Combining Interferometric and Single Dish Data

NAIC/NRAO Single Dish School on Radio Astronomy 2011

Juergen Ott
The Need for Interferometry

The resolution of a telescope depends on the wavelength and its diameter

$$\theta \sim \frac{\lambda}{D}$$

Solution (1) build larger telescopes!
The Need for Interferometry
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Prize Time !!!
The Need for Interferometry
The Need for Interferometry

The resolution of a telescope depends on the wavelength and its diameter

$$\Theta \sim \frac{\lambda}{D}$$

Solution (1) build larger telescopes!
But telescopes are substantial structures and there are engineering, physical and monetary limits

Solution (2) go to smaller wavelengths!
The Need for Interferometry
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The resolution of a telescope depends on the wavelength and its diameter

$$\Theta \sim \frac{\lambda}{D}$$

**Solution (1)** build larger telescopes!
But telescopes are substantial structures and there are engineering, physical and monetary limits

**Solution (2)** go to smaller wavelengths!
Ok, for some applications but the strength of the continuum changes, as well as the physics of the emission mechanisms. No way to change the frequency when one is after specific spectral lines

**Solution (3)** interferometry!
Cheat when building a telescope, build many and use Fourier Optics theorems
Single dish: diameter is responsible for sensitivity, field of view, resolution

Interferometer: takes this apart

D for resolution, synthesized beam

D’ for dish diameter (field of view) primary beam

NxD’ for sensitivity
The Need for Interferometry
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Interferometry

Interferometry decouples the aperture properties of a single dish

• The sensitivity is given by the number of antennas times their area

• The resolution is given by the largest distance of antennas (synthesized beam)

• The field of view is given by the beam of a single antenna (corresponding to the single dish resolution; primary beam)

• The largest angular scale that can be imaged is given by the shortest distance of antennas
Interferometry

• Every pair of telescopes (every baseline) is sensitive to one spatial frequency, i.e., a wave pattern on the sky.

• The projected distance as seen from the source defines a point in the Fourier domain, the uv domain.

• For each uv point, a phase and amplitude is measured.

• The uv points determine what is measured, the phase/amplitude define the strength and displacement of that signal.

• The distribution of uv points determine the image quality.
Interferometry uv-plane
Interferometry – fringe, phase amplitude

Signal in antenna 1

Signal in antenna 2

Lag spectrum (fringe)

Amplitude

Phase
<table>
<thead>
<tr>
<th>uv domain</th>
<th>Image domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visibility</td>
<td>Spatial frequency</td>
</tr>
<tr>
<td>Angle $\psi$ toward the uv point</td>
<td>Direction of wave vector $k$</td>
</tr>
<tr>
<td>Length of Vector</td>
<td>Spatial frequency</td>
</tr>
<tr>
<td>Phase $\phi$ of visibility</td>
<td>Offset from the central pixel (phase center)</td>
</tr>
<tr>
<td>Amplitude $</td>
<td>V</td>
</tr>
</tbody>
</table>
Dirty Beam Shape and N Antennas

2 Antennas
Dirty Beam Shape and N Antennas

3 Antennas
Dirty Beam Shape and N Antennas

4 Antennas
Dirty Beam Shape and N Antennas

5 Antennas

![Diagram of dirty beam shape and 5 antennas](image-url)
Dirty Beam Shape and N Antennas

6 Antennas
Dirty Beam Shape and N Antennas

7 Antennas
Dirty Beam Shape and N Antennas

8 Antennas
Dirty Beam Shape and N Antennas

8 Antennas x 6 Samples
Dirty Beam Shape and N Antennas

8 Antennas x 30 Samples
Dirty Beam Shape and N Antennas

8 Antennas x 60 Samples
Dirty Beam Shape and N Antennas

8 Antennas x 120 Samples
Dirty Beam Shape and N Antennas

8 Antennas x 240 Samples
Dirty Beam Shape and N Antennas

8 Antennas x 480 Samples
Interferometry

• How to get a lot of visibilities?
• Solution (1) build a lot of antenna (pairs)

ALMA: 50 antennas = \( \frac{n(n-1)}{2} = 1225 \) pairs!

• Solution (2) aperture synthesis, let the earth rotation do the trick
  (Nobel prize in physics 1974, Sir Martin Ryle)
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A-config 0.7-36km

B-config 0.2-11km

C-config 0.04-4km

D-config 0.04-1km

Surface brightness sensitivity
Let’s zoom in!
No matter what you do,
There’s always a central hole

Let’s zoom in!
uv-coverage

• What’s the problem with the hole?
• It’s the short baselines
• They define the largest spatial frequencies, or the largest angular scales that an interferometer is sensitive to
• The field of view is given by the beam of a single antenna
• The largest angular scale is given by the shortest distance between 2 antennas \( \theta < \frac{2 \lambda}{d_{\text{min}}} \)
• Single antenna diameter < shortest distance
• Field of view > largest sensitive scale
• Extreme: full flux in field of view is given by the central pixel in the uv-coverage

Short spacings & zero spacings problem
Short Spacing Problem
The short-spacing problem

UV plane

ideal

Central hole

Typical interferometer

PSF

Braun & Walterbos (1985)

Negative ‘bowl’
The short (zero)-spacing problem

UV plane

PSF

ideal

Central hole

Typical interferometer

Braun & Walterbos (1985)

Negative ‘bowl’
Mosaicing helps!
Ekers & Rots Theorem

- Extended this formalism to interferometers to show that an interferometer doesn’t just measure angular scales $\theta = \frac{\lambda}{b}$ it actually measures $\frac{\lambda}{b - D} < \theta < \frac{\lambda}{b + D}$
Hole is smaller due to the smearing of the uv-tracks, but still there.
The zero-spacing problem

The dirty map extended to infinity contains exactly a flux of zero!
uv-coverage

• **Solution (1) deconvolution**
  • Take dirty map → find strongest source → remove dirty beam → write in table
  • Fit dirty beam with Gaussian → clean beam
  • Convolve table of positions and strengths with clean beam
CLEAN

$T^D(x,y)$

CLEAN model

restored image

residual map
dirty Image slice FFT FFT-slice

Few iterations

more iterations
uv-coverage

- **Solution (I) deconvolution**
- Take dirty map → find strongest source → remove dirty beam → write in table
- Fit dirty beam with Gaussian → clean beam
- Convolve table of positions and strengths with clean beam

- **CLEAN:**
  - Makes nice images
  - Removes sidelobes
  - Interpolates visibilities
  - Removes gaps → gives a flux to the components (Σsine waves have zero flux)
  - Extrapolates the short and zero spacing → estimates flux to full image
uv-coverage

- **Solution (I) deconvolution**
- ‘classic’ CLEAN assumes that the image is composed of a number of point sources!
- This works to some extent but is clearly an issue when trying to decompose extended emission by δ-functions
- Other deconvolution methods that work better for extended structure:
  - Multi-scale clean: go for different widths in addition to δ-functions
  - Maximum entropy: maximize a “quality of fit” value between a model and the data

\[ \mathcal{N} = - \sum_i I_i \ln \left( \frac{I_i}{M_i e} \right) \]
**uv-coverage**

- CLEAN **EXTRAPOLATES** to the central short and zero spacings
- Can we measure those instead?
- Yes!

- **Solution (2) Short/zero spacing correction**

- Short spacings due to minimum telescope distance, but what about a single dish?

![Diagram](image_url)
uv-coverage

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uv-coverage

Interferometer

Both are measured,
Now combine!

Single Dish
Practical aspects:

• What single dish to use?

• To cover all uv-ranges, one needs to observe at all spacings. So best to use a single dish that is larger than the minimum separation of interferometer antennas, e.g.
  EVLA-C/D + GBT
  EVLA-B + Arecibo
  ATCA + Parkes
  Plateau de Bure + Pico Veleta
  CARMA + LMT
Interferometry

- ALMA approach:
  - 12m antennas + 7m antennas +
  - 12m antennas that operate as SD
Practical aspects:

• How deep?

• Surface brightnesses need to be adjusted, not point source sensitivity. Like go to the same sensitivity in K.

• SB sensitivity proportional to the beam area. So the single dish is usually not nearly as long per point as the interferometer observations.

• Interferometer Antenna beam (fov) > SD beam > Interferometer synthesized beam

• BUT fov of SD is much smaller, so mapping may take longer than interferometer maps
Practical aspects:

- Can I use the autocorrelations of a single antenna in an array to be used as the single dish for the SSC?

- There will be a gap in the uv-domain as the diameter of the single dish is smaller than the minimum baseline

- However, the zero spacing may be covered, at least in theory

- Practically, a single dish needs a different observation technique, e.g. position switching, an interferometer stays on source interrupted by phase calibrator scans. The SD should also map the source where the interferometer may not

- On-the-fly mosaicking may be the solution, where the autocorrelations map the source and the reference can be constructed by a running median. We are at the beginning of testing such telescope modes
Methods – (1) Feathering

• Corrections:

\[ f = \frac{S_{\text{int}}}{S_{sd}} \quad \alpha = \frac{\Omega_{\text{int}}}{\Omega_{sd}} \]

Calibration adjustment \hspace{1cm} \text{beam/SB scaling}
Methods – (I) Feathering

- All radio images are weighted/convolved by the radio beams!

\[ I' = I \otimes B \]

\[ \Rightarrow \text{FFT}(I') = \text{FFT}(I) \times \text{FFT}(B) \]

\[ \text{FFT}(I) = \frac{\text{FFT}(I'_{\text{int}})}{\text{FFT}(B_{\text{int}})} \]

\[ \text{FFT}(I) = \frac{\text{FFT}(I'_{\text{SD}})}{\text{FFT}(B_{\text{SD}})} \]

\[ \text{FFT}(I) = \frac{\text{FFT}(I'_{\text{int}})}{\text{FFT}(B_{\text{int}})} \]

\[ \text{FFT}(I) = \frac{\text{FFT}(I'_{\text{SD}})}{\text{FFT}(B_{\text{SD}})} \]

= in the overlap region

\[ \text{FFT}(I) = \frac{\text{FFT}(I'_{\text{int}})}{\text{FFT}(B_{\text{int}})} \]

\[ \text{FFT}(I) = \frac{\text{FFT}(I'_{\text{SD}})}{\text{FFT}(B_{\text{SD}})} \]
Methods – (1) Feathering

• Practically: weigh the visibilities along the FFT of the synthesized beam (synthesized beam is the large scale Gaussian over a uv-coverage)

\[
V_{comb}(k) = w'(k)V_{int}(k) + fw''(k)V'_{sd}(k)
\]

\[
w'(k) + w''(k) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{\theta^2_{int}k^2}{4\ln 2}\right)
\]
Example of Fourier plane combination: McClure-Griffiths et al.
Methods – (2) Feathering w/o weighting

- Clean interferometric dataset & replace extrapolated center by SD visibilites, then FFT
- Deconvolve SD map from SD beam
- Convolve it with Int. beam
- Replace inner uv hole with SD values
- FFT into image

Weiss et al.
Methods – (3) ‘Linear’ Combination in Image Domain

Combined image:

\[ I_{\text{tot}} = w_{\text{int}} I_{\text{int}} + w_{\text{sd}} f_{\text{sd}} I_{\text{sd}} \]

Weights:

\[ w_{\text{int}} = \frac{\Omega_{\text{sd}}}{\Omega_{\text{int}} + \Omega_{\text{sd}}} \]
\[ w_{\text{sd}} = \frac{\Omega_{\text{int}}}{\Omega_{\text{int}} + \Omega_{\text{sd}}} \]

Interferometric
Dirty image
and beam
SD image
and beam
Combination

Stanimirovic et al.

\[ \text{DECONVOLVE} \]

deconvolve
Example of image-plane combination:
SMC in HI at $V_h=130$ km/s
(Stanimirovic et al 1999)
Methods – (4) SSC during Deconvolution

• One can use the SD image as a starting model for the deconvolution. Works best with multi-scale-cleaning or maximum-entropy

• What it does: clean will not extrapolate anymore but intrapolate!
Methods – (4) Joint Deconvolution

• Deconvolve both images simultaneously, with 2 conditions for improving the quality (entropy)

\[ \mathcal{K} = - \sum_i I_i \ln \left( \frac{I_i}{M_i e} \right) \]

Subject to (1)

\[ \sum_i \left\{ I_{\text{int}}^D - B_{\text{int}} \ast I \right\}_i^2 < N \sigma_{\text{int}}^2 \]

(2)

\[ \sum_i \left\{ I_{\text{sd}}^D - \frac{B_{\text{sd}} \ast I}{f_{\text{sd}}} \right\}_i^2 < M \sigma_{\text{sd}}^2 \]
Feather in uv-domain

Linear combination of images

Model in deconvolution

Joint deconvolution
Summary

• Interferometric imaging lack spatial sensitivity on large scales (including the full flux over an image) (minimum baseline)
• SD lacks the high resolution an interferometer can get to (maximum baseline)
• Lack of inner $uv$-points create bowls in the image
• CLEAN extrapolates fluxes and structures into images which sometimes work sometimes not. Regular CLEAN assumes point sources (bad) more sophisticated methods like MEM and multi-scale clean can do better
• Even better than extrapolation is a SD measurement to determine the very inner $uv$-points
• The beams play an important role – know well you SD beam! Also calibrate your data well!
• Methods of combination incl. uv-domain, image domain, and deconvolution at different stages