Point Source Detection Thresholds for Global Fringe Fitting

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1 Introduction

The use of a global fringe fitting technique for a homogenous array can reduce the detection threshold for a point source relative to that for a baseline based method[2]. The purpose of this memo is to analyse the relationship between the number of (identical) antennas in the array and the threshold for point-source detection. In this analysis it will be assumed that the search windows, and therefore the minimum signal-to-noise ratio (SNR), are the same in all cases. An attempt is made to determine the scaling of the threshold level with the number of antennas.

2 Analysis of "baseline stacking"

In this analysis it will be assumed that the detection threshold is set by a minimum SNR and that the SNR is inversely proportional to the error in the determination of the phase. It is also assumed that the error distribution of the measured phases is zero mean and of finite extent. The problem can then be redescribed as determining the scaling of the error in the estimates of the antenna phases with the number of antennas in the array. It is assumed that all baselines have identical sensitivities.

Interferometers are sensitive only to differences in antenna phase, delay and rate, and so these values are usually determined relative to a "reference" antenna whose value is customarily set to zero. In a "global" or coherent fringe fitting scheme the antenna phase, delay and rate are determined for each antenna from all the data; as the number of baselines increases rapidly with increasing number of antennas, the minimum detectable point source flux density decreases. The uncertainty of the determination of each antenna phase with respect to the reference antenna can be used as a measure of the SNR.

As shown in [2] it is possible to use the antenna-based nature of the errors to "stack" linear combinations of the phases on various baselines to simulate the phase of another baseline. This memo will use this conceptual framework to estimate the scaling of the phase uncertainty with number of antennas. Consider antenna a, reference antenna r and any other antenna j. The phase on baseline ar can be estimated as:

$$\Phi_{ar} = \Phi_{ar}$$

$$\phi_a - \phi_r = (\phi_a - \phi_r)$$

or using two baselines,

$$\Phi_{ar} = \Phi_{aj} + \Phi_{jr}$$

$$\phi_a - \phi_r = (\phi_a - \phi_j) + (\phi_j - \phi_r)$$

In the two-baseline case the ϕ_j term cancels, leaving only the phases of a and r. One possible global fringe fitting scheme is to determine Φ_{ar} from an average of all the one- and two-baseline combinations involving antennas a and r. Using this technique we can estimate the variance of the estimated phase using the following linearized error propagation analysis relationship [1]:

$$y = f(x_1, ...x_n)$$

$$\sigma_y^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}\sigma^2_{x_i,x_j}$$

Assuming that the errors are uncorrelated this reduces to:

$$\sigma_y^2 = \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 \sigma^2_{x_i}$$

We can use this relationship to examine the error propagation using the one- and two-baseline estimates of Φ_{ar}. Not all of these measures are equally sensitive; if a single baseline is assumed to have unit variance then the two-baseline estimates will have twice unit variance. Therefore, in the averaging we want to weight inversely as the variance of the measure. The average includes a single baseline of unit weight and $n-2$ baselines of half weight for an n antenna array. The partial derivatives of the weighted average w.r.t. each of the components is the ratio of the component weight to the total weight $= 1 + (n-2)/2 = n/2$. The variance of the estimate of Φ_{ar} in units of the baseline variance is:

$$\sigma^2_{avg} = \left(\frac{2}{n}\right)^2 + 2(n-2)\left(\frac{1}{n}\right)^2 = \frac{2}{n}$$

Therefore, using one- and two-baseline combinations the decrease in threshold sensitivity is a factor of $\sqrt{n/2}$.

Using the one- and two-baseline combinations to make a "composite" baseline does not use all the information available. In the formulation above, baselines jr contribute unit variance to the analysis. If instead of using the single jr baseline in the analysis, the "composite" jr baseline is used, the variance of the new composite ar baseline will be reduced. This is the equivalent of including the three baseline combination of [2]. As was shown above, the variance of a composite jr baseline is $2/n$. (This isn't quite right as this includes the aj baseline which is included elsewhere in the analysis).

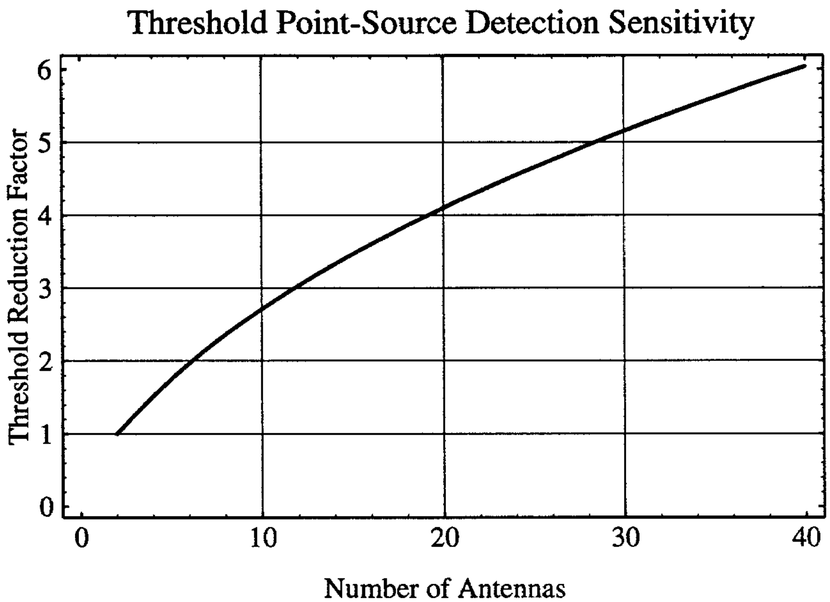

Figure 1: This figure shows the factor by which the point-source detection threshold is reduced by global fringe fitting as a function of the number of antennas in a homogenous array.

Using the "composite" jr baselines in the expression for the variance of the composite Φ_{ar} gives:

$$\sigma^2_{avg} = \left(\frac{2}{n}\right)^2 + \left(1 + \frac{2}{n}\right)(n-2)\left(\frac{1}{n}\right)^2 = \frac{1}{n} + \frac{4}{n^2} - \frac{4}{n^3}$$

In the limit of large n this reduces to $1/n$ or a reduction in the detection threshold by a factor of $\sqrt{n}$. Figure 1 shows the detection threshold reduction factor as a function of the number of antennas.

3 Discussion

The above analysis shows that a substantial reduction in the threshold point source detection level for a homogenous array can be obtained using a coherent fringe fitting method. For the 10 element VLBA this can reduce the detection threshold by a factor of ≈ 2.7 relative to a single baseline.

This analysis depends on the source being unresolved or at least having insignificant baseline-dependent structure phase. Baseline-dependent structure phase will be noiselike in that its effect will be to corrupt the composite baseline sums. If the structure of the source is known, its effects can be removed by dividing the observed visibilities by the Fourier transform of the source brightness distribution. This procedure will not restore the SNR lost to resolution, and so the analysis above cannot be directly applied in this case as it assumed that the variance of the phase on each baseline was the same.

References

[1] A. Hald. *Statistical Theory with Engineering Applications.* John Wiley & Sons, New York, 1952.

[2] F. R. Schwab and W. D. Cotton. "Global fringe search techniques for VLBI". *Astron. J*, 88:688–694, 1983.