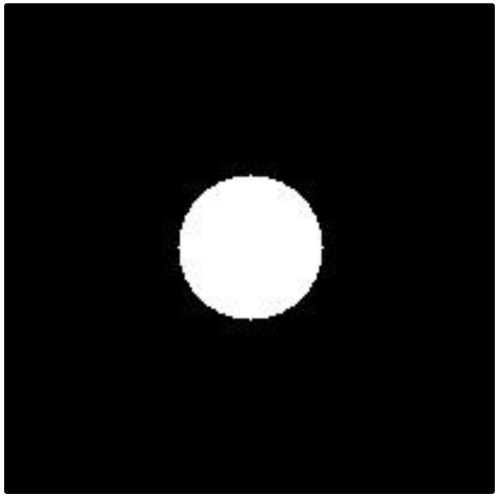

**Rules of the Fourier Transform:**

Narrow features transform to wide features (and vice versa)

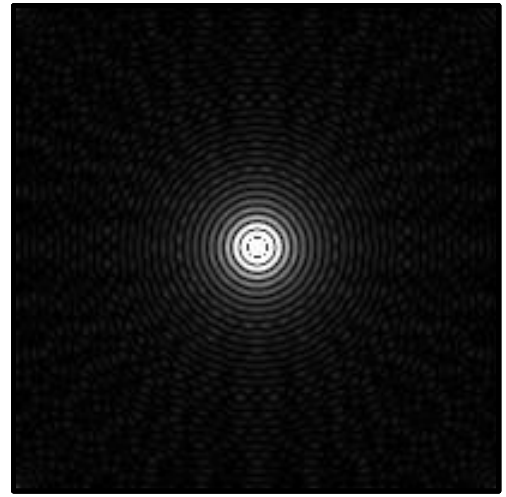
# Examples of 2D Fourier Transforms

**T(x,y)** | **V(u,v) amplitude**

**Uniform Disk**

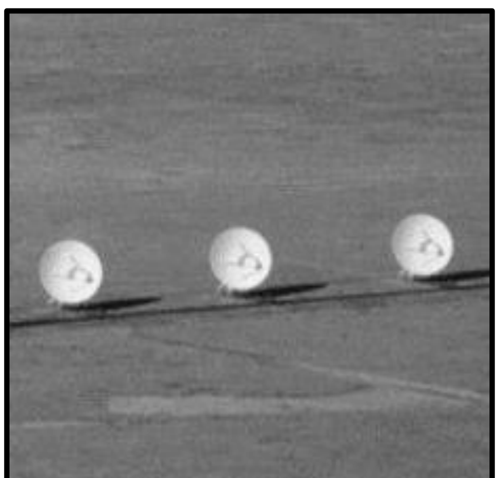


**FT**



**Bessel Function**

**VLA Antennas**



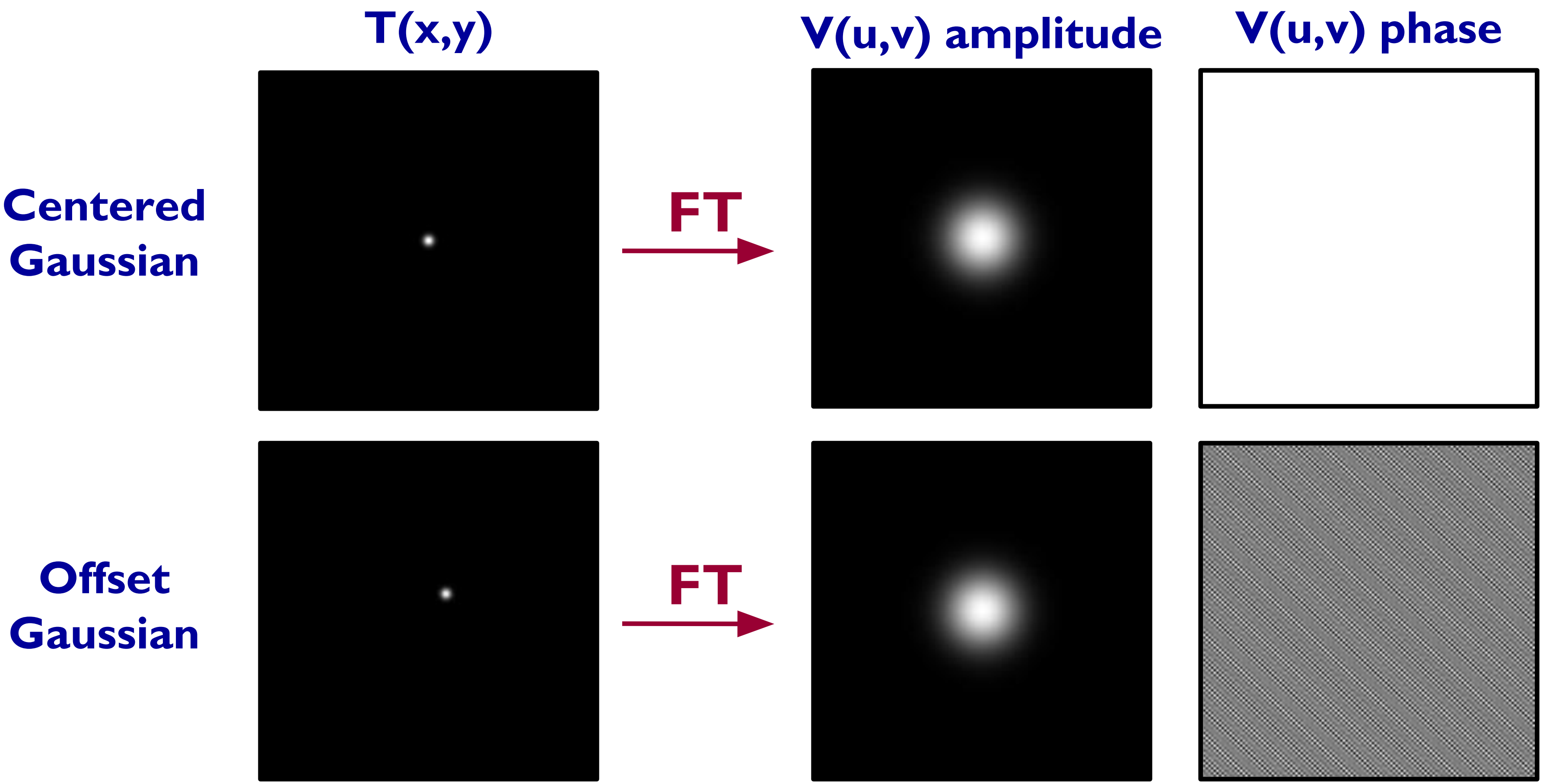
**FT**

**Bessel Function!**

**Rules of the Fourier Transform:**

Sharp features (edges) result in many high spatial features

# Examples of 2D Fourier Transforms




**Rules of the Fourier Transform:**

Amplitude tells you 'how much' of a spatial frequency

Phase tells you 'where' the spatial frequency is

# Basics of Aperture Synthesis

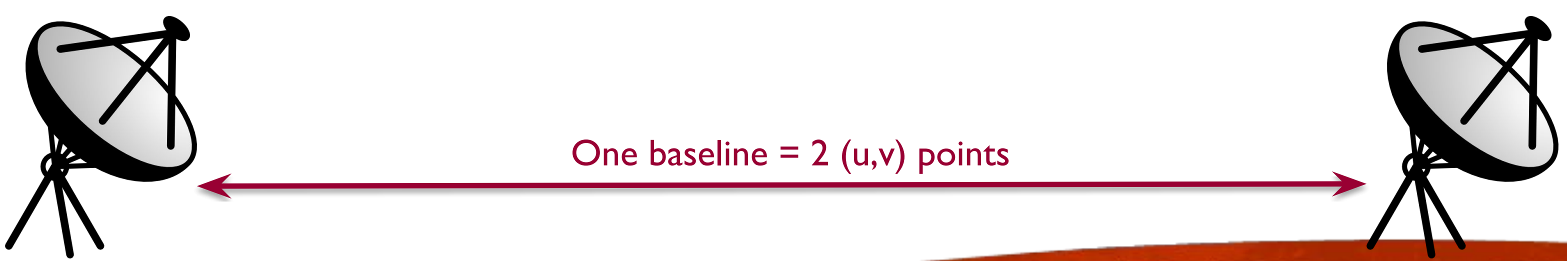
**Idea:** Sample $V(u,v)$ at enough $(u,v)$ points using distributed small aperture antennas to synthesize a large aperture antenna of size $(u_{max}, v_{max})$

One pair of antennas = one baseline

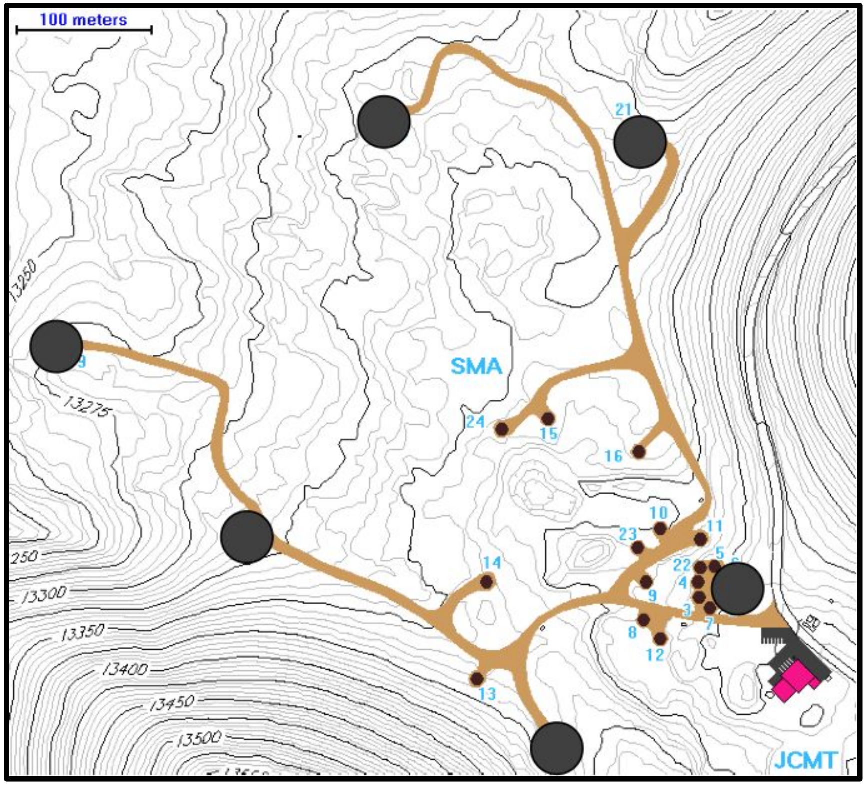
For **N antennas**, we get **N(N-1) samples** at a time

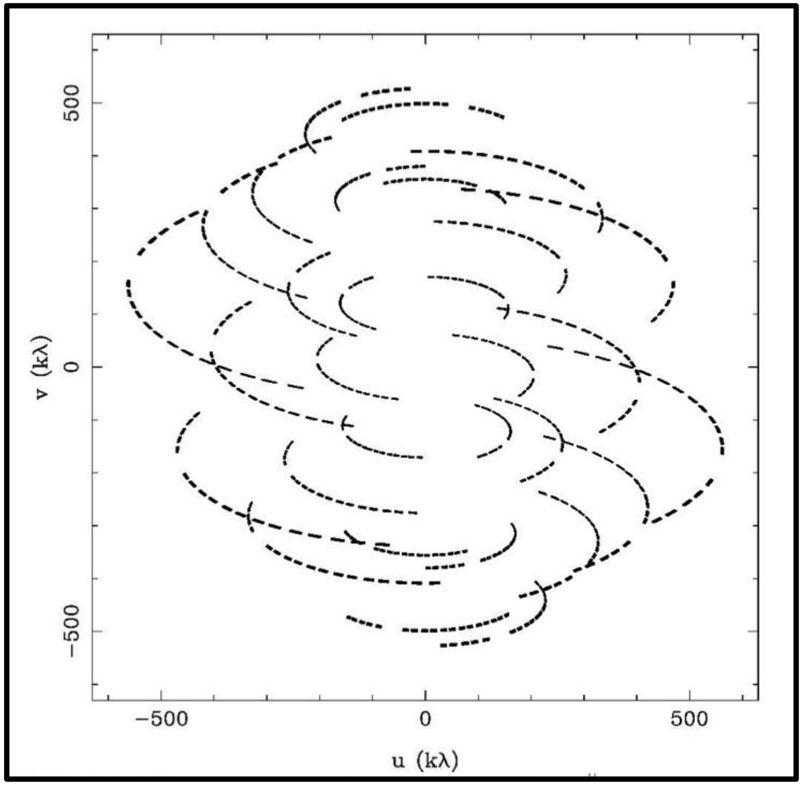
**How do we fill in the rest of the (u,v) plane?**

1. Earth's rotation
2. Reconfigure physical layout of N antennas



# (u,v) Plane Sampling



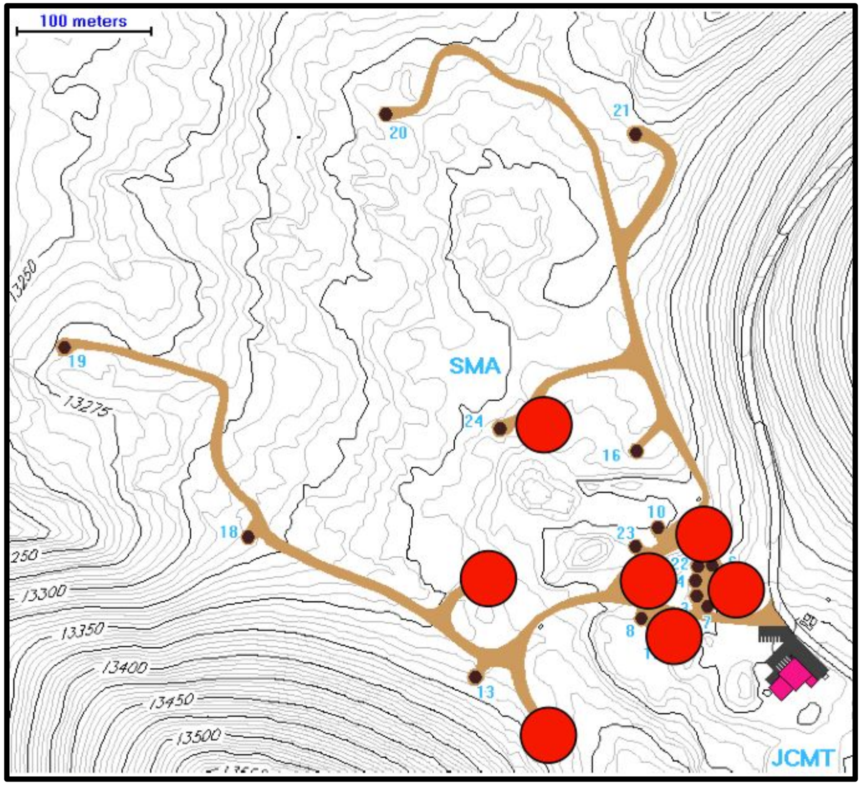



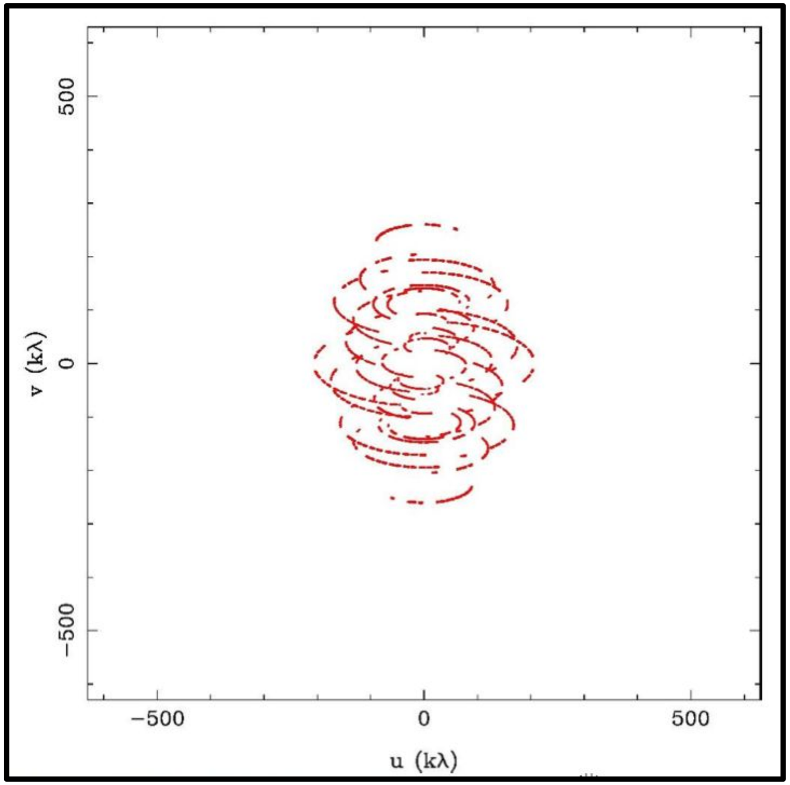

**Very Extended SMA configuration**

(most extended baselines)

345 GHz, DEC = +22

# (u,v) Plane Sampling



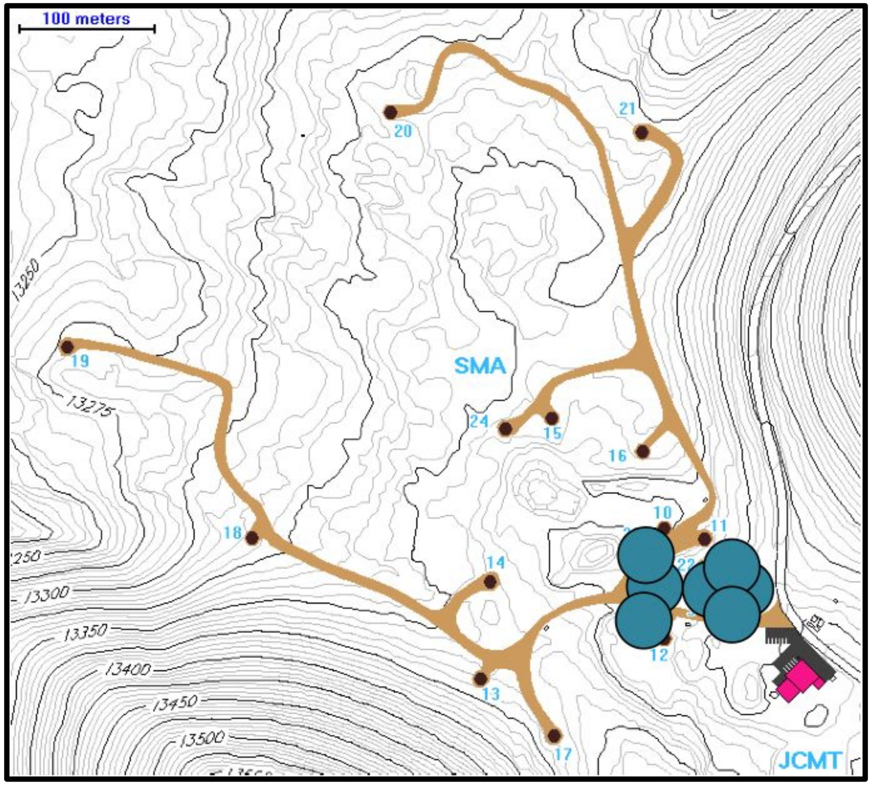



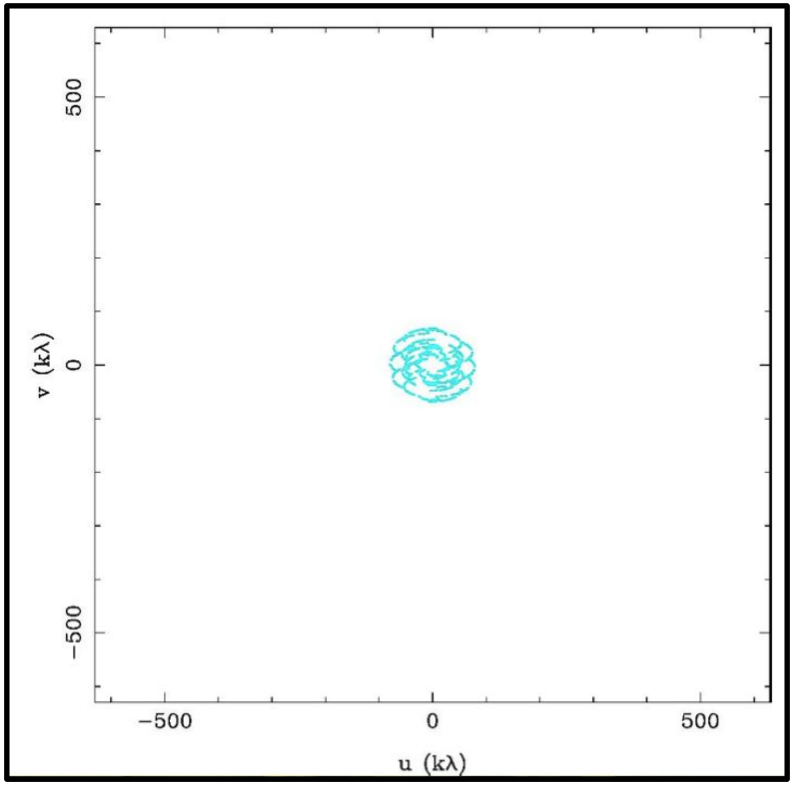

**Extended SMA configuration**

(extended baselines)

345 GHz, DEC = +22

# (u,v) Plane Sampling



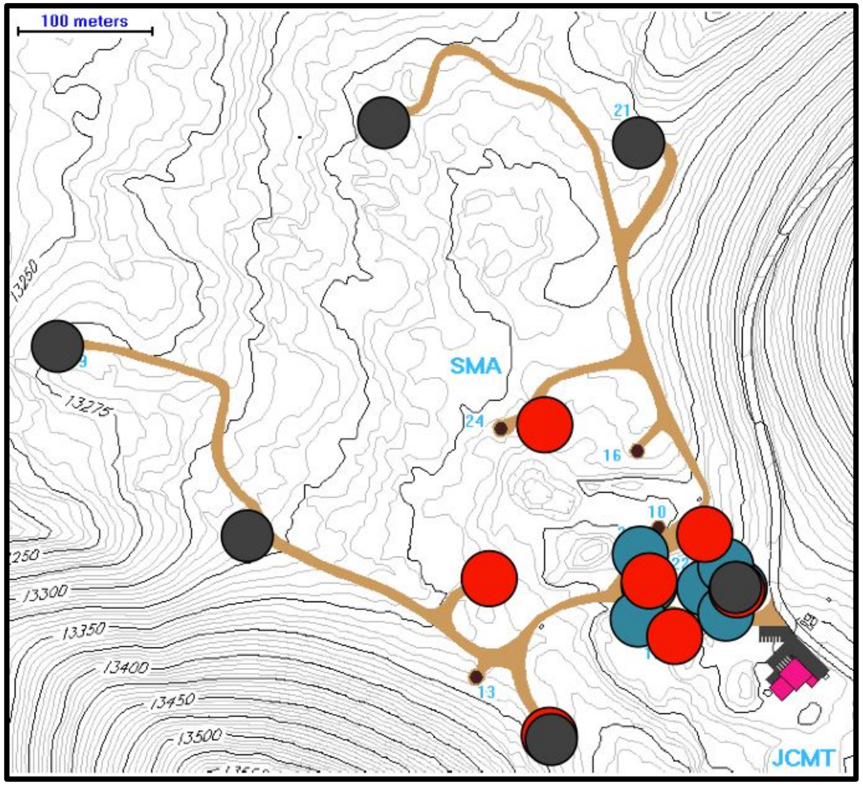



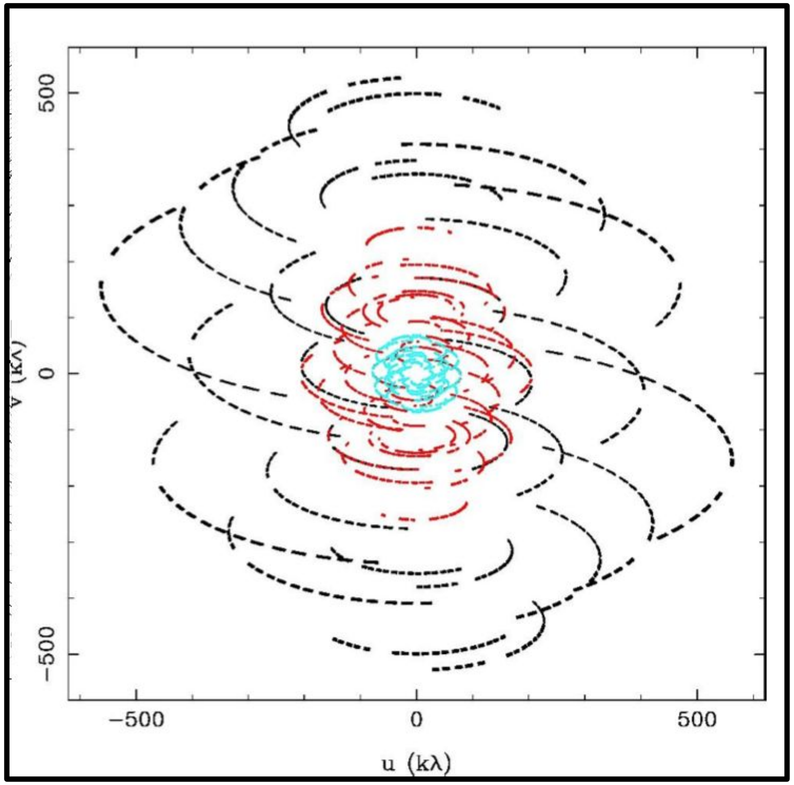

**Compact SMA configuration**

(compact baselines)

345 GHz, DEC = +22

# (u,v) Plane Sampling

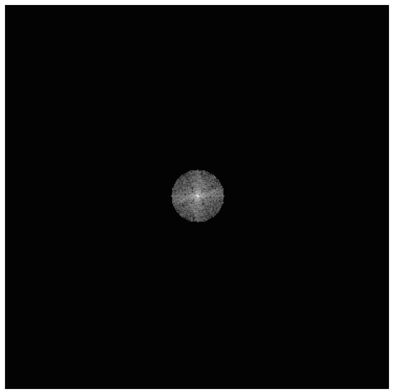
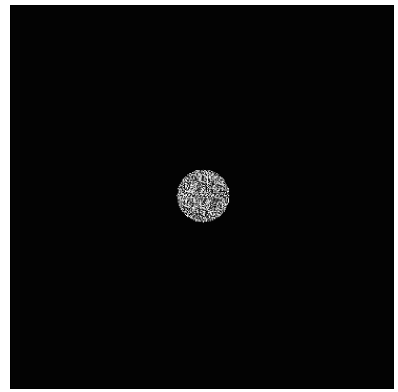
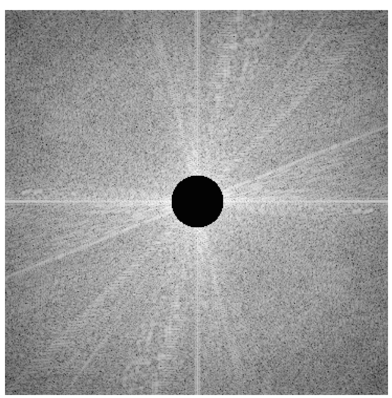
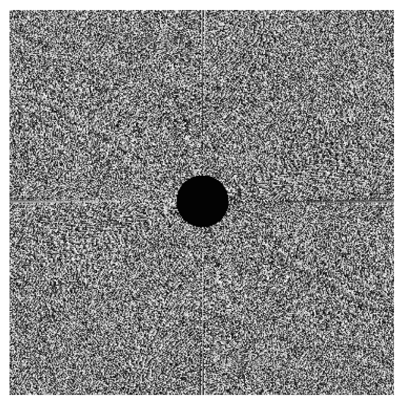



**Combine multiple configurations to get the most complete coverage of the (u,v) plane**

# Implications of (u,v) Coverage

**What does it mean if our (u,v) coverage is not complete?**

| | V(u,v) amplitude | V(u,v) phase | | T(x,y) |
|---|---|---|---|---|
| Missing High Spatial Frequencies |  |  | FT → |  |
| Missing Low Spatial Frequencies |  |  | FT → |  |

# Characteristic Angular Scales

**Angular resolution of telescope array:**

$\sim \lambda/B_{max}$ ($B_{max}$ = longest baseline)

**Maximum angular scale:**

$\sim \lambda/B_{min}$ ($B_{min}$ = shortest distance between antennas)

**Field of view (FOV):**

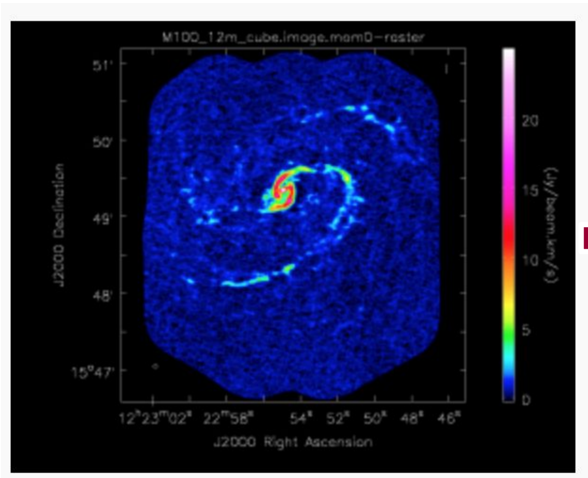
$\sim \lambda/D$ (D = antenna diameter)

*Sources more extended than the FOV can be observed using multiple pointing centers in a mosaic

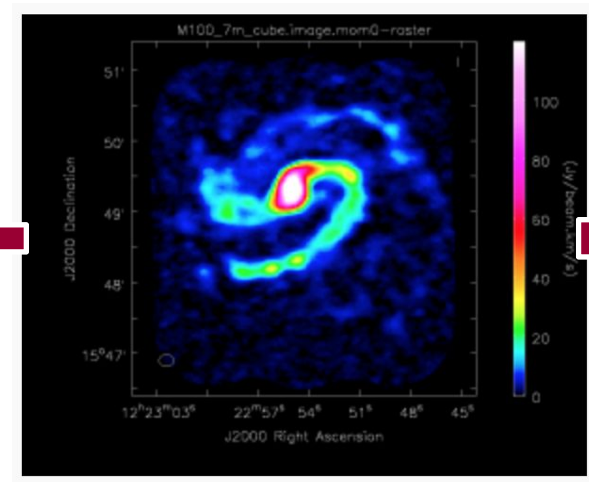
**An interferometer is sensitive to a range of angular sizes: $\lambda/B_{max} < \theta < \lambda/B_{min}$**

# Characteristic Angular Scales: M100

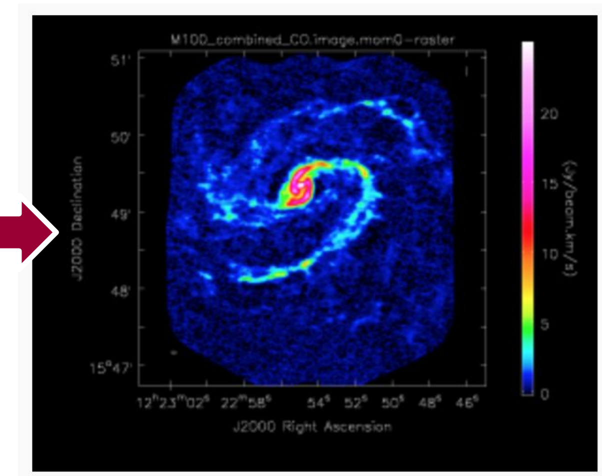
ALMA 12m




ACA 7m




Combined




ALMA 12m shows smaller spatial scales (denser, clumpier emission)
ACA 7m data shows larger spatial scales (diffuse, extended emission)

**To get both — you need a combined image!**

# Interferometry: Spatial Scales

- The **sensitivity** is given by the number of antennas times their area
- The **field of view** is given by the beam of a single antenna (corresponding to the resolution for a single dish telescope or the primary beam)
- The **resolution** is given by the largest distance between antennas (called the synthesized beam)
- The **largest angular scale** that can be imaged is given by the shortest distance between antennas

# Angular Scales — A Proposal Tip!

**Interferometers act as spatial filters - shorter baselines are sensitive to larger targets, so remember …**

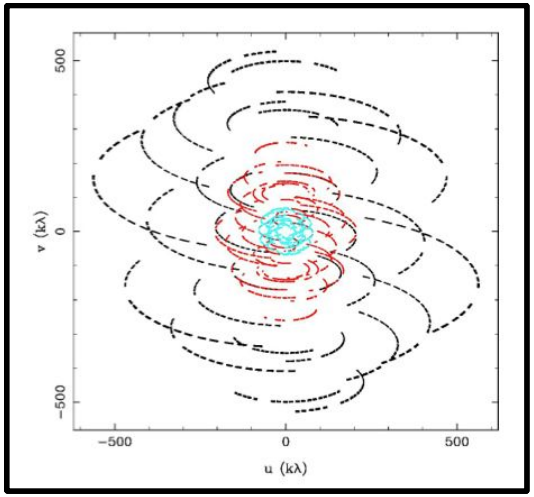
Spatial scales larger than the smallest baseline cannot be imaged ($\theta_{MRS}$)

Spatial scales smaller than the largest baseline cannot be resolved ($\theta_{res}$)

| | Band | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | Frequency (GHz) | 100 | 150 | 185 | 230 | 345 | 460 | 650 | 870 |
| Config. | | | | | | | | | |
| 7-m | $\theta_{res}$ (arcsec) | 12.5 | 8.35 | 6.77 | 5.45 | 3.63 | 2.72 | 1.93 | 1.44 |
| | $\theta_{MRS}$ (arcsec) | 66.7 | 44.5 | 36.1 | 29.0 | 19.3 | 14.5 | 10.3 | 7.67 |
| C-1 | $\theta_{res}$ (arcsec) | 3.38 | 2.25 | 1.83 | 1.47 | 0.98 | 0.735 | 0.52 | 0.389 |
| | $\theta_{MRS}$ (arcsec) | 28.5 | 19.0 | 15.4 | 12.4 | 8.25 | 6.19 | 4.38 | 3.27 |
| C-2 | $\theta_{res}$ (arcsec) | 2.30 | 1.53 | 1.24 | 0.999 | 0.666 | 0.499 | 0.353 | 0.264 |
| | $\theta_{MRS}$ (arcsec) | 22.6 | 15.0 | 12.2 | 9.81 | 6.54 | 4.9 | 3.47 | 2.59 |
| C-3 | $\theta_{res}$ (arcsec) | 1.42 | 0.943 | 0.765 | 0.615 | 0.41 | 0.308 | 0.218 | 0.163 |
| | $\theta_{MRS}$ (arcsec) | 16.2 | 10.8 | 8.73 | 7.02 | 4.68 | 3.51 | 2.48 | 1.86 |
| C-4 | $\theta_{res}$ (arcsec) | 0.918 | 0.612 | 0.496 | 0.399 | 0.266 | 0.2 | 0.141 | 0.106 |
| | $\theta_{MRS}$ (arcsec) | 11.2 | 7.5 | 6.08 | 4.89 | 3.26 | 2.44 | 1.73 | 1.29 |
| C-5 | $\theta_{res}$ (arcsec) | 0.545 | 0.363 | 0.295 | 0.237 | 0.158 | 0.118 | 0.0838 | 0.0626 |
| | $\theta_{MRS}$ (arcsec) | 6.7 | 4.47 | 3.62 | 2.91 | 1.94 | 1.46 | 1.03 | 0.77 |
| C-6 | $\theta_{res}$ (arcsec) | 0.306 | 0.204 | 0.165 | 0.133 | 0.0887 | 0.0665 | 0.0471 | 0.0352 |
| | $\theta_{MRS}$ (arcsec) | 4.11 | 2.74 | 2.22 | 1.78 | 1.19 | 0.892 | 0.632 | 0.472 |
| C-7 | $\theta_{res}$ (arcsec) | 0.211 | 0.141 | 0.114 | 0.0917 | 0.0612 | 0.0459 | 0.0325 | 0.0243 |
| | $\theta_{MRS}$ (arcsec) | 2.58 | 1.72 | 1.4 | 1.12 | 0.749 | 0.562 | 0.398 | 0.297 |
| C-8 | $\theta_{res}$ (arcsec) | 0.096 | 0.064 | 0.0519 | 0.0417 | 0.0278 | - | - | - |
| | $\theta_{MRS}$ (arcsec) | 1.42 | 0.947 | 0.768 | 0.618 | 0.412 | - | - | - |
| C-9 | $\theta_{res}$ (arcsec) | 0.057 | 0.038 | 0.0308 | 0.0248 | 0.0165 | - | - | - |
| | $\theta_{MRS}$ (arcsec) | 0.814 | 0.543 | 0.44 | 0.354 | 0.236 | - | - | - |
| C-10 | $\theta_{res}$ (arcsec) | 0.042 | 0.028 | 0.0227 | 0.0183 | 0.0122 | - | - | - |
| | $\theta_{MRS}$ (arcsec) | 0.496 | 0.331 | 0.268 | 0.216 | 0.144 | - | - | - |

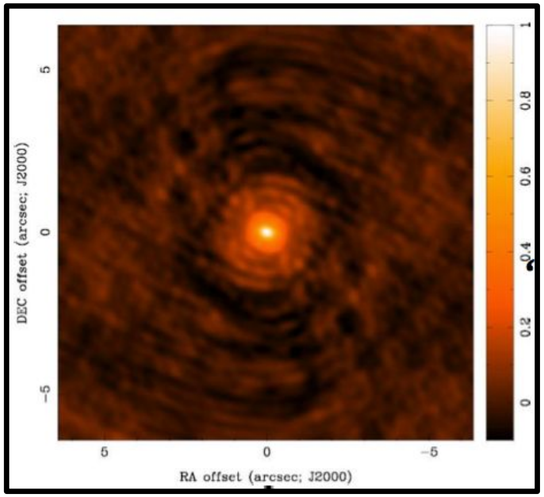
# The Dirty Beam

**S(u,v)**

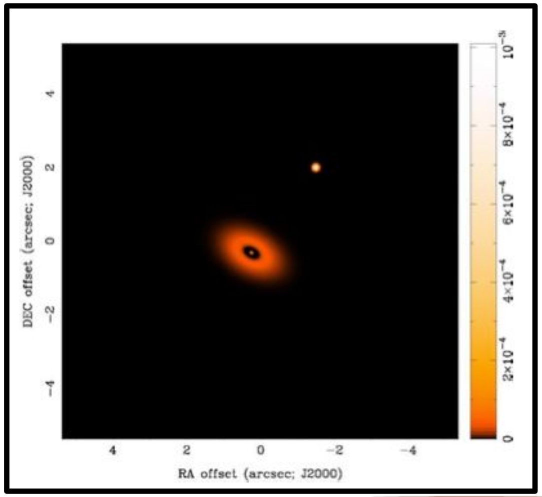



**FT**

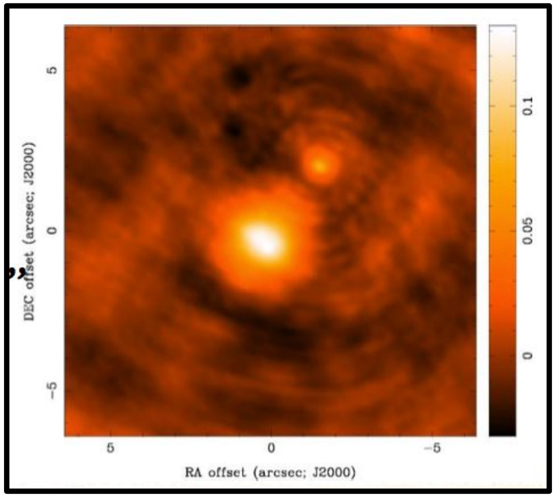
**s(x,y)**
**“Dirty Beam”**




*** (Convolution)**

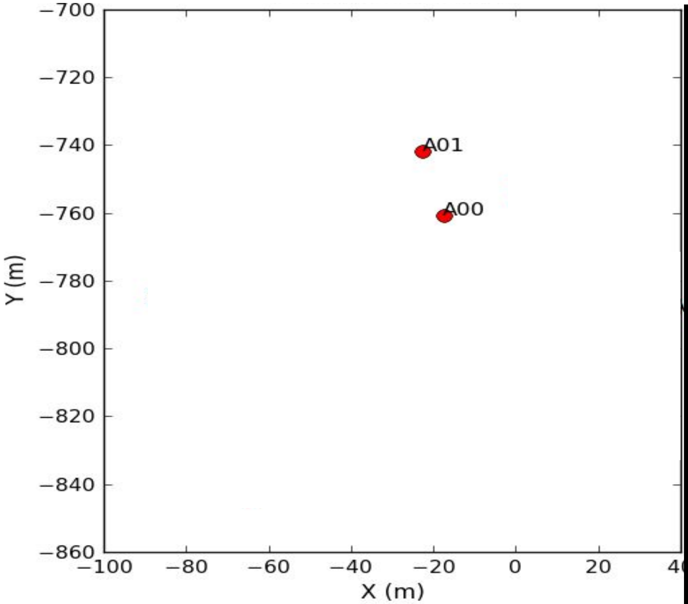


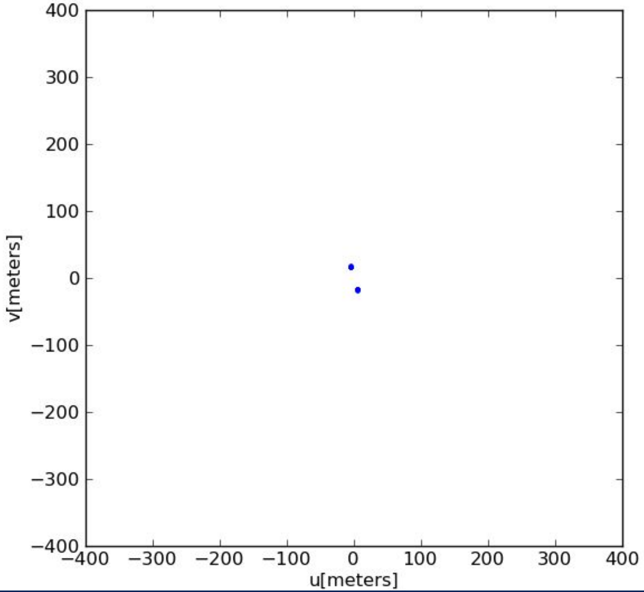

**T(x,y)**

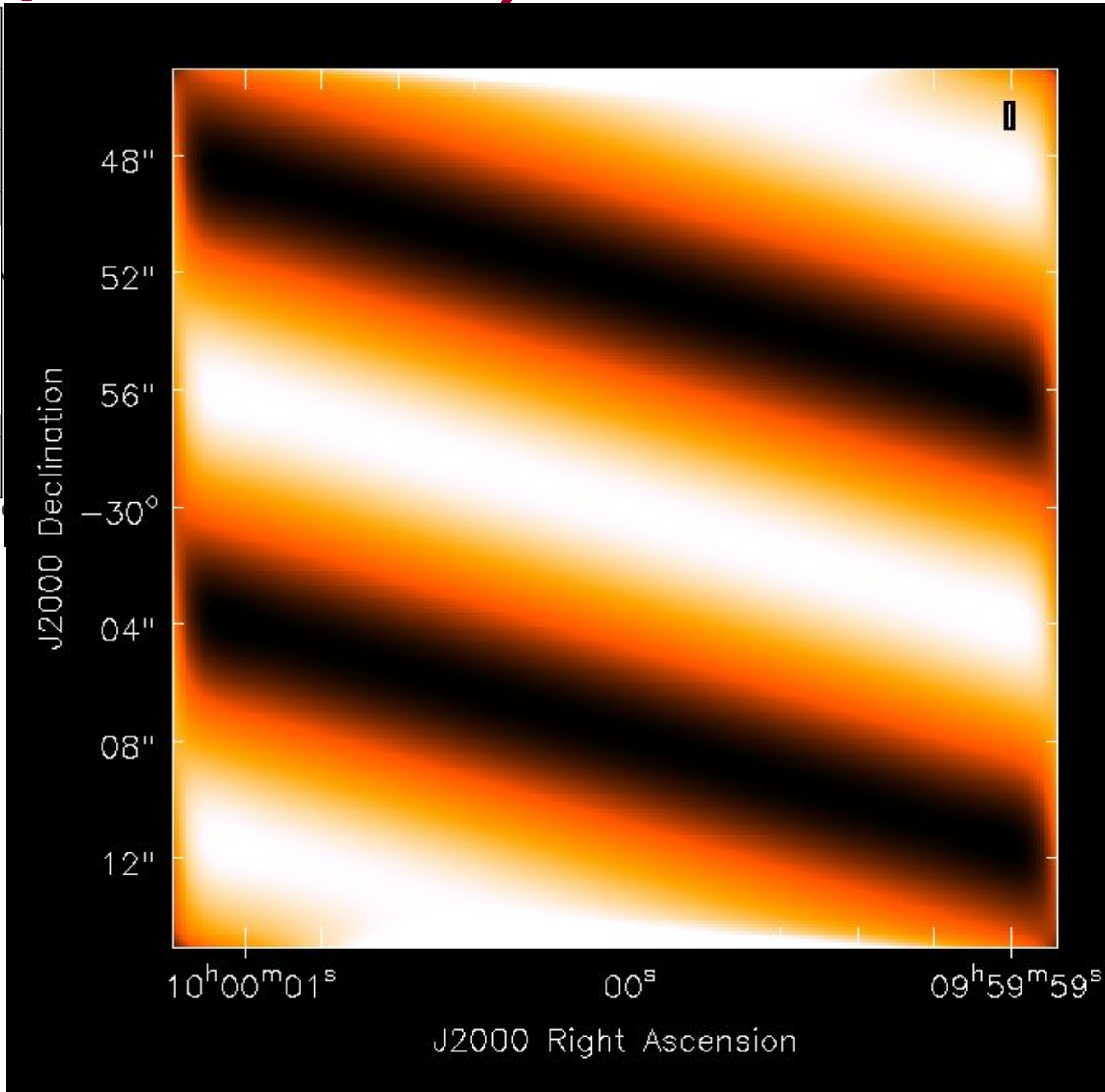



**$T_D(x,y)$**
**“Dirty Image”**

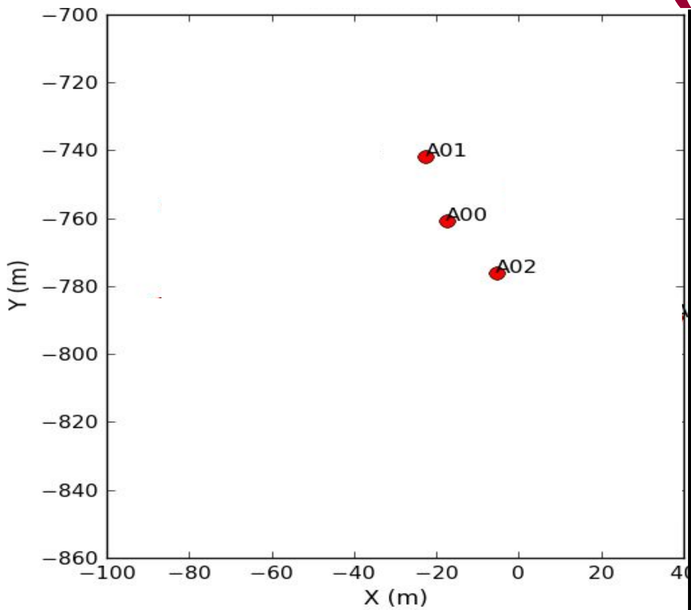
# Example: Fringe pattern with 2 Antennas (one baseline)

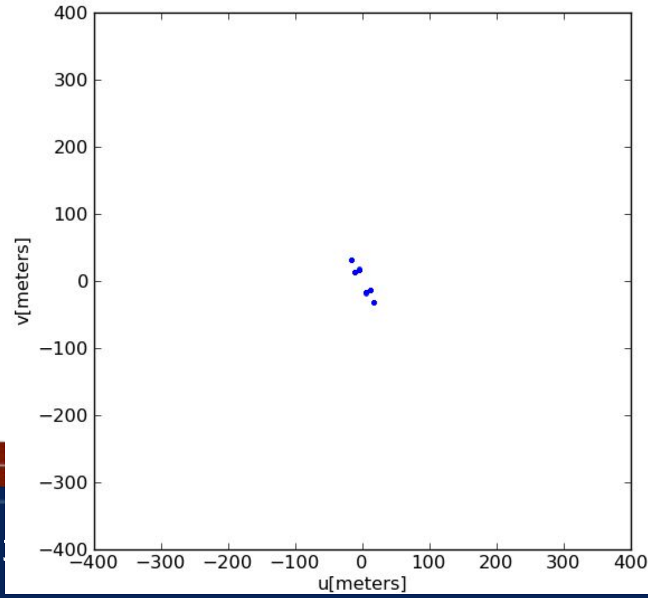


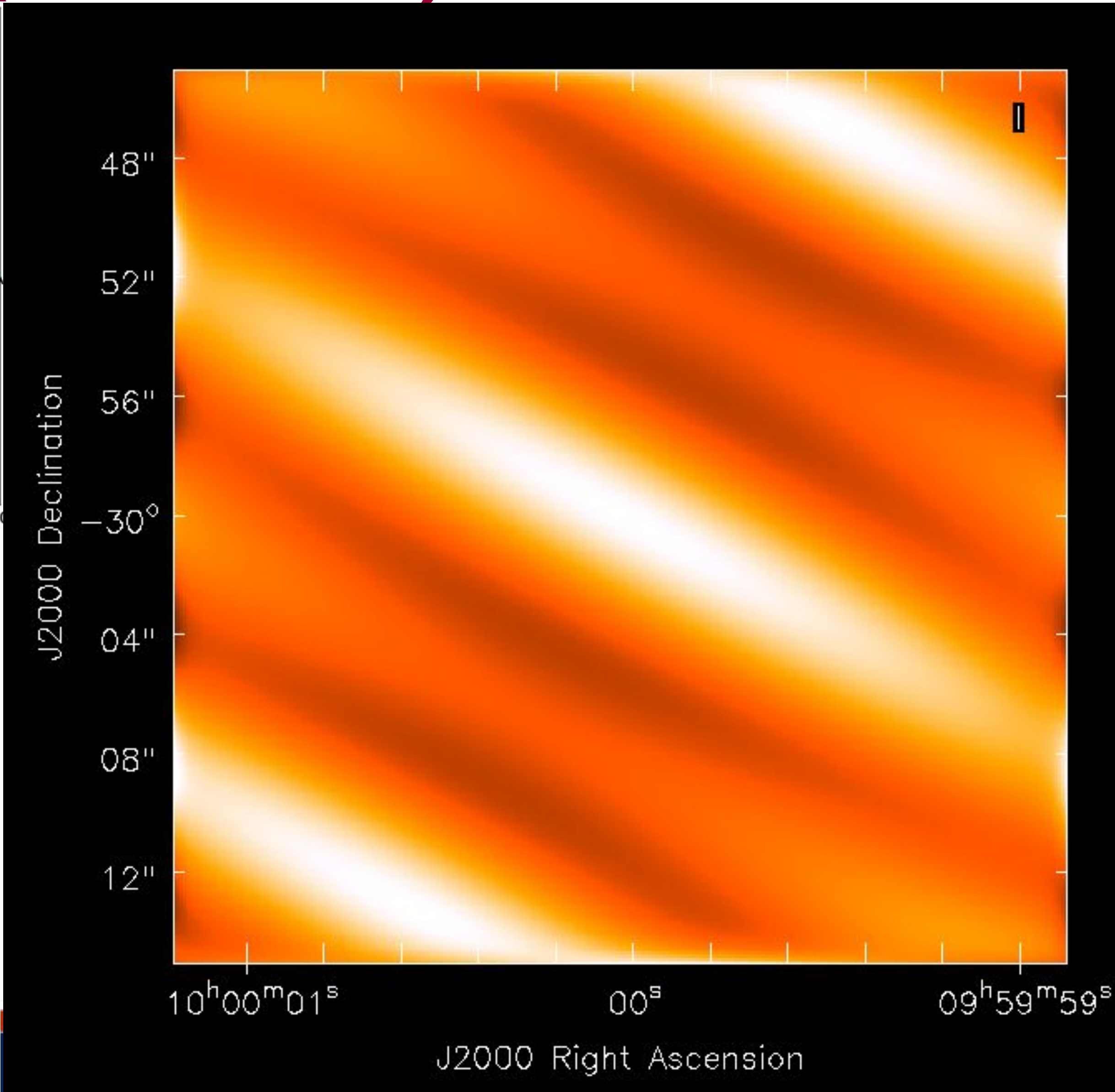



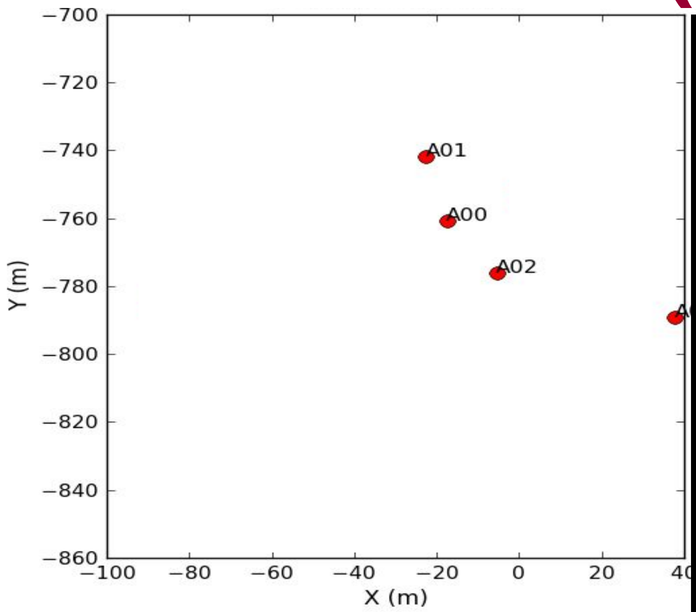
# Example: Fringe pattern with 3 Antennas (3 baselines)

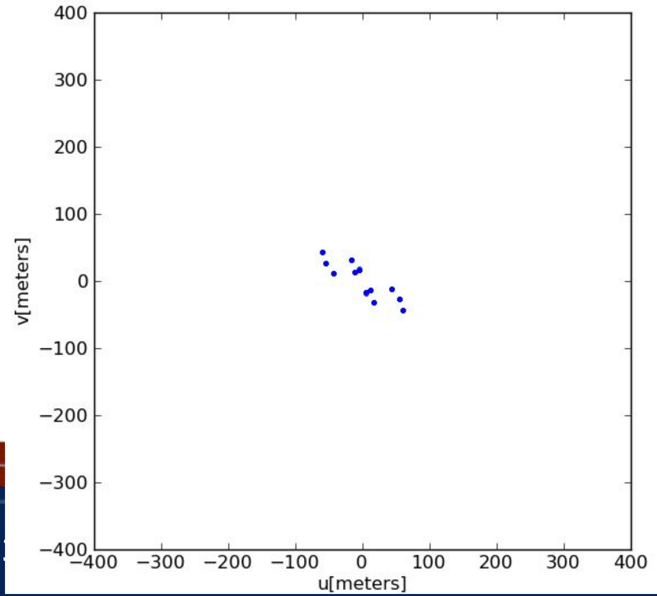


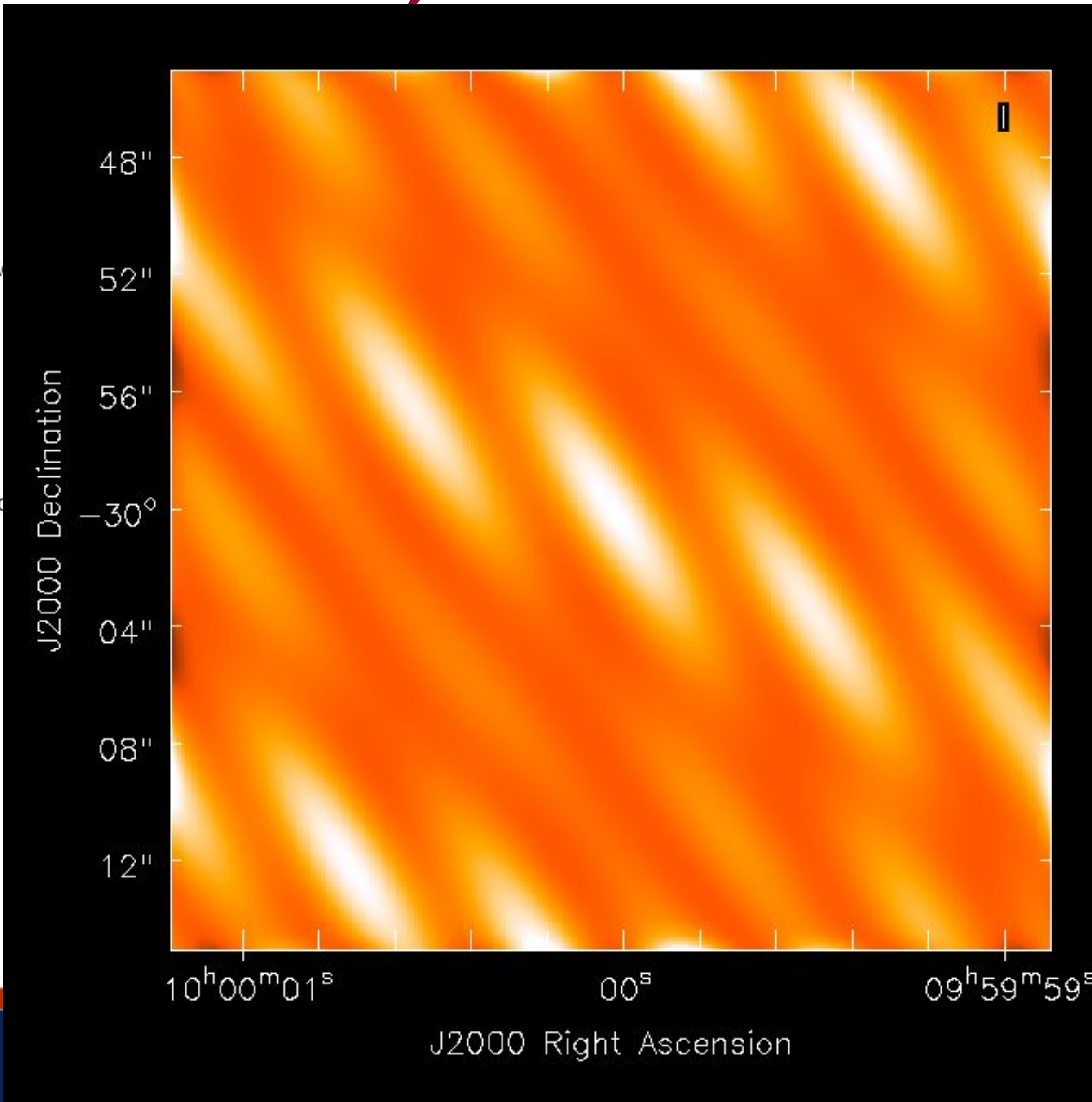



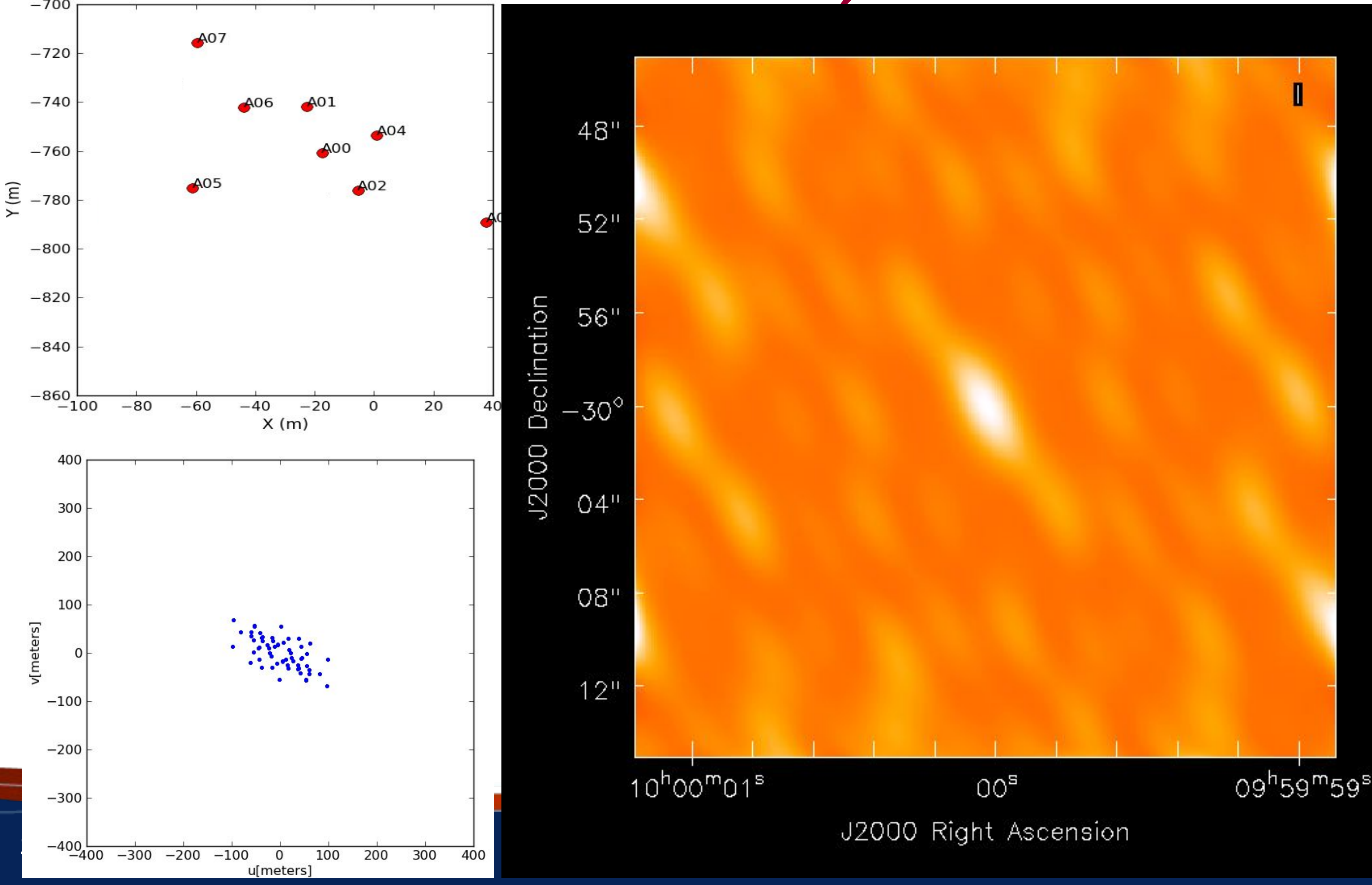
# Example: Fringe pattern with 4 Antennas (6 baselines)



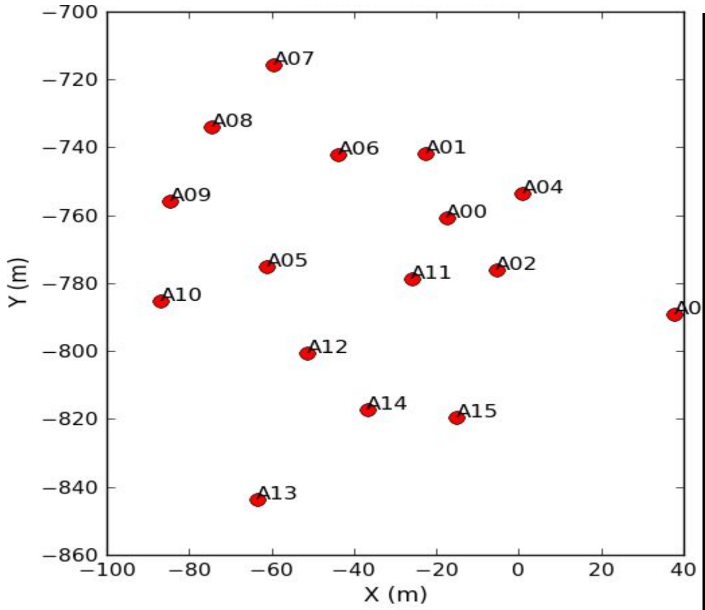



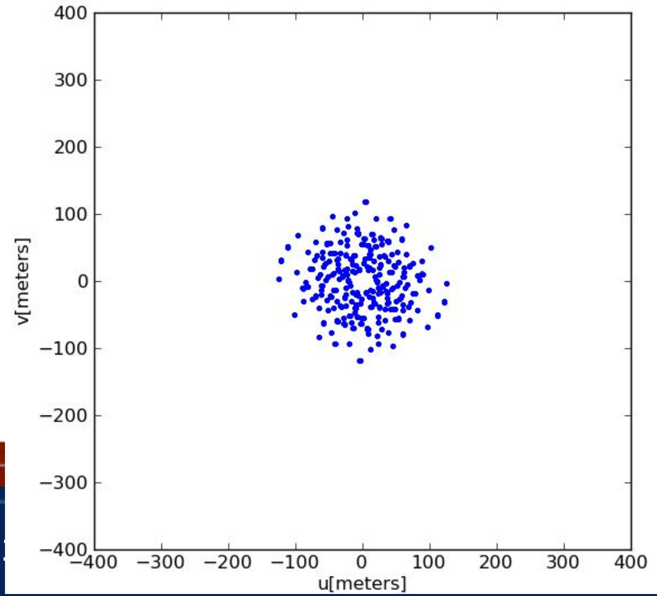



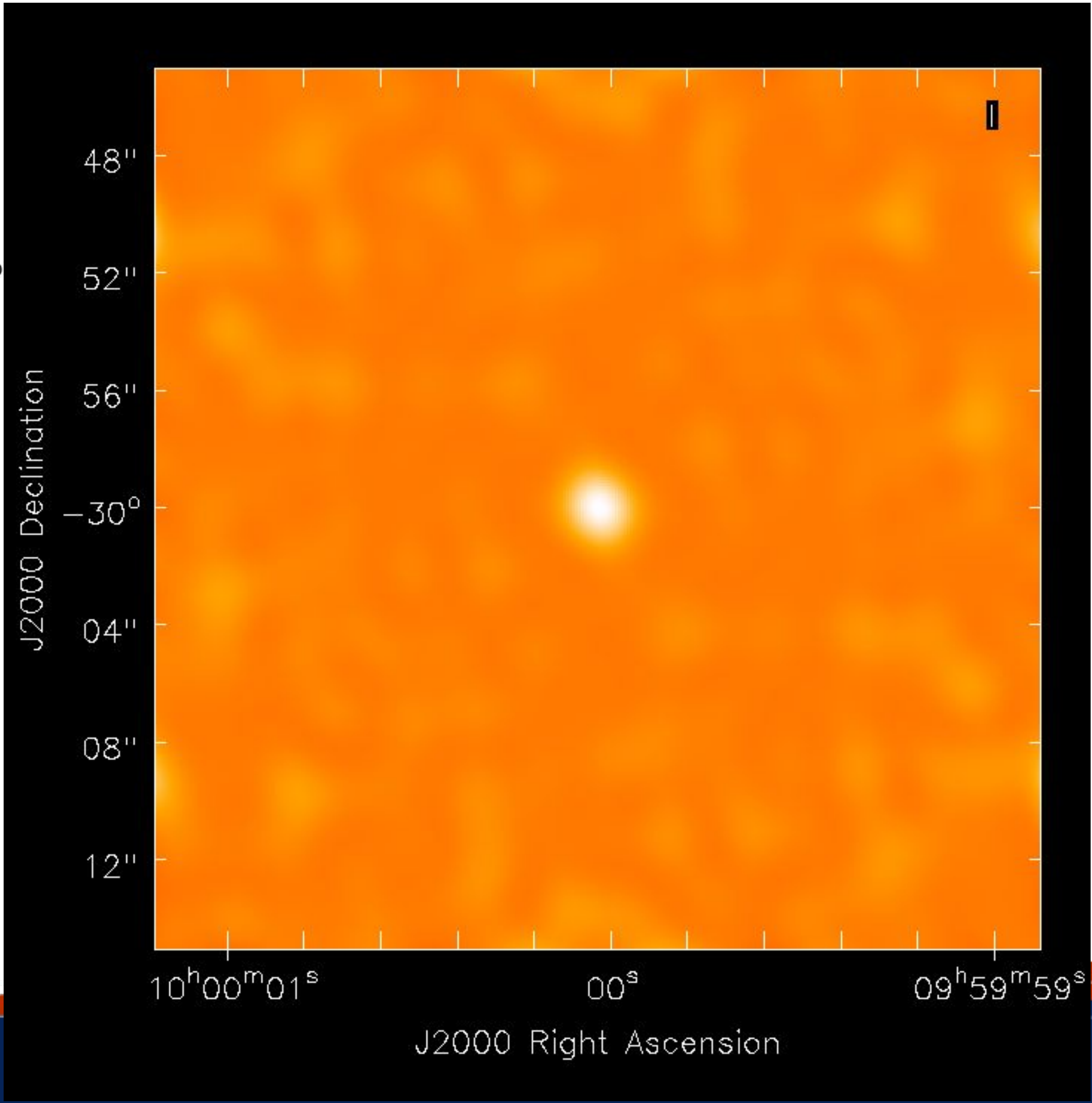
# Example: Fringe pattern with 8 Antennas (28 baselines)



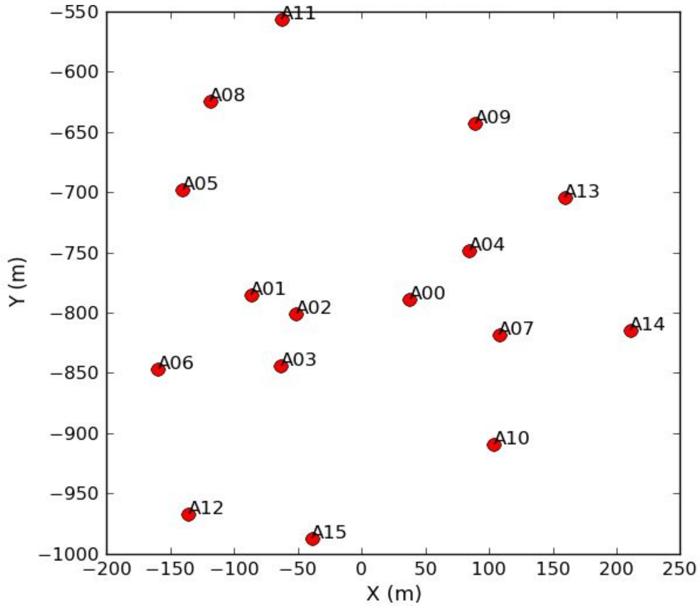
# 16 Antennas – Compact Configuration

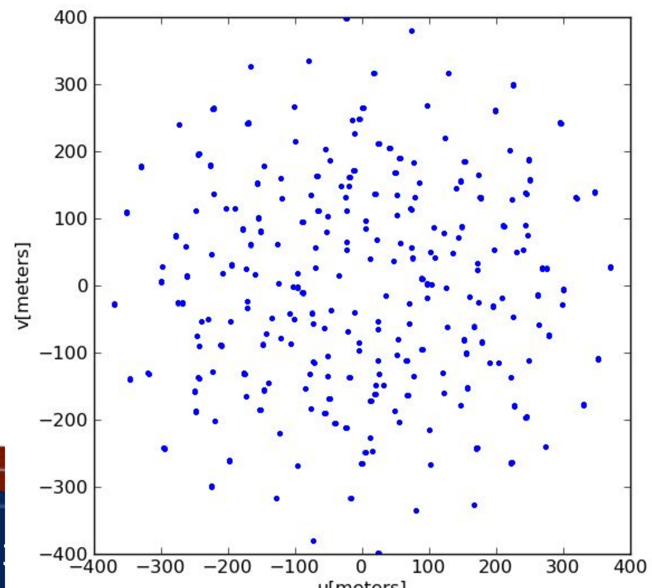


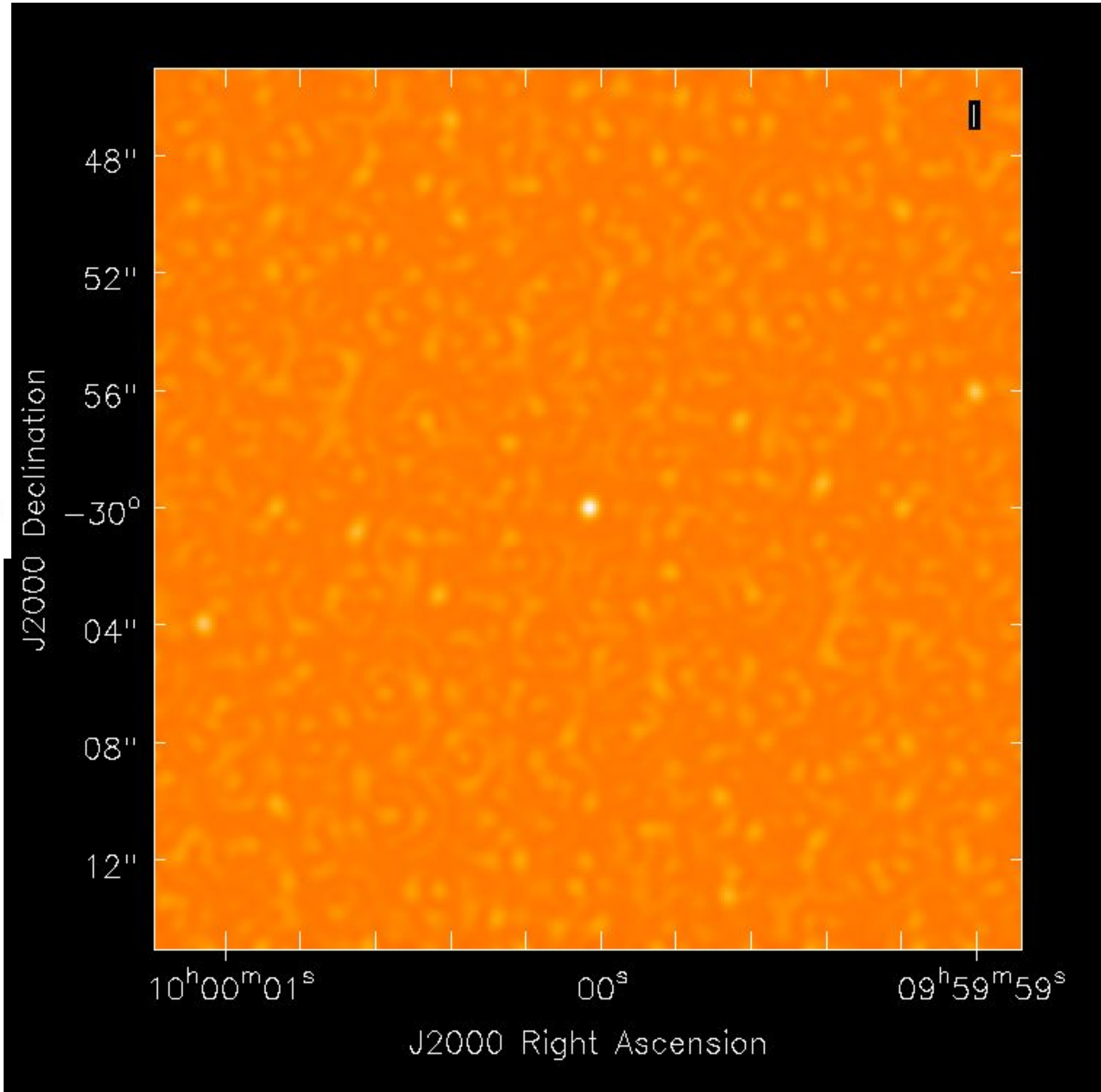



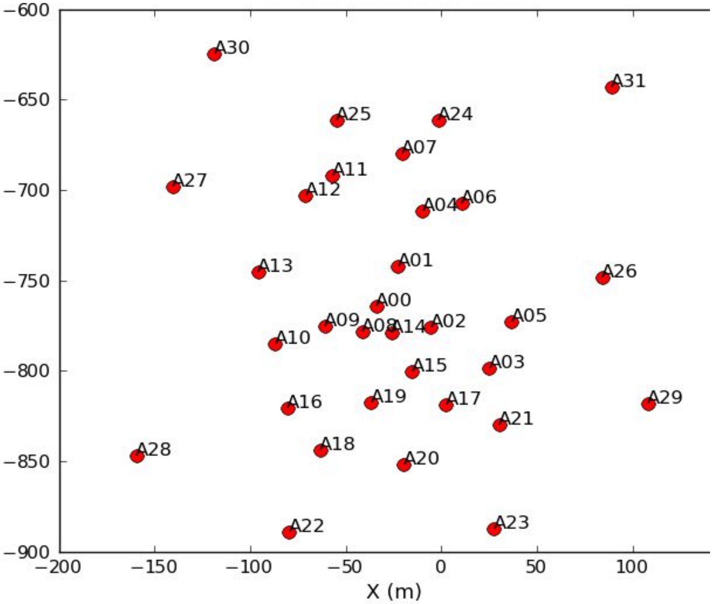
# 16 Antennas – Extended Configuration

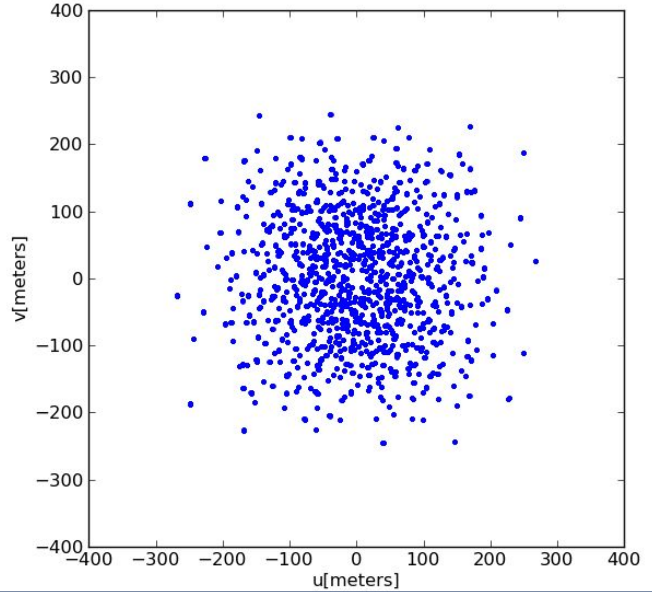


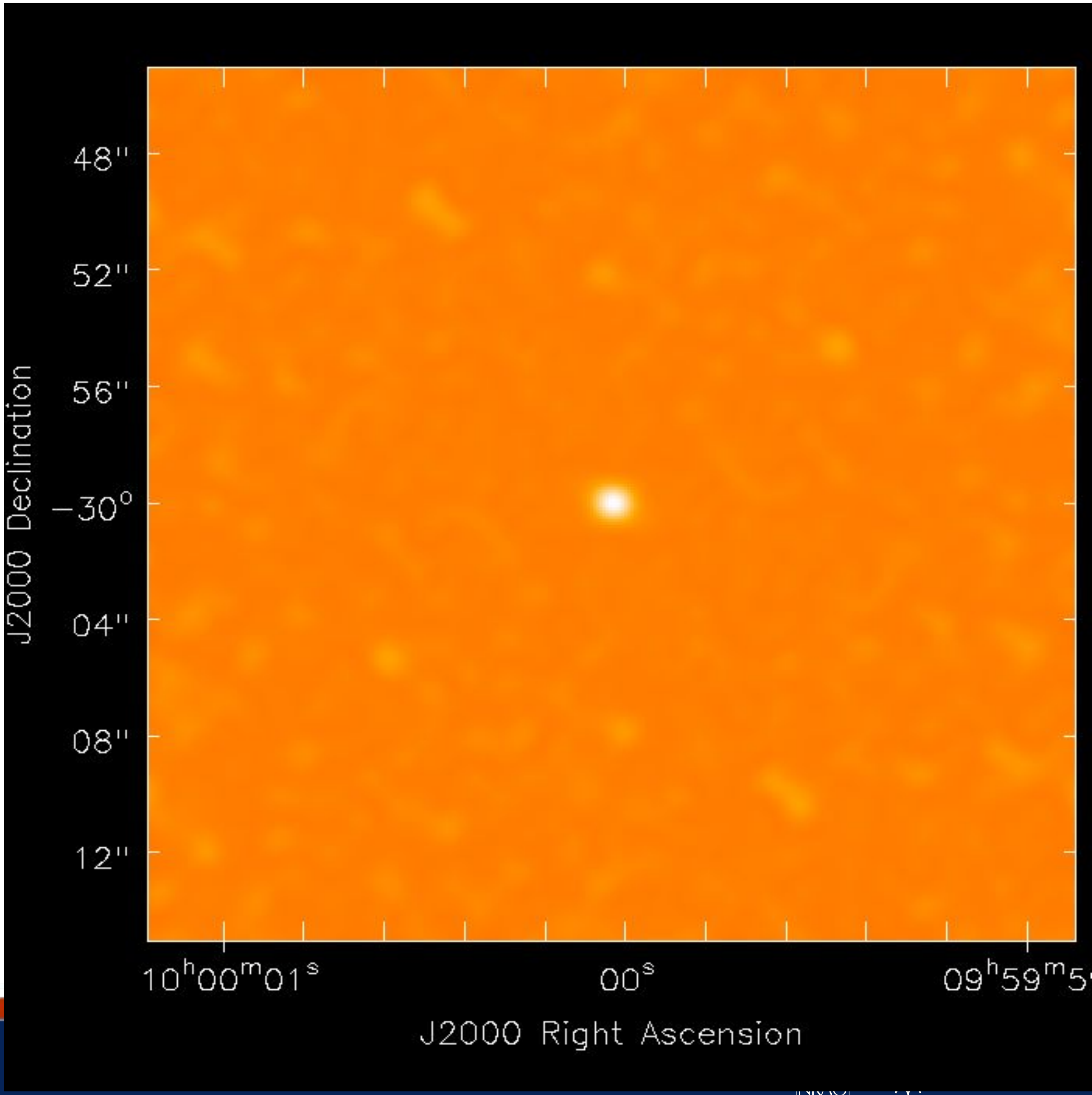



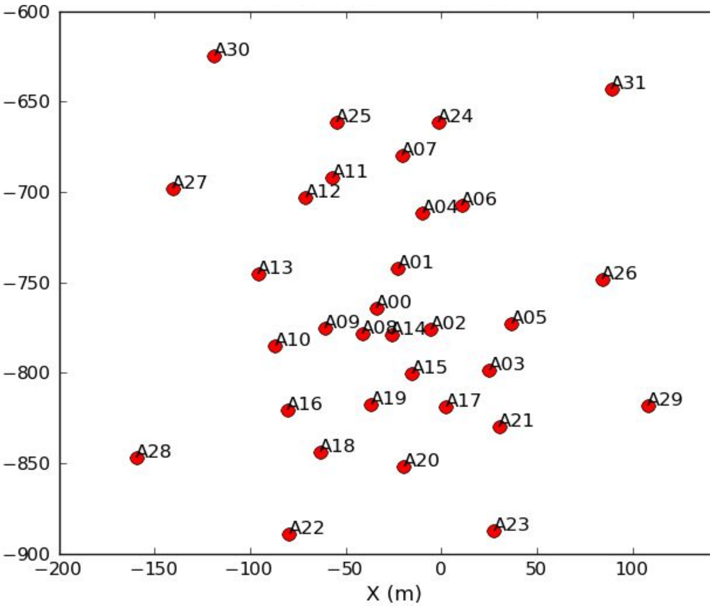
# 32 Antennas – Instantaneous

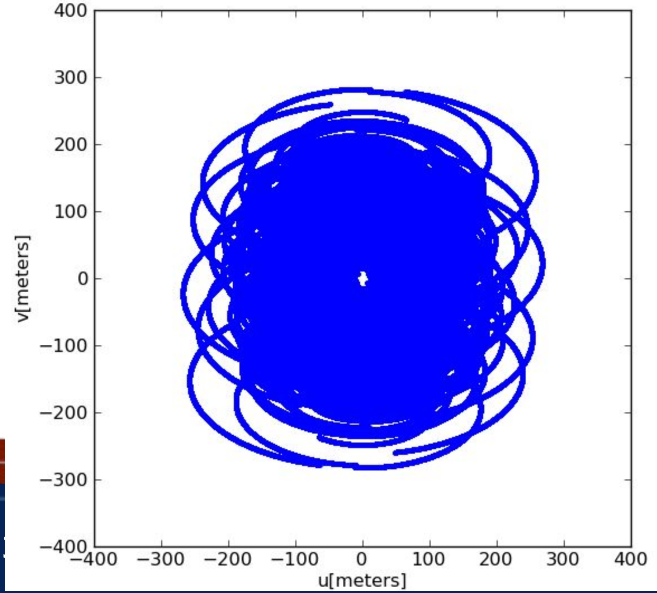


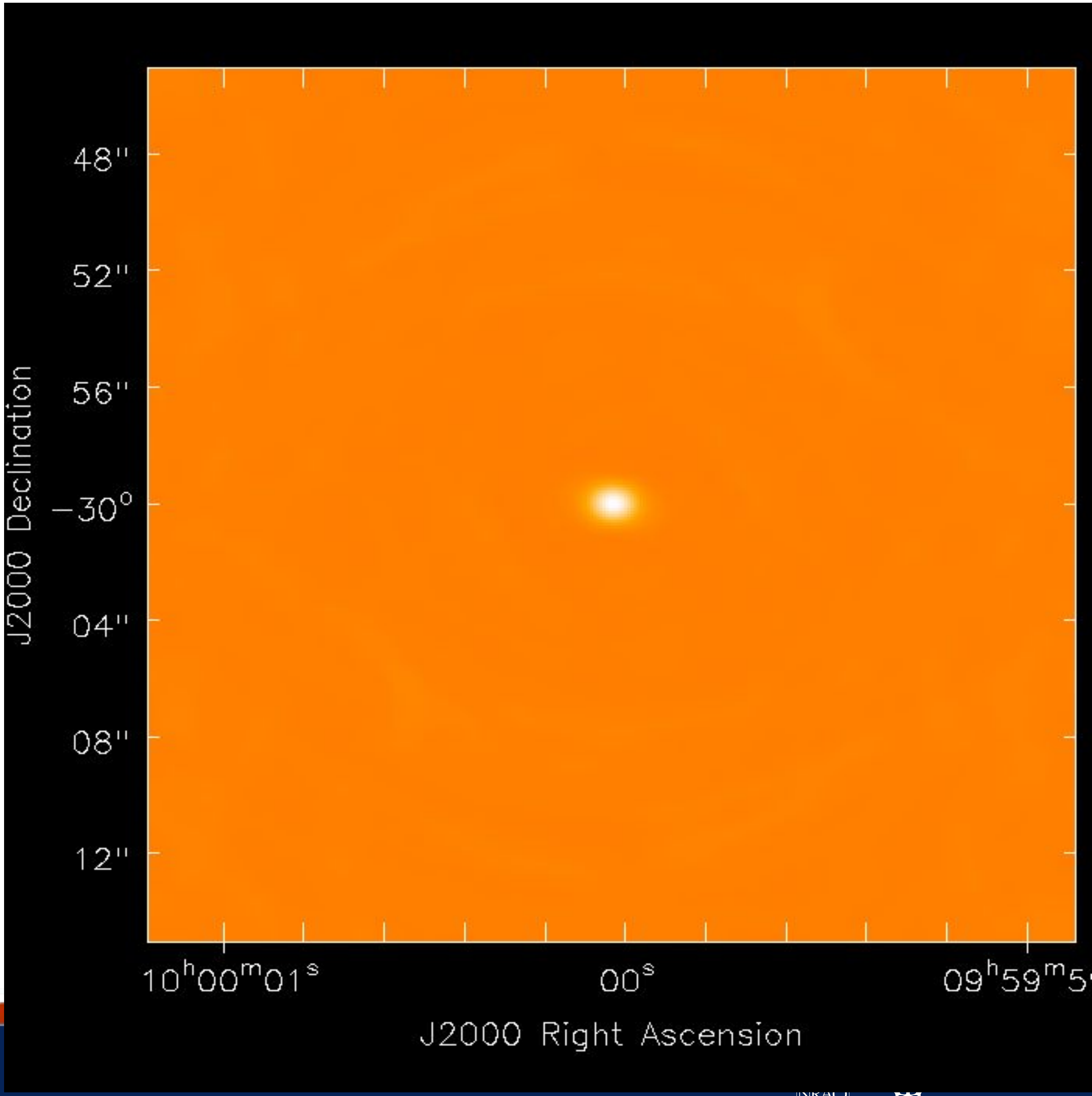



# 32 Antennas – 8 hours

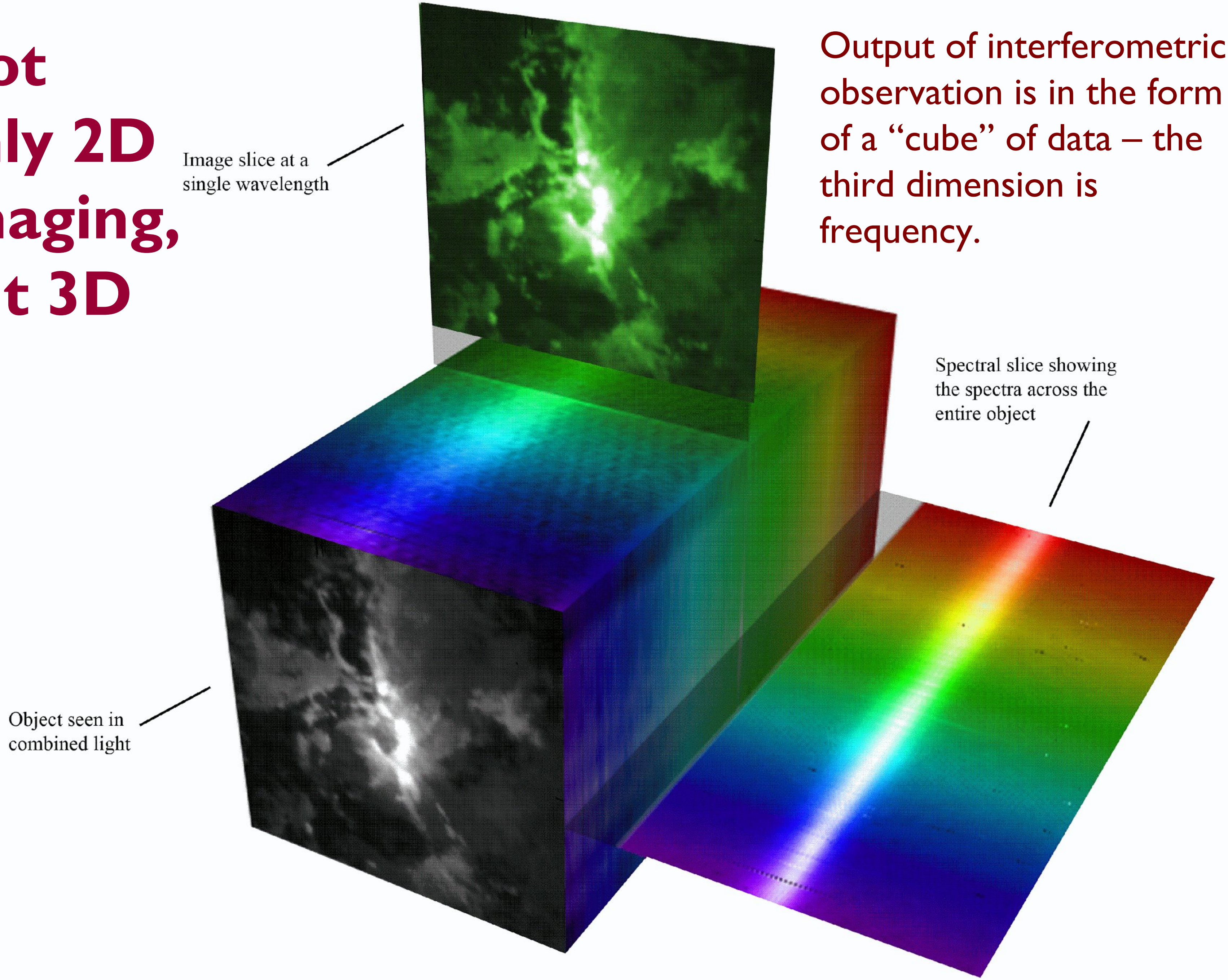


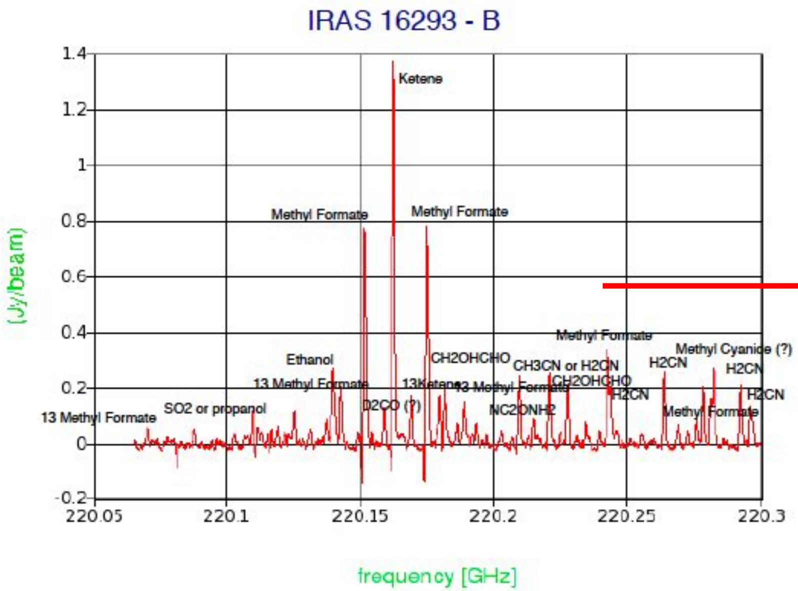
# Not only 2D imaging, but 3D

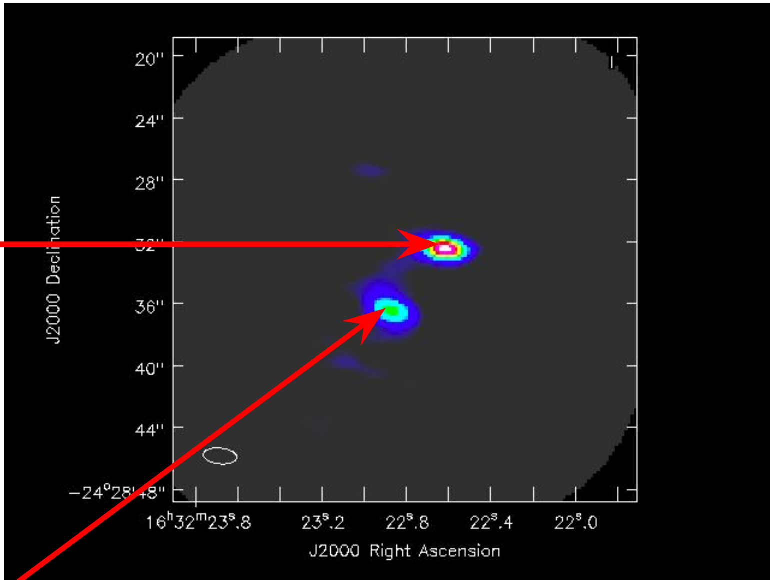
Output of interferometric observation is in the form of a "cube" of data – the third dimension is frequency.



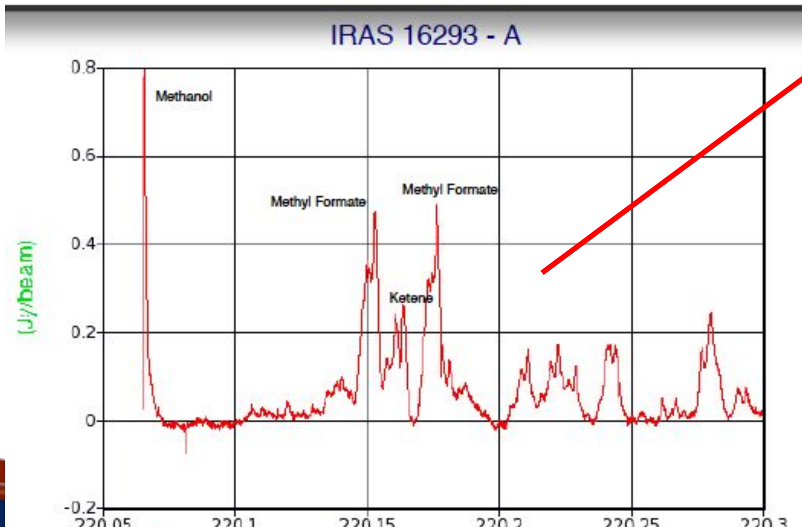
# Sometimes the most interesting science lies in the third dimension

Band 6




J. Turner & ALMA CSV team

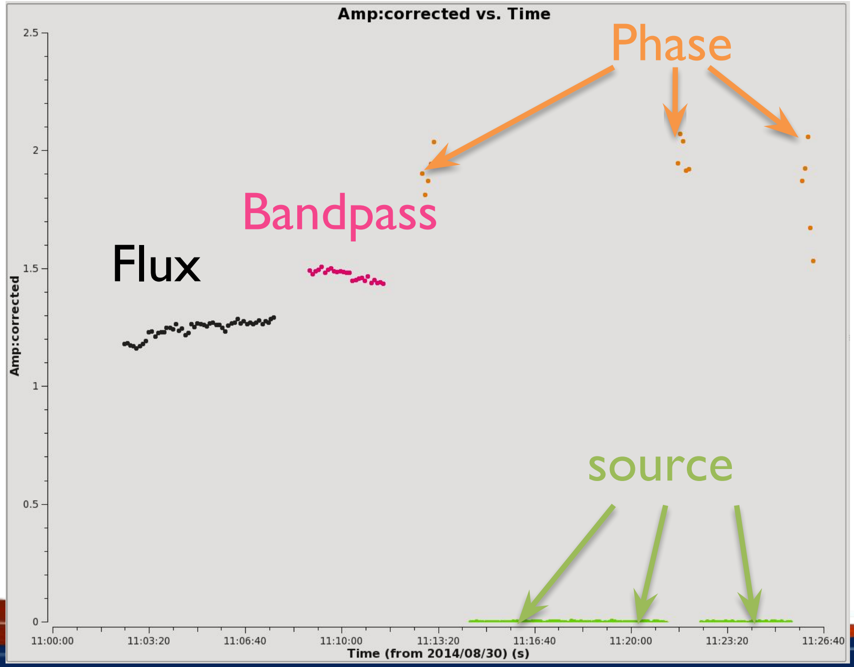


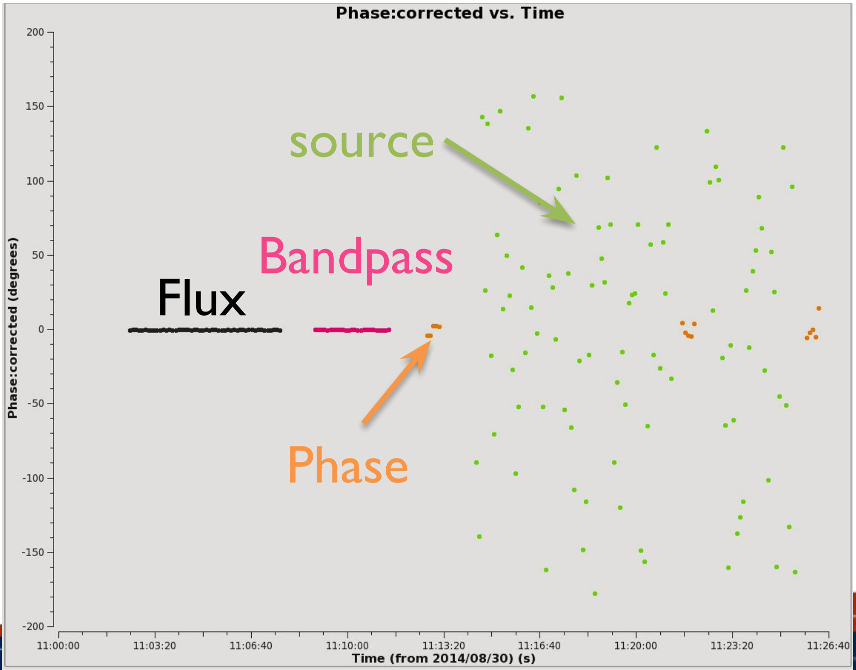

## Young Low Mass Stars: IRAS16293

- Note narrow lines toward preprotostellar core B (top) with infall apparent in methyl formate and ketene lines.

# A Brief Word on Calibration

- Interferometers measure visibilities, i.e., the amplitude and phase of the cross-correlated signals between pairs of antennas, as a function of time and frequency.
- We calibrate these data by determining the complex gains (amplitude and phase), the frequency response (bandpass) and flux scale for each antenna.



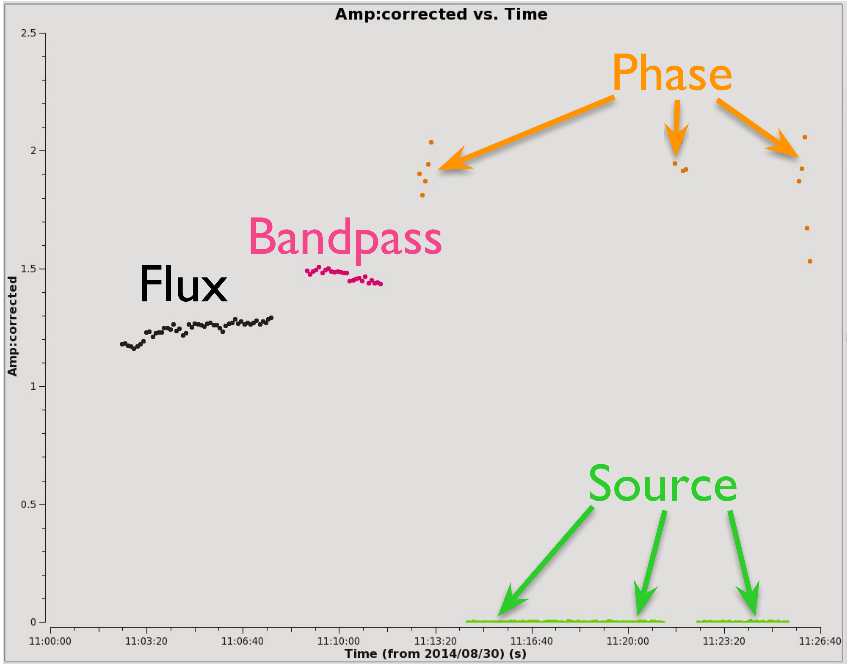
# A Brief Word on Calibration

**Calibration requirements (Handled by ALMA):**

**Gain calibrator**
Bright quasar near science target
Solves for atmospheric and instrumental variations with time

**Bandpass calibrator**
Bright quasar
Fixes instrumental effects and variations vs frequency

**Absolute flux calibrator**
Solar system object or quasar
Used to scale relative amplitudes to absolute value



# Calibration Process

Calibration is the effort to measure and remove the time-dependent and frequency-dependent atmospheric and instrumental variations.

Steps in calibrating interferometric data:
(Note: You don't have to worry about these in your observational set up!)

- Bandpass calibration (correct frequency-dependent telescope response)
- Phase and amplitude gain calibration (remove effects of atmospheric water vapor and correct time-varying phases/amplitudes)
- Set absolute flux scale

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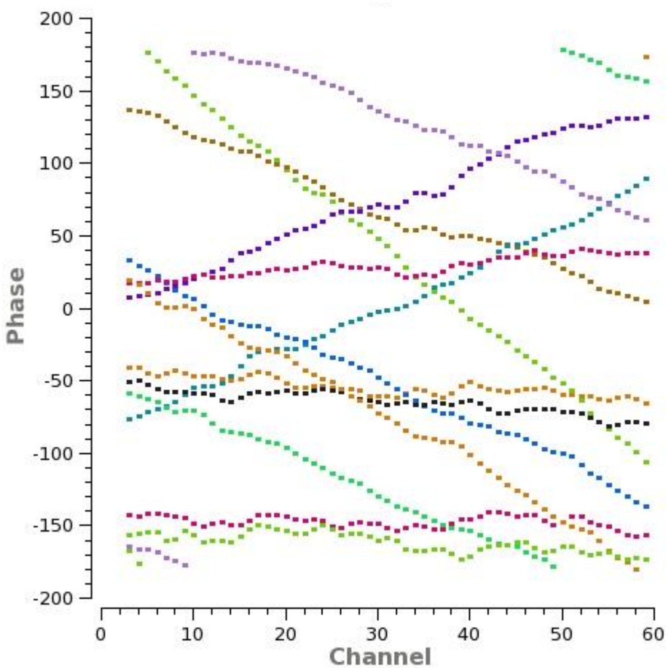
# Additional Slides

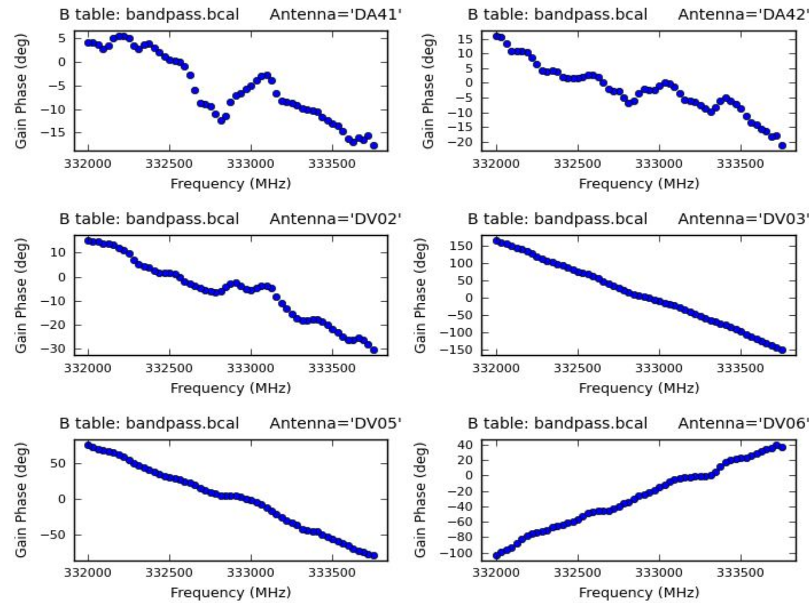
# Bandpass Calibration: Phase

* Analogous to optical "flat fielding" + bias subtraction for each antenna.
* Primarily correcting for frequency dependent telescope response (i.e. in the correlator/spectral windows)
* Done once in an SB, uses bright point sources like quasars
* Typically, baseline responses are inverted to antenna-based correction

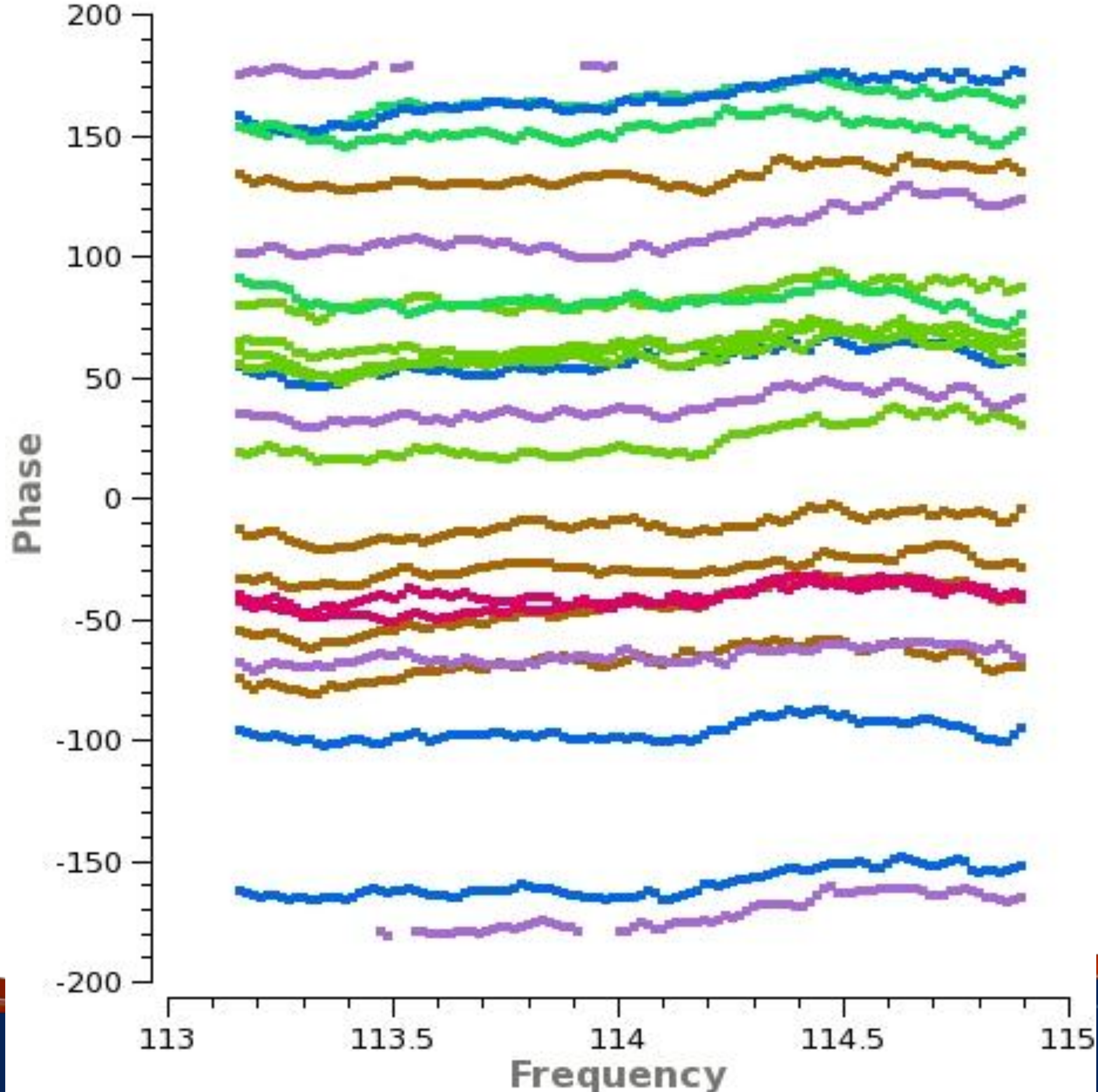
Baselines to one antenna



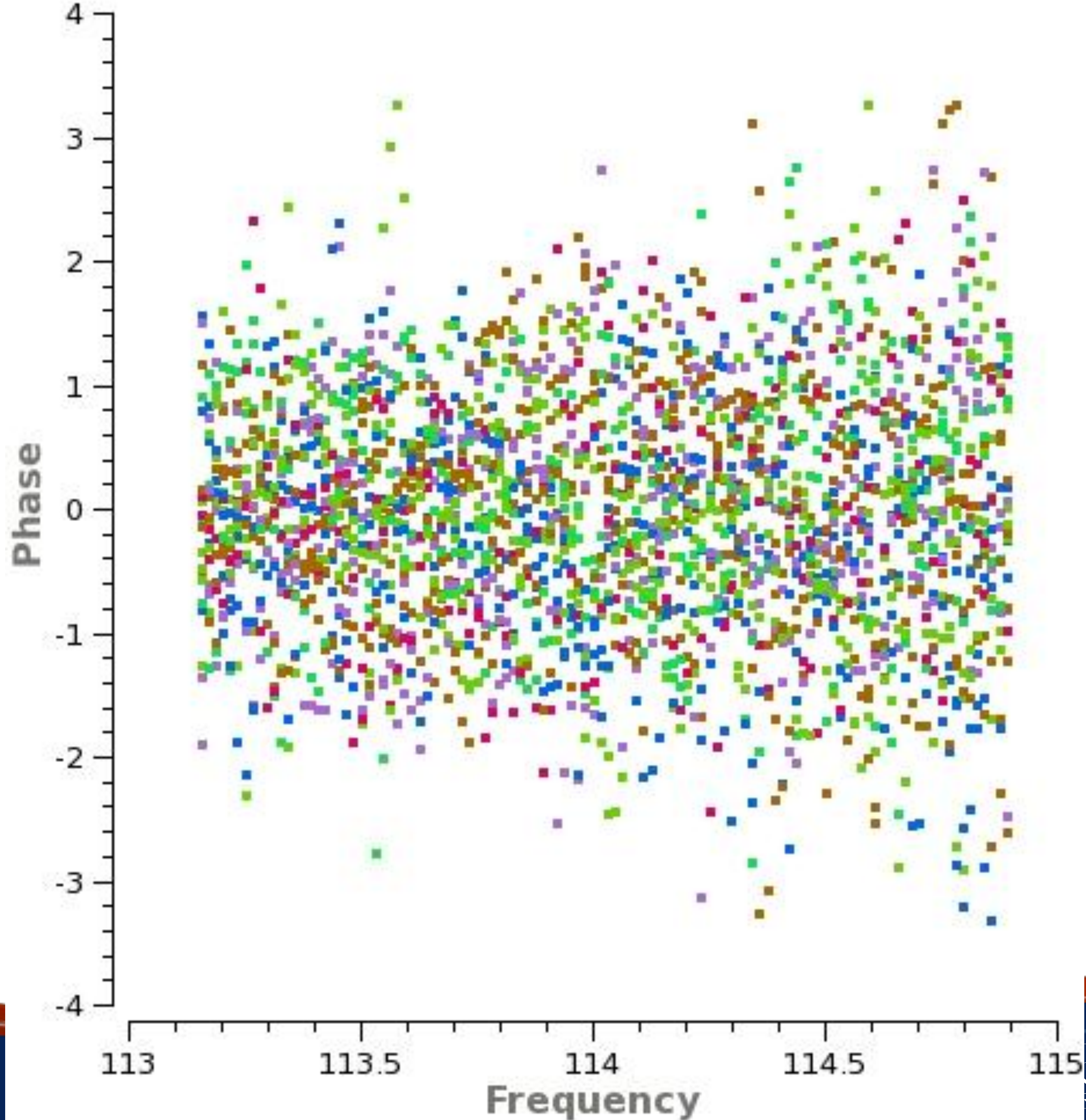

Antenna-based Bandpass Solutions



# Bandpass Phase vs. Frequency (Before)

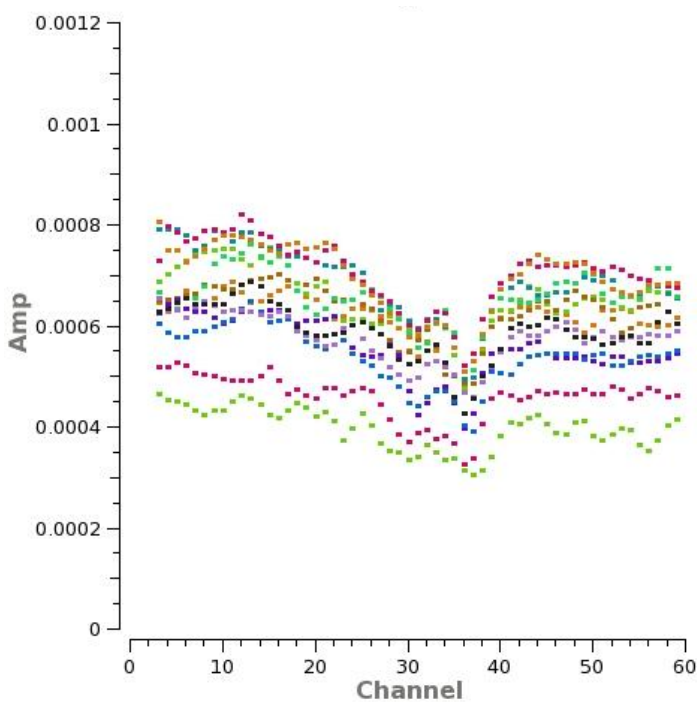


# Bandpass Phase vs. Frequency (After)

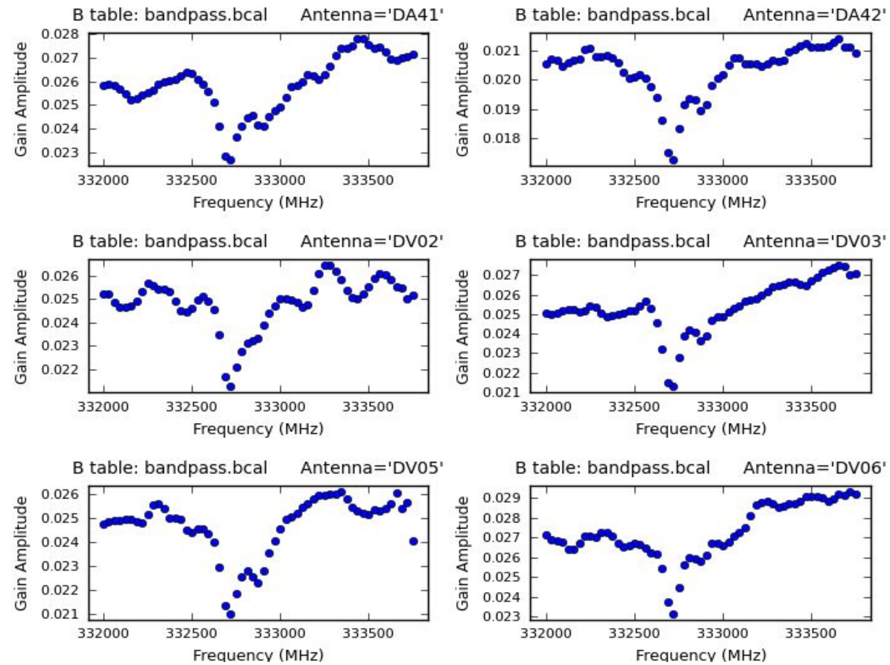


# Bandpass Calibration: Amplitude

Baselines to one antenna




Amplitude Before Bandpass Calibration

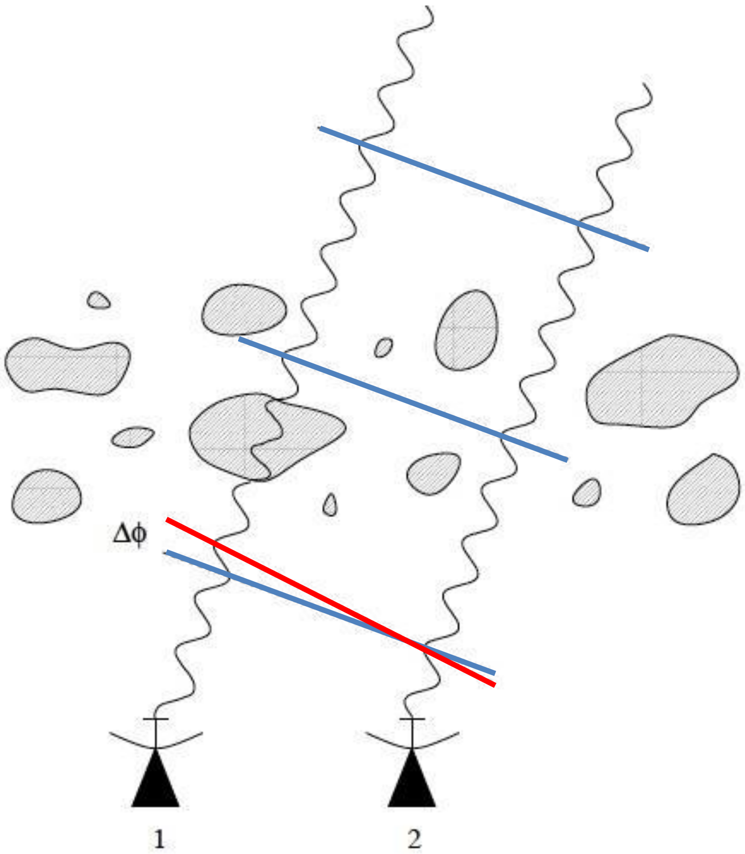



Bandpass solutions for individual antennas

# Atmospheric Phase Correction

- Variations in the amount of precipitable water vapor cause phase fluctuations that result in:
  - Low coherence (loss of sensitivity)
  - Radio "seeing" of 1arcsec at 1mm
  - Anomalous pointing offsets
  - Anomalous delay offsets

Patches of air with different water vapor content (and hence index of refraction) affect the incoming wave front differently.



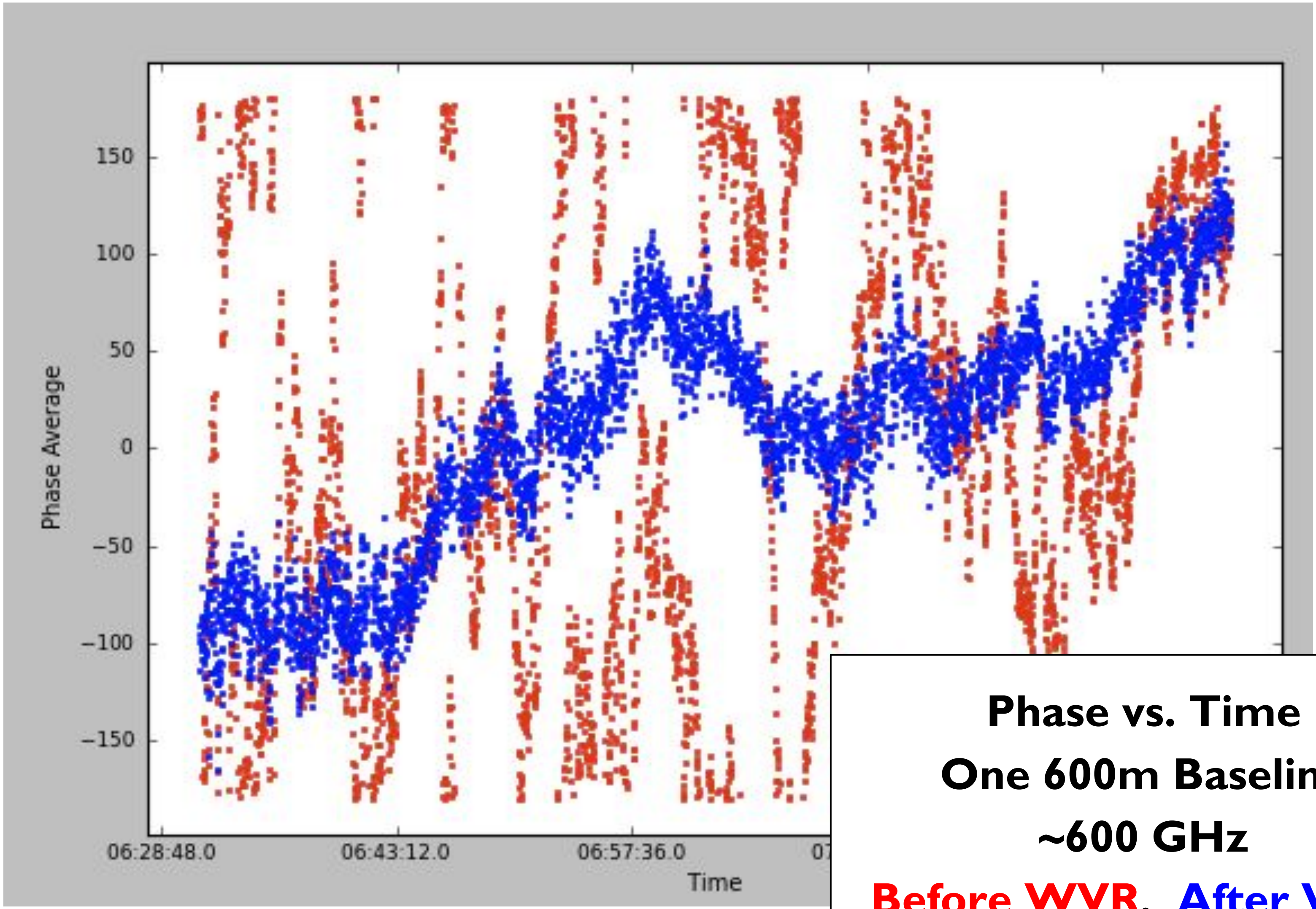
# Phase & Amplitude Gain Calibration

Determines the variations of phase and amplitude over time

- First pass is atmospheric correction from Water Vapor Radiometers readings

- Final correction from gain calibrator (point source near to target) that is observed every few minutes throughout the observation (analogous to repeat trips to a standard star)

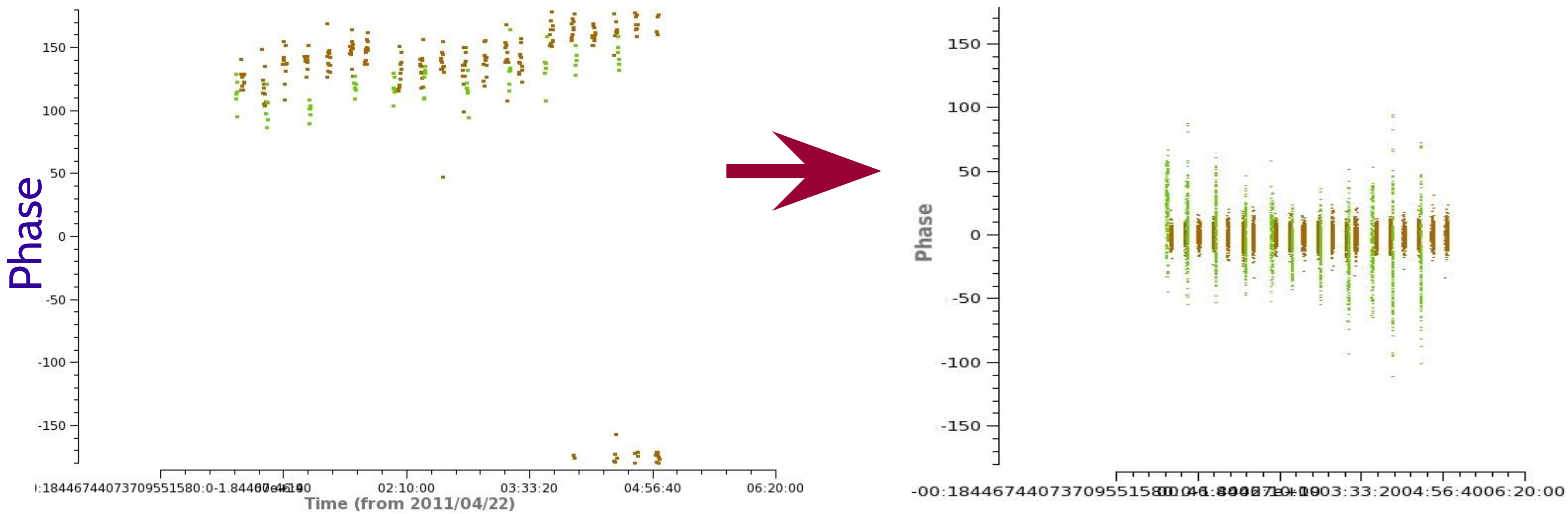
# Water Vapor Correction on ALMA




**Phase vs. Time**
**One 600m Baseline**
**~600 GHz**
**Before WVR, After WVR**

# Phase Calibration

The phase calibrator must be a point source close to the science target and must be observed frequently. This provides a model of atmospheric phase change along the line of sight to the science target that can be compensated for in the data.




Time

Corrected using point source model

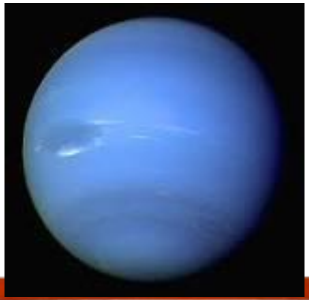
# Flux (or Amplitude) Calibration

Two Steps:

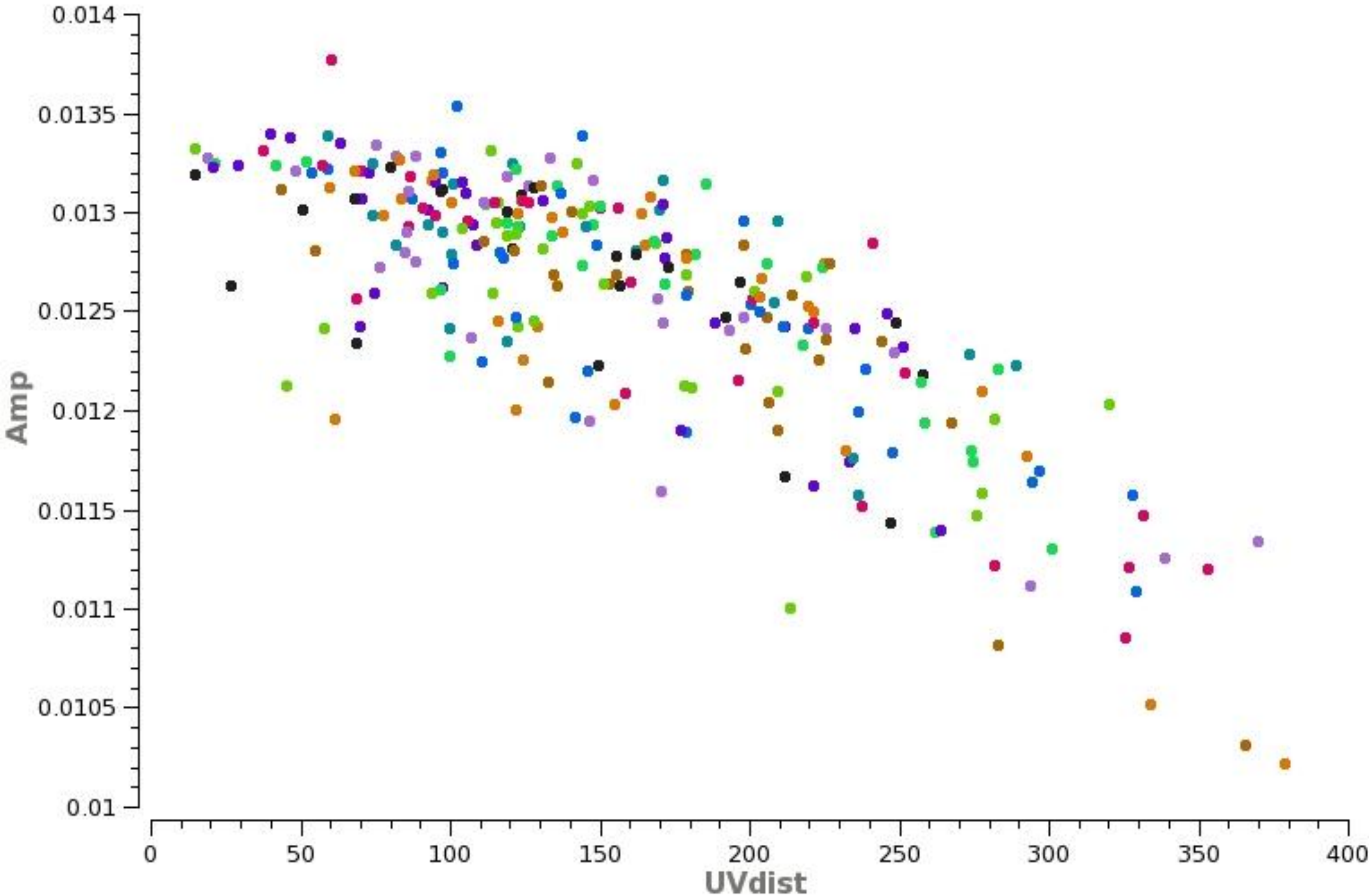
1. Use calibration devices with known temperatures (hotload and ambient load) to measure System Temperature frequently.

2. Use a source of known flux to convert the signal measured at the antenna to common unit (Janskys). If the source is resolved, or has spectral lines, it must be modeled very well.

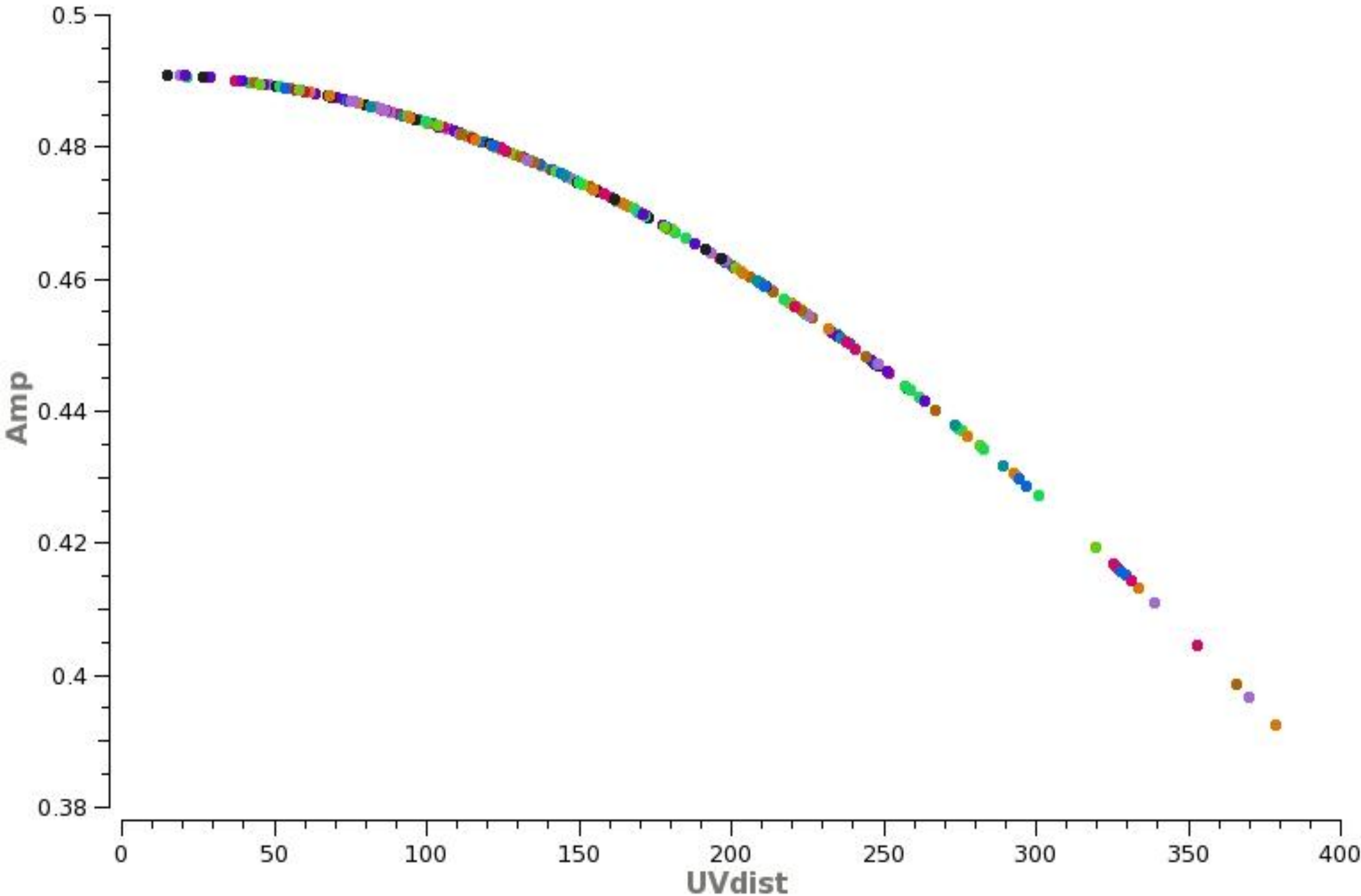
The derived amplitude vs. time corrections for the flux calibrator are then applied to the science target.



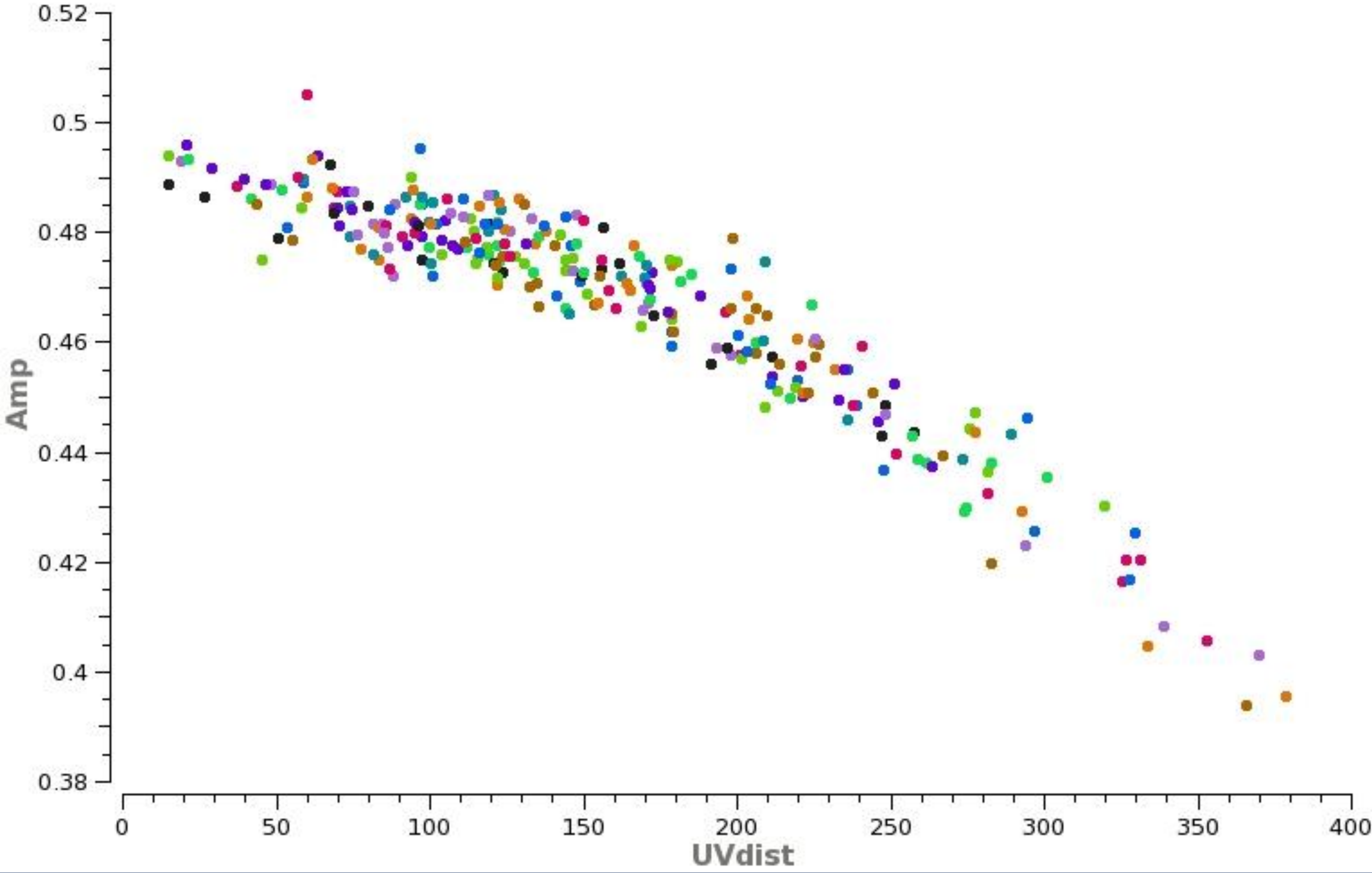
# Amp-Calibrators Amp vs. uv-distance (Before)



# Amp-Calibrators Amp vs. uv-distance (Model)



# Amp-Calibrators Amp vs. uv-distance (After)



# Good Future References

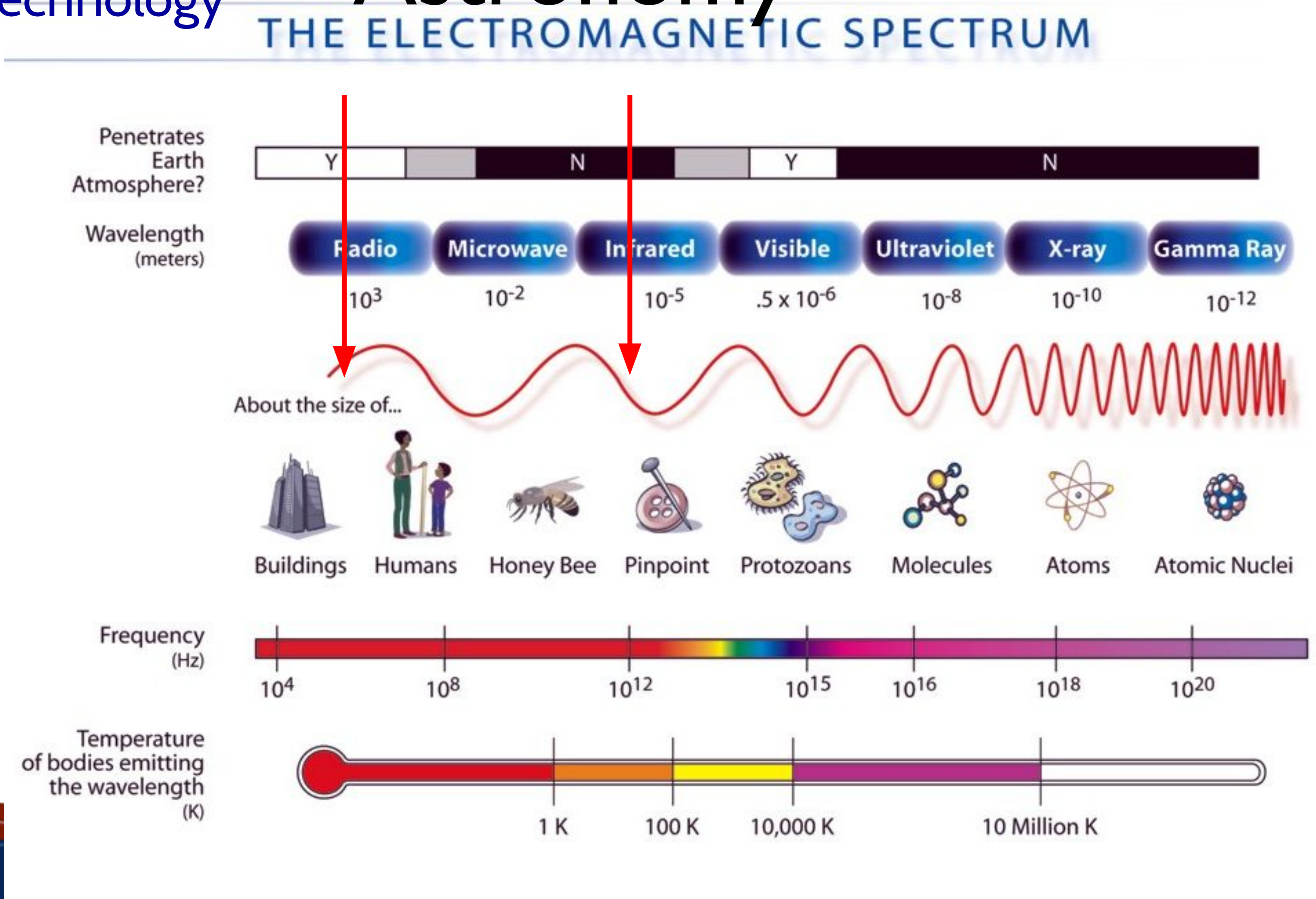
Thompson, A.R., Moran, J.M., Swensen, G.W. 2017 "Interferometry and Synthesis in Radio Astronomy", 3rd edition (Springer)
http://www.springer.com/us/book/9783319444291

Perley, R.A., Schwab, F.R., Bridle, A.H. eds. 1989 ASP Conf. Series 6 "Synthesis Imaging in Radio Astronomy" (San Francisco: ASP)
www.aoc.nrao.edu/events/synthesis

IRAM Interferometry School proceedings
www.iram.fr/IRAMFR/IS/IS2008/archive.html

# Radio Astronomy

Now used to refer to most telescopes using heterodyne technology



# What is heterodyne?

In a heterodyne receiver, observed sky frequencies are converted to lower frequency signals by mixing with a signal artificially created by a Local Oscillator. The output can then be amplified and analyzed more easily while retaining the original phase and amplitude information.

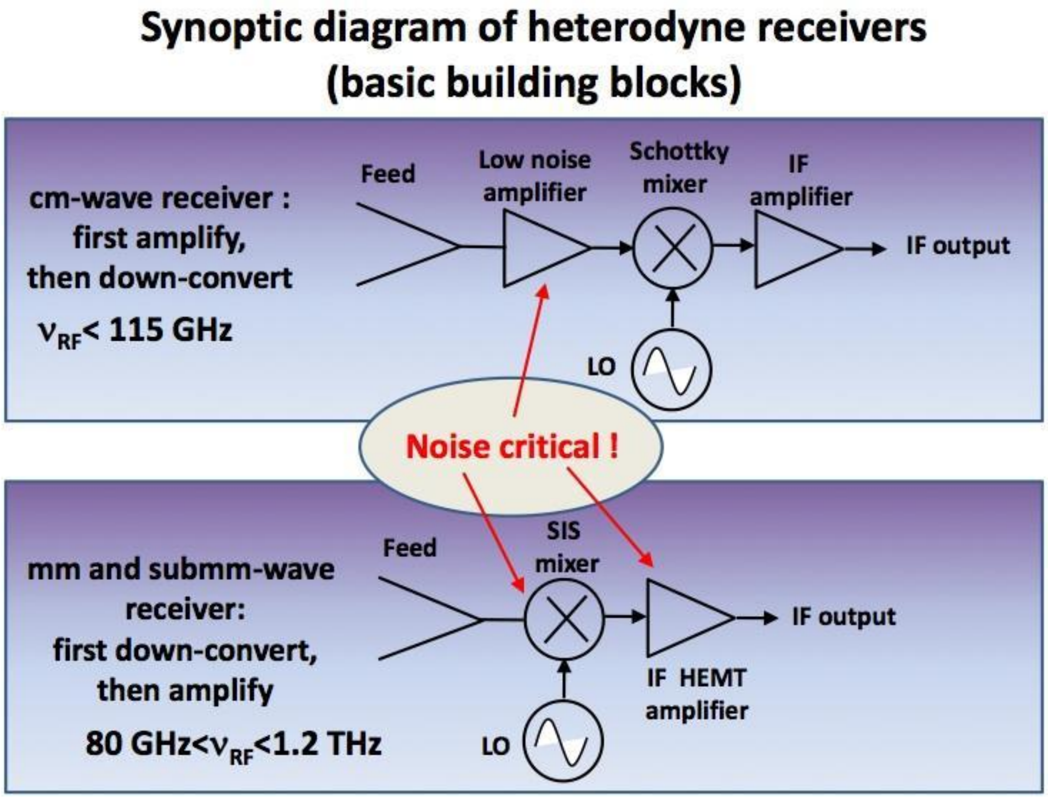


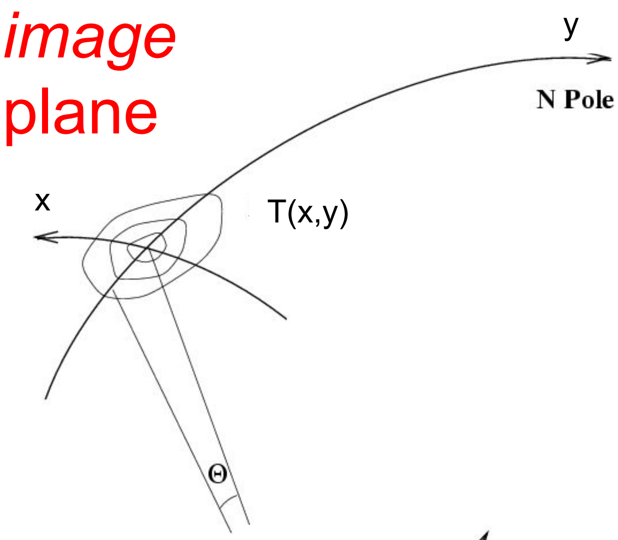

Image from Alessandro Navarrini (IRAM)

# The Fourier Transform relates the measured interference pattern to the radio intensity on the sky

Fourier

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dx dy$$

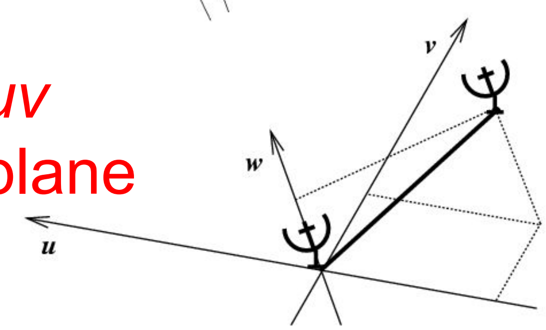
*image* plane



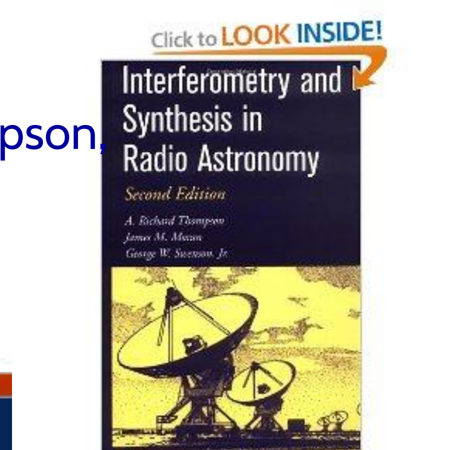

Image

$$T(x,y) = \int\int V(u,v) e^{-2\pi i(ux+vy)} du dv$$

*uv* plane

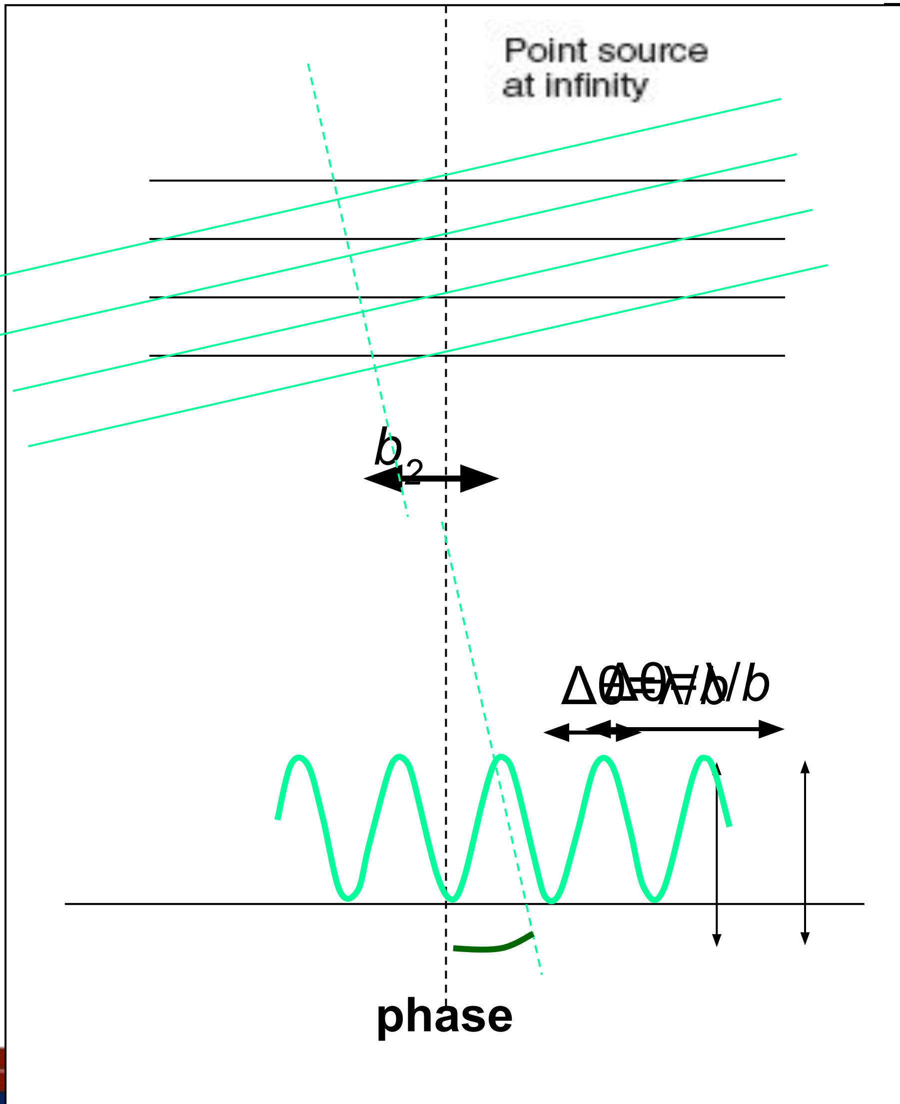


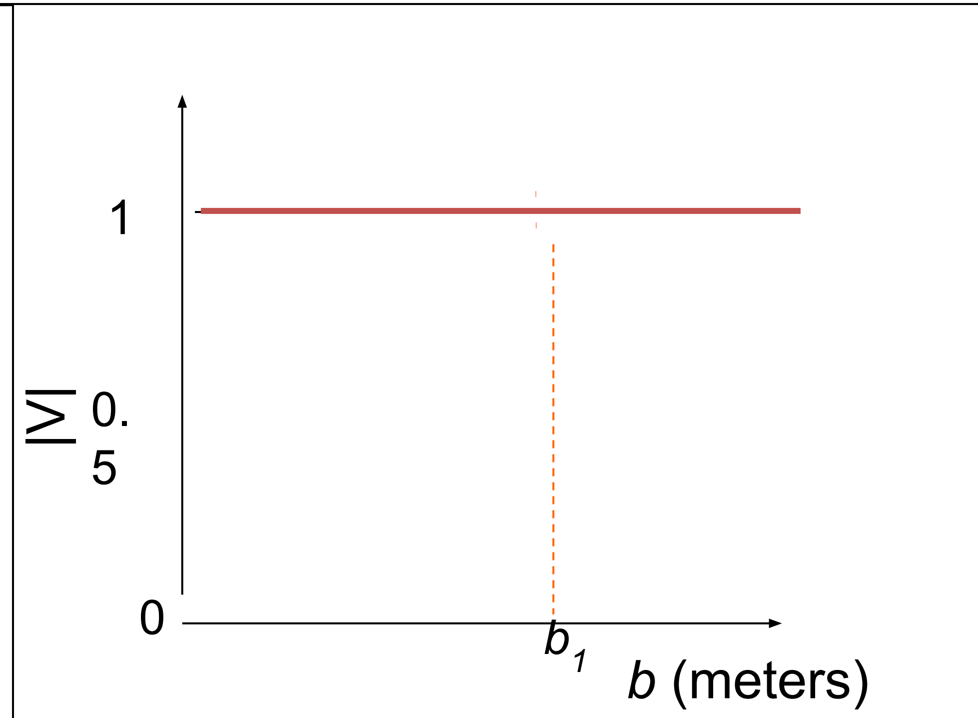

(for more info, see e.g. Thompson, Moran & Swenson)



# Visibility and Sky Brightness

Graphic courtesy Andrea Isella



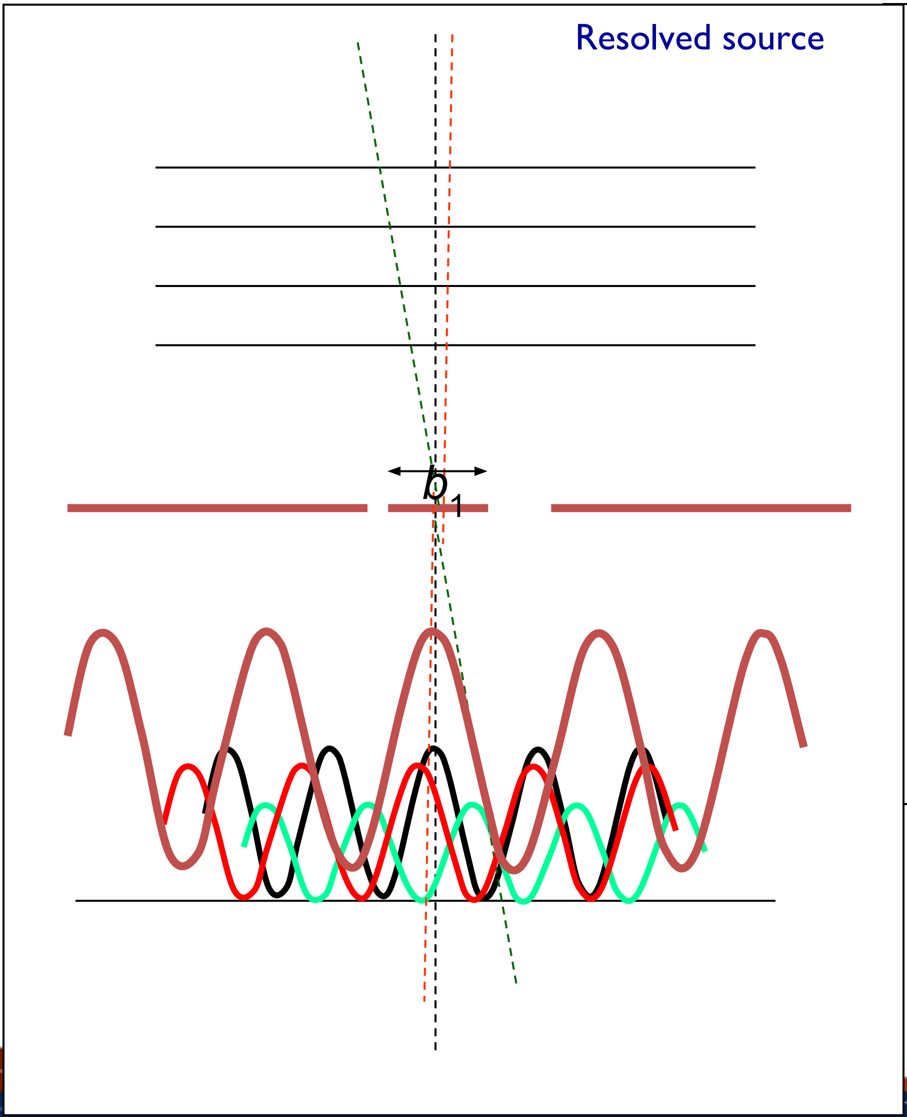

$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

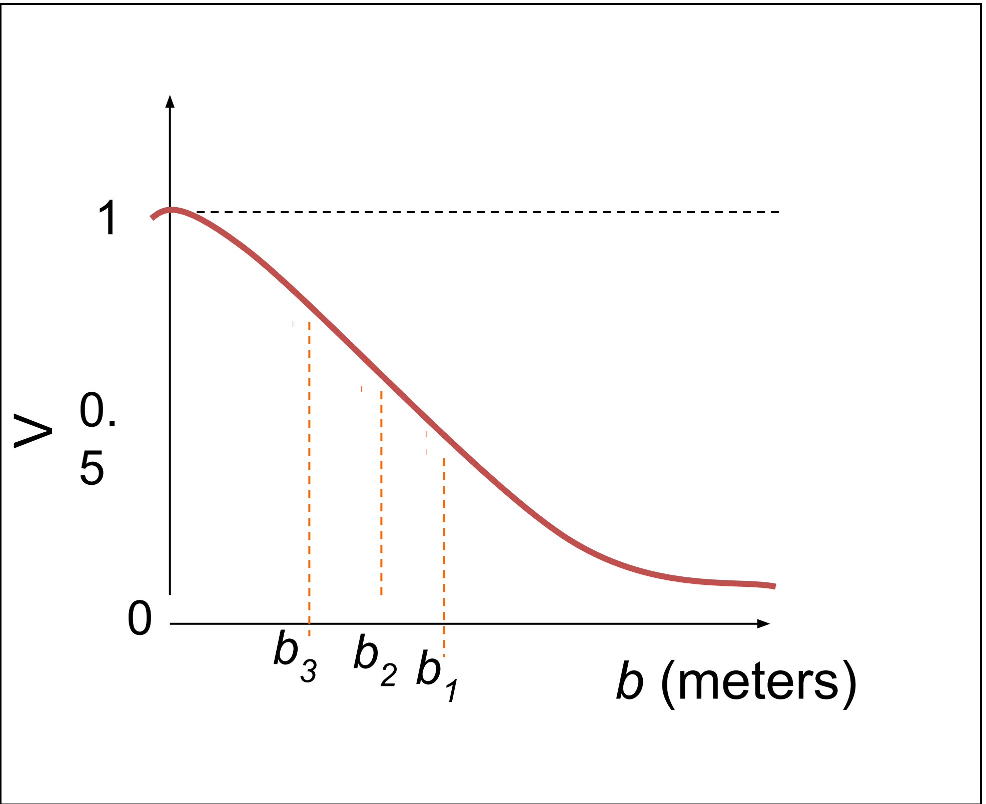
The visibility is a **complex** quantity:

- **amplitude** tells "how much" of a certain frequency component
- **phase** tells "where" this component is located

# Visibility and Sky Brightness

Graphic courtesy Andrea Isella

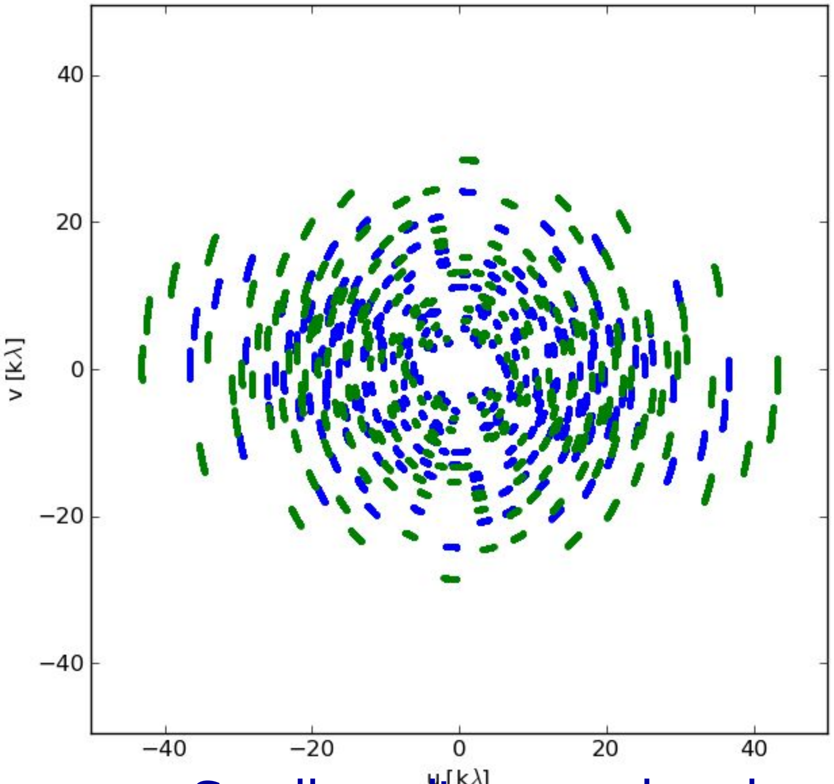


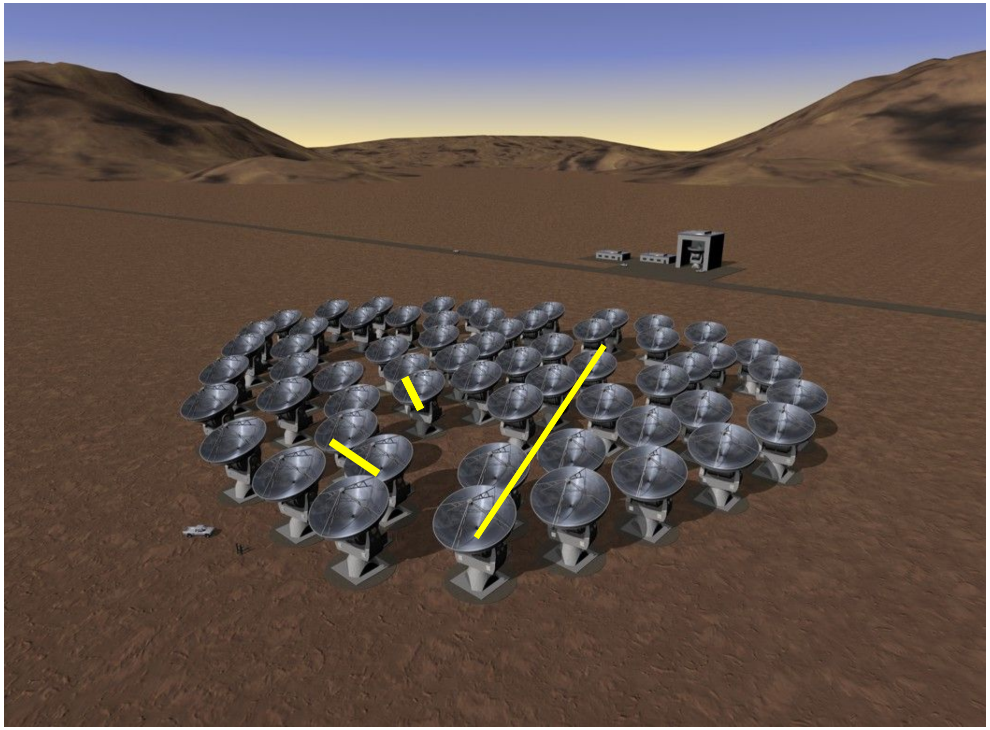




$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

# Sampling Function

Each antenna pair samples only one spot; the array cannot sample the entire Fourier/uv domain resulting in an **imperfect image**



Small uv-distance: short baselines (measure extended emission)
Long uv-distance: long baselines (measure small scale emission)
Orientation of baseline also determines orientation in the uv-plane

# uv coverage: why the central hole?

- The central hole in the sampling of the uv plane arises due to **short baselines**
- The largest angular scale that an interferometer is sensitive to is given by the shortest distance between 2 antennas.
- The field of view is given by the beam of a single antenna.
- A single antenna diameter will always be < the shortest distance between two antennas.
- So the field of view is always > the largest angular scale
- If your source is extended, you will always have some flux at short spacings (i.e. extended emission) that is not recovered.
- **Solutions:** We can extrapolate to these shorter spacings after our observations are taken (more on this tomorrow!) or we can fill in the information with 7m observations or ultimately single dish data.