AN HEURISTIC INTRODUCTION TO RADIOASTRONOMICAL POLARIZATION

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OUTLINE

- Polarization: the unique probe of magnetic fields in the Universe
- Quantifying polarization with Stokes parameters
- Radio beats optical: We measure all Stokes parameters simultaneously
- The Mueller Matrix relates the CAL to the SKY
- The receiver system introduces calibratable effects in both gain and phase
- Beam effects: squint, squash, more distant sidelobes—Arecibo and GBT
- My website: a "paradise" of tutorials and documentation
- Three things to remember

WHAT PRODUCES RADIO POLARIZATION?

From magnetic fields:

- Cyclotron emission—electrons spiraling in a *magnetic field.* Polarization is *strong* (like 100%).
- Synchrotron emission—relativistic electrons spiraling in a *magnetic field.* Polarization is *strong* (like 70%), but it's often reduced by Faraday-rotation effects.
- mm-wave and IR emission from *magnetic*ally aligned dust grains. Polarization is typically a few %, plus-or-minus.
- Goldreich/Kylafis effect: spectral-line scattering of anisotropic radiation by molecules in a *magnetic field.* Polarization is typically a few %, plus-or-minus.
- Zeeman splitting of spectral lines by atoms/molecules in a *magnetic field.* Polarization is *very weak* (like 0.1% if you're lucky).

<u>Not</u> from magnetic fields:

- Blackbody Emission from a solid planetary surface (remember the "Brewster angle"?). Polarization can be *strong* (up to 100%).
- Scattering of radiation (remember the polarization of blue sky? Polarization can be *strong* (up to 100%).

Note the prevalence of the *magnetic field* in producing polarization!

Why do we care about cosmic magnetism?

FORCES ON INTERSTELLAR GAS:

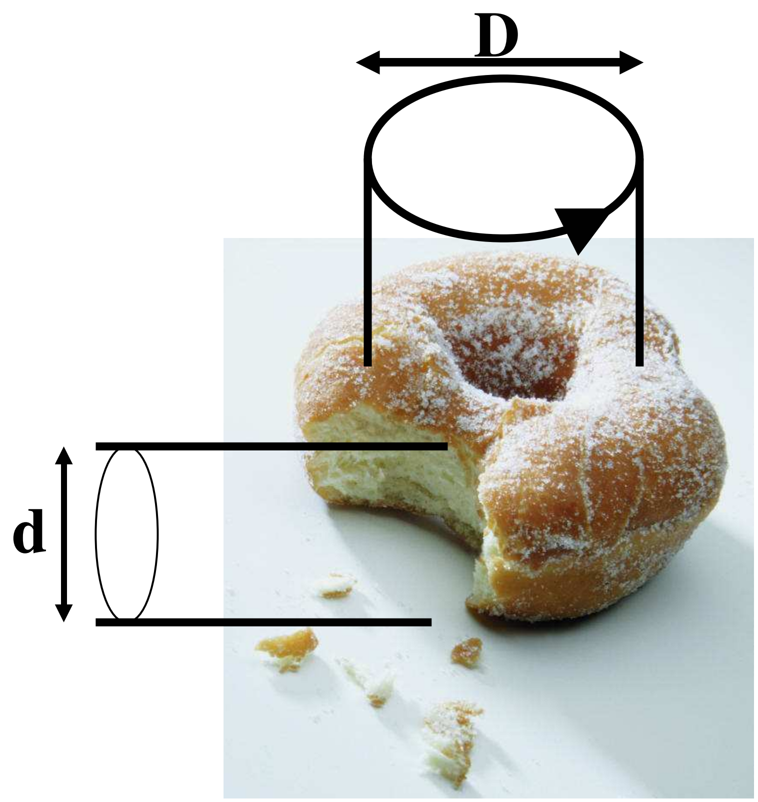
- **Gravity**
- **Pressure**
- **Magnetic Fields**

Consider a current loop in the ISM:

Inductance L ~ D

Resistance ~ D/d² ~ 1/D [d ~ D]

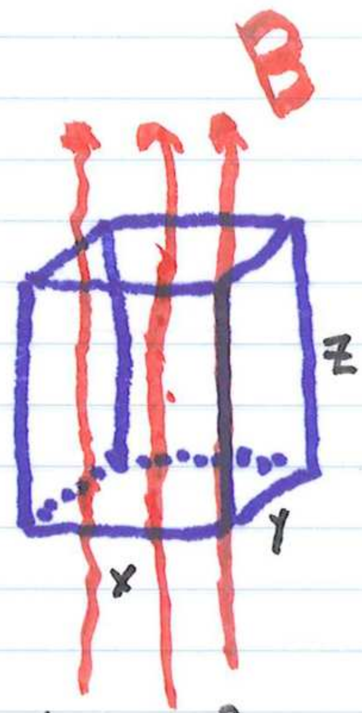
Time constant = L/R ~ D²



Interstellar structures are BIG.
Even for low fractional ionization, time constant is long, so we have

FLUX FREEZING

Magnetic Pressure and Tension…a simple approach.



$$W_B = \frac{B^2}{8\pi} xyz$$

$$\frac{dW_B}{dz} = \frac{B^2}{8\pi} xy \quad - \quad \underline{\text{TENSION}}$$

$$\frac{dW_B}{dx} = \left[\frac{2B}{8\pi}\frac{dB}{dx} + \frac{B^2}{8\pi}\right] yz$$

$$= -\frac{B^2}{8\pi} yz \quad - \quad \underline{\text{PRESSURE}}$$

SOME CANONICAL PRESSURES in the DIFFUSE ISM (P/k in units of cm^{-3} K)

Thermal pressure (typical): P/k = nT ≈ 3000 (Jenkins/Tripp)

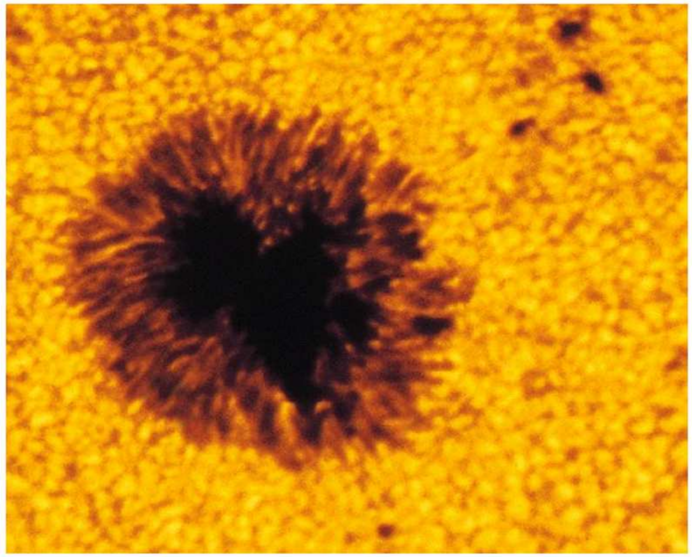
Turbulent Pressure (typical) 10000

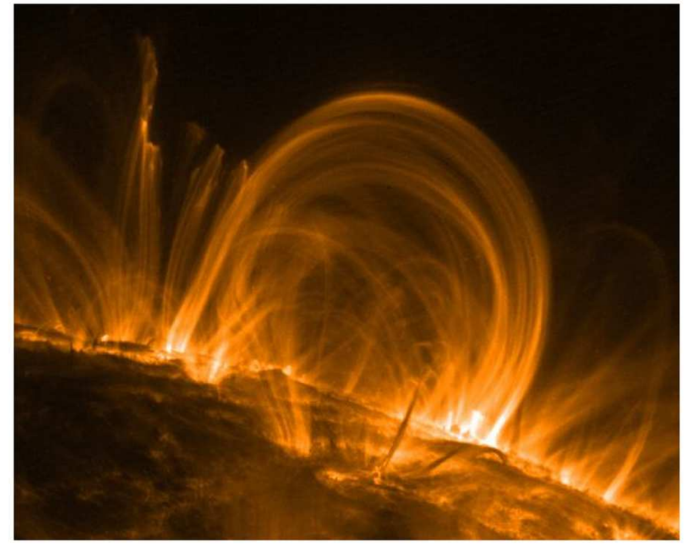
Magnetic Pressure (B = 6 μG) 10000

Galactic Hydrostatic Pressure (Boulares & Cox) 30000

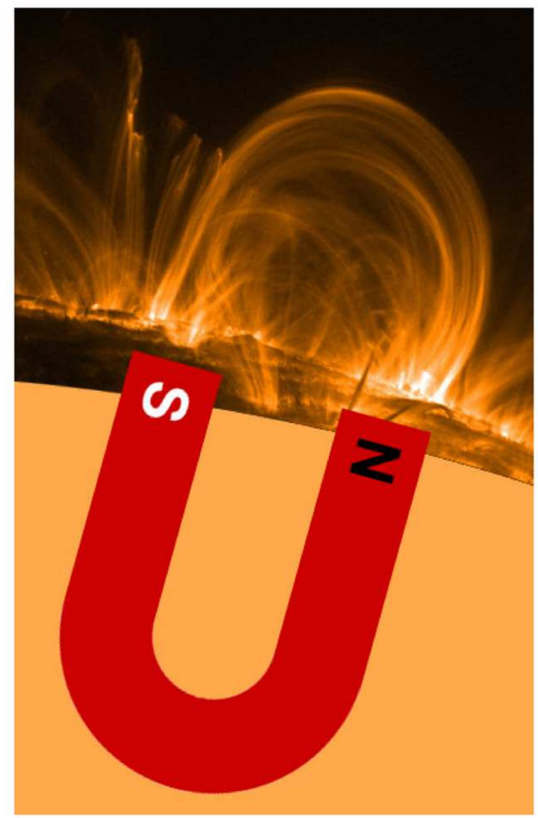
Cosmic Magnetism is a MAJOR FORCE

The interaction of the 3 forces—gravity, pressure, magnetism—produces fascinating and complex phenomenae and is not much understood, in either the terrestrial or astronomical context!





Gravity, Pressure, and Magnetic equilibrium. Solar prominences, sunspots, nice and stable!



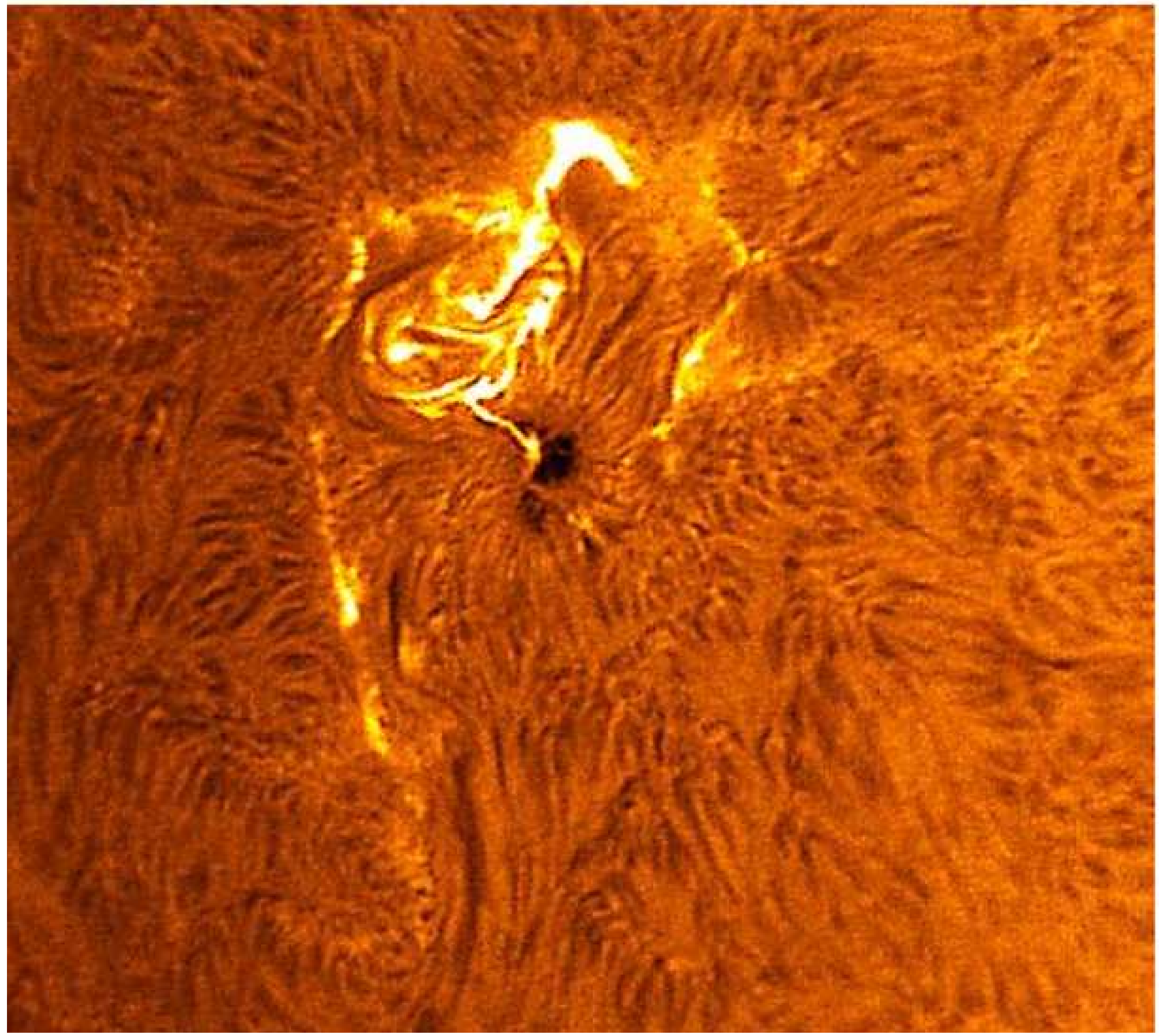
“FLUX FREEZING” – the magnetic lines more with the gas. And vice-versa. Nice ordered flows give nice ordered fields; or, if the field is strong, nice ordered fields give nice ordered flows.

But there can be...

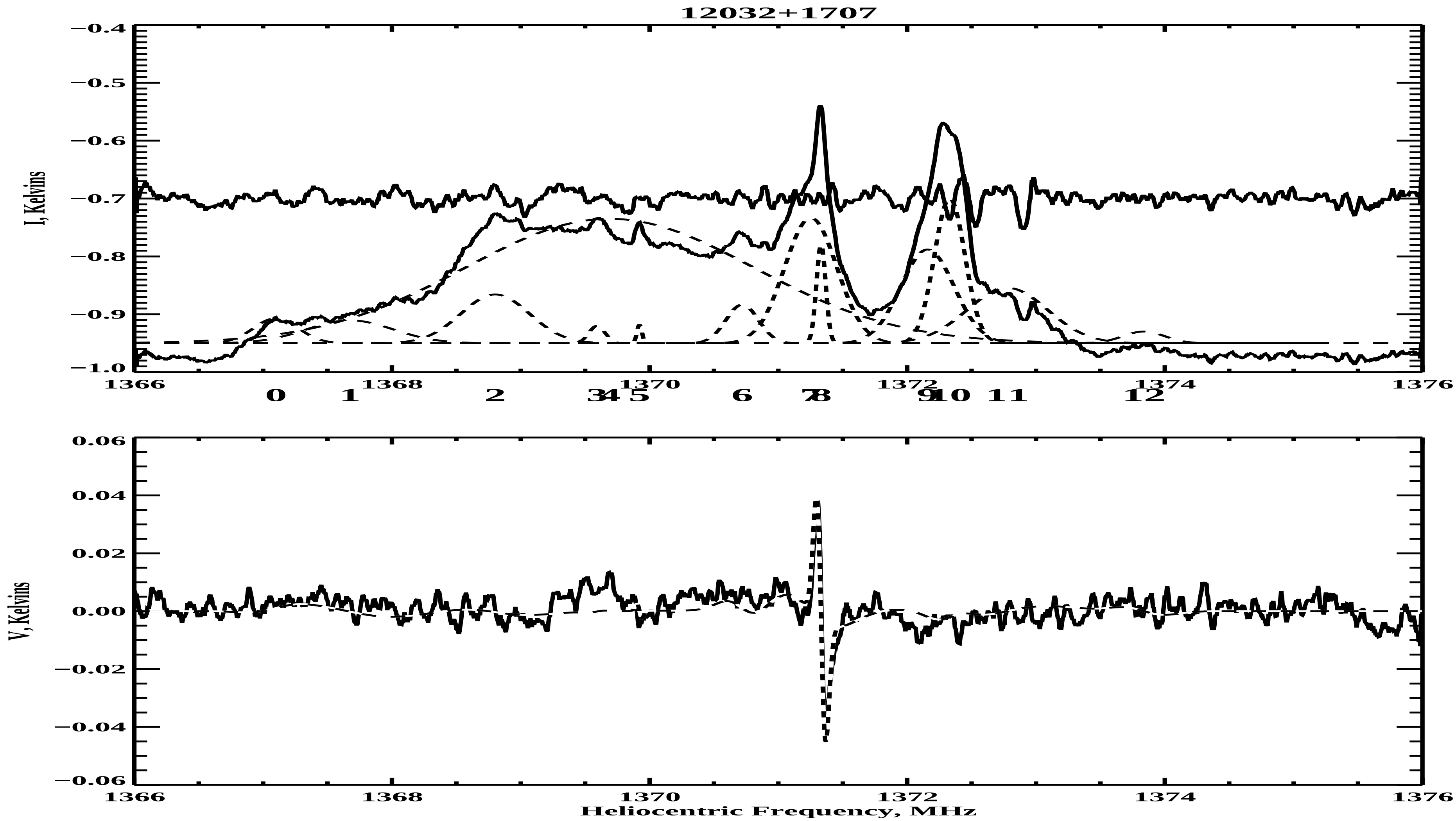
TURBULENCE!!

and INSTABILITY!!

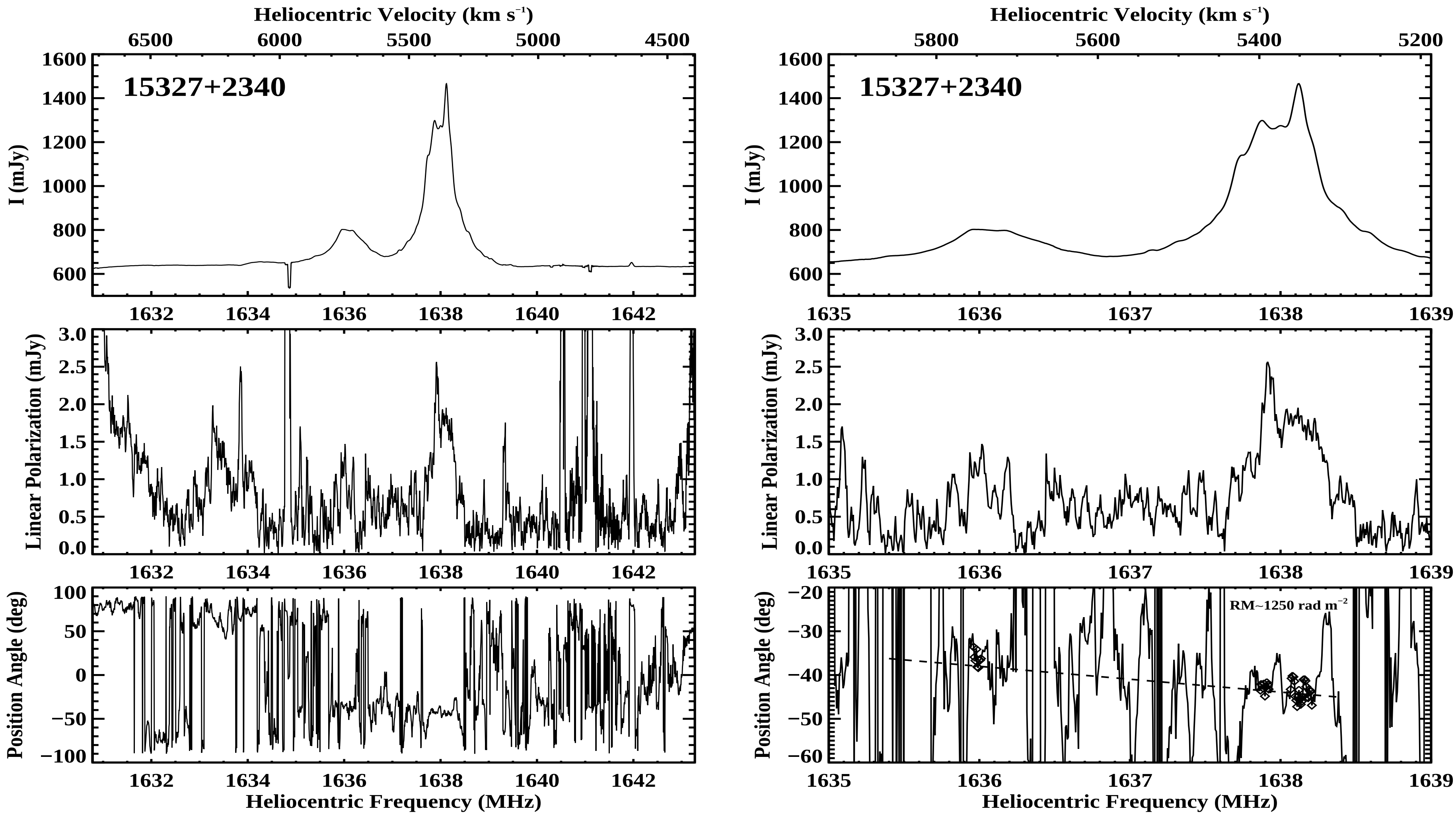
SOLAR FLARE – the N and S poles attract! NOT so nice and stable!



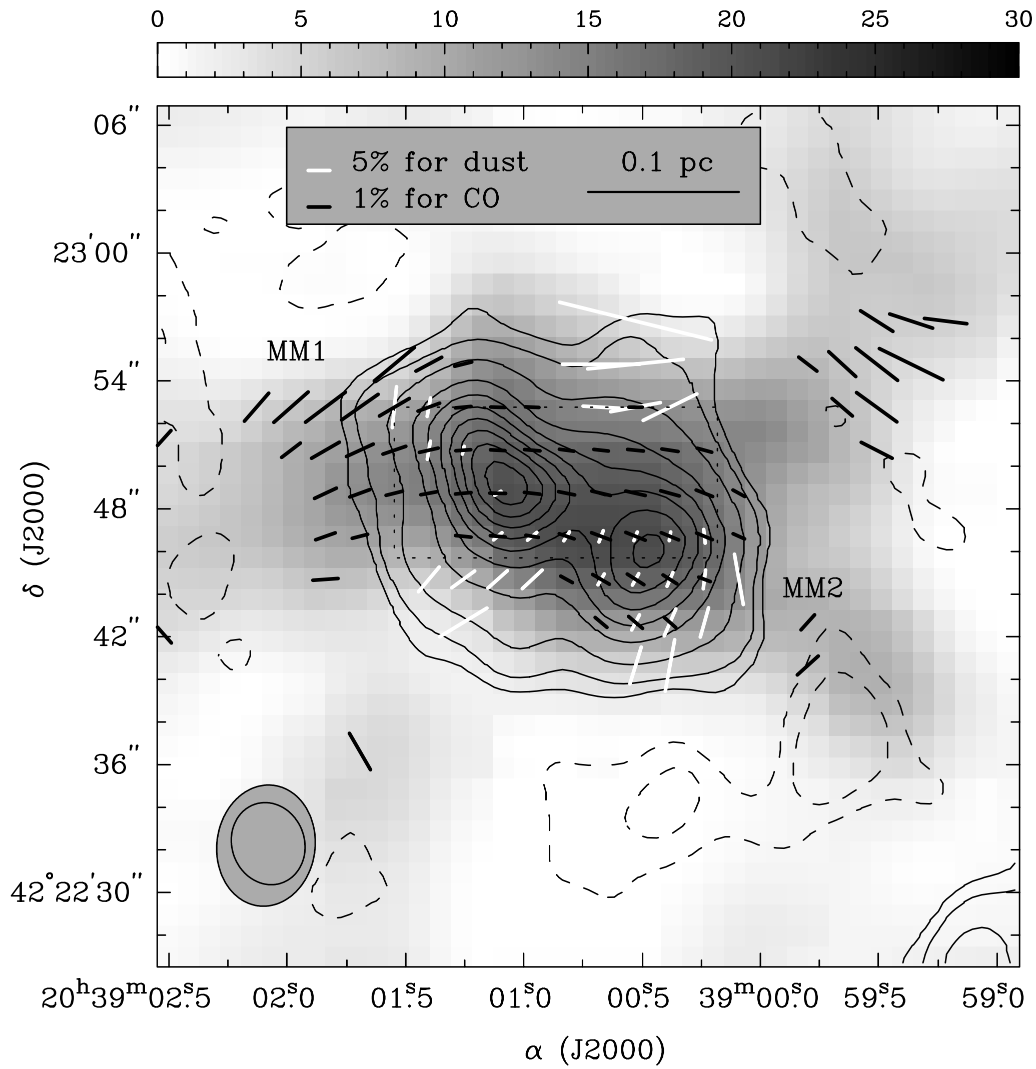
Spectral line CIRCULAR polarization: Zeeman Splitting of an OH Megamaser in a distant ULIRG



Spectral line LINEAR polarization of OH Megamaser emission in a less-distant ULIRG



LINEAR polarization of mm-wave dust emission and CO 2-1 in a protostellar cloud



LINEAR polarization of planetary thermal emission (theory)

Figure 1 shows the distribution of T_{D_0} on the planet's disk. The distribution of $T_{(D_0+\pi/2)}$ is that of T_{D_0} rotated $\pi/2$.

In practice, more precise results for the polarization can be obtained by measuring

$$\Delta T_p = T_{D_0} - T_{(D_0+\pi/2)},$$

since the differential measurement is easier to make than an absolute one. A typical contour plot of this quantity is shown in

However, resolution is obtainable indirectly by the use of interferometer techniques. Theoretical fringe amplitudes for a conventional two-element interferometer may be found from the expression:

Fringe visibility

$$= \frac{\left|\int_0^R T_B(x) \cos (2\pi a x/\lambda D) dx\right|}{\int_0^R T_B(x)\, dx} \qquad (6)$$

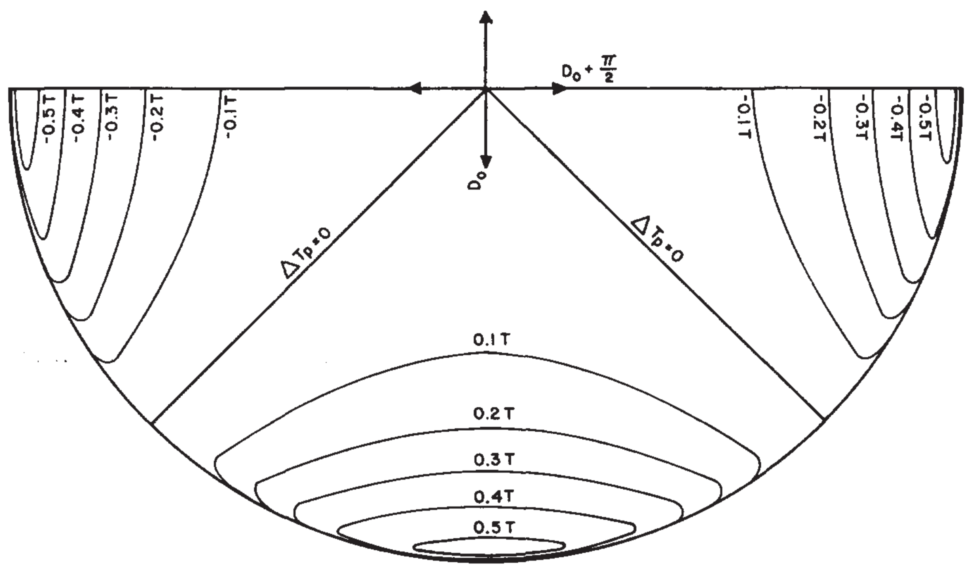


FIG. 2. Typical contour plot of $\Delta T_p = T_{D_0} - T_{(D_0+\pi/2)}$; $\epsilon = 5$.

LINEAR polarization of lunar thermal emission (observation by the 300 foot)

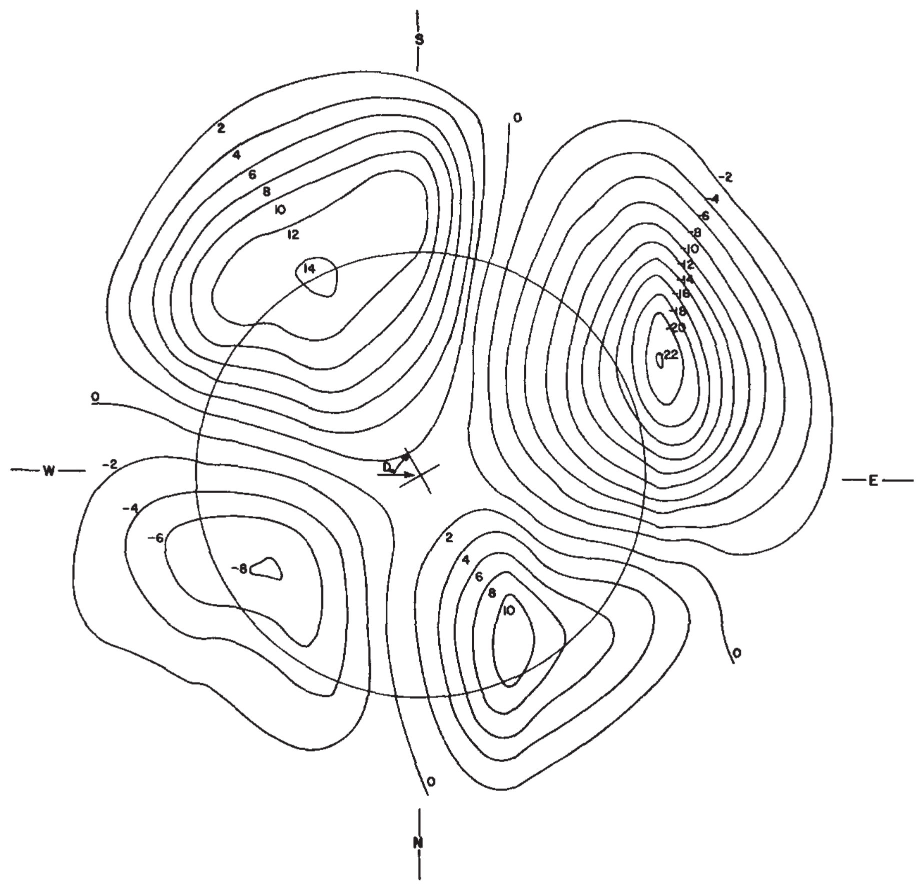
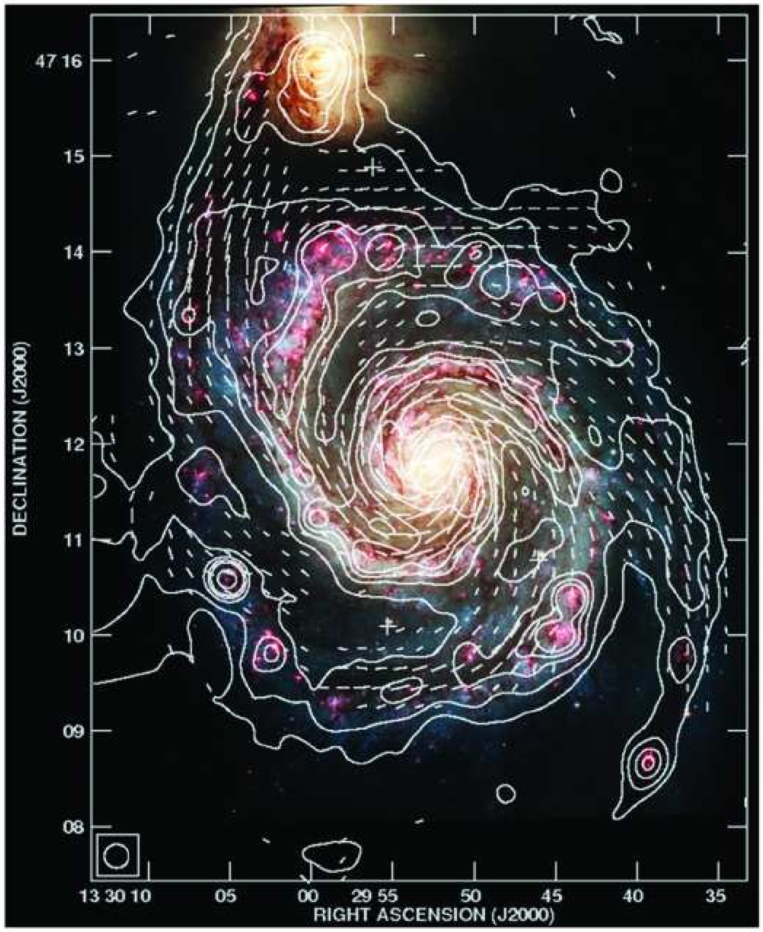
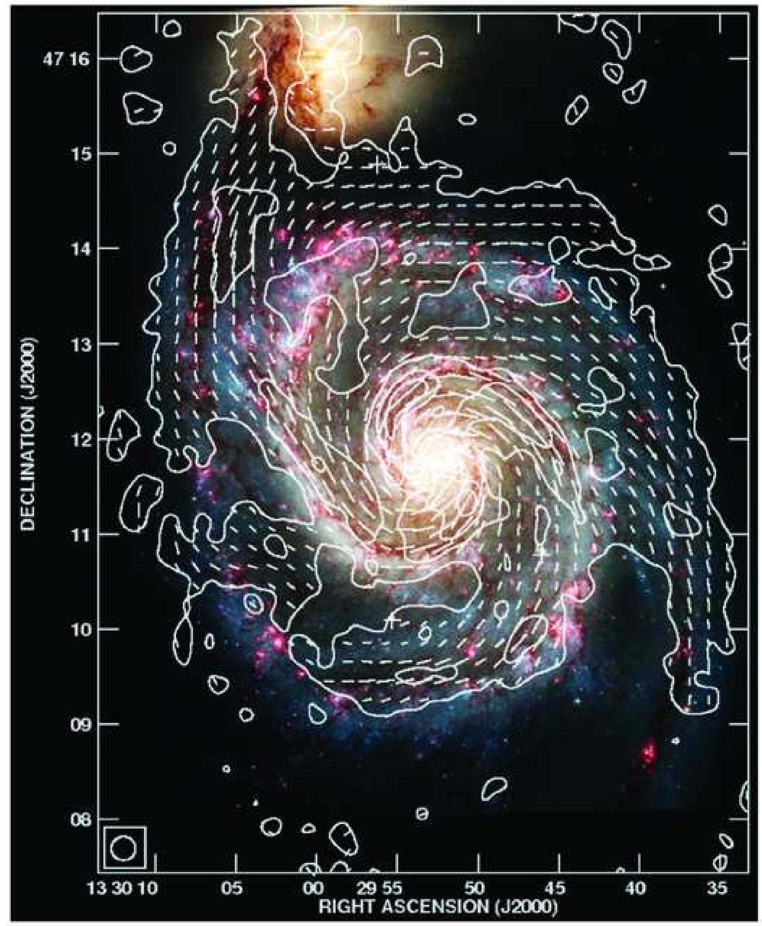


FIG. 6a. The Moon. Contours of $\Delta T'_p$ observed on June 22 and June 28.

Linear polarization of synchrotron emission from M51—a strong spiral density wave and synchrotron emission/polarization





Total Intensity— max in arms

Polarized Intensity-- max between arms

ASTRONOMY & ASTROPHYSICS
SUPPLEMENT SERIES

MAY II **1996**, PAGE 161

Astron. Astrophys. Suppl. Ser. **117,** 161-165 (1996)

Understanding radio polarimetry.
III. Interpreting the IAU/IEEE definitions of the Stokes parameters

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Received September 26; accepted October 22, 1995

Abstract. — In two companion papers (Paper I, Hamaker et al. 1996; Paper II, Sault et al. 1996), a new theory of radio-interferometric polarimetry and its application to the calibration of interferometer arrays are presented. To complete our study of radio polarimetry, we examine here the definition of the Stokes parameters adopted by Commission 40 of the IAU (1974) and the way this definition works out in the mathematical equations. Using the formalism of Paper I, we give a simplified derivation of the frequently-cited 'black-box' formula originally derived by Morris et al. (1964). We show that their original version is in error in the sign of Stokes V, the correct sign being that given by Weiler (1973) and Thompson et al. (1986).

Key words: methods: analytical — methods: data analysis — techniques: interferometers — techniques: polarimeters — polarization

1. Introduction

In a companion paper (Hamaker et al. 1996, Paper I) we have presented a theory that describes the operation of a polarimetric radio interferometer in terms of the properties of its constituent elements and in doing so unifies the heretofore disjoint realms of radio and optical polarimetry. In a second paper (Sault et al., Paper II) we apply this theory along with theorems borrowed from optical polarimetry to the problem of calibrating an interferometer array such as an aperture-synthesis telescope.

In practical applications, the theory must be supplemented by precise definitions of the coordinate frames and the Stokes parameters that are used. This problem was first addressed by the Institute of Radio Engineers in 1942; the most recent version of their definition was published in 1969 (IEEE 1969). For radio-astronomical applications, the IAU (1974) endorses the IEEE standard, supplementing it with definitions of the Cartesian coordinate frame shown in Fig. 1 and of the sign of the Stokes parameter V.

Most published work on actual polarimetric interferometer observations infers the source's Stokes-parameter brightness distributions from a formula derived by Morris et al. (1964). Weiler (1973) rederives their result, agreeing except for the sign of Stokes V. Thompson et al. (1987) include his version in their textbook, even though they suggest in their wording that they agree with Morris et al. Clearly the situation needs to be clarified; starting from a complete interpretation of the definitions, we are in a good position to do so. We shall show Weiler's version indeed to be the correct one.

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2. The Stokes parameters in a single point in the field

The definition of the Stokes parameters most frequently found in the literature is in terms of the auto- and cross-correlations of the x and y components of the oscillating electrical field vectors in a Cartesian frame whose z axis is along the direction of propagation. Following the notation of Paper I, we represent the components of the electric field by their time-varying complex amplitudes $e_x(t), e_y(t)$. The Stokes parameters are then customarily defined by (e.g. Born & Wolf; Thompson et al. 1986):

$$\begin{aligned} I &= <|e_x|^2 + |e_y|^2> \\ Q &= <|e_x|^2 - |e_y|^2> \\ U &= 2<|e_x||e_y|\cos\delta> \\ V &= 2<|e_x||e_y|\sin\delta> \end{aligned} \quad (1)$$

Their equation (1):

$$
\begin{aligned}
I &= <|e_x|^2 + |e_y|^2> \\
Q &= <|e_x|^2 - |e_y|^2> \\
U &= 2<|e_x||e_y|\cos\delta> \\
V &= 2<|e_x||e_y|\sin\delta>
\end{aligned}
$$

(The four **STOKES PARAMETERS**). They look *awfully* complicated...

But it's not *that* complicated!

Stokes parameters are linear combinations of power measured in *orthogonal polarizations*. There are four:

$$I = E_X^2 + E_Y^2 = E_{0^\circ}^2 + E_{90^\circ}^2$$
$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$
$$U = E_{45^\circ}^2 - E_{-45^\circ}^2$$
$$V = E_{LCP}^2 - E_{RCP}^2$$

We like to write the *Stokes vector*

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} .$$

STOKES PARAMETERS: BASICS

$$I = E_X^2 + E_Y^2 = E_{0^\circ}^2 + E_{90^\circ}^2$$

$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$

$$U = E_{45^\circ}^2 - E_{-45^\circ}^2$$

$$V = E_{LCP}^2 - E_{RCP}^2$$

The first, Stokes I, is total intensity. It is the sum of *any two orthogonal* polarizations[1].

The second two, Stokes Q and U, completely specify linear polarization.

The last, Stokes V, completely specifies circular polarization.

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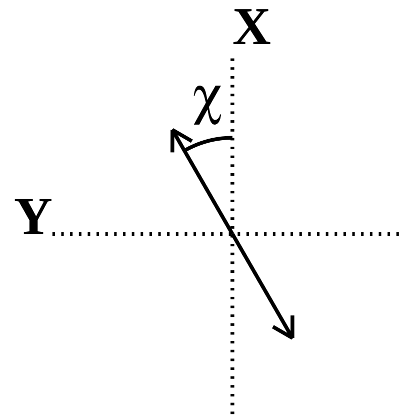
[1] Some ill-advised people (like at the VLA) define I as the *average* instead of the *sum*. BE CAREFUL!

CONVENTIONAL LINEAR POL PARAMETERS

$$\frac{Q}{I} = p_{QU}\cos(2\chi)$$

$$\frac{U}{I} = p_{QU}\sin(2\chi)$$



FRACTIONAL LINEAR POLARIZATION:

$$p_{QU} = \left[\left(\frac{Q}{I}\right)^2 + \left(\frac{U}{I}\right)^2\right]^{1/2}$$

POSITION ANGLE OF LINEAR POLARIZATION:

$$\chi = 0.5\tan^{-1}\frac{U}{Q}$$

REMEMBER THIS # 1: AVERAGING LINEAR POLARIZATIONS!!!

Suppose you average two polarization observations together:

Observation 1 has $p = 13.6\%$ and $\chi = 2^\circ$

Observation 2 has $p = 13.7\%$ and $\chi = 178^\circ$

NOTE THAT THE POSITION ANGLES AGREE TO WITHIN 4 DEGREES.

If you average p and χ, you get $p = 13.65\%$ and $\chi = 90^\circ$.

========= **THIS IS INCORRECT!!!!!!!!!** =========

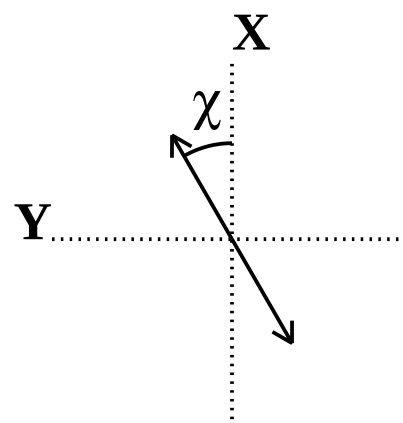
There is only one *proper* way to combine polarizations, and that is to use the Stokes parameters. The reason is simple: because of conservation of energy, powers add and subtract.

What you must **always** do is convert the polarizations and position angles to Stokes parameters, average the Stokes parameters, and convert back.

REMEMBER THIS # 2: TERMINOLOGY MATTERS!!!

HEY!!! LINEAR POLARIZATION "DIRECTION" ??

Look at the figure again:



THERE'S NO ARROWHEAD ON THAT "VECTOR"!! That's because it's the angle 2χ, not χ, that's important.

Moral of this story:

- **NEVER** say "linear polarization **DIRECTION**".
- **INSTEAD**, always say "linear polarization **ORIENTATION**".

OTHER CONVENTIONAL POLARIZATION PARAMETERS

FRACTIONAL CIRCULAR POLARIZATION:

$$p_V = \frac{V}{I}$$

TOTAL FRACTIONAL POLARIZATION:

$$p = \left[\left(\frac{Q}{I}\right)^2 + \left(\frac{U}{I}\right)^2 + \left(\frac{V}{I}\right)^2\right]^{1/2}$$

If both p_{QU} and p_V are nonzero, then the polarization is *elliptical*.

THE (NON) SENSE OF CIRCULAR POLARIZATION

How is Right-hand Circular Polarization defined?

- If you're a physicist: clockwise as seen by the *receiver*.
- If you're an electrical engineer: the IEEE convention, clockwise as seen by the *transmitter*. *Hey!!! what does the receiver see???*
- If you're a radio astronomer: the technical roots are in microwave engineering, so it's the IEEE convention. Probably!! *You'd better check with your receiver engineers!* Or, to be *really* sure, *measure it yourself* by transmitting a helix from a known vantage point (and remember that V changes sign when the signal reflects from a surface!).
- If you're an optical astronomer: you read it off the label of the camera and you have no idea (your main goal is the grant money, so getting the science right is too much trouble).

THE (NON) SENSE OF STOKES V

OK. . . Now that we have RCP straight, how about Stokes V?

- If you're a physicist: $\underline{V = RCP - LCP}$.
- If you're an electrical engineer: there's no IEEE convention. Radio astronomers' convention is, historically, from Kraus (e.g. his "ANTENNAS" or his "RADIO ASTRONOMY"): $\underline{V = LCP - RCP}$. *Hey! With Kraus's definition of V, do physicists and engineers agree???*
- If you're an official of the International Astronomical Union (IAU): The IAU uses the IEEE convention for RCP. . . ,
  and it defines $\underline{V = RCP - LCP}$, meaning that, for V, the IAU *differs from both the physicist and the Kraus convention*!.

IS ALL THIS PERFECTLY CLEAR?

WE'RE NOT THE ONLY ONES WHO ARE CONFUSED! In his thesis, Tim Robishaw traced historical use of V by astronomers in his thesis. Lets take a look:

(separate pdf file).

TABLE 2.4

CIRCULAR POLARIZATION AND STOKES V CONVENTIONS IN RADIO OBSERVING

Reference (1)	IEEE (2)	Stokes V (3)	IAU (4)	Note (5)	Excerpts & Comments (6)
Textbooks					
Pawsey & Bracewell (1955)	Yes	...	...	...	Cite IRE definition.
van de Hulst (1957)	No	$a^2 \sin 2\beta$	No	1	For classical LCP, $\tan\beta = -1$ and $V < 0$.
Piddington (1961)	Yes	...	...	...	Only a description of handedness.
Steinberg & Lequeux (1963)	...	...	...	...	
Kraus (1966)	Yes	$S_L - S_R$	No	1	Cites IRE definition.
Christiansen & Högbom (1969)	...	$B_{\rm rc} - B_{\rm lc}$	Yes	1	Unable to find any convention for RCP or LCP.
Rybicki & Lightman (1979))	No	RCP − LCP[a]	No	1	Use physics convention for handedness; define $V > 0$ for non-IEEE RCP.
Shu (1991)	...	...	...	...	Refrains from taking a stance: "Considerable room for confusion on conventions exists here; so if you need to measure circular polarization, always check on the sign conventions being used by other people."
Stutzman (1993)	Yes	LCP − RCP	No	2	Follows (Kraus 1966).
Rohlfs & Wilson (1996)	Yes	RCP − LCP	Yes	2	Description of circular polarization matches IEEE; statement of Stokes V equivalent to IAU.
Tinbergen (1996)	Yes	$I_{\rm rc} - I_{\rm lc}$	Yes	1	Does no favors by dancing around V issue: "Given the apparently contrary conventions of radio and 'traditional' optical astronomers, I hesitate to recommend either." Points out an IAU convention exists. In Fig. 2.1 caption, adopts IEEE and IAU conventions for circular. States elliptical polarization Jones vector using IEEE convention on p. 59., then on p. 61 defines $V = I_{\rm rc} - I_{\rm lc}$.
Burke & Graham-Smith (1997)	Yes	$RCP - LCP$	Yes	1	Explicitly state both IEEE and IAU conventions.
Jackson (1998)	No	$a_+^2 - a_-^2$	Yes	1	Here a_+^2 corresponds to *positive helicity*, which is LCP in the classical physics convention.
Thompson et al. (2001)	Yes	RCP − LCP	Yes	2	Explicitly state both IEEE and IAU conventions.
Articles					
Cohen (1958)	Yes	$I_l - I_r$	No	1	Cites IRE definition.
Morris et al. (1963)	Yes	L.H. − R.H.	No	1	Cite Pawsey & Bracewell (1955) and radio convention in defining L.H. and R.H.
Barrett & Rogers (1966)	Yes	LCP − RCP	No	1	Cite IEEE definition, but not Stokes V sense; results match Raimond & Eliasson (1969), who used IEEE and defined $V =$ LCP − RCP.
Coles et al. (1968)	Yes	$T(\rm R) - T(\rm L)$	Yes	1	"$T(\rm R)$ and $T(\rm L)$ are the temperatures for right- and left-circular polarization, defined with respect to the direction of propagation."
Ball & Meeks (1968)	Yes	$T(R) - T(L)$	Yes	1	"We consider right circular polarization as rotation of the electric vector in a clockwise sense when viewed along the direction of propagation. This is the standard radio definition, which is opposite to the definition used in optics."
Verschuur (1968)	...	RCP − LCP	...	2	States that "difference between right- and left-hand polarization incident on the feed" is plotted in text and Fig. 1 caption. Incorrectly infers negative fields in Cas A absorption: should be positive.
Davies et al. (1968)	Yes	L − R	No	1	Caption of Fig. 1 states "polarization is plotted as left minus right hand (IRE definition) incident on the telescope." Also, frequent calibration was performed by radiating a circularly polarized CW signal into the horn. Derive a positive field for Cas A.

TABLE 2.4—*Continued*

Reference (1)	IEEE (2)	Stokes V (3)	IAU (4)	Note (5)	Excerpts & Comments (6)
Verschuur (1969a)	···	RCP − LCP	Yes	2,3,4	States Zeeman spectrum is "difference between right- and left-hand polarization." Later formulates spectrum as $LH_j - RH_j$. Then in Fig. 3 caption says V is "right-hand minus left-hand polarization, incident on the dish."
Verschuur (1969b)	···	RCP − LCP	Yes	2,3,4	V plotted as "right-minus-left hand polarization incident on the telescope."
Raimond & Eliasson (1969) ..	Yes	$I_{\rm left} - I_{\rm right}$	No	1	No explicit IEEE mention, but "Right-hand polarization is defined as a clockwise rotation of the electric vector when viewed along the direction of propagation."
Coles & Rumsey (1970)	Yes	$T(R) - T(L)$	Yes	1,3	1665 MHz W49 V spectrum matches that of Ball & Meeks (1968).
Brooks et al. (1971)	···	RCP − LCP	Yes	2,3,4	Used Parkes to meaure "right-hand minus left-hand polarization profile" toward Ori A; observe the same profile as Verschuur (1969b).
Weiler (1973)	Yes	RCP − LCP	Yes	2	Interferometric treatment of WSRT polarization: "$+V$ corresponds to Right Hand Circular Polarization (IRE Definition)."
Conway (1974)	Yes	RCP − LCP	Yes	2	Points out the "deplorable confusion" surrounding sign and terminology of circular polarization work.
IAU (1974)	Yes	RCP − LCP	Yes	2	The original IAU definition: "right-handed" circular = "positive" V.
Crutcher et al. (1975)	···	RCP − LCP	Yes	2	The "Zeeman profile" is "difference between right and left circularly polarized power spectra."
Verschuur (1979)	Yes	$I_{\rm RH} - I_{\rm LH}$	Yes	1	
Crutcher et al. (1981)	···	···	···	···	Computed the "Stokes parameter V spectrum." No definition. Can't be inferred from field signs because no detections were made, only upper limits.
Troland & Heiles (1982a)	···	LCP − RCP	No	2,4	Only mention the "Stokes parameter V spectrum" is the "difference between the line profiles detected in opposite senses of circular polarization." For inferred positive fields in emission lines, V must have been defined as LCP − RCP(i).
Heiles & Troland (1982)	···	RCP − LCP	Yes	2,4	State V is the "difference between right-hand and left-hand circular polarization." Derived positive fields for both emission and absorption consistent with right minus left.
Bregman et al. (1983)	···	LCP − RCP	No	2,3,4	Interferometric Cas A observations show V matching Davies et al. (1968) and Heiles & Troland (2004), therefore V = LCP − RCP. Curious because they use WSRT and cite Weiler (1973), who define RCP − LCP, in calibration discussion.
Crutcher & Kazes (1983)	···	LCP − RCP	No	2,3	V is "the difference between the left and right polarization spectra." If truly LCP − RCP, derived fields should be *positive* for OH absorption in Ori B. Incorrectly infer a negative field.
Heiles & Stevens (1986)	···	RHC − LHC	Yes	1	Only mentioned in figure captions. Measure positive field for Ori B, opposite to Crutcher & Kazes (1983).

TABLE 2.4—*Continued*

Reference (1)	IEEE (2)	Stokes V (3)	IAU (4)	Note (5)	Excerpts & Comments (6)
Troland et al. (1986)	···	LHC − RHC	No	1,4	V sense only listed on plot. Negative field in Ori A OH absorption consistent with 21 cm Zeeman and consistent with V = LHC − RHC. However, they claim they "obtained an H I Zeeman effect spectrum with the Nançay telescope" and that "it is similar to" those of Verschuur (1969b) and Brooks et al. (1971): however, the latter two papers show RHC − LHC and have Stokes V flipped in sense from this result.
Kazes & Crutcher (1986)	···	RCP − LCP	Yes	4	No V definition; must be RCP − LCP for the inferred negative fields in W22 absorption.
Schwarz et al. (1986)........	···	LCP − RCP	No	3,4	Interferometric Cas A observations show V matching Davies et al. (1968), Bregman et al. (1983), and Heiles & Troland (2004), therefore V = LCP − RCP.
Crutcher et al. (1987)........	···	RCP − LCP	Yes	2	Define V as "right minus left circularly polarized power."
Heiles (1987)...............	Yes	RCP − LCP	Yes	2	"There is confusion with regard to signs." Points out IEEE convention. "In Zeeman splitting, if the RHC component is observed at a higher frequency then the magnetic field points towards the observer." This is true for emission lines if V = RCP − LCP.
Heiles (1988)...............	···	LCP − RCP	No	4	Only states "V is the difference between two circular polarizations." Positive field in emission in Fig. 2 consistent with V = LCP − RCP.
Kazes et al. (1988)..........	···	RCP − LCP	Yes	4	No V definition; for derived positive field in S106 absorption, must have plotted RCP − LCP.
Heiles (1989)...............	Yes	LCP − RCP	No	1	"V is the difference between the two circular polarizations, equal to left-hand *minus* right-hand circular (LCP-RCP, in the IEEE definition, in which RCP is defined as a clockwise rotation of the electric vector as seen from the transmitter of the radiation, Kraus 1966). This convention has also been used in several of our previous publications (Troland & Heiles 1982b; Heiles & Troland 1982; Heiles 1988)." This is not true of Heiles & Troland (1982).
Goodman et al. (1989).......	Yes	RCP − LCP	Yes	2,4	"The Stokes V spectrum is a display of the right minus left cicularly polarized signals. The sense of our left and right circular polarizations follows the IEEE definition; that is, right circular polarization is clockwise rotation of the electric vector as seen from the transmitter of radiation."
Fiebig & Güesten (1989)	···	$T_{\rm B}$(RHC) $-T_{\rm B}$(LHC)	Yes	1	Cite Heiles (1987) for definition of V; luckily for them, this is one of the places Heiles states the proper IAU definition.
Güsten & Fiebig (1990)	···	RCP − LCP	Yes	2	V is "difference between the right- and left-handed circularly polarized emission."
Kazes et al. (1991)..........	···	LCP − RCP	No	4	LCP − RCP is inferred from derived field signs.
Heiles et al. (1993)..........	···	RCP − LCP	Yes	2	The Stokes V spectrum is observed "by subtracting the left from the right circular polarization."
Crutcher et al. (1993)........	···	$T_r - T_l$	Yes	1	
Davies (1994)	···	LCP − RCP	No	2,3,4	$B_{\parallel}$ obtained from "left minus right polarization."
Goodman & Heiles (1994)...	···	RCP − LCP	Yes	1,4	
Güesten et al. (1994)........	···	LHC − RHC	No	1	
Verschuur (1995a)	···	RCP − LCP	Yes	2	V plots show "right-hand−left-hand circular

TABLE 2.4—*Continued*

Reference (1)	IEEE (2)	Stokes V (3)	IAU (4)	Note (5)	Excerpts & Comments (6)
Verschuur (1995b)	···	RCP − LCP	Yes	2	V displayed "as right-hand minus left-hand circular polarization."
Hamaker & Bregman (1996)	Yes	RCP − LCP	Yes	2	An authoritative account of IEEE and IAU circular conventions and their use in radio interferometric polarization. Find disturbing result: all interferometric "radio polarimetry work has until now been based on a formula published first by (Morris et al. 1964)." This complex 'black-box' formula, when decoded, yields V = LCP − RCP. Thompson et al. (2001) points out that (Morris et al. 1964) "predates the IAU definition and follows an earlier convention." (The 1986 edition claimed they were in agreement with (Morris et al. 1964).) Weiler (1973) rederived the black-box formula and found that it did not comply with the IAU convention.
Heiles (1996)	Yes	$LCP - RCP$	No	1	"In the present and our previous papers, we follow the definition of Stokes parameters given by Kraus (1966), to wit: (1) We use the IEEE definition, in which left circular polarization (LCP) rotates clockwise as seen by the receiving antenna; and (2) $V =$ LCP − RCP."
Troland et al. (1996)	···	$R - L$	Yes	1,4	Only mention $V = R - L$ in figure captions. Mention sign of ρ Oph detection incorrectly reported as positive in Crutcher et al. (1993).
Crutcher et al. (1996)	···	$T_{\rm R} - T_{\rm L}$	···	1	State that $V \equiv T_{\rm R} - T_{\rm L}$, but inferred field directions from Stokes V spectra are either incorrect or the Stokes V spectra are truly plotted as $T_{\rm L} - T_{\rm R}$.
Crutcher et al. (1999)	···	$L - R$	No	1	State that "middle panels show the observed $V = L - R$ spectrum." However, inferred negative fields for emission features only possible if $V = R - L$.
Crutcher & Troland (2000)	···	LCP − RCP	No	4	V is not defined, but a positive field is inferred for emisison in L1544; V must be plotted as LCP − RCP. Astronomical and ground-based circular polarization calibration was performed.
Uchida et al. (2001)	···	LCP − RCP	No	2	"Stokes-V spectrum (the difference between the left- and right-circularly polarized spectra)."
Heiles (2001)	Yes	$i_{\rm LCP} - i_{\rm RCP}$	No	1	Explicitly follows IEEE convention for LCP. States that Tinbergen (1996) defines V "opposite ours and, also, the conventional one used by radio astronomers." Former is true, latter false.
Bourke et al. (2001)	···	$I_{\rm RCP} - I_{\rm LCP}$	Yes	1	Inferred positive fields Ori B absorption consistent with RCP − LCP.
Heiles (2002)	···	$E^2_{LCP} - E^2_{RCP}$	No	1	Incorrectly labels Stokes V as "the IEEE definition."
Heiles & Troland (2004)	···	LCP − RCP	No	2,3,4	V formed "by subtracting RCP from LCP." Checked polarization sense with a calibration helix.

Notes.—IEEE RCP and LCP conventions are generously assumed for all papers for which no circular polarization convention was stated. (1) Stokes V explicitly defined mathematically; (2) Stokes V definition and conformity with IAU convention are inferred from text, no formulation given; (3) Stokes V definition is inferred from plot of Stokes V and its comparison to the Stokes V plot of an author who has defined Stokes V; (4) Stokes V definition is inferred from plot of Stokes V and the derived sign of a detected magnetic field.

[a]Here, classical RCP is equivalent to the IEEE LCP.

PULSAR ASTRONOMERS vs. THE WORLD

The WORLD considers positive velocies as moving away from us. This is like the IEEE definition of circular polarization; we regard ourselves as the transmitter.

Similarly, the WORLD considers positive MAGNETIC FIELDS as pointing AWAY FROM us.

But...PULSAR ASTRONOMERS???

They define positive magnetic fields as producing positive Faraday Rotation. A positive Faraday rotation results from a magnetic field that points TOWARDS us. So...

PULSAR ASTRONOMERS consider positive magnetic fields as pointing TOWARDS us...opposite to the WORLD view !!!

RADIOASTRONOMICAL FEEDS

Feeds are normally designed to approximate pure linear or circular—known as *native linear* or *native circular*.

Generally speaking, native linear feeds are intrinsically accurate and provide true linear. However, *native circular feeds are less accurate and their exact polarization response is frequency dependent.*

At the *GBT*:

- Feeds below 8 GHz are native linear.
- Feeds above 8 GHz are native circular. For the 8-10 GHz receiver, the response changes from pure circular at 8 GHz to 14% elliptical at 10 GHz.

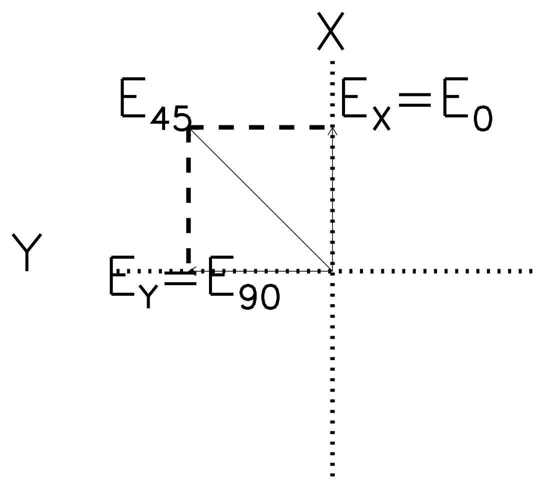
At *ARECIBO*:

- Feeds at 1-2 GHz and 4-6 GHz are native linear. In fact, most feeds are native linear.
- However, a couple are native circular. At Arecibo, circular is achieved with waveguide turnstile junctions, which can be tuned to produce very accurate polarization at the center frequency. However, these are narrow band devices: the feeds become increasingly elliptical, changing to linear and back again over frequency intervals $\sim$ 100 MHz!

REAL RADIO ASTRONOMERS MEASURE ALL STOKES PARAMETERS SIMULTANEOUSLY!

Extracting two orthogonal polarizations provides *all* the information; you can synthesize *all* other E fields from the two measured ones!

Example: Sample (E_X, E_Y) and synthesize E_{45} from (E_X, E_Y):



To generate E_{45}, add (E_X, E_Y) with no phase difference.

To generate E_{LCP}, add (E_X, E_Y) with a 90° phase difference.

CARRYING THROUGH THE ALGEBRA FOR THE TWO LINEARS ...

It's clear that
$$E_{45^\circ} = \frac{E_{0^\circ} + E_{90^\circ}}{\sqrt{2}}$$

$$E_{-45^\circ} = \frac{E_{0^\circ} - E_{90^\circ}}{\sqrt{2}}$$

Write the two linear Stokes parameters:

$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$

$$U = E_{45^\circ}^2 - E_{-45^\circ}^2 = 2E_X E_Y$$

STOKES $\boldsymbol{U}$ IS GIVEN BY THE TWICE THE CROSS-CORRELATION $\boldsymbol{2E_X E_Y}$

To get V, throw a 90° phase factor into the correlation.

DOTTING THE I'S AND CROSSING THE T'S GIVES. . .

Carrying through the algebra and paying attention to complex conjugates and extracting the real part of the expressions yields (for sampling linear polarization (X, Y):

$$I = \langle E_X \overline{E_X} \rangle + \langle E_Y \overline{E_Y} \rangle \equiv \mathbf{XX} + \mathbf{YY} \tag{1a}$$

$$Q = \langle E_X \overline{E_X} \rangle - \langle E_Y \overline{E_Y} \rangle \equiv \mathbf{XX} - \mathbf{YY} \tag{1b}$$

$$U = \langle E_X \overline{E_Y} \rangle + \langle \overline{E_X} E_Y \rangle \equiv \mathbf{XY} + \mathbf{YX} \tag{1c}$$

$$V = i(\langle E_X \overline{E_Y} \rangle - \langle \overline{E_X} E_Y \rangle) \equiv \mathbf{XY} - \mathbf{YX} \tag{1d}$$

The overbar indicates the complex conjugate. These E products are time averages, as indicated by the $\langle\rangle$ brackets.

SIMILARLY FOR NATIVE CIRCULAR POLARIZATION:

We can express the Stokes parameters in terms of any two orthogonal polarizations. Specifically, many radioastronomical feeds, especially for VLBI, provide (approximately) orthogonal circular polarizations E_R and E_L instead linear ones E_X and E_Y. For these, equations 1 become:

$$I = \langle E_R \overline{E_R} \rangle + \langle E_L \overline{E_L} \rangle \equiv \mathbf{RR} + \mathbf{LL} \tag{2a}$$

$$V = \langle E_R \overline{E_R} \rangle - \langle E_L \overline{E_L} \rangle \equiv \mathbf{RR} - \mathbf{LL} \tag{2b}$$

$$Q = \langle E_R \overline{E_L} \rangle + \langle \overline{E_R} E_L \rangle \equiv \mathbf{RL} + \mathbf{LR} \tag{2c}$$

$$U = i(\langle E_R \overline{E_L} \rangle - \langle \overline{E_R} E_L \rangle) \equiv \mathbf{RL} - \mathbf{LR} \tag{2d}$$

The overbar indicates the complex conjugate. These E products are time averages, as indicated by the $\langle\rangle$ brackets.

REMEMBER THIS # 3: CROSSCORRELATIONS BEAT DIFFERENCES!!!

IMPORTANT FACT for NATIVE LINEAR FEEDS::

Stokes I and Q are the sum and difference of *self-products*. These *self-products* are large—equal to the full system temperature—so Q is the difference between two large numbers, and is correspondingly inaccurate.

Stokes U and V are sums and differences of *cross products*. These *cross products* are small—in fact, in the absence of polarization they should equal *ZERO*! So...

For small polarizations (the usual case for spectral lines), *cross* products are *much less subject to error* than self-products.

COROLLARY:

To accurately measure small *linear* polarization, use a dual *circular* feed (for which Q and U are cross products); to accurately measure small *circular* polarization, use a dual *linear* feed (for which V is a cross product).

THE MUELLER MATRIX: 3 SIMPLE EXAMPLES

For a device, such as a dual-polarized feed, the Mueller matrix relates the *output* Stokes parameters S_{out} to the *input* ones S_{in}. Suppose, for example, that a perfect native linear dual-polarized feed has its probes oriented vertically and horizontally. Then its voltage outputs reflect the vertical and horizontal E fields E_X and E_Y and, following equation 1 we can write

$$S_{out} = \mathbf{M} \cdot S_{in}$$

where M is the unitary matrix.

If we rotate that feed by 45°, we have Q and U interchange, together with a sign change as befits rotation:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{3}$$

(3). And for a dual linear feed rotated 90°, which reverses the signs of Q and U:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{4}$$

(4). A perfect native circular dual-polarized feed for which $V = \mathbf{RR} - \mathbf{LL}$:

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} . \tag{5}$$

(5) As an alt-az telescope tracks a source, the feed rotates on the sky by the *parallactic angle* PA_{az} (What's *that*?).

$$\mathbf{M_{SKY}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2PA_{az} & \sin 2PA_{az} & 0 \\ 0 & -\sin 2PA_{az} & \cos 2PA_{az} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{6}$$

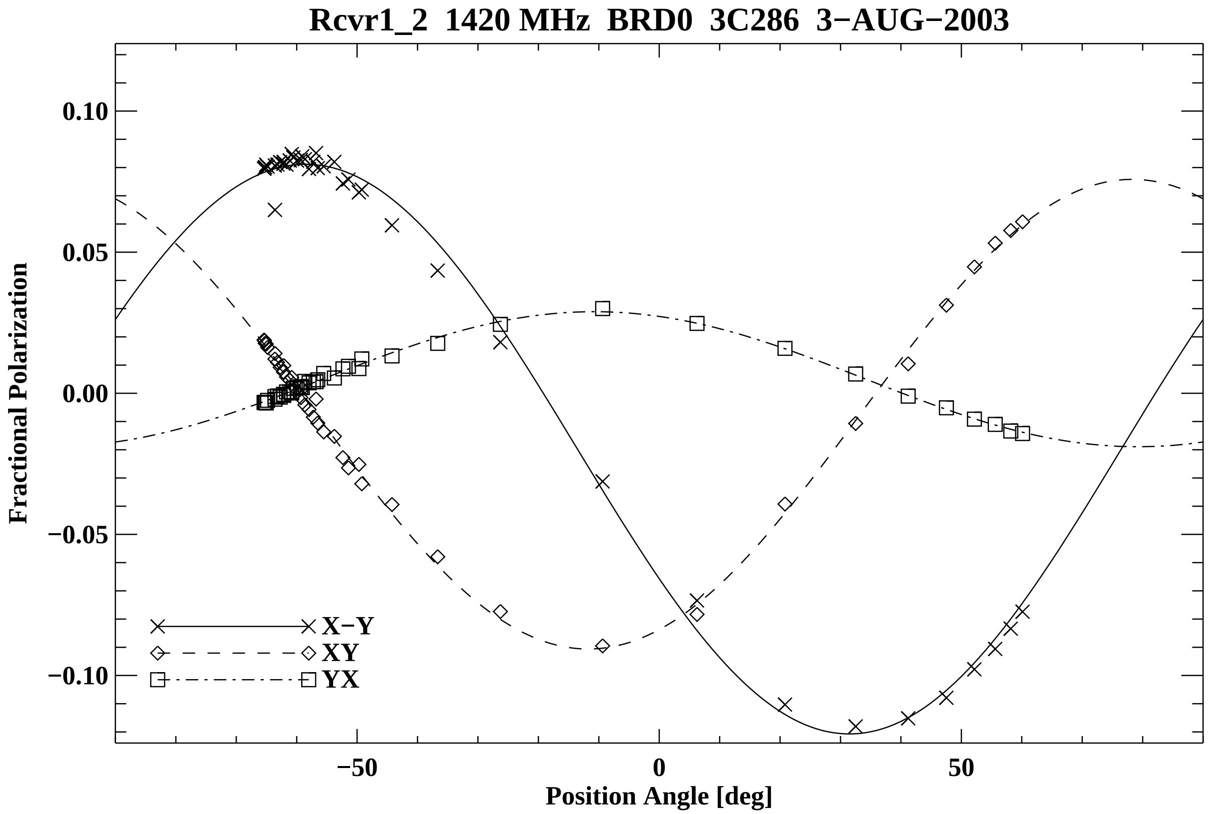
The central 2×2 submatrix is, of course, nothing but a rotation matrix. $\mathbf{M_{SKY}}$ doesn't change I or V.

For a perfect native linear feed oriented such that its Mueller matrix **M** unitary, as we track a linearly polarized source across the sky the parallactic angle PA changes. This should produce:

- $Q \propto [\cos 2(PA_{AZ} + PA_{SRC})]$ centered at zero;
- $U \propto [\sin 2(PA_{AZ} + PA_{SRC})]$ centered at zero;
- $V = 0$ (most sources have zero circular polarization).

The extent to which these idealizations are not realized defines the degree to which the Mueller matrix elements are nonzero.

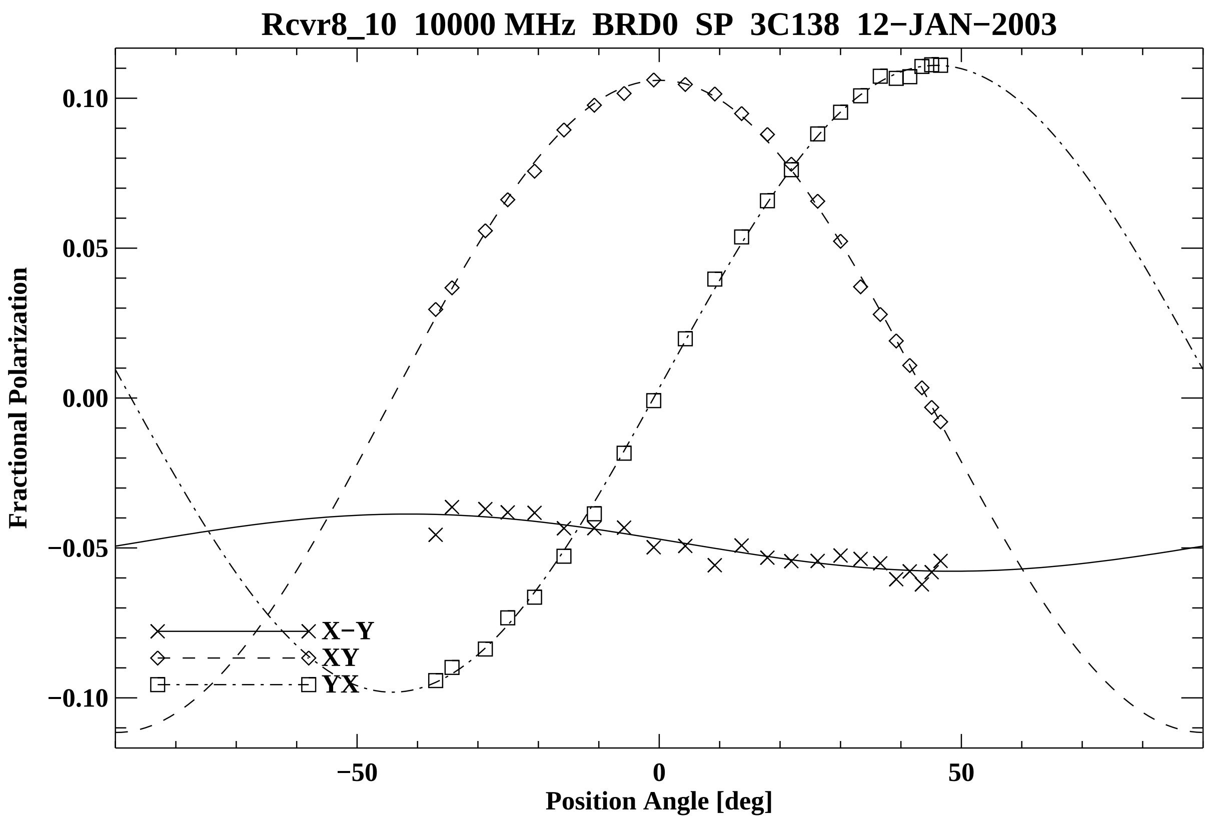
Let's look at some *real data*: first native linear, then native circular.



DELTAG = −0.040 ± 0.016
PSI = −16.0 ± 4.8
ALPHA = +0.2 ± 2.4
EPSILON = 0.004 ± +0.004
PHI = +162.1 ± 50.3
Q_{SRC} = −0.040 ± 0.006
U_{SRC} = −0.085 ± 0.006
POL_{SRC} = +0.094 ± 0.000
PA_{SRC} (UNCORRECTED FOR M_{ASTRO}**) = −57.7 ± 0.0**
NR GOOD POINTS: X−Y = 41 XY = 42 YX = 42 / 42
SCAN 26

Mueller Matrix:

1.0000	−0.0198	−0.0085	0.0026
−0.0198	1.0000	0.0002	0.0075
−0.0074	−0.0021	0.9612	0.2760
0.0050	−0.0073	−0.2760	0.9611



DELTAG = −0.096 ± 0.004
PSI = +0.0 ± 0.0
ALPHA = −47.6 ± 0.5
EPSILON = +0.004 ± 0.001
PHI = +113.5 ± 18.2
Q_{SRC} = −0.002 ± 0.001
U_{SRC} = +0.107 ± 0.001
POL_{SRC} = +0.107 ± 0.001
PA_{SRC} (UNCORRECTED FOR M_{ASTRO}**) = +45.47 ± 0.38**
NR GOOD POINTS: X−Y = 23 XY = 23 YX = 23 / 23

Mueller Matrix:

1.0000	0.0107	−0.0028	0.0475
−0.0482	−0.0903	0.0001	−0.9959
−0.0028	−0.0000	1.0000	0.0000
0.0064	0.9959	0.0000	−0.0900

THE SINGLE MATRIX FOR THE RADIOASTRONOMICAL RECEIVER

The observing system consists of several distinct elements, each with its own Mueller matrix. The matrix for the whole system is the product of all of them. Matrices are not commutative, so we must be careful with the order of multiplication.

$$\mathbf{M_{TOT}} = \begin{bmatrix} 1 & (-2\epsilon \sin\phi \sin 2\alpha + \frac{\Delta G}{2} \cos 2\alpha) & 2\epsilon \cos\phi & (2\epsilon \sin\phi \cos 2\alpha + \frac{\Delta G}{2} \sin 2\alpha) \\ \frac{\Delta G}{2} & \cos 2\alpha & 0 & \sin 2\alpha \\ 2\epsilon \cos(\phi + \psi) & \sin 2\alpha \sin\psi & \cos\psi & -\cos 2\alpha \sin\psi \\ 2\epsilon \sin(\phi + \psi) & -\sin 2\alpha \cos\psi & \sin\psi & \cos 2\alpha \cos\psi \end{bmatrix} .$$

NOTE: The Mueller matrix has 16 elements, but *ONLY 7 INDEPENDENT PARAMETERS*. <u>The matrix elements are not all independent</u>.

$\boldsymbol{\Delta G}$ is the error in relative intensity calibration of the two polarization channels. It results from an error in the relative cal values $(T_{cal A}, T_{cal B})$.

$\boldsymbol{\psi}$ is the phase difference between the cal and the incoming radiation from the sky (equivalent in spirit to $L_X - L_Y$ on our block diagram..

$\boldsymbol{\alpha}$ is a measure of the voltage ratio of the polarization ellipse produced when the feed observes pure linear polarization.

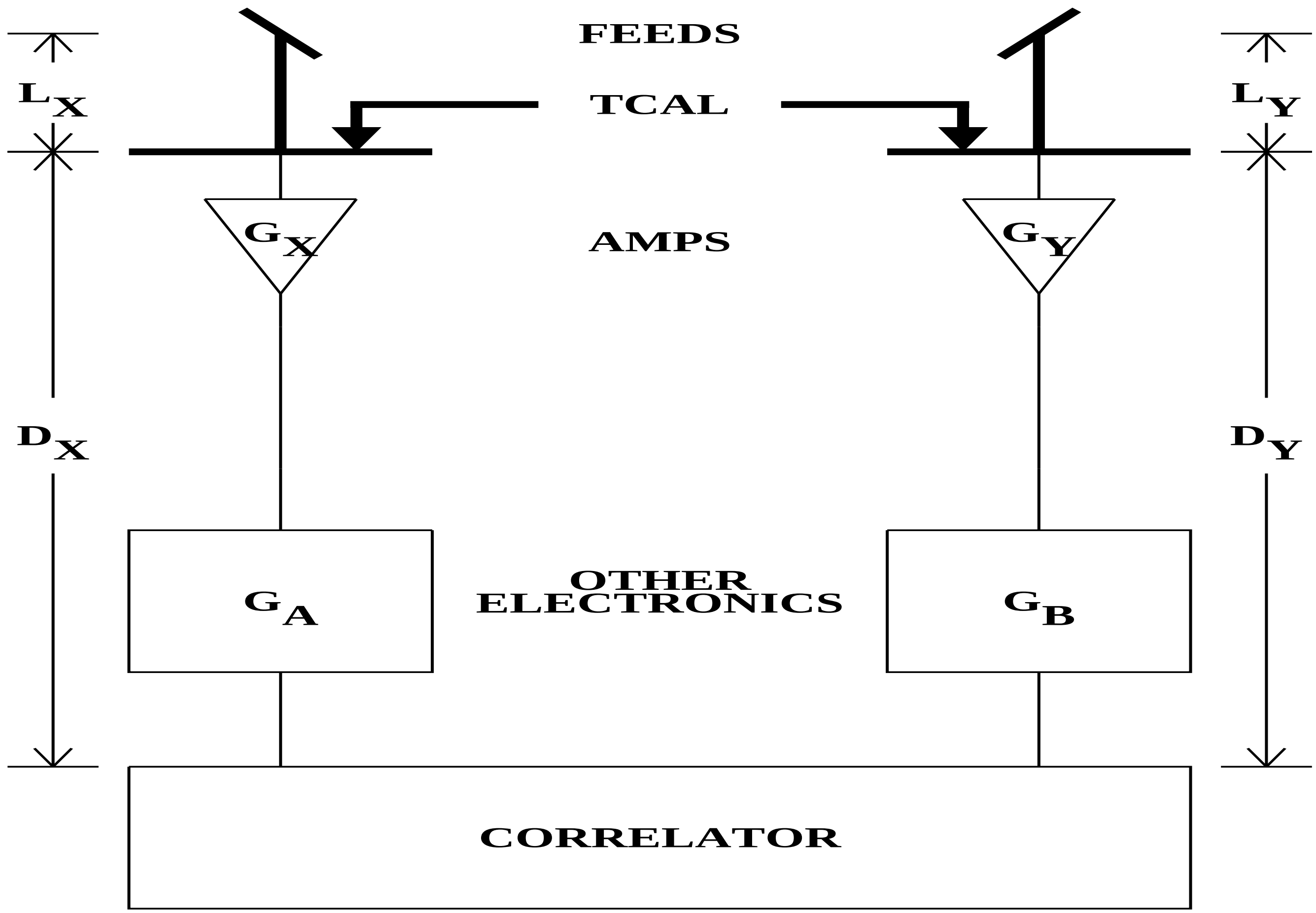
$\boldsymbol{\chi}$ is the relative phase of the two voltages specified by α.

$\boldsymbol{\epsilon}$ is a measure of imperfection of the feed in producing nonorthogonal polarizations (false correlations) in the two correlated outputs.

$\boldsymbol{\phi}$ is the phase angle at which the voltage coupling ϵ occurs. It works with ϵ to couple I with (Q, U, V).

$\boldsymbol{\theta_{astron}}$ is the angle by which the derived position angles must be rotated to conform with the conventional astronomical definition.

THE MUELLER MATRIX REFERS <u>ONLY</u> TO THE THICK CIRCUIT BELOW (RELATES CAL TO SKY)



THE THIN CIRCUIT IS EASIER: NO COUPLING! EACH CHANNEL HAS A <u>GAIN</u> AND <u>PHASE</u>

THE CAL...OUR CONVENIENT INTENSITY AND PHASE REFERENCE

We determine the Mueller matrix elements by comparing the cal deflection with the deflection of an astronomical source of known polarization (i.e., a "polarization calibrator"; the best is 3C286). **THIS CALIBRATES THE THICK CIRCUIT**

The signal and cal share common paths below its injection point (thin circuit). The thin circuit affects both equally. We use the cal to determine the properties of the thin circuit, minute-by-minute (or whatever).

THE THIN CIRCUIT HAS ACTIVE ELEMENTS AND IS THUS TIME-VARIABLE

Amplifier gains are *complex*: **amplitude (GAIN)** and **PHASE**.

The **GAIN** calibration is just like *ordinary nonpolarized observations*.

The **PHASE** calibration. . . It's the **PHASE DIFFERENCE** that matters. Cable lengths introduce phase delays. Cables are never identical! Since

$$\Delta\phi = \frac{2\pi(D_X - D_Y)}{\lambda} \,,$$

THE PHASE DIFFERENCE DEPENDS ON FREQUENCY!

$$\frac{d\Delta\phi}{df} = \frac{2\pi\ \Delta D}{c} \approx 0.3\ \frac{\text{rad}}{\text{MHz}}$$

(At Arecibo; it's almost as large for the GBT). This corresponds to

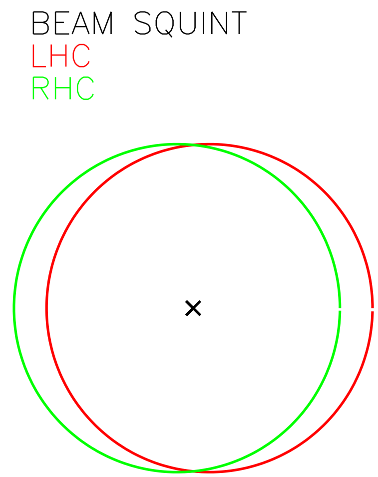
$$\Delta D \approx 20\ \text{m} \quad !!!!!$$

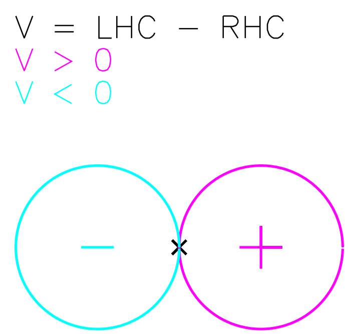
THIS HAS CONSEQUENCES!

- You must include the frequency dependence when you calibrate your data. This is a bit tricky.

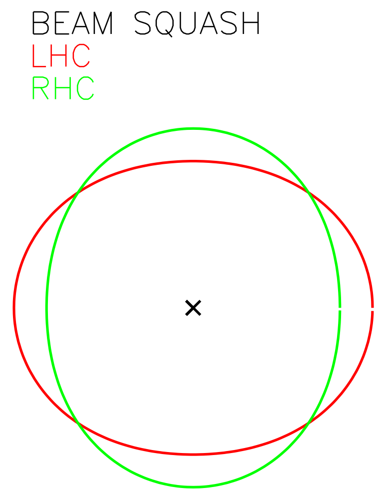
- You cannot do continuum polarization over significant bandwidths without including $\frac{d\Delta\phi}{df}$. In particular, continuum observing usually uses large bandwidth—and the phase can easily wrap over multiple 2π intervals. **DISASTER! EVEN POLARIZED SOURCES WILL APPEAR UNPOLARIZED!!**

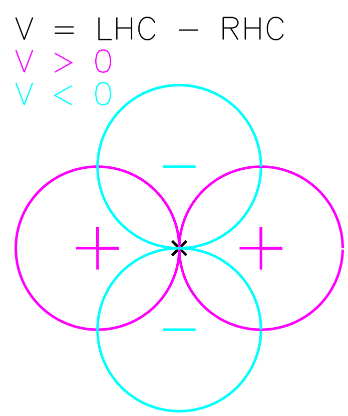
POLARIZED BEAM EFFECTS: BEAM SQUINT

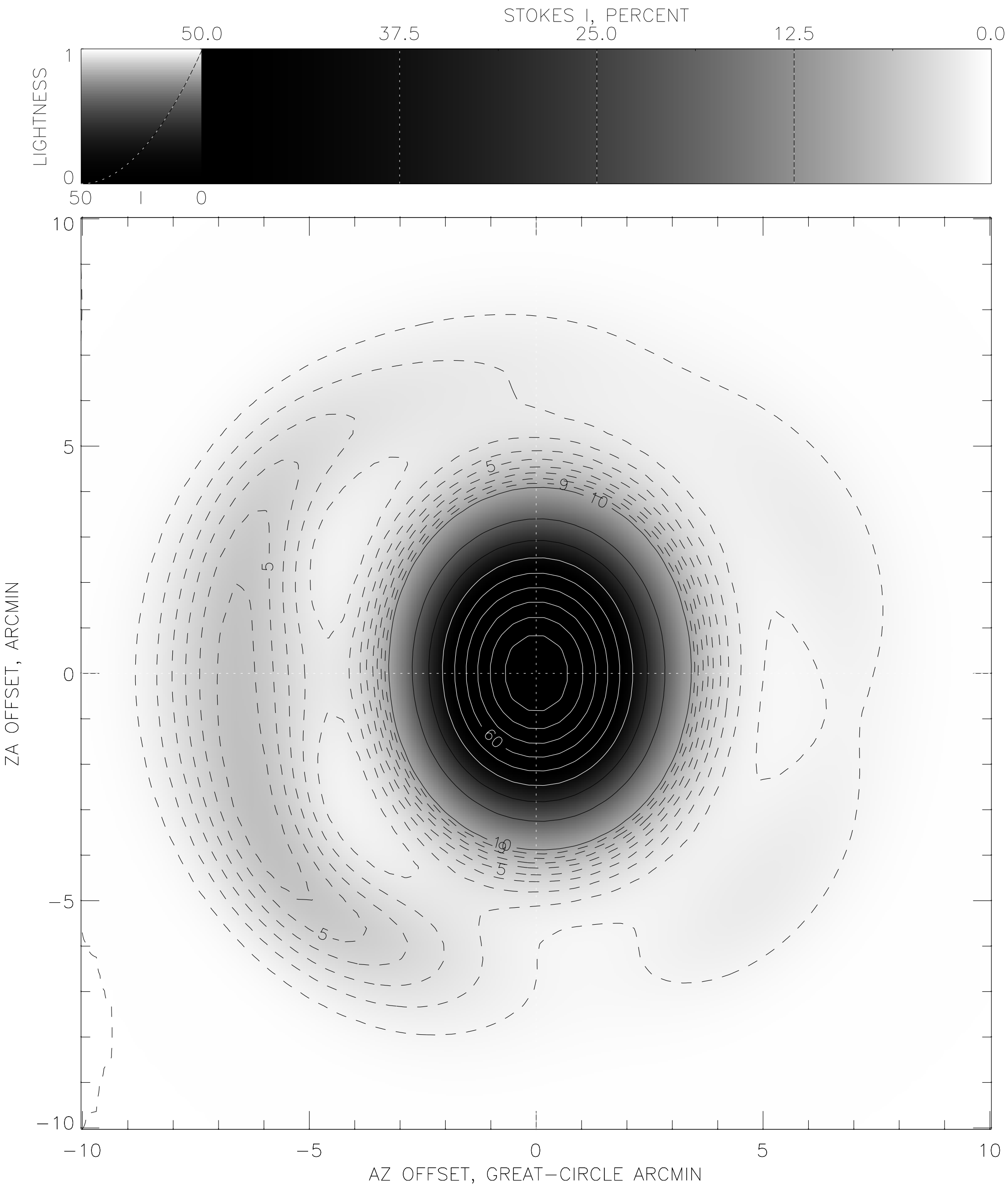


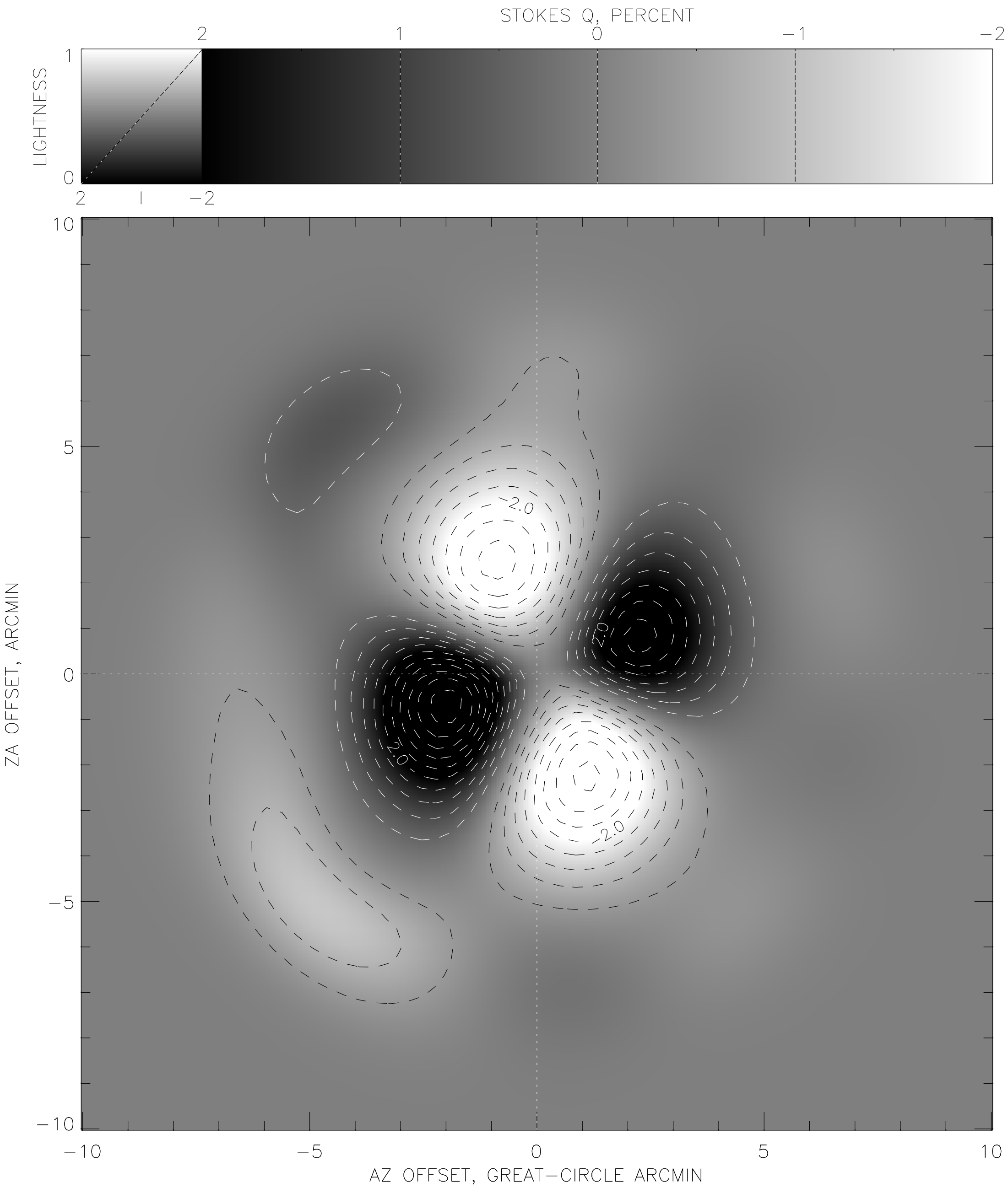


POLARIZED BEAM EFFECTS: BEAM SQUASH






STOKES I, PERCENT
50.0
37.5
25.0
12.5
0.0
LIGHTNESS
1
0
50
I
0
ZA OFFSET, ARCMIN
AZ OFFSET, GREAT−CIRCLE ARCMIN
10
5
0
−5
−10
−10
−5
0
5
10
5
9
10
60


STOKES Q, PERCENT
2
1
0
−1
−2
LIGHTNESS
1
0
2
I
−2
ZA OFFSET, ARCMIN
AZ OFFSET, GREAT−CIRCLE ARCMIN
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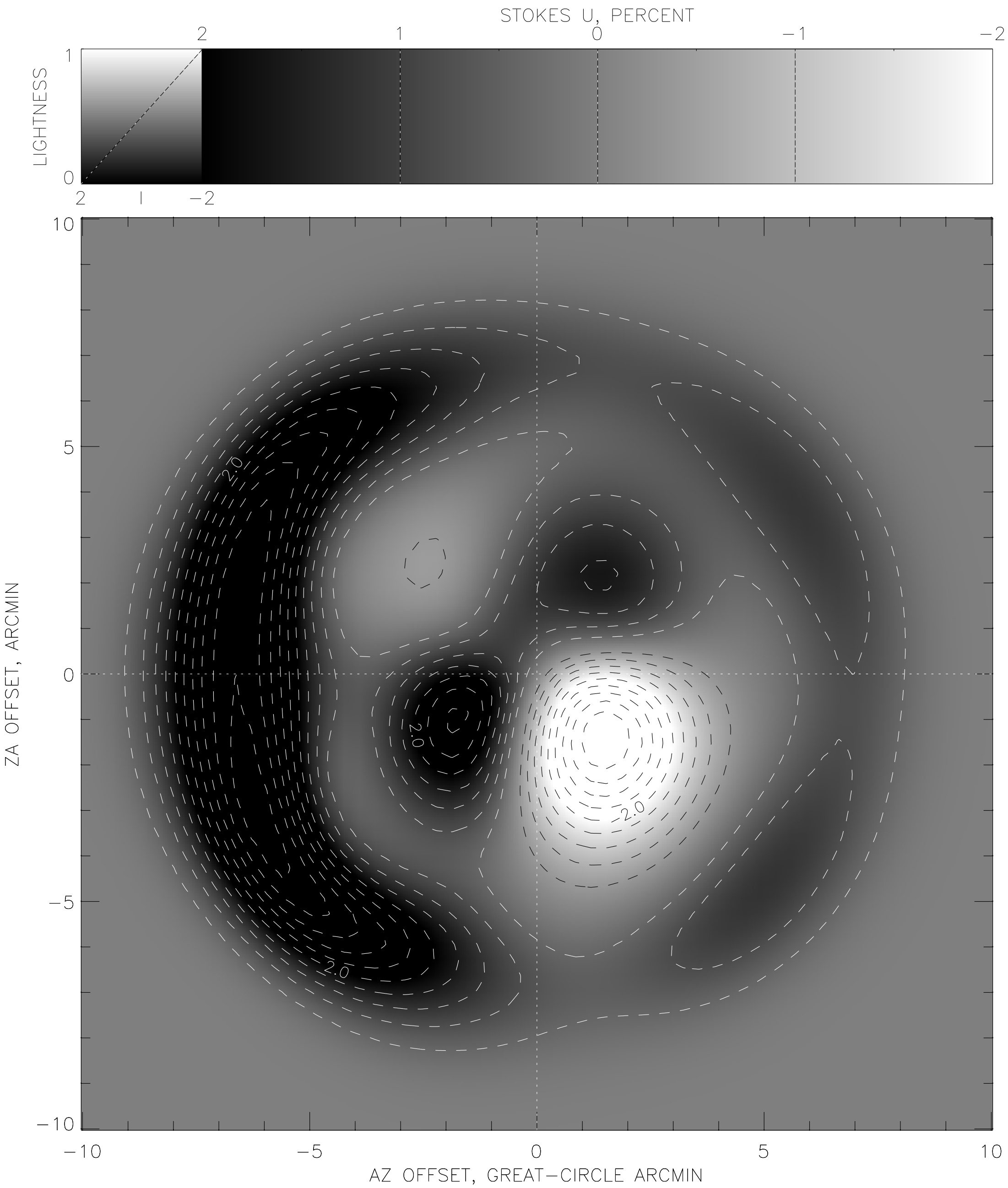
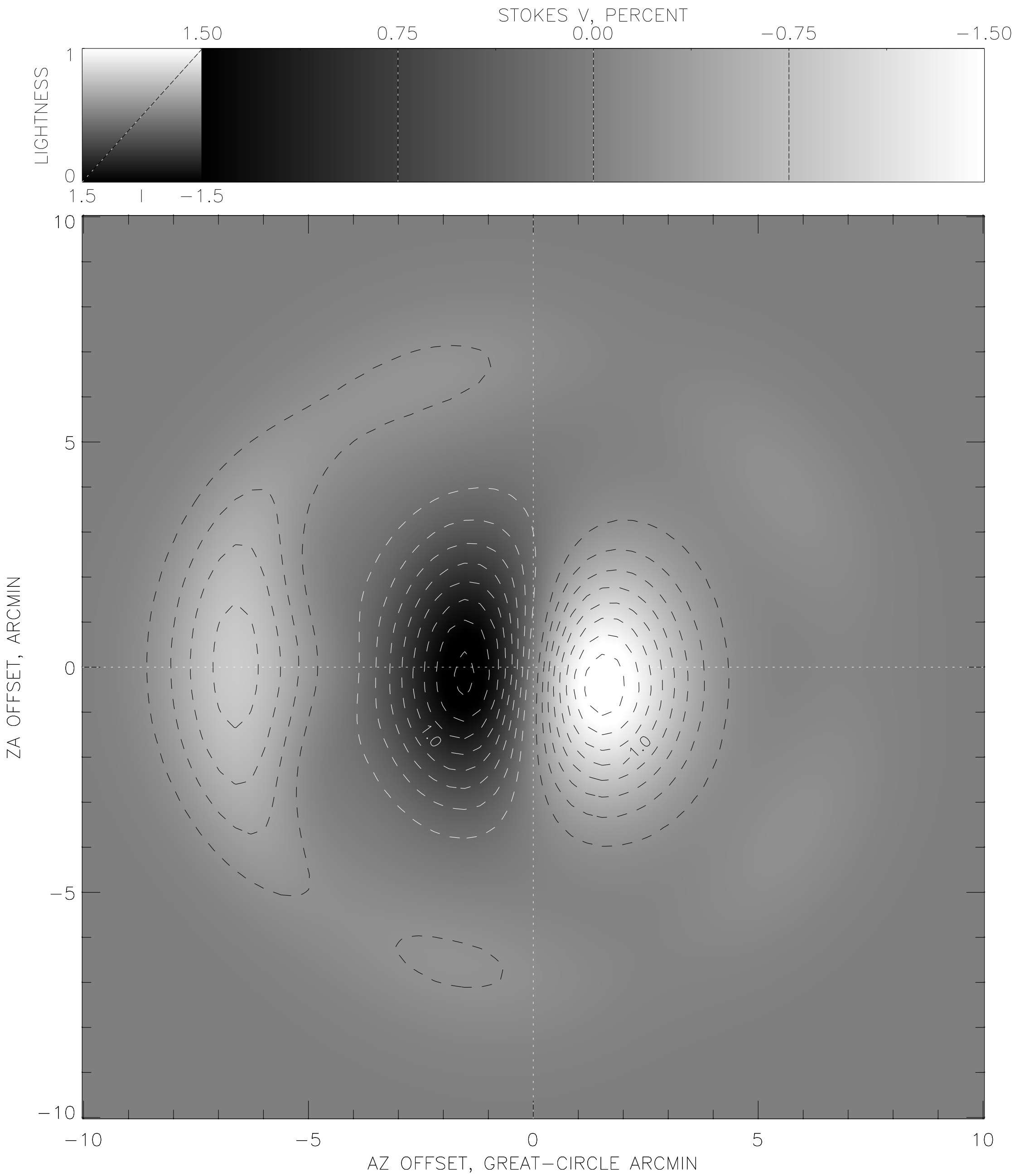
5
0
−5
−10
−10
−5
0
5
10
2.0
2.0
2.0
2.0


STOKES U, PERCENT
2
1
0
−1
−2
LIGHTNESS
1
0
2
I
−2
ZA OFFSET, ARCMIN
10
5
0
−5
−10
AZ OFFSET, GREAT−CIRCLE ARCMIN
−10
−5
0
5
10
2.0


STOKES V, PERCENT
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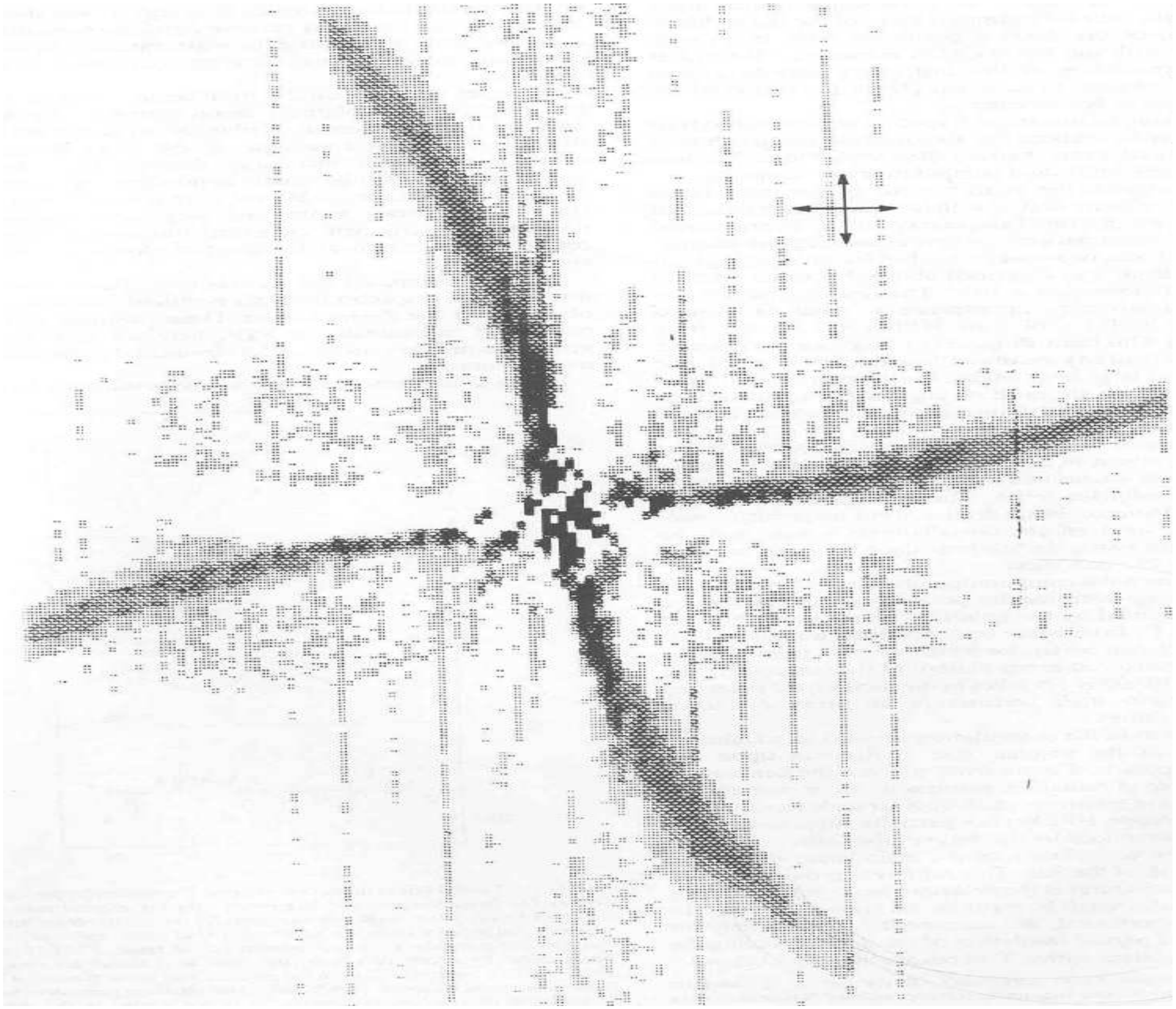
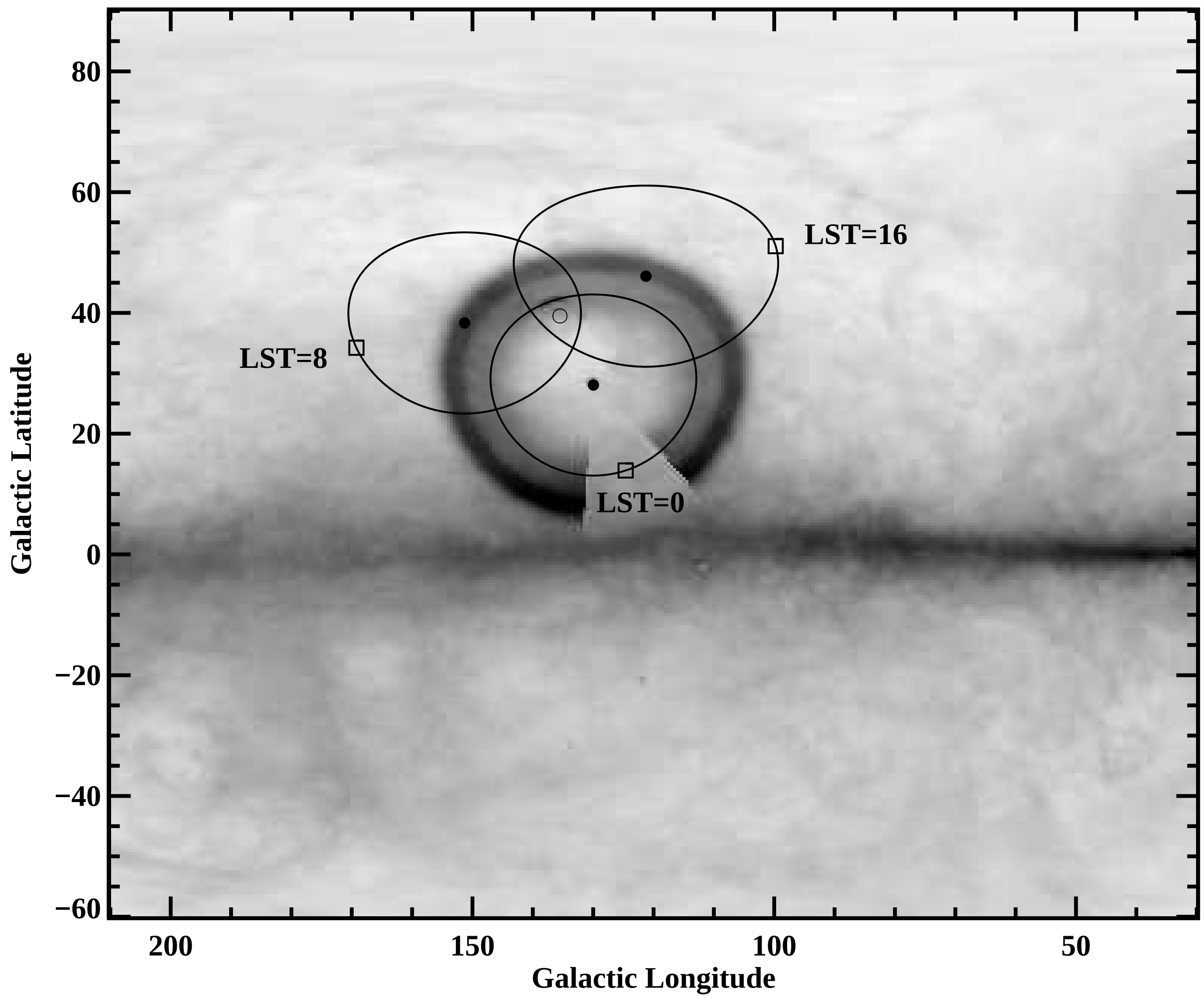
0.75
0.00
−0.75
−1.50
LIGHTNESS
1
0
1.5
I
−1.5
ZA OFFSET, ARCMIN
10
5
0
−5
−10
1.0
1.0
AZ OFFSET, GREAT−CIRCLE ARCMIN

POLARIZED BEAM EFFECTS: DISTANT SIDELOBES



(Stokes V from the Hat Creek 85-footer. Image is $120^\circ \times 120^\circ$)

Even the GBT is not sidelobe-free. Here's an approximate image of the secondary spillover in Stokes I—and there are also serious near-in lobes. All are *highly polarized!!!* (Robishaw & Heiles 2009, PASP, 121, 272)



THE EFFECT ON ASTRONOMICAL POLARIZATION MEASUREMENTS

Large-scale features have spatial structure of *Stokes I*. Sidelobes in Stokes Q, U, and V see this structure. The polarized beam structure interacts with the Stokes I derivatives to produce **FAKE RESULTS** in the *polarized* Stokes parameters (Q, U, V).

Correcting for these effects is a complicated business. First, you have to measure them; they are weak, so this is difficult. (At the GBT, Robishaw and Heiles (2009) used the Sun.) They may well be time variable, particularly at Arecibo where the telescope geometry changes as the telescope tracks. Finally, the polarized sidelobes rotate on the sky as the parallactic angle changes—and distant sidelobes might see the ground instead of the sky.

IT'S REALLY HARD TO ACCURATELY MEASURE POLARIZATION OF EXTENDED EMISSION!!

MY WEBSITE: SOME DOCUMENTATION.

You might find my website useful; it contains instructional handouts and practical IDL software.

`http://astro.berkeley.edu/~heiles/`

It has sections on (a partial list):

- *Radio Astronomical Techniques and Calibration* [specific intensity, spectral lines, polarization, characterizing the telescope beam (including "Spider scans"), LSFS (Least-Squares Frequency Switching)]
- *Handouts on Numerical Analysis* [Least squares, Fourier, Wavelets]
- *Principles of Imaging and Projections* [Four tutorials, including use of color]
- *IDL Procedures and Instructional Handouts* [Introductory tutorial; datatypes]
- Downloading my set of IDL procedures

Tim Robishaw and I, with a lot of help from Amanda Kepley, have developed a package called RHSTK ('Robishaw and Heiles Stokes'). We have melded this package into `gbtidl` and, to a lesser extent, Arecibo. For several years we have been generating two coherent practical writeups of this software.