Wide-Band wide-field Imaging – sky domain algorithms

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Outline :

- What is wideband imaging ?
- Cube imaging / Multi-frequency synthesis
- Frequency-dependent primary beam correction
- Many examples of what works when

Wide-band Imaging Sensitivity

Frequency Range :	(1 –2 GHz)	(4 –8 GHz)	(8 –12 GHz)
Bandwidth : $\nu_{max} - \nu_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : $\nu_{max} : \nu_{min}$	2 : 1	2 : 1	1.5 : 1
Fractional Bandwidth : $(\nu_{max} - \nu_{min}) / \nu_{mid}$	66%	66%	40%

Broad-band receivers increase the 'instantaneous' imaging sensitivity of an instrument

Continuum sensitivity :
(at field-center)

$$\sigma_{cont} \propto \frac{T_{sys}}{\sqrt{N_{ant}(N_{ant}-1)\ \delta\tau\,\delta\nu}}$$

50 MHz → 2 GHz   Theoretical sensitivity improvement : $\sqrt{\frac{2\,GHz}{50\,MHz}} \approx 6$ times.

In practice, effective broadband sensitivity for imaging depends on bandpass shape, data weights, and regions of the spectrum flagged due to RFI (radio-frequency interference).

Use narrow-band channels – avoid bandwidth smearing

In the early days of continuum-observing, only one visibility was computed across the entire bandwidth of the receiver, and attributed to the reference (or middle) frequency ν_0. Delay-tracking was also done only at ν_0 .

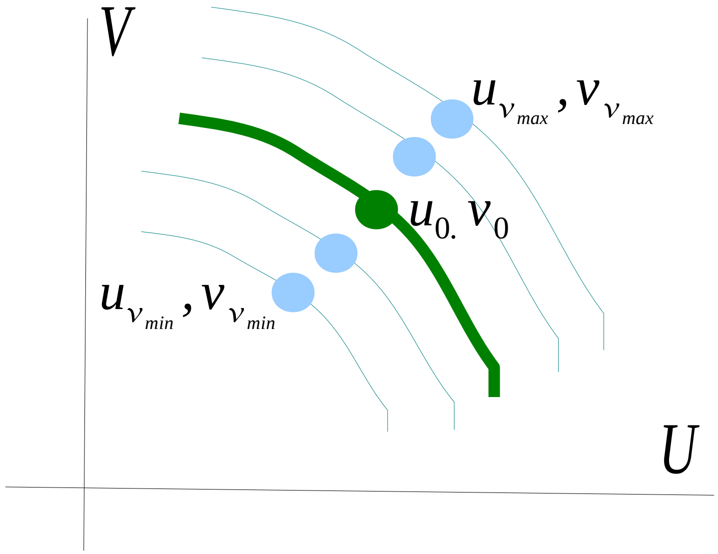
The visibility $V(u_\nu)$ is mistakenly mapped to $u_0 = \frac{b\,\nu_0}{c} = \frac{\nu_0}{\nu} u_\nu$

Similarity theorem of Fourier-transforms :

=> A radial shift in the source position, with frequency.

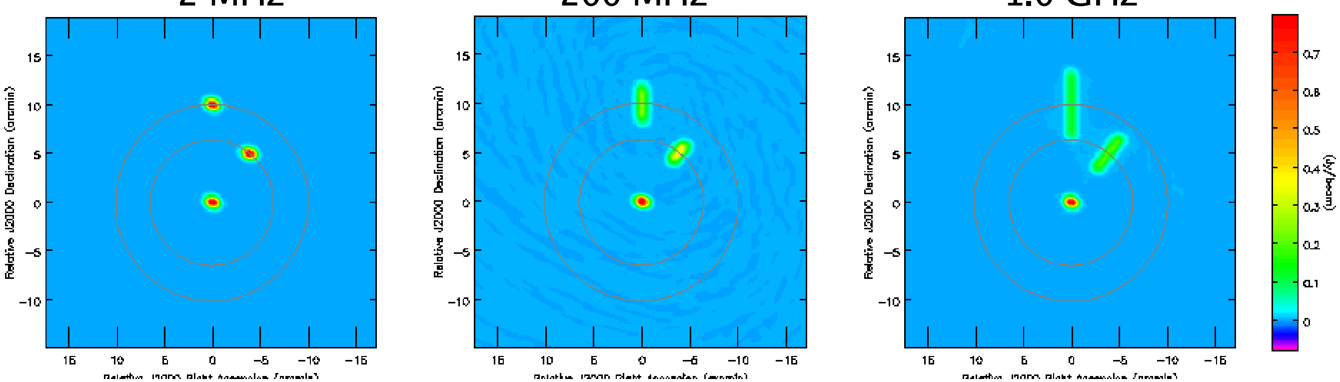
=> Radial smearing of the brightness-distribution

Note : Excessive channel-averaging has a similar effect.



An (exaggerated) example of bandwidth-smearing with a 1-2 GHz signal......

2 MHz 200 MHz 1.0 GHz



Bandwidth Smearing Limits at 1.4 GHz

33 MHz (VLA D-config),
10 MHz (VLA C-config),
3 MHz (VLA B-config),
1 MHz (VLA A-config)

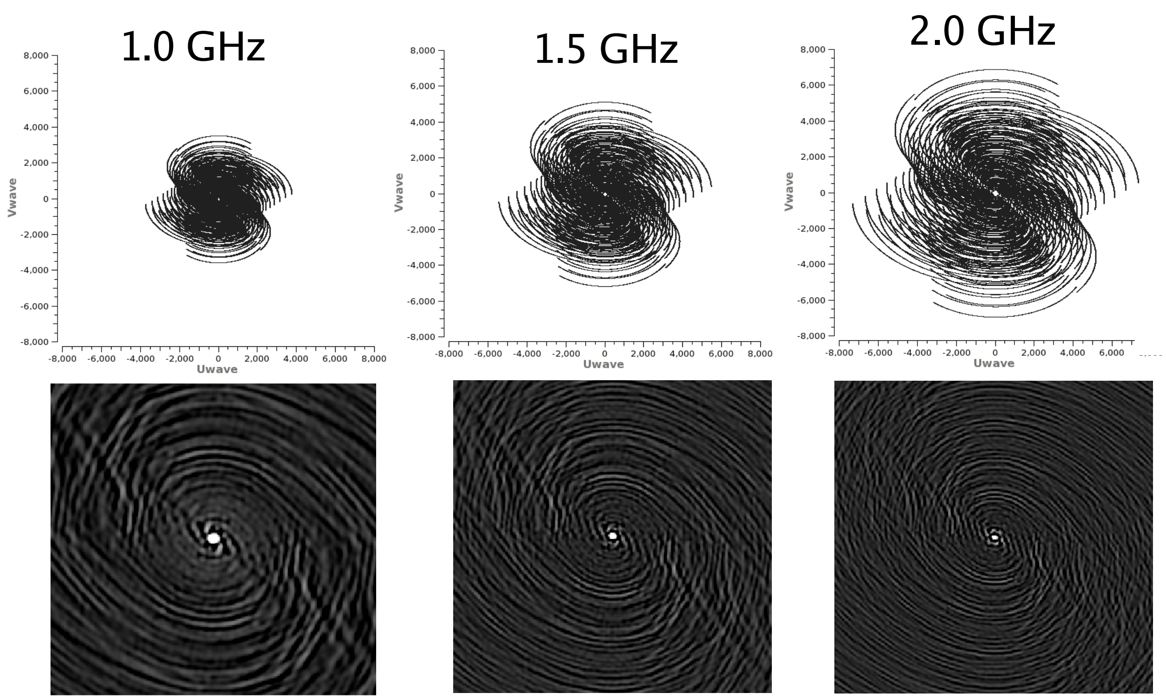
Contours represent 5 and 10 arcmin distances from the phase-center.

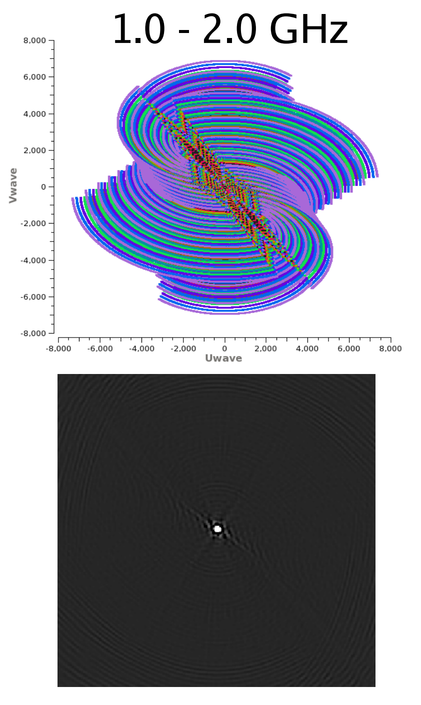
Frequency-dependent UV-coverages and PSFs

Spatial-frequency coverage and imaging properties change with frequency :

- Angular-resolution increases at higher frequencies
- Sensitivity to large scales decreases at higher frequencies
- Wideband UV-coverage has fewer gaps => lower Psf sidelobe levels

$$S(u,v)_\nu = \frac{\vec{b}}{\lambda} = \frac{\vec{b}\,\nu}{c}$$



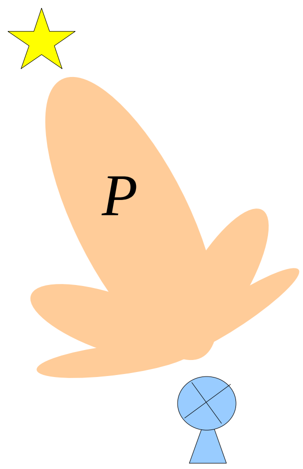


But, when the source intensity varies with frequency, different channels measure the visibility function of different sky-brightness distributions

$$V(u_\nu, v_\nu) = \iint I(l,m,\nu)\, e^{2\pi i (u_\nu l + v_\nu m)}\, dl\, dm$$

=> Need to model the spectrum as part of the image reconstruction

Frequency Dependent Antenna response (Primary-Beam)



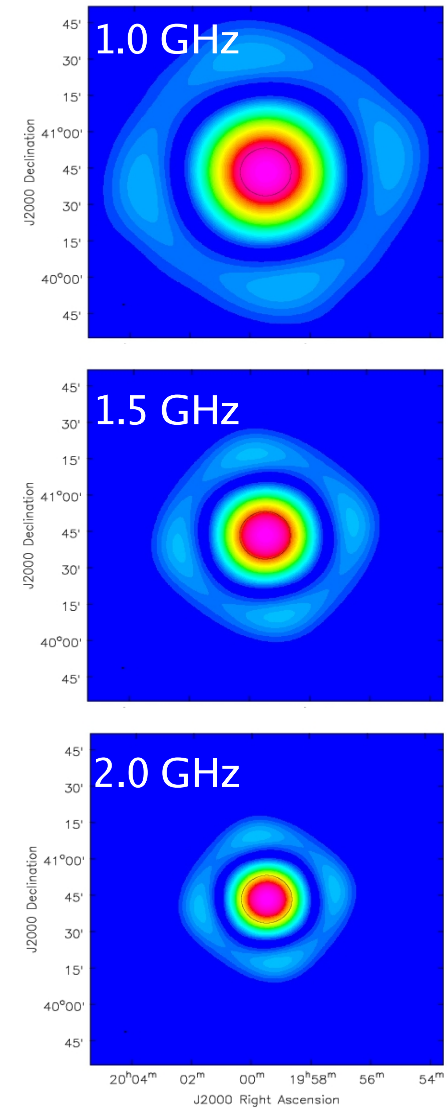
Primary-beam scales with frequency

$$HPBW_{\nu} = \frac{\lambda}{D} = \frac{c}{\nu D}$$

Bandpass calibration does **not** correct for off-axis gains or their frequency-dependence.

The average effect in the image-domain is a multiplication by an artificial PB-spectrum

EVLA Primary Beams



=> Away from the pointing center, the Primary Beam introduces an artificial 'spectral index' on the measured sky : $\alpha_{observed} = \alpha_{sky} + \alpha_{PB}$

About -0.4 at the PB=0.8 (6 arcmin from the center for VLA 1.5 GHz)
About -1.4 at the HPBW (15 arcmin from the center for VLA 1.5 GHz)

What is wide-band Imaging ?

– Use broad-band receivers to increase instantaneous continuum sensitivity

-- Measure visibilities in many narrow-band channels to avoid bandwidth-smearing

-- Use multi-frequency-synthesis

--- to increase the uv-coverage used in deconvolution and image-fidelity
--- to make images at the angular-resolution allowed by the highest frequency

– Account for the sky spectrum

--- by modeling and reconstructing the spectrum as well as the intensity
--- by flattening it out (bandpass self-calibration) –if there is only one source...

– Account for the frequency-dependent off-axis gains of the antennas

--- by including the PB-spectrum in the sky-spectrum model
--- by applying wide-field imaging techniques to eliminate the PB frequency dependence during imaging (gridding).

Wideband Imaging Options

(1) Make images for each channel / SPW separately.

- Signal-to-noise ratio : one SPW
- Angular resolution varies with SPW (smooth to lowest)
- Imaging fidelity may change across SPWs
- Primary beam correction can be done per SPW

Cube imaging will suffice for sources with simple spatial structures, and where the added uv-coverage, sensitivity and angular resolution is not required for the target science.

(2) Combine all frequencies during imaging (MFS : multi-frequency synthesis)

- Signal-to-noise ratio : all SPWs
- Angular resolution is given by the highest frequency
- Imaging fidelity is given by the combined uv-coverage
- Wideband PB correction is required (average gain and spectrum)

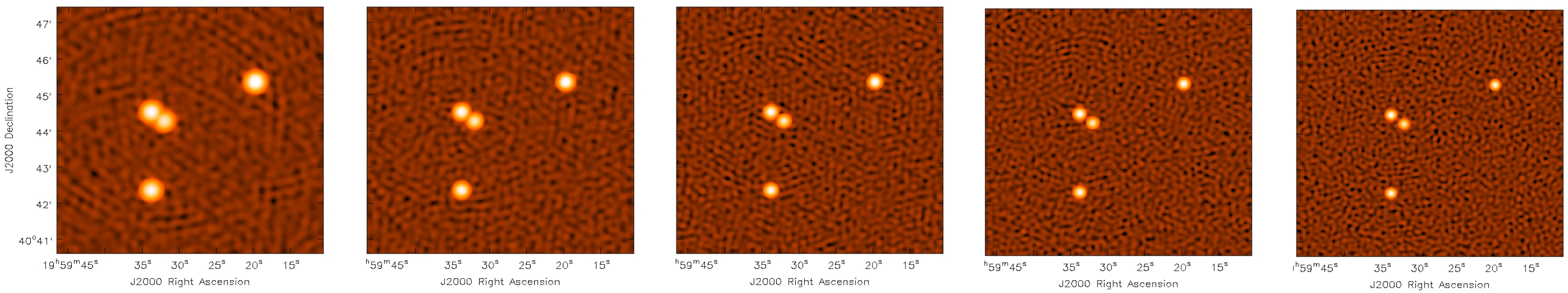
Multi-frequency-synthesis is needed to fully utilize the wideband uv-coverage and sensitivity during image reconstruction.

The frequency dependence of the sky and instrument must be taken into account

Single-channel vs MFS imaging – Angular Resolution

Simulated Example : 3 flat-spectrum sources + 1 steep-spectrum source (1-2 GHz)

Images made separately at different frequencies between 1 and 2 GHz

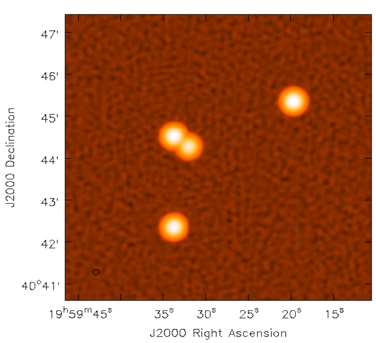


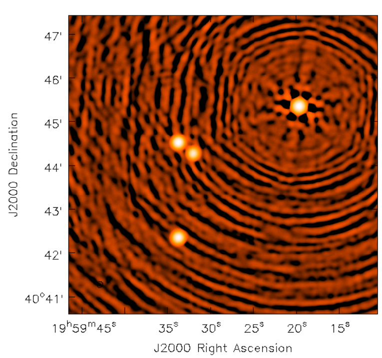
Combine all single-frequency images (after smoothing)

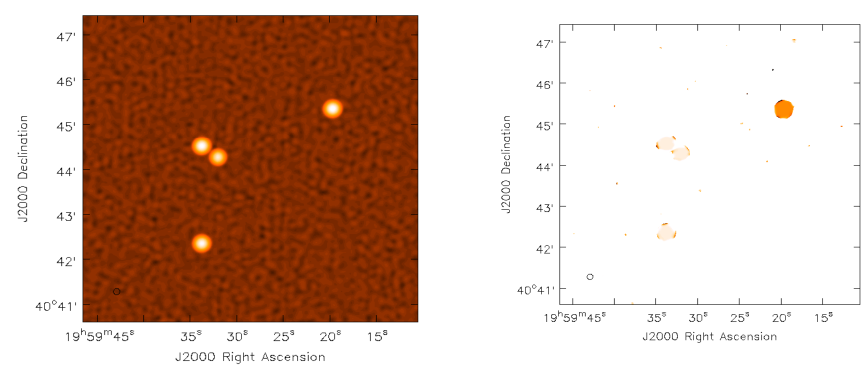
Use all UV-coverage together, but ignore spectra

Use all UV-coverage together
+ Model and fit for spectra too

Output : Intensity and Spectral-Index







=> Imaging with a spectrum model : higher angular resolution + continuum sensitivity.

Continuum Imaging : (multi-scale) multi-frequency-synthesis

Sky Model : Collection of multi-scale flux components whose amplitudes follow a polynomial in frequency

$$I_\nu^{sky} = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t \quad \text{where} \quad I_t = \sum_s [I_s^{shp} * \underline{I_{s,t}}]$$

Algorithm : Linear least squares + deconvolution

Parameters : mode='mfs', nterms=2, reffreq='1.5GHz', multiscale=[0,6,10]

Data Products : Taylor-Coefficient images $I_{0,}^m I_{1,}^m I_{2,}^m \ldots$ that represent the observed spectrum

Interpretation :

- As a power-law (spectral index and curvature) $I_\nu = I_{\nu_0} \left(\frac{\nu}{\nu_0} \right)^{\alpha + \beta \log(\nu/\nu_0)}$

$$I_0^m = I_{\nu_0} \qquad I_1^m = I_{\nu_0} \alpha \qquad I_2^m = I_{\nu_0} \left(\frac{\alpha(\alpha-1)}{2} + \beta \right)$$

- PB-correction : Model the average PB-spectrum with a Tayor-polynomial, and do a post-deconvolution Polynomial-Division

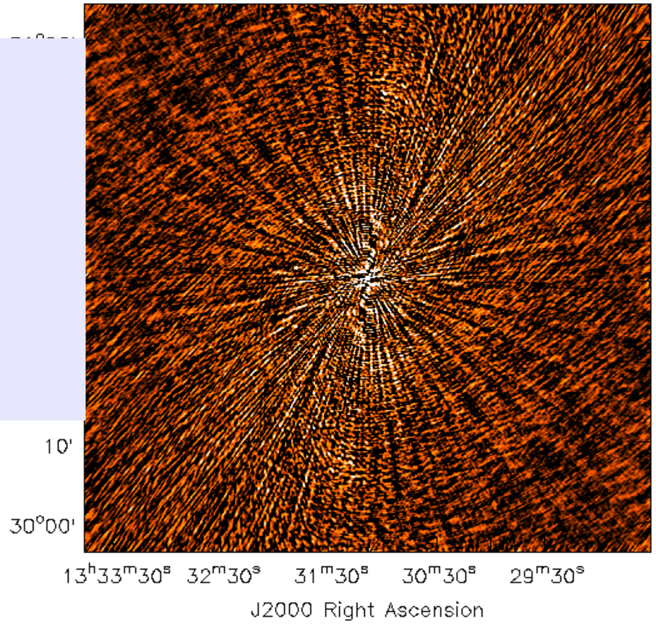
$$\frac{(I_{0,}^m I_{1,}^m I_{2,}^m \ldots)}{(P_{0,} P_{1,} P_{2,} \ldots)} = (I_{0,}^{sky} I_{1,}^{sky} I_2^{sky} \ldots)$$

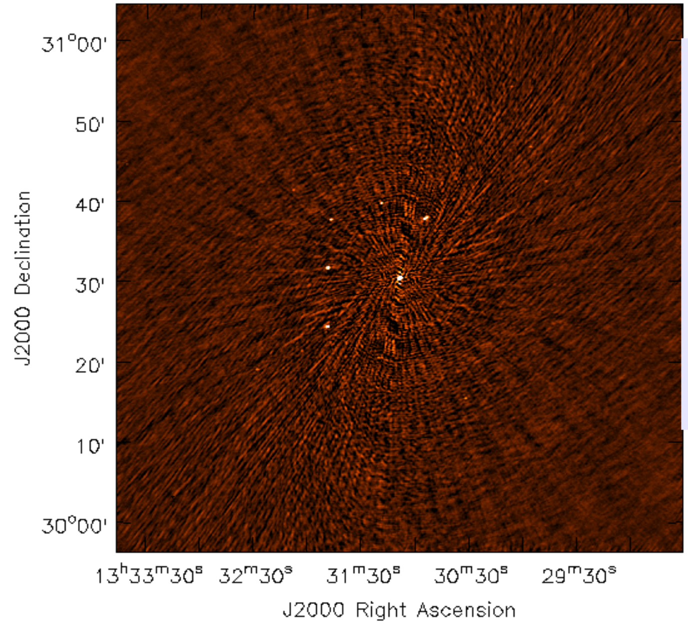
Dynamic-range with MS-MFS : 3C286 example : Nt=1,2,3,4

NTERMS = 1

Rms :
9 mJy -- 1 mJy

DR :
1600 -- 13000





NTERMS = 2

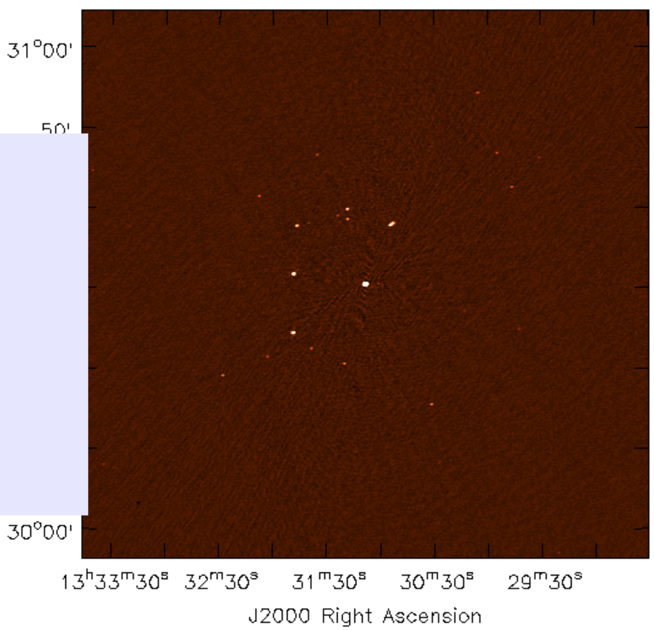
Rms :
1 mJy -- 0.2 mJy

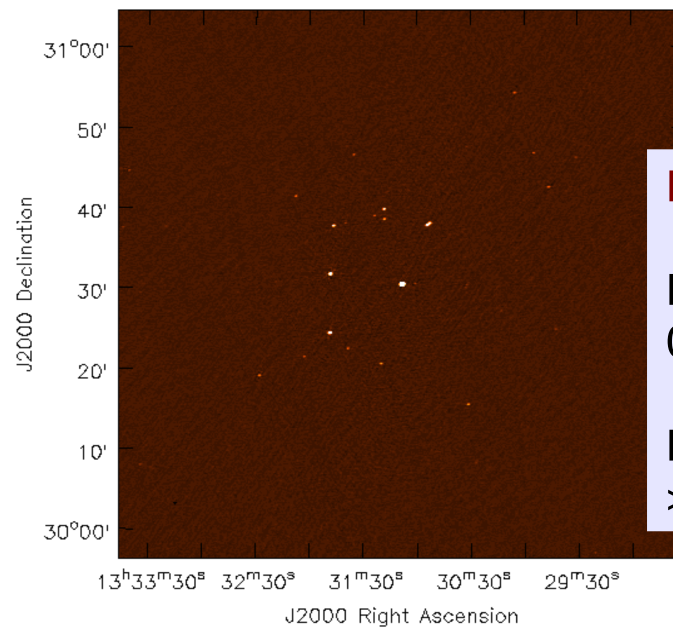
DR :
10,000 -- 17,000

NTERMS = 3

Rms :
0.2 mJy -- 85 uJy

DR :
65,000 -- 170,000





NTERMS = 4

Rms
0.14 mJy -- 80 uJy

DR :
>110,000 -- 180,000

Errors in polynomial fitting + Imaging (empirical)

For a 1 Jy point source with spectral index of -1.0 ...

– If spectra are ignored during MFS imaging => Errors increase with bandwidth.

Dynamic-range limits for VLA uv-coverage (natural)

1-2 GHz => ~ 1000
1-3 GHz => few 100

– If spectra are modeled + High signal-to-noise => Need higher-order polynomials to fit a power-law

1 term (flat spectrum) => peak intensity error of 0.1 (on 1 Jy)
2 terms (linear spectrum) => peak intensity error of 0.02, spectral index error of 0.1
3 terms (quadratic spectrum) => intensity error of 0.0001, spectral index error of 0.05

– If spectra are modeled + Low signal-to-noise => Higher-order polynomials give more errors

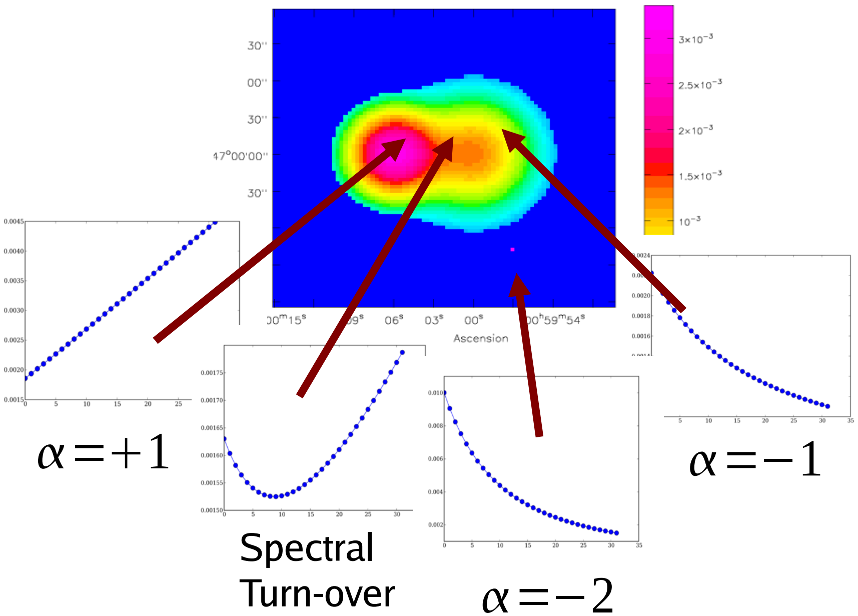
The following situations give similar error on spectral index (~ 0.1) for a point source....

L-Band + C-Band : 1-8 GHz : Sources with signal-to-noise ratio of 10~20
L-Band only (1-2 GHz) or C-Band only (4-8 GHz) : Sources with SNR ~ 40

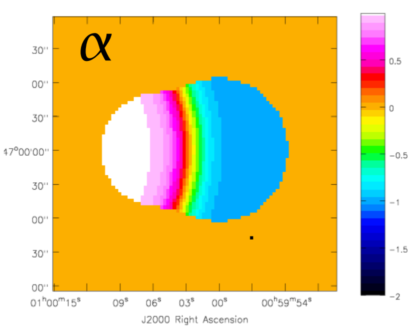
For extended emission, spectral index errors <= 0.2 only for SNR > 100......

Example of wideband-imaging on extended-emission

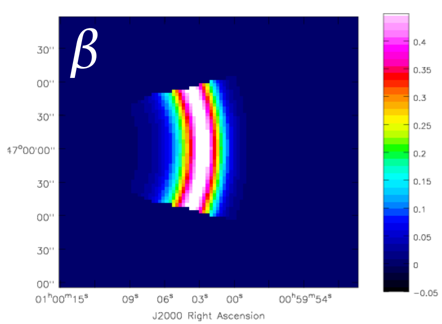
Intensity Image



Average Spectral Index

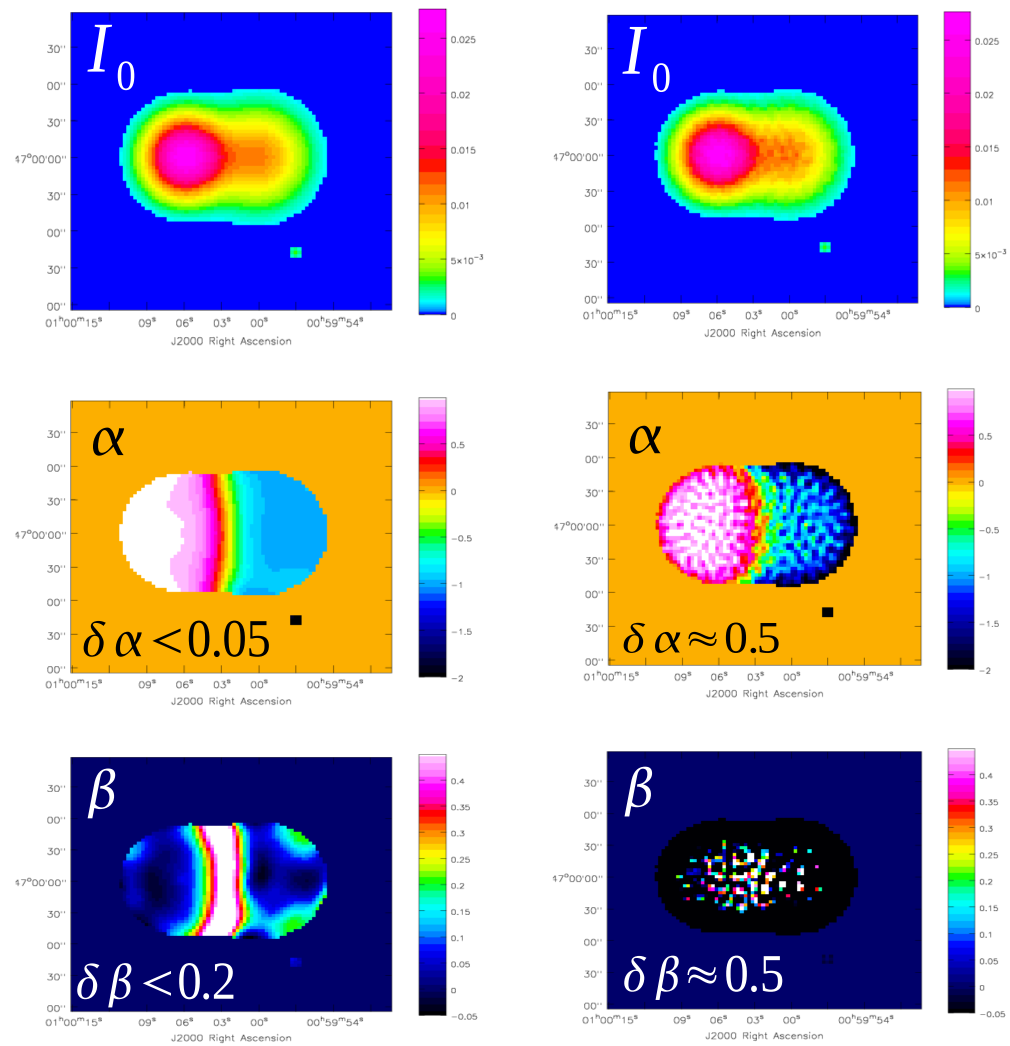


Gradient in Spectral Index



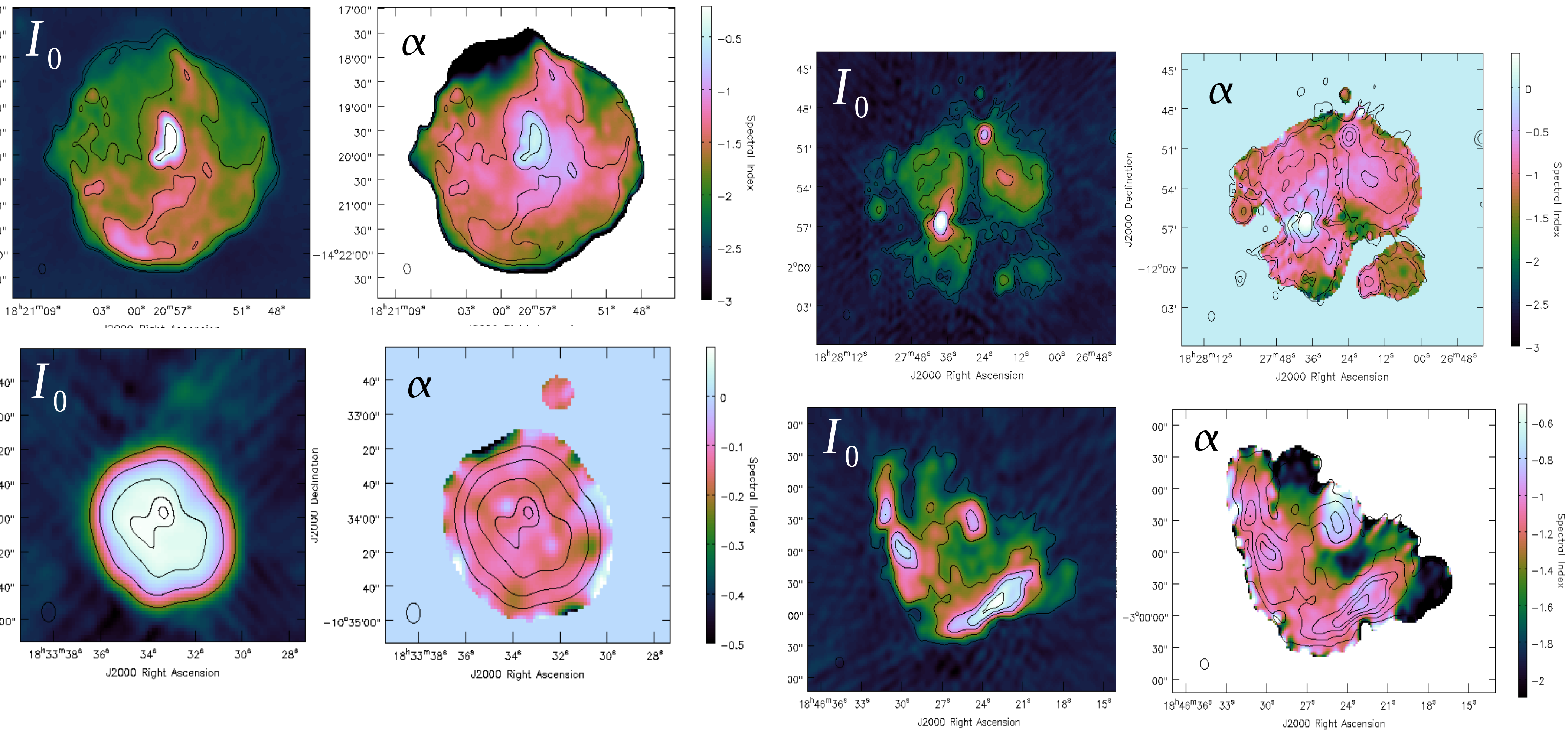
MFS

multi-scale **point-source**



=> For extended emission - spectral-index error is dominated by 'division between noisy images'
– a multi-scale model gives better spectral index and curvature maps

Extended emission – SNR example (a realistic expectation)



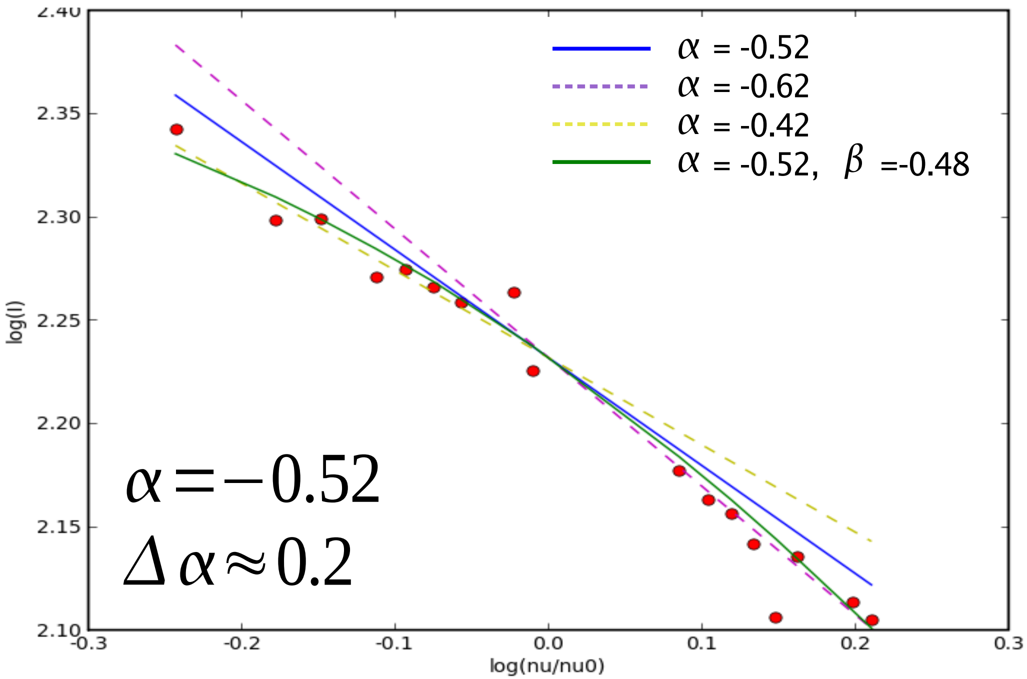
These examples used nterms=2, and about 5 scales.

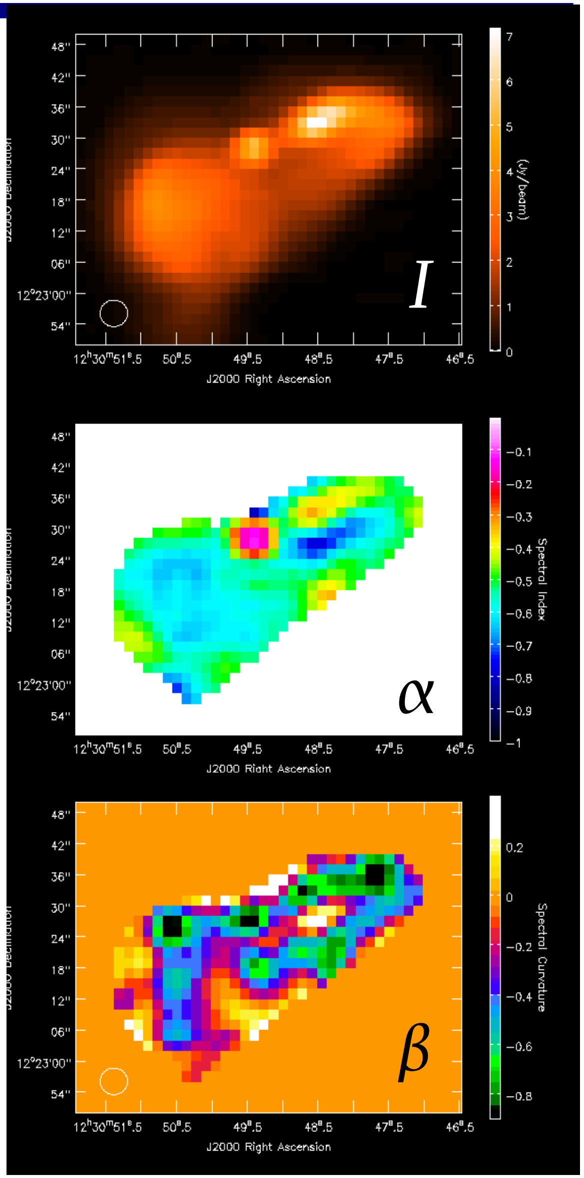
=> Within 1-2 Ghz and 4-8 GHz, can tell-apart regions by their spectral-index (+/- 0.2) if SNR>100.
(this accuracy will increase with wider bandwidths –1-3 GHz CABB)

=> These images have a dynamic-range limit of few x 1000 ---> residuals are artifact-dominated

Spectral Curvature

Data : 10 VLA snapshots at 16 frequencies (1.2 –2.1 GHz)





From existing P-band (327 MHz), L-band(1.42 GHz) and C-band (5.0 GHz) images of the core/jet

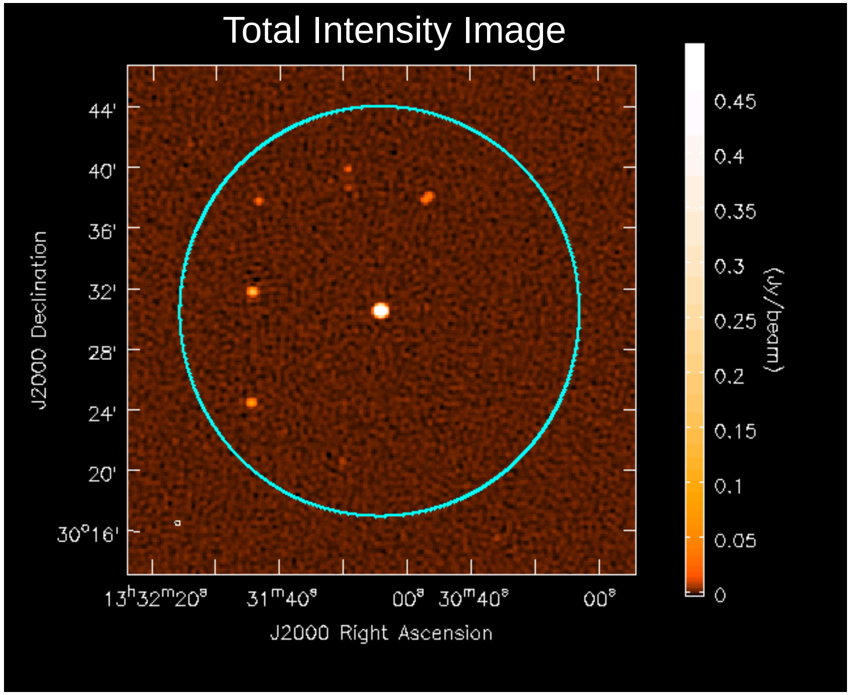
P-L spectral index : -0.36 ~ -0.45
L-C spectral index : -0.5 ~ -0.7

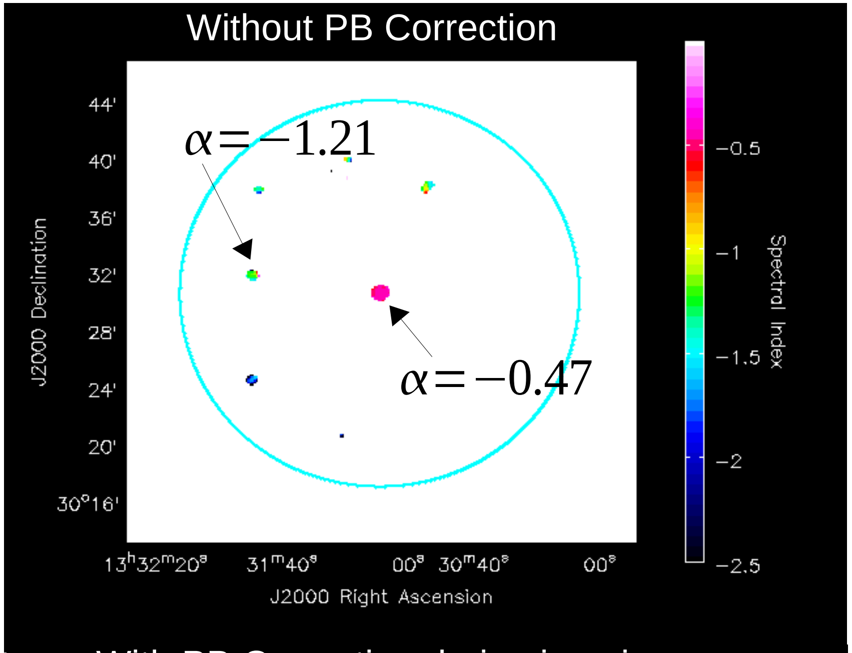
=> Need SNR > 100 to fit spectral index variation ~ 0.2 (at the 1-sigma level ...)

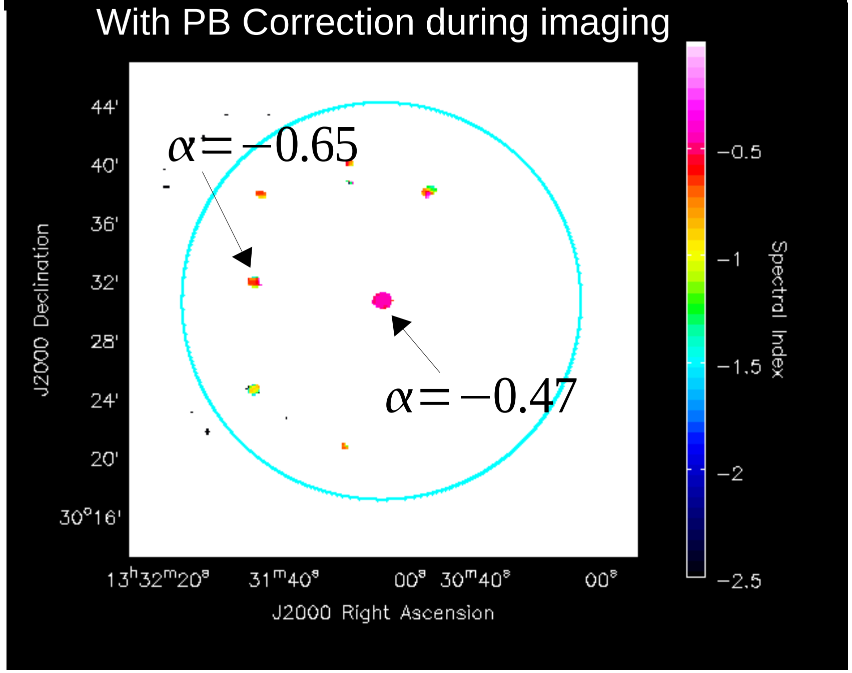
=> Be very careful about interpreting β

Example of Imaging with wide-band PB (artificial spectrum)

3C286 field , C-config , L-band (30min)







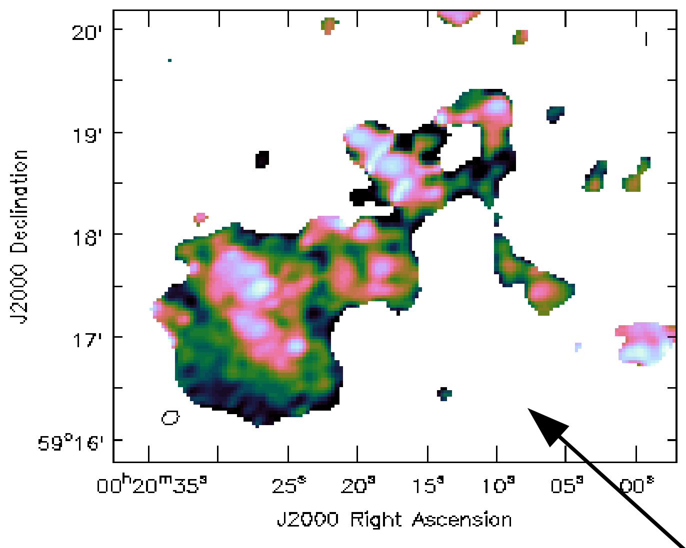
Post-deconvolution polynomial-division of the model spectrum by the PB-spectrum

Verified spectral-indices by pointing directly at one background source.

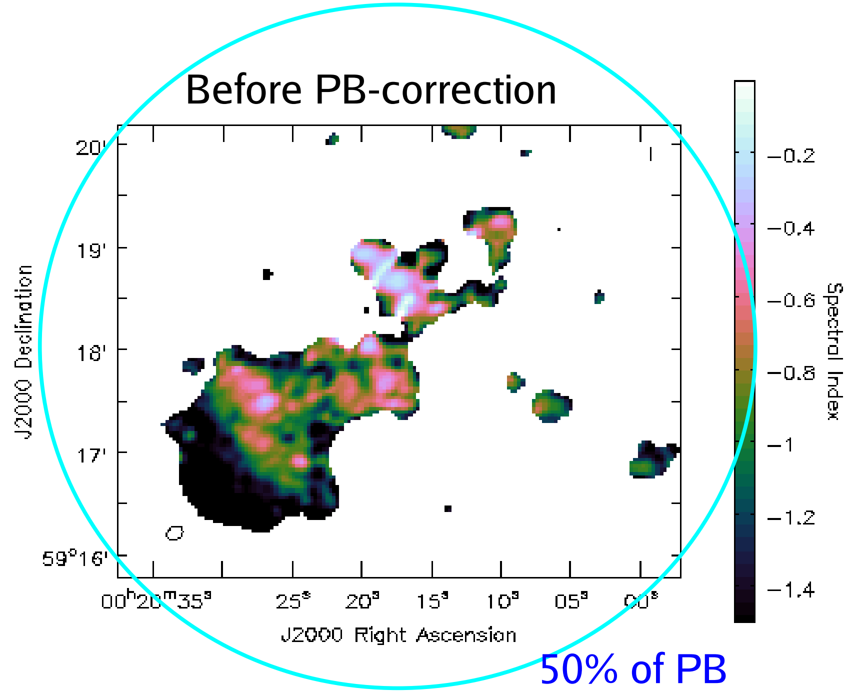
Obtained $\delta\alpha$ = 0.05 to 0.1 for SNR of 1000 to 20

Continuum (MS-MFS) vs Cube Imaging (with PB-correction)

After PB-correction



Before PB-correction

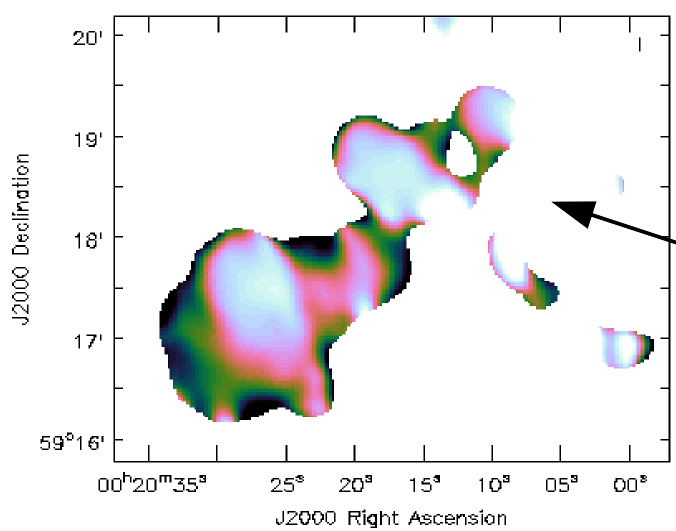


50% of PB

IC10 Dwarf Galaxy :

Spectral Index across C-Band.

Dynamic-range ~ 2000 (~ noise-limited image obtained)



MS-MFS :

Result of wide-band PB-correction after MT-MS-MFS.

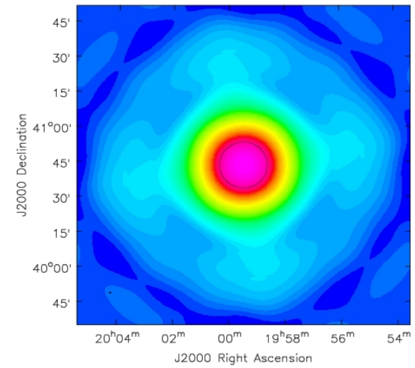
Cube :

Spectral-index map made by PB-correcting single-SPW images smoothed to the lowest resolution.

Other wide-field issues (primary-beam, w-term, mosaicing)

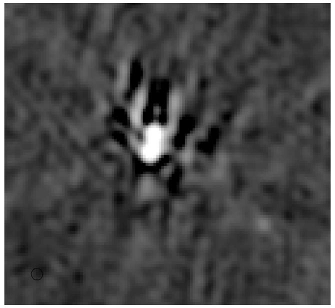
Wide-Band Imaging often requires Wide-field imaging techniques.

"Primary Beam" : The antenna-primary beam introduces a time-varying spectrum in the data.



Any algorithm that works with time-averaged images will not suffice for high dynamic-range.

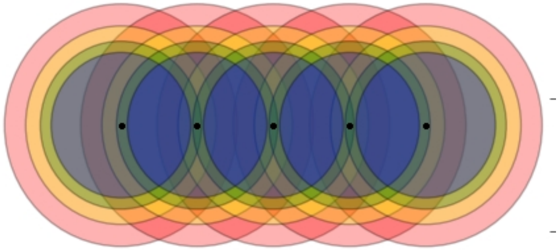
=> Need to reduce/eliminate the PB frequency dependence before time-averaging...



"W-term " : Also a frequency-dependent instrumental effect.

Narrow-band w-projection algorithm works for wide-band.

"Mosaicing" : Make observations with multiple pointing and delay-tracking centers. Combine the data during (or after) image-reconstruction.



Single-pointing wide-band-imaging ideas will work for mosaics too.

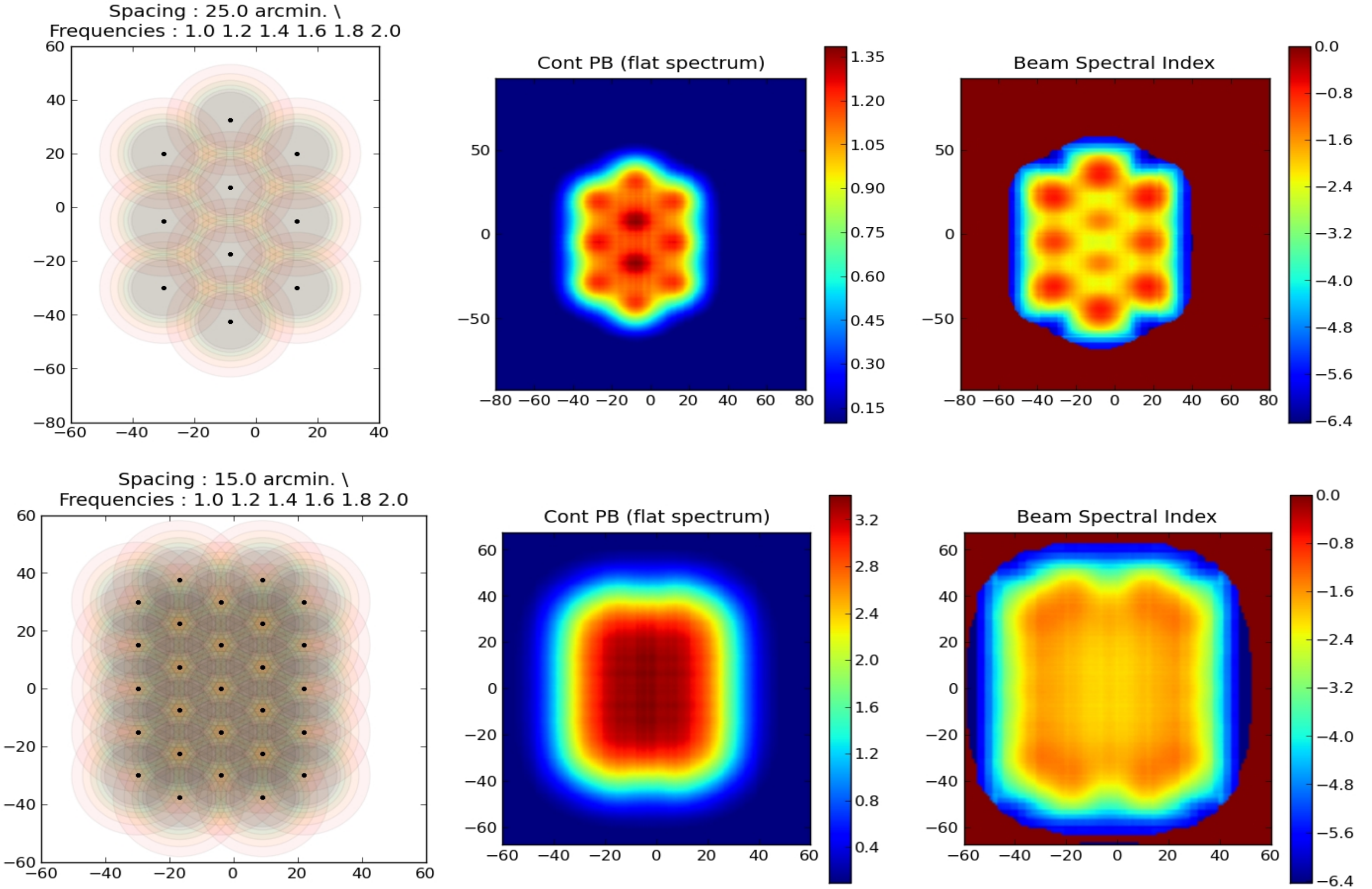
Wideband Primary Beams – Mosaic

For single pointings, the wideband PB spectrum is relevant only away from the pointing center.

For mosaics, the wideband PB spectrum must be accounted for all over the mosaic field of view

25 arcmin spacing (1-2 GHz)

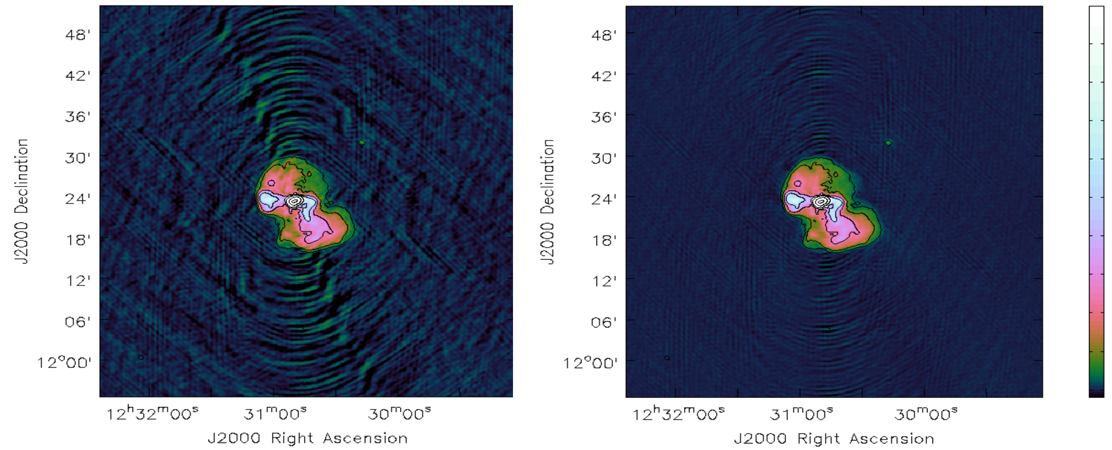
15 arcmin spacing (1-2 GHz)



Using Wide-Band Models for other processing....

WideBand Model : $I^m_{0,} I^m_{1,} I^m_{2,} \ldots$

Evaluate spectrum $I_\nu^{sky} = \sum_t I_t \left(\frac{\nu - \nu_0}{\nu_0} \right)^t$

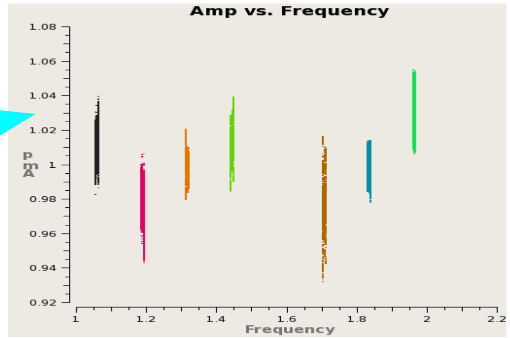


(1) Wide-Band Self-Calibration

– Can be used on target source, after initial calibration per spw.

– Can use it on the calibrator itself to bootstrap the model.

Amplitudes of bandpass gain solutions......



(2) Continuum Subtraction

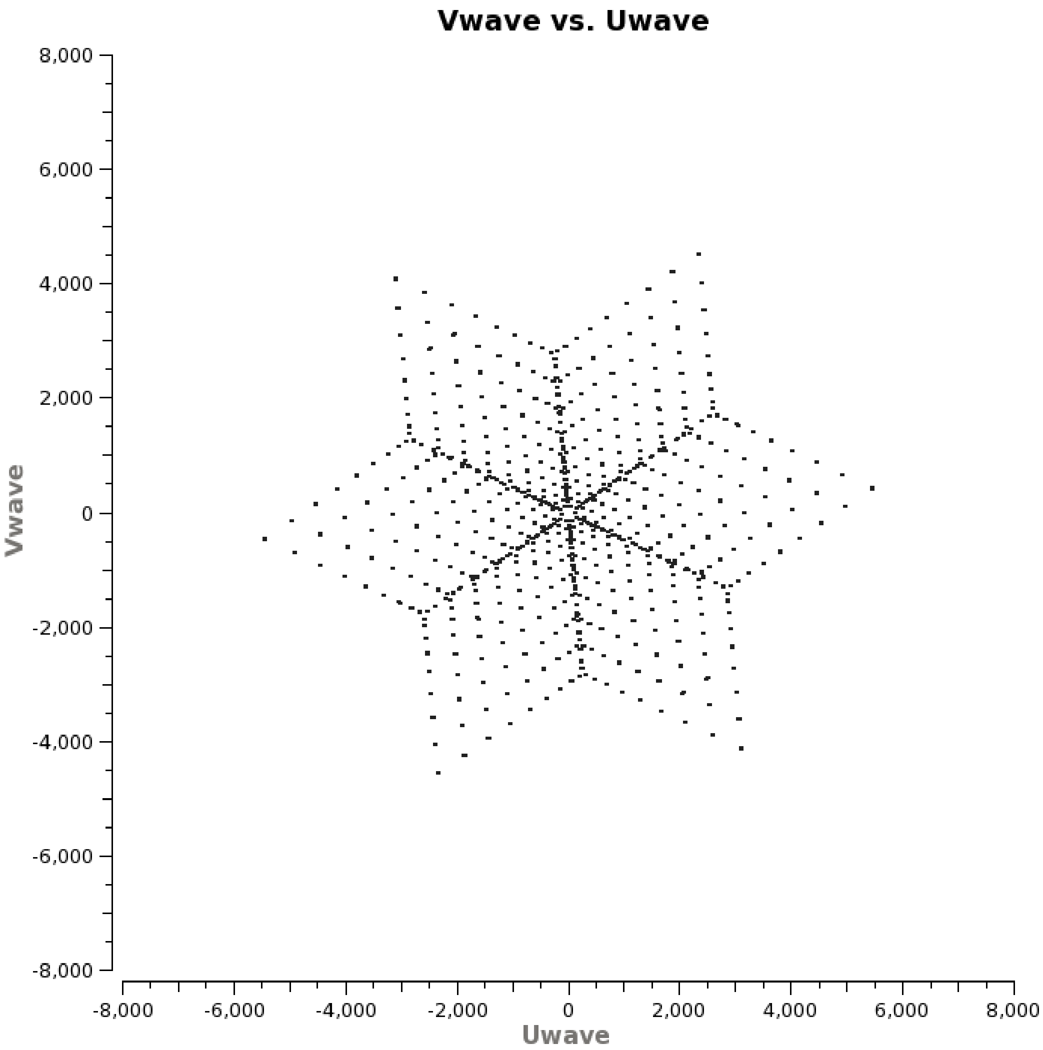
-- De-select frequency channels in which your spectral-lines exist.
– Make a wide-band image model of the continuum intensity and spectra
– Predict model-visibilities over **all** channels
-- Subtract these model visibilities from the data

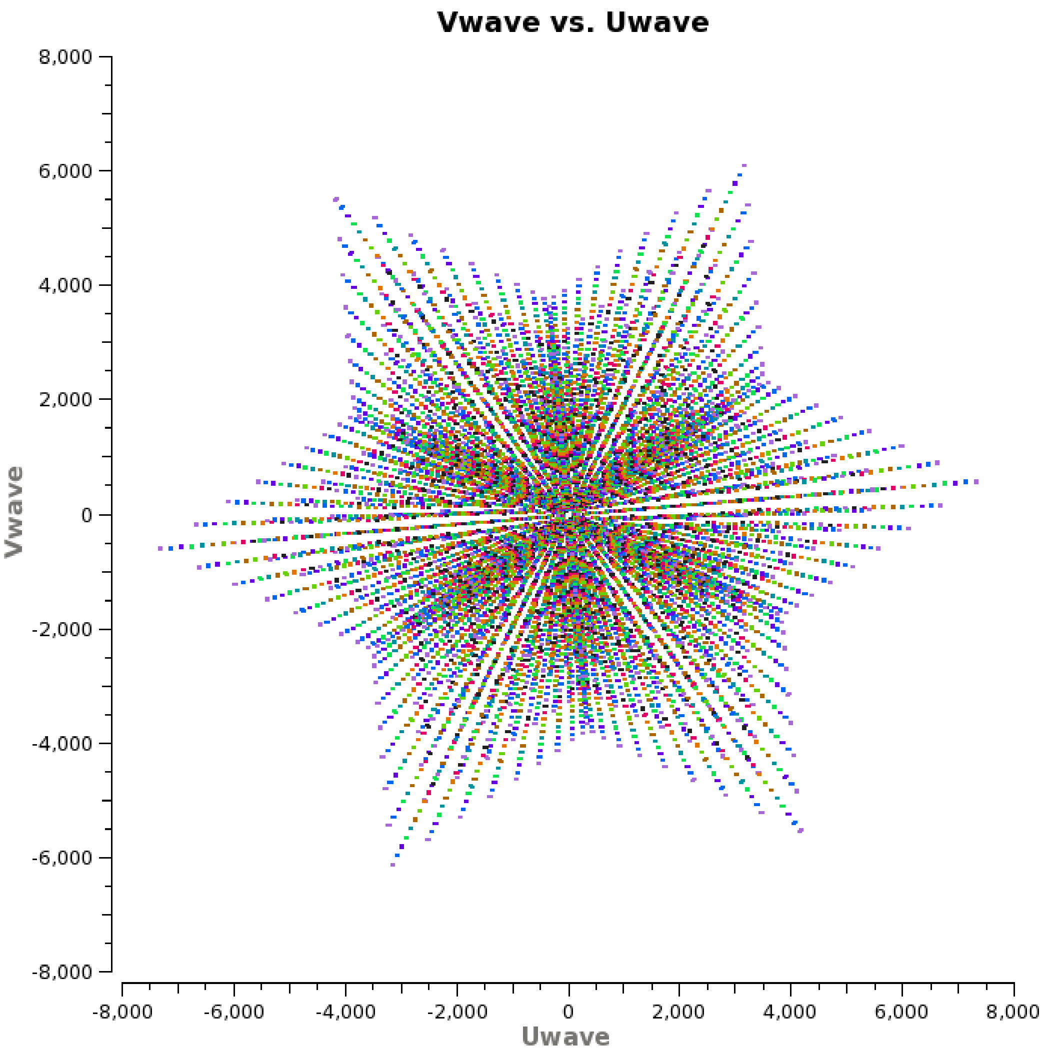
(3) Combination with single-dish data

– Use Taylor-coefficient images made from single-dish images, as a starting model

Multi-Frequency-Synthesis : Snapshot

Observing tip.....

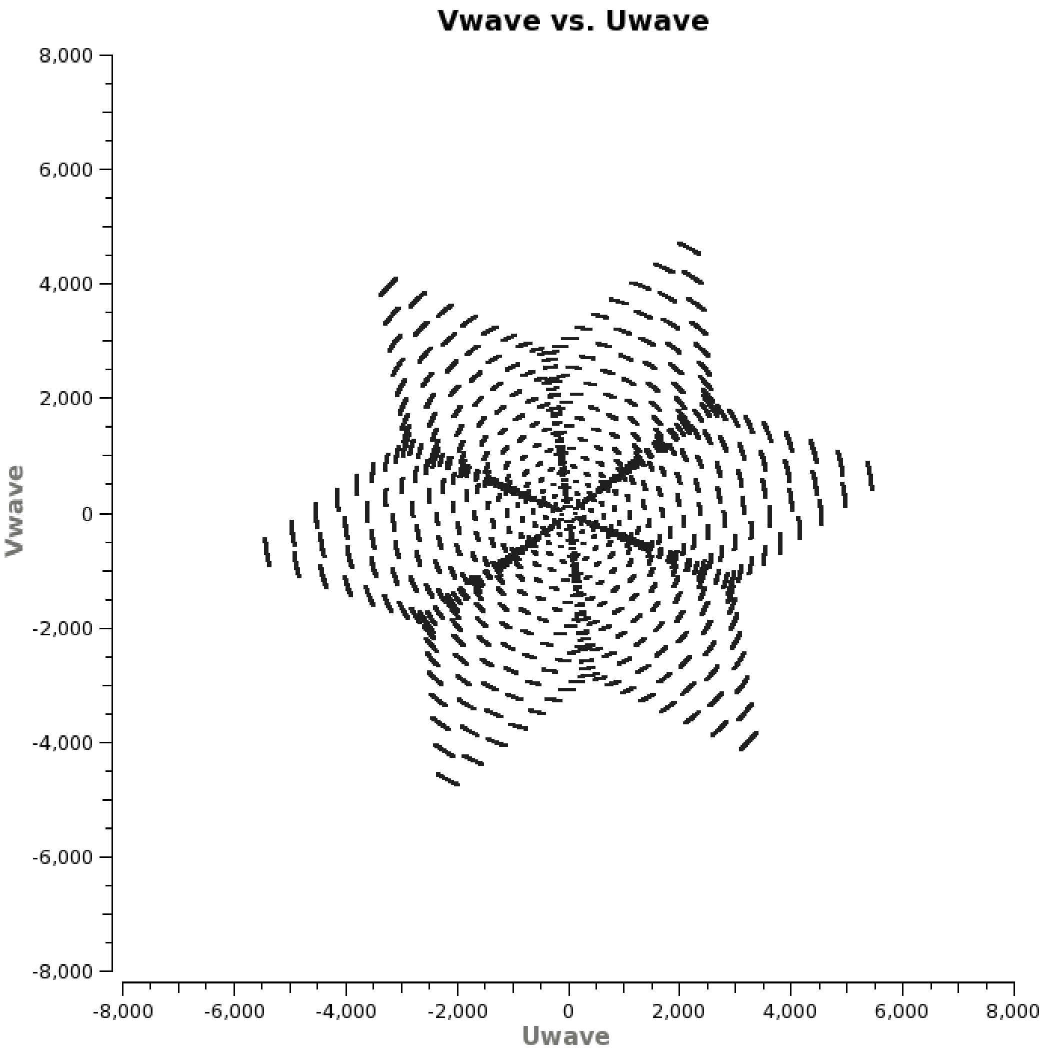


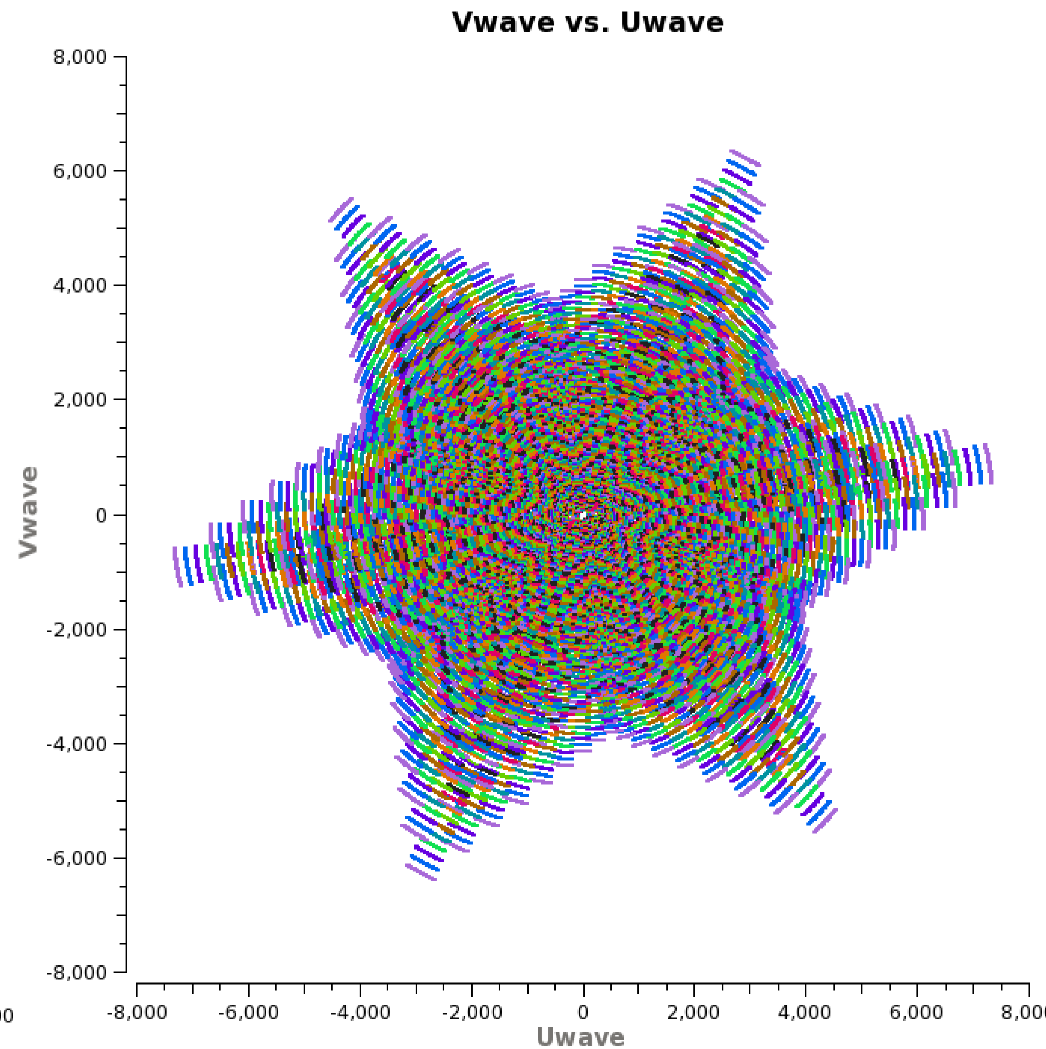


Wideband UV-coverage fills the UV-plane radially.....

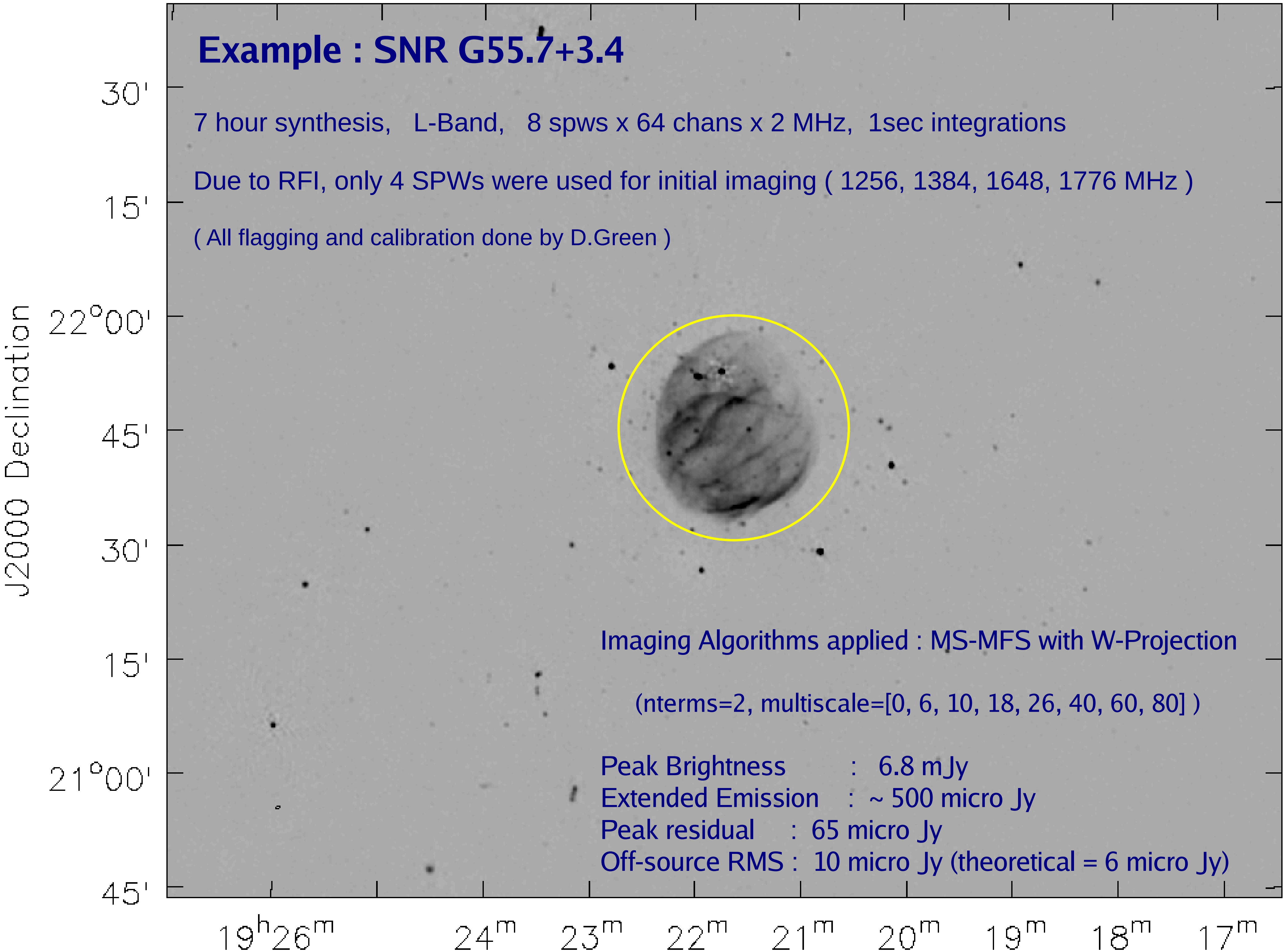
Multi-Frequency-Synthesis : 30 min

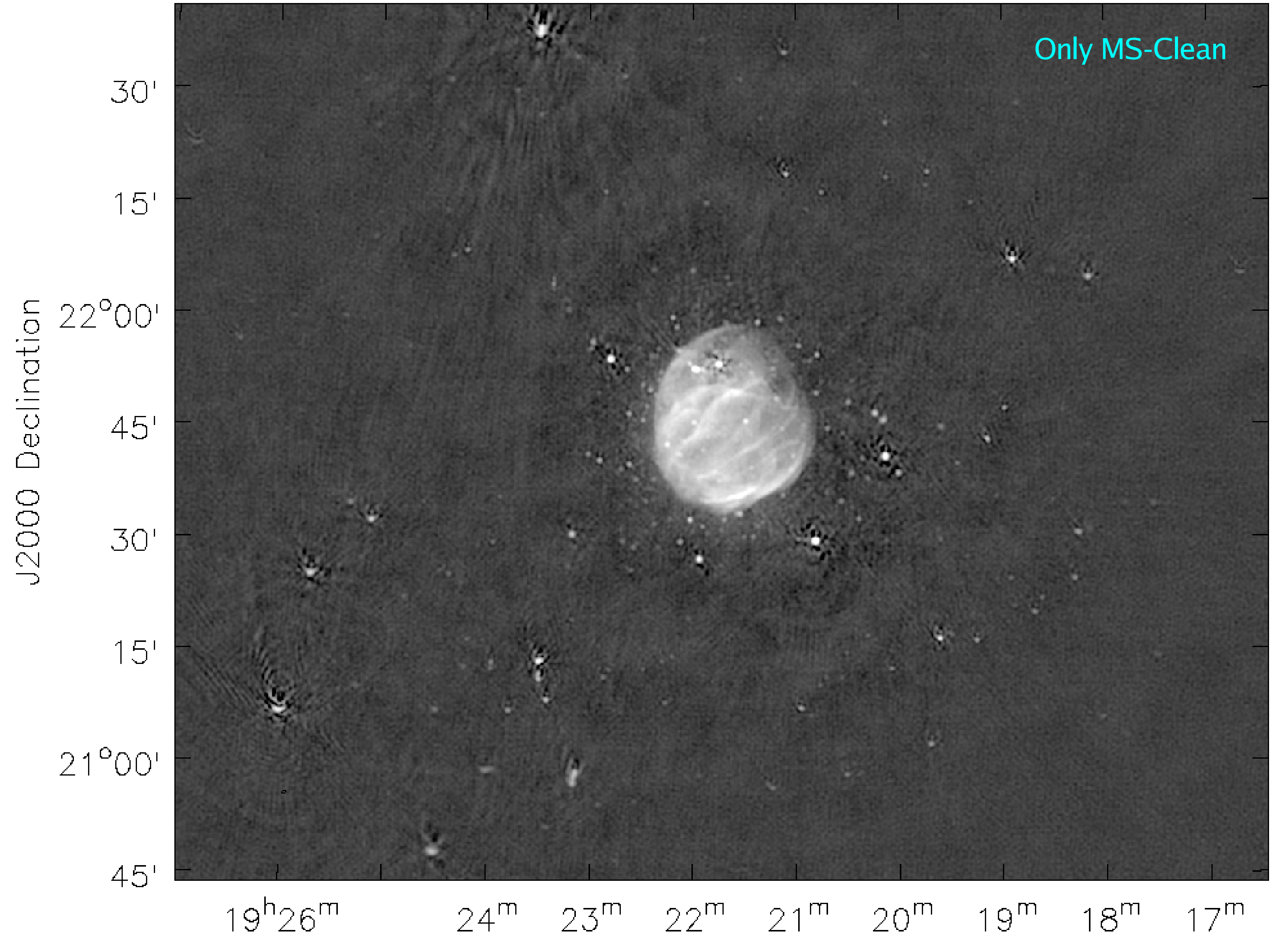
Observing tip.....

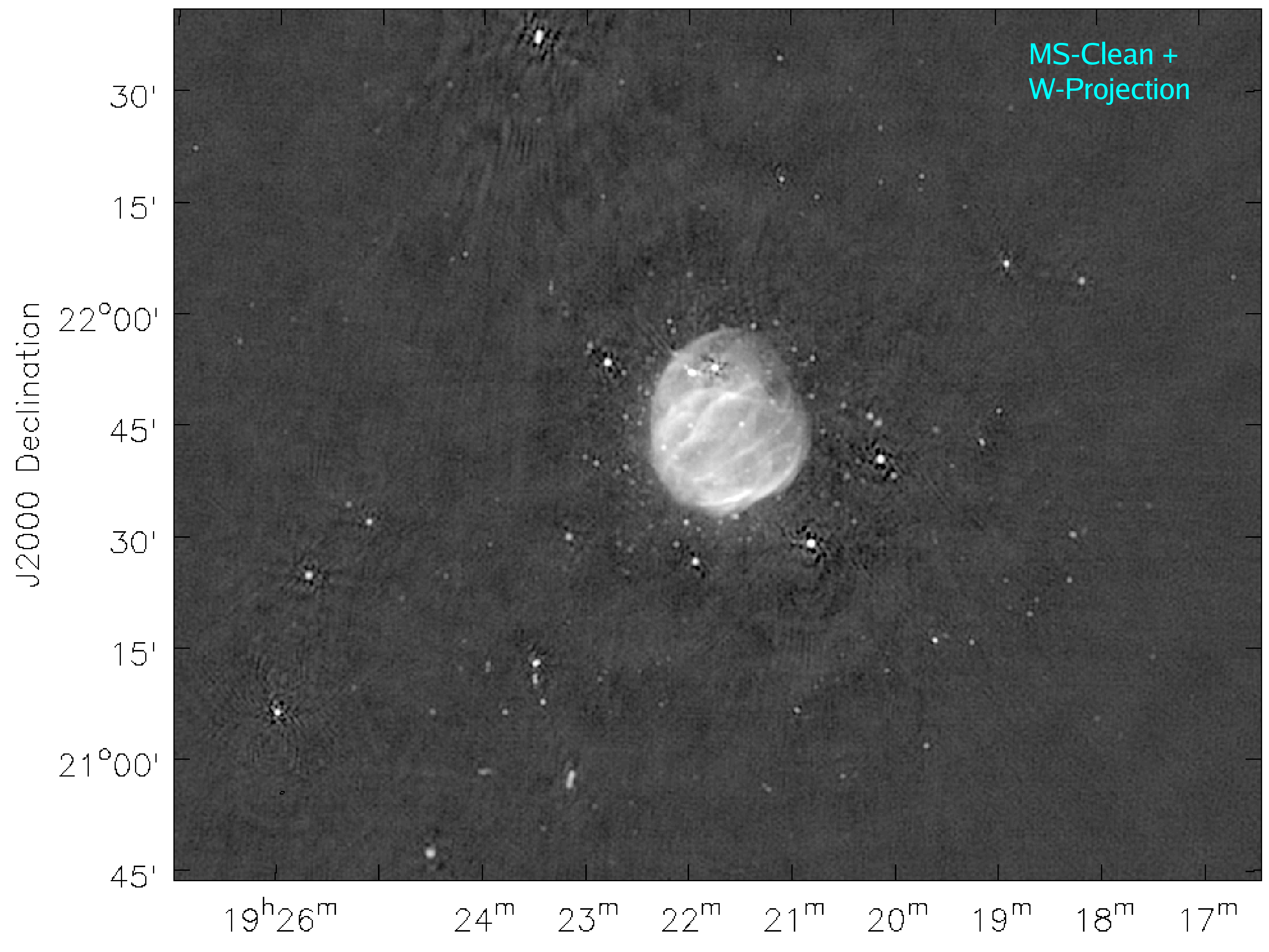




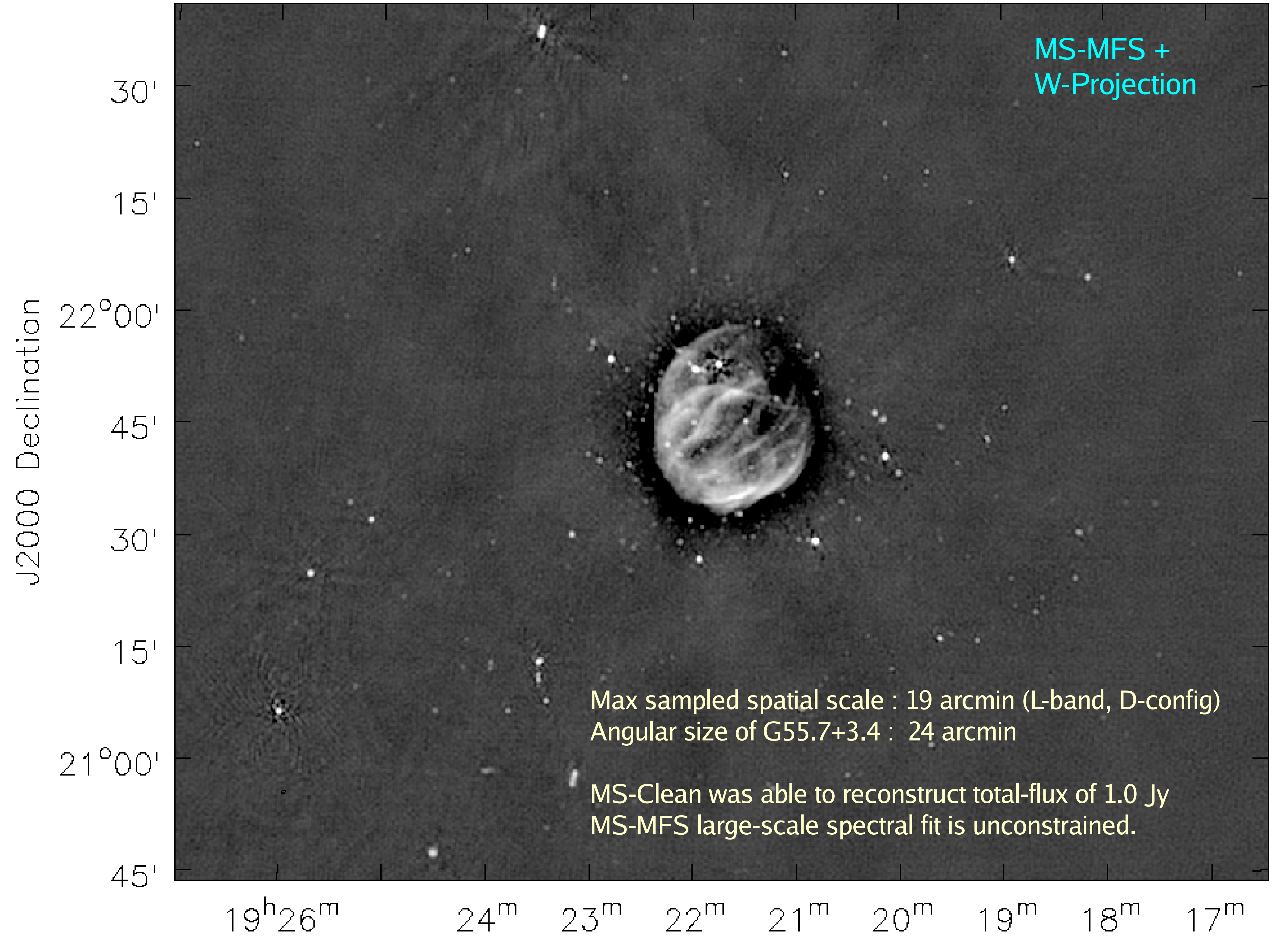
Small time-increments generate good uv-filling => Plan wideband observations in small time-chunks, spread out in time to cover more spatial-frequencies at-least once.

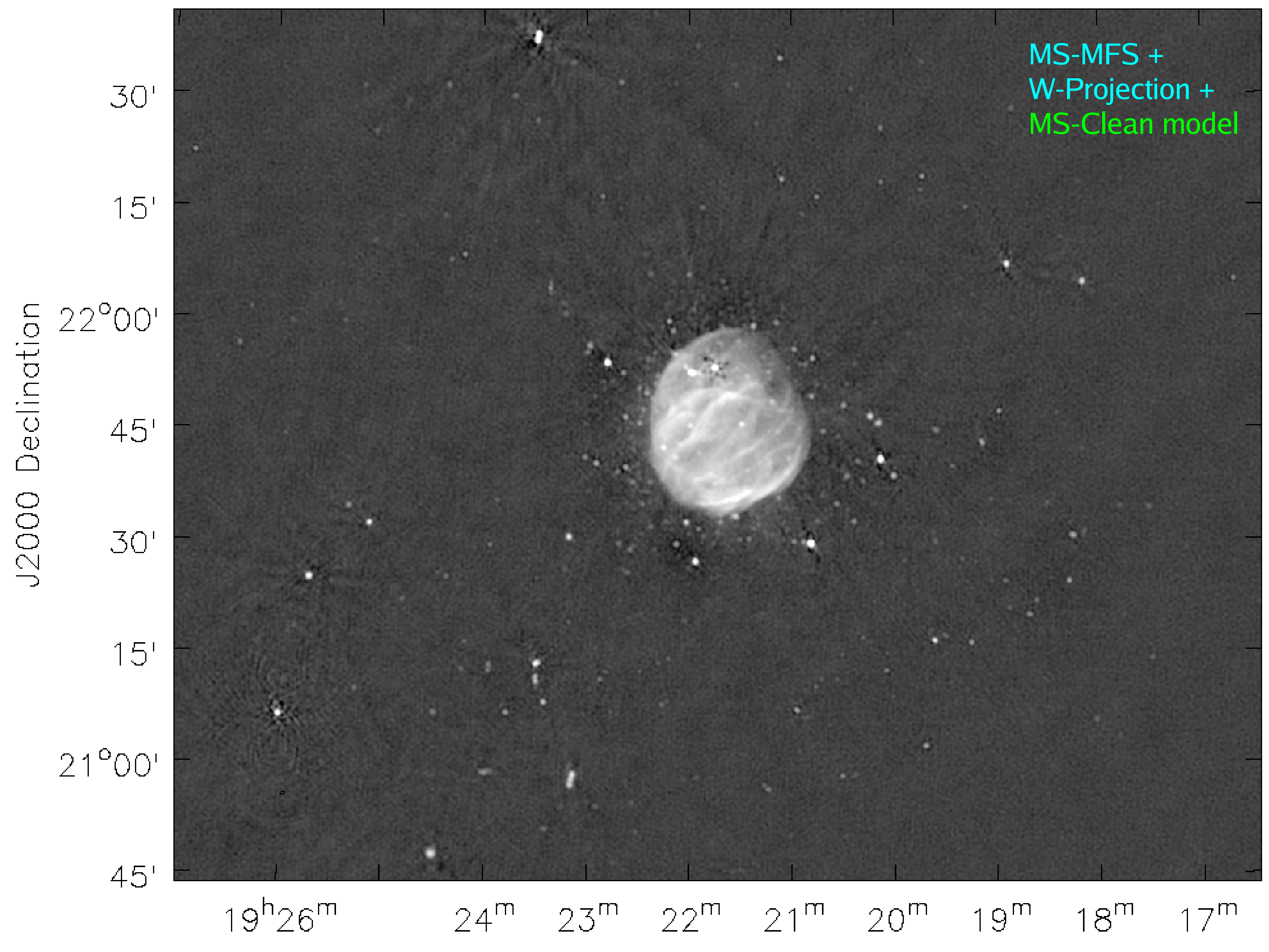

Example : SNR G55.7+3.4
7 hour synthesis, L-Band, 8 spws x 64 chans x 2 MHz, 1sec integrations
Due to RFI, only 4 SPWs were used for initial imaging (1256, 1384, 1648, 1776 MHz)
(All flagging and calibration done by D.Green)
Imaging Algorithms applied : MS-MFS with W-Projection
(nterms=2, multiscale=[0, 6, 10, 18, 26, 40, 60, 80])
Peak Brightness : 6.8 mJy
Extended Emission : ~ 500 micro Jy
Peak residual : 65 micro Jy
Off-source RMS : 10 micro Jy (theoretical = 6 micro Jy)
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m


Only MS-Clean
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m



MS-Clean +
W-Projection
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m


MS-MFS +
W-Projection
Max sampled spatial scale : 19 arcmin (L-band, D-config)
Angular size of G55.7+3.4 : 24 arcmin
MS-Clean was able to reconstruct total-flux of 1.0 Jy
MS-MFS large-scale spectral fit is unconstrained.
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m


MS-MFS +
W-Projection +
MS-Clean model
J2000 Declination
30'
15'
22°00'
45'
30'
15'
21°00'
45'
19h26m
24m
23m
22m
21m
20m
19m
18m
17m

Wide-field sensitivity because of wide-bandwidths

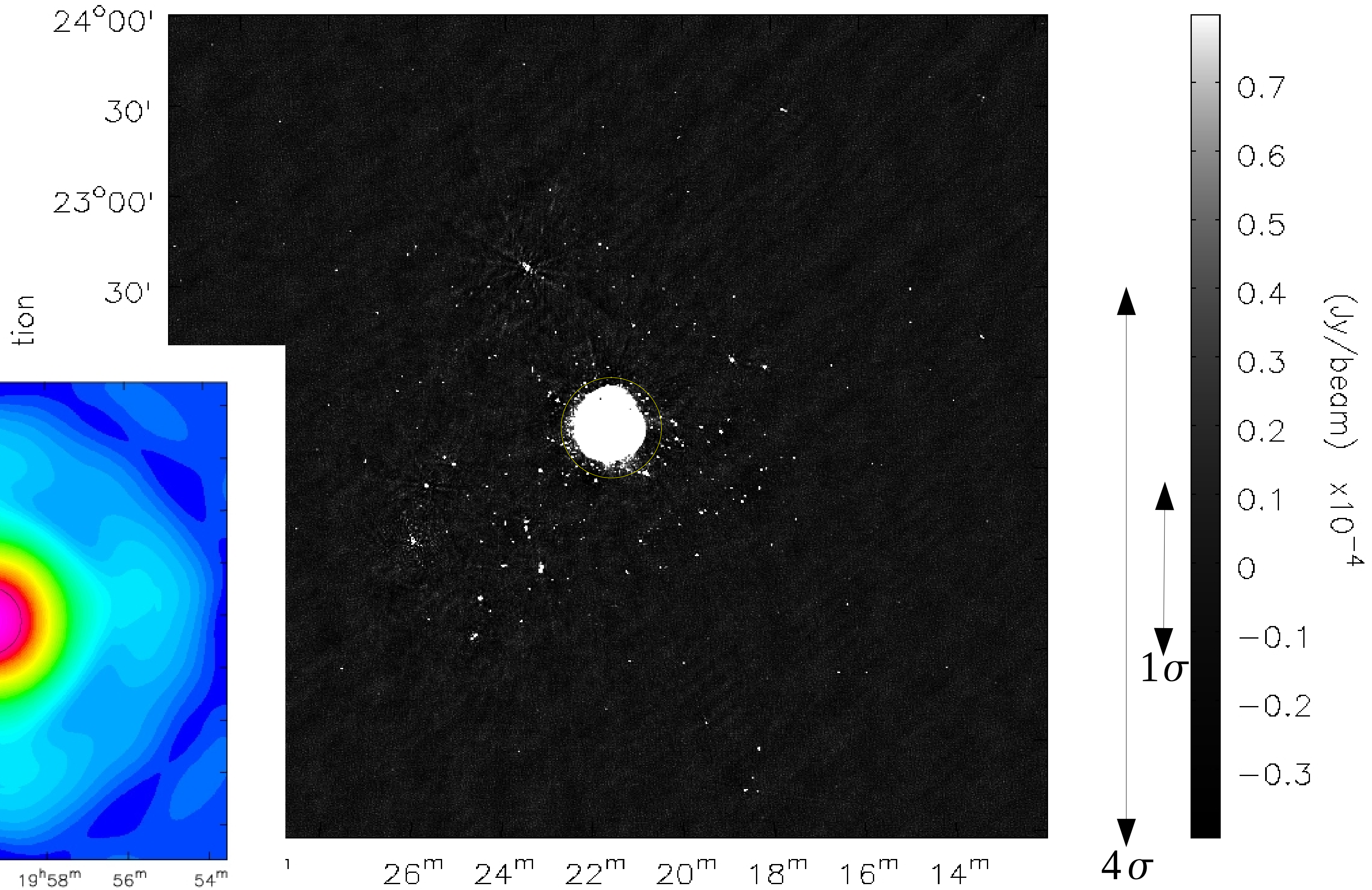
G55.7+3.4 : Galactic supernova remnant : 4 x 4 degree field-of-view from one EVLA pointing

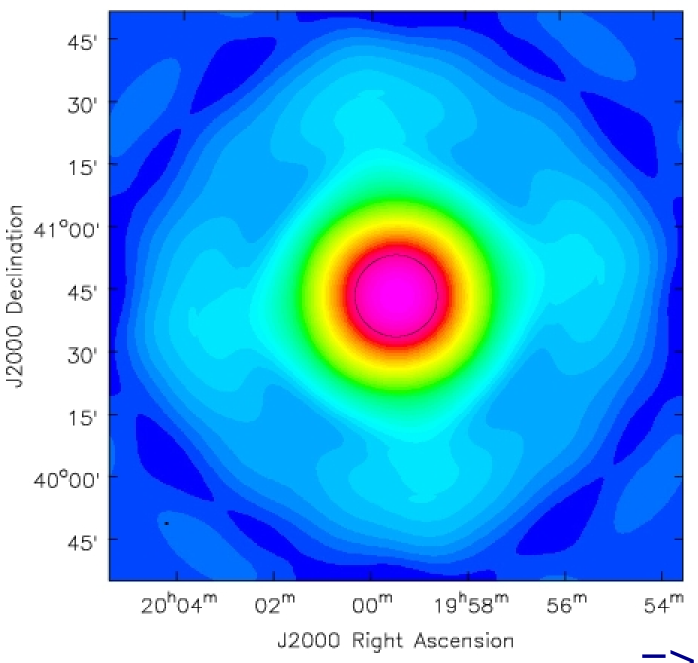
1 Jy total flux

24 arcmin

(PB: 30 arcmin)

10 micro Jy RMS





=> Need to combine wide-field imaging techniques with wideband..

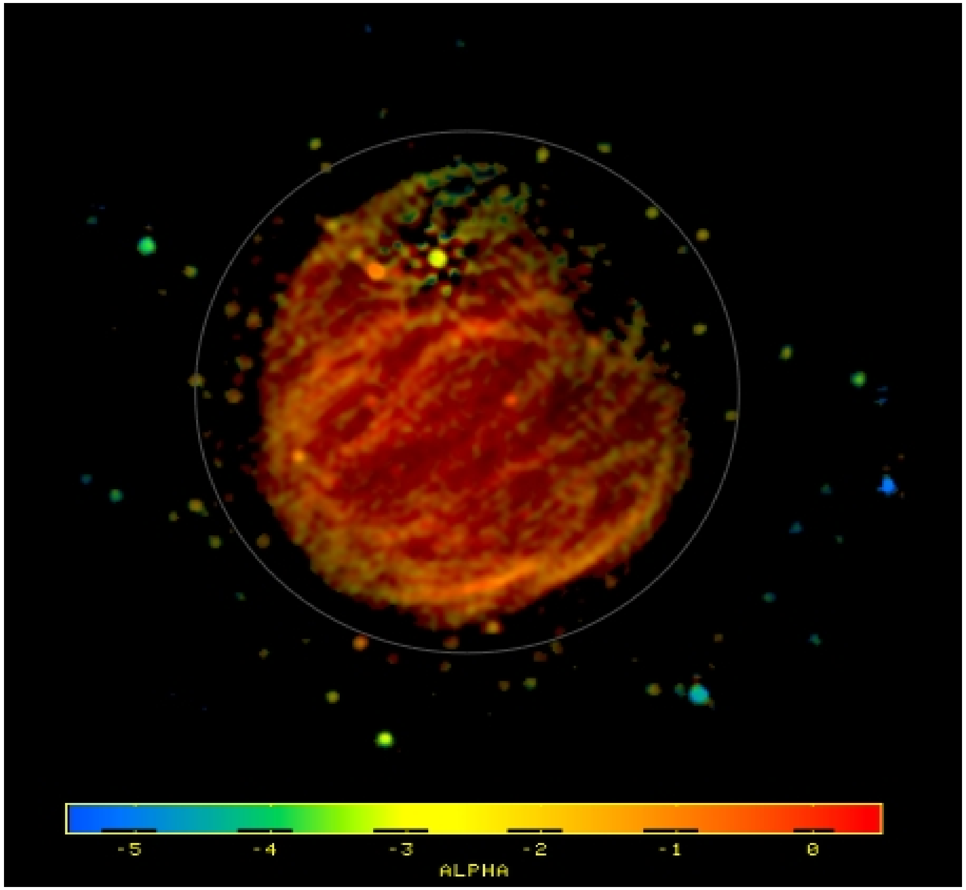
Full-beam imaging of sky spectral index – SNR G55.7+3.4 (EVLA D-config L-band)

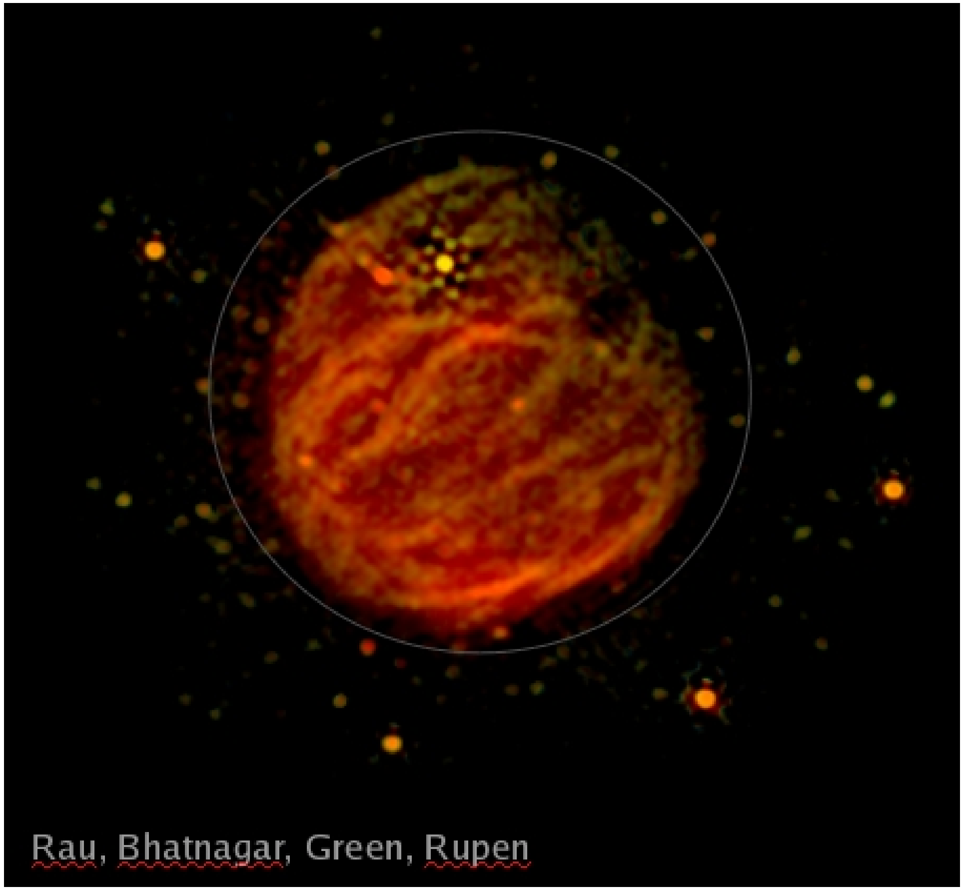
Multi-term imaging with standard gridding.

=> Wideband PB causes artificial steepening

Multi-term imaging with Wideband A-Projection

=> No artificial steepening due to the PB





– Intensity-weighted Spectral Index maps (red : -0.4 to blue : -5.5)

– Image reconstruction via the MT-MFS wideband imaging algorithm with multi-scale support
+ wideband A-Projection gridding to correct for beam squint and frequency dependence.

Summary

Broad-Band Receivers

↓

Cube-Imaging (or per SPW) will suffice for a quick-look.

↓

Multi-Frequency-Synthesis for better sensitivity

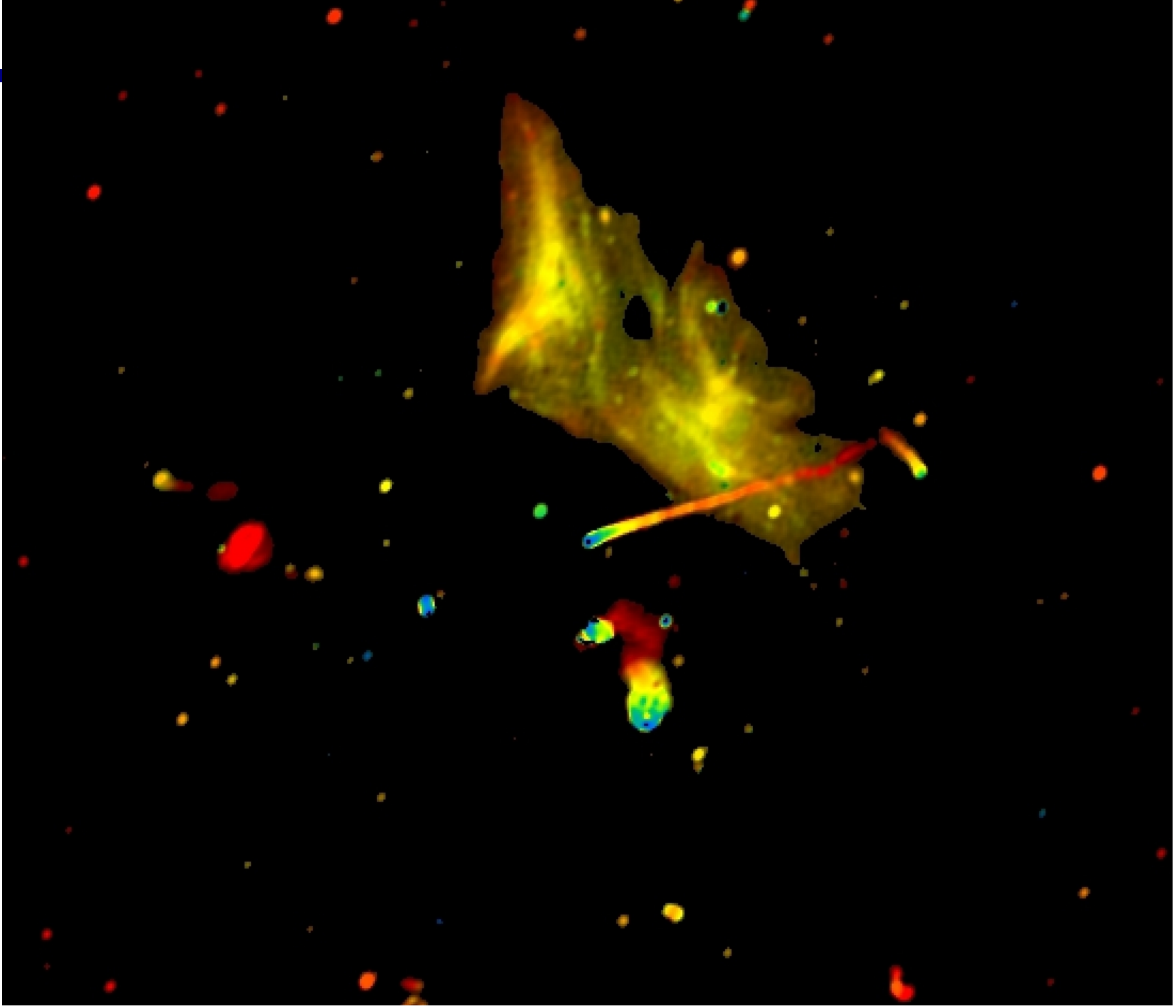
↓

Reconstruct Intensity and Spectrum during Imaging

↓

Wide-band primary beam correction

Image from Frazer Owen : Intensity-weighted Spectral Index of Abell 2256



If this is done correctly, you could get increased imaging sensitivity (over wide fields), high-fidelity high dynamic-range reconstructions of both spatial and spectral structure, all from a single wide-band observation. Remember, however, that you will not always need everything.

Wideband imaging capabilities in CASA – Now, and coming...

Cube Imaging :

- Per channel imaging with point source and multi-scale methods, and w-projection
- Post-deconvolution PB correction (divide final image by PB).
- Linear mosaics using the output of imaging
- Joint mosaic deconvolution within imaging (no w-projection)
- Tasks to smooth planes to a common resolution, and to collapse along frequency.

MFS Imaging :

- Wideband imaging with point source and multi-scale methods
- Choice of the order of the polynomial used to model the spectrum
- Post-deconvolution wideband PB-correction with multi-term imaging
- Linear mosaics using the output of wideband imaging (single and multi-term)
- Joint mosaics with MFS gridding (not for multi-term)

Coming...... (needed mainly for high dynamic-range imaging)

- Time variable, full-polarization wide-band PB corrections with W-projection (DD-corrections)
- Combined application of DD-corrections for mosaics
- Joint mosaics for MFS multi-term imaging with pre- and post-deconvolution PB correction