An investigation of the conversion function from the true correlation to measured one if the input signals are quantizated differently (two- and four- levels quantization)

Leonid Kogan

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ABSTRACT

The function that converts digital measurements to the equivalent analog correlations has been analyzed for different levels of quantization of the input signals (two- and four- levels) for both XF and FX correlators. The nonlinear effect of the fringe stop procedure as well as a fractional bit shift correction has been analyzed. It is shown that the nonlinear effect of these procedures linearises the common conversion function. Different conversion functions need to be used in the XF case for auto and cross-correlation spectrum restoration.

I. INTRODUCTION

Amplitude quantization of the input signals is widely used in correlation data reduction in particular during the reduction of an interferometric observation. When the signals are normal random processes (as occurs for practically all applications) it has been shown (Van Vleck, 1943) that in the case of clipping of the input signals there is a strong relation between the correlation of the digitized signals **r** and the correlation of the input analog signals- ρ

$$r = \frac{2}{\pi}\cdot\arcsin(\rho) \qquad \rho = \sin\left(\frac{\pi}{2}\cdot r\right) \qquad \textbf{(1)}$$

In other cases (two to four and four to four levels in particular) it is impossible to find solution for the conversion function at the elementary functions. The different functions have been used for approximation of the conversion function at the case of the same level quantization (three or four) for both correlated signals at some range of ρ (D'Addario et al., 1984, Kulkarni and Heiles, 1980, Swab, 1986). The VLBA terminals have both two- and four- level options at the quantizer. So it is necessary to analyze the conversion function for all three possible pairs of signal quantization at the sites of the interferometers.

When we analyze the auto correlation of the signal the conversion functions $\rho(\mathbf{r})$ or $\mathbf{r}(\rho)$ are used to resolve completely the problem of restoring the original correlation from correlation of digitized signal. But in the VLBI case there are two additional phenomena which are not linearly dependent on the input correlation value. There are the fringe stopping procedure and a fractional bit shift correction. These two effects do not give the opportunity to apply correctly the conversion function for the whole range of input correlation ($0<\rho<1$). Barry Clark was the person who pointed this out. Let us illustrate this problem for the limiting case $\rho=1$. The output of the correlator is described by the formula (2) in the case of analog input signals:

$$out = \rho(\tau)\cos(2\pi ft+\varphi) \qquad \textbf{(2)}$$

where $\rho(\tau)$ is the cross correlation function dependent upon the spectrum of the input signals;
f is the fringe rate.

Suppose $\tau=0$ and $\rho(0)=1$. Then the graph of (2) is the sine wave with amplitude equal 1. This wave is transformed to the triangular one having been corrected by conversion function **r=(2/π)arcsin(out)**, corresponding to one bit digitizing of the input signals (Fig. 1). We receive $(2/\pi)^2\approx 0.4$ for the amplitude instead of expected 0.5 after the fringe stopping procedure is applied to the triangular wave. This effect is absent for small input correlation because the conversion function is linear in this case. The same type of problem occurs for the fractional bit shift correction. This procedure works for analog input signals with limited spectrum and sampling frequency equal to or more than the Nyquist rate. But it can not work completely to be applied to the cross correlation of the digitized signals because their spectrum is not limited. In this work we estimate the influence of all these effects.

II. THE CONVERSION FUNCTION ANALYSIS

1. THE APPROACH TO THE PROBLEM

The symmetrical case of four level quantization of the signal can be described by the next formula

$$\epsilon = \begin{cases} +n & if\ \ \varepsilon \ge v\cdot\sigma \\ +1 & if\ \ 0 \le \varepsilon < v\cdot\sigma \\ -1 & if\ \ -v\cdot\sigma \le \varepsilon < 0 \\ -n & if\ \ \varepsilon < -v\cdot\sigma \end{cases} \qquad \textbf{(3)}$$

where σ is the variance of the random process $\varepsilon(\mathbf{t})$,
v is the threshold of the digitizer measured at number of σ,
n is the ratio of the two upper levels.

We suppose here that the digitizers of the interferometer terminals are symmetrical so the mean of ε is equal zero as well as the mean of ε. Let's $\mathbf{v}_1$ and $\mathbf{v}_2$ are the thresholds of the first and second digitizer respectively. Assume that $\varepsilon_1(\mathbf{t})/\sigma_1$ and $\varepsilon_2(\mathbf{t})/\sigma_2$ are jointly Gaussian, stationary random processes with crosscorrelation coefficient ρ which we'd like to determine from the measurements of crosscorrelation coefficient of digitized signals $\mathbf{r}$. Then their probability density function is determined by the next formula:

$$P(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left[-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}\right] \qquad \textbf{(4)}$$

The mean of output of the digit correlator is determined (Swab, 1986) by the next equation:

$$\begin{aligned} r(\rho) &= \langle \epsilon_1 \cdot \epsilon_2 \rangle = \\ &(n-1)^2[L(v_1,v_2,\rho)+L(v_1,-v_2,\rho)+L(-v_1,v_2,\rho)+L(-v_1,-v_2,\rho)] \\ &+2(n-1)[L(v_1,0,\rho)+L(-v_1,0,\rho)+L(v_2,0,\rho)+L(-v_2,0,\rho)] \\ &+\frac{2}{\pi}\arcsin(\rho)+C \end{aligned} \qquad \textbf{(5)}$$

where **L(h,k,ρ)** is the bivariate normal integral:

$$L(h,k,\rho) = \int_h^\infty \int_k^\infty P(x,y,\rho)\,dx\,dy \qquad \textbf{(6)}$$

The expression for the derivative **dL/dρ** can be received following Swab, 1986. Another method of evaluating this derivative is shown at the Appendix of this Memo.

$$\frac{dL(h,k,\rho)}{d\rho} = P(h,k,\rho) \qquad \textbf{(7)}$$

Having substituted (7) at (5) we receive the formula for the derivative **dr/dρ**

$$\begin{aligned} \frac{dr}{d\rho} = \frac{1}{\pi\sqrt{1-\rho^2}} \Bigg[&(n-1)^2\left[\exp\left(-\frac{v_1^2-2\rho v_1 v_2+v_2^2}{2(1-\rho^2)}\right)+\exp\left(-\frac{v_1^2+2\rho v_1 v_2+v_2^2}{2(1-\rho^2)}\right)\right] \\ &+2(n-1)\left[\exp\left(-\frac{v_1^2}{2(1-\rho^2)}\right)+\exp\left(-\frac{v_2^2}{2(1-\rho^2)}\right)\right]+2\Bigg] \end{aligned} \qquad \textbf{(8)}$$

The expression (8) will be extensively used in the next parts of the memo to analyze the behavior of the functions **r(ρ)** and **ρ(r)**. When $\mathbf{v_1} = \mathbf{v_2} = \mathbf{v}$ we receive the case of FOUR TO FOUR levels which was investigated rather well in the past. If we put $\mathbf{v_1=0}$ we'll receive particular case of (3) - two level digitizer with levels +n and -n. So when $\mathbf{v_1=0}$ and $\mathbf{v_2=v}$ we receive the case of TWO to FOUR levels. For calculating ρ corresponding to an arbitrary value of **r** we suggest to carry out digital integrating of **dr/dρ** along the **r** axis:

$$\rho(r_k) = \sum_{i=1}^{k} \frac{1}{\frac{dr}{d\rho}(\rho_{i-1})} \Delta r, \qquad \Delta r = \frac{r_{max}}{N}, \qquad r_k = \Delta r \cdot k \qquad k = 1, 2 \ldots N \tag{9}$$

The maximum possible error of this approximation is determined by the next inequality:

$$|\rho(k) - \rho| \le \sum_{i=1}^{k} \left| \frac{d}{d\rho}\left(\frac{1}{\frac{dr}{d\rho}}\right) \frac{1}{\frac{dr}{d\rho}} \right|_{max} \cdot \left(\frac{r_{max}}{N}\right)^2 = |z(0) - z(\rho)| \frac{1}{N} \tag{10}$$

where function $\mathbf{z}(\rho)$ is determined by the next equation if we normalized the maximum value of the correlation coefficient to 1:

$$z(\rho) = \frac{r_{max}^2}{2} \left(\frac{1}{\frac{dr}{d\rho}}\right)^2 \tag{11}$$

The table of values $\rho(\mathbf{r_k})$ has to be calculated in advance before the correlation process. The values of $\rho(\mathbf{r_k})$ are obtained for $\mathbf{r_k}$ with equal steps. So the only thing which the correlation program has to make to convert $\mathbf{r}$ into ρ is to calculate the number $\mathbf{k}$ and take the correspond element of the table. Number of terms at the integrating sum provided the given accuracy is determined by the equation (10).

What about $\mathbf{r_{max}}$? The derivative $\mathbf{dr/d}\rho$ is positive for the whole domain $\mathbf{0<\rho<1}$. So the function $\mathbf{r}(\rho)$ is increasing one and maximum value of $\mathbf{r}$ corresponds to maximum value of $\rho=1$. But the case $\rho=1$ corresponds to absolutely coincided random process' $\varepsilon_1(\mathbf{t})$ and $\varepsilon_2(\mathbf{t})$. For such a case it is not difficult to calculate correlation coefficient. For FOUR TO FOUR levels digitizers maximum value of the correlation coefficient is determined by the next equation (Swab, 1986):

$$r_{m44} = n^2 - (n^2 - 1)\, erf\left(\frac{v}{\sqrt{2}}\right) \tag{12}$$

For TWO TO FOUR levels digitizers maximum value of the correlation coefficient is determined by the next equation:

$$r_{m24} = n^2 - (n^2 - n)\, erf\left(\frac{v}{\sqrt{2}}\right) \tag{13}$$

We have evaluated the function $\mathbf{z}(\rho)$ using (11), (12), and (13) for the both FOUR TO FOUR and TWO TO FOUR cases and for optimal value of $\mathbf{n}$ = 3.336 and $\mathbf{v}$ = 0.9816. The graph of this function is shown at Fig. 1.

Maximum value of $|\mathbf{z}(\rho)-\mathbf{z}(0)|$ is equal 0.64 for the both cases. Having substituted this maximum value at (10) we find the expression for the maximum possible error of the original correlation coefficient versus number of points at the integral sum N:

$$|\rho(k) - \rho| \leq = \frac{0.64}{N} \qquad \mathbf{(14)}$$

So we have to take N > 6400 if we desire to estimate function $\rho(\mathbf{r})$ with accuracy better than 10^{-4} for all possible value of ρ from 0 to 1. It is not so serious problem because the table of function $\rho(\mathbf{r})$ is calculated only once. The number of points at the table can be rather less than N. The number of points N can be taken less providing the same accuracy if we limit the maximum value of ρ. The value of number of points N can be chosen using the graphs at Fig. 2. and equation (10). The relative error $[\rho(\mathbf{k}) - \rho]/\rho$ is limited by the **0.64/N** also because graph of the function $\mathbf{z}(\rho)$ is located lower then function $\mathbf{0.64}\rho$ for all possible ρ.

2. OPTIMAL VALUE FOR THE RATIO OF UPPER LEVELS **n** AND THRESHOLD **v**

Let's consider the case of low correlation coefficient ρ. Really only this case has a sense because optimization of signal to noise ratio is not actual for the case of a large correlation coefficient. Just to compare the signal/noise ratio for TWO TO FOUR and FOUR TO FOUR levels correlator we deduce general formulas for signal to noise ratio. For small correlation coefficient signal (output of the digital correlator) can be determined as linear function with a slope $\mathbf{dr/d\rho(\rho=0)}$. Then signals level are determined from (12) by the next expressions:

$$\begin{aligned} S_{44} &= \frac{2}{\pi}\left[(n-1)\exp\left(-\frac{v^2}{2}\right)+1\right]^2\rho, \qquad \textit{two- four} \\ S_{24} &= \frac{2}{\pi}n\left[(n-1)\exp\left(-\frac{v^2}{2}\right)+1\right]\rho, \qquad \textit{four- four} \end{aligned} \qquad \mathbf{(15)}$$

Analyzing the noise we consider the correlation coefficient ρ equal zero and then the random processes ε_1 and ε_2 are becoming independent. So square of noise variance is equal $\langle(\varepsilon_1\varepsilon_2)^2\rangle = \langle(\varepsilon_1)^2\rangle \langle(\varepsilon_2)^2\rangle$. For two levels digitizer $\langle(\varepsilon)^2\rangle = \mathbf{n}^2$. For four levels digitizer $\langle(\varepsilon)^2\rangle$ is equal maximal value of output of the FOUR TO FOUR correlator-$\mathbf{r}_{m44}$. Finally we can write the next equations for the noise variance:

$$(N_{24}) = n\sqrt{(N_{44})} = \sqrt{n^2 - (n^2-1)\,erf\left(\frac{v}{\sqrt{2}}\right)} \qquad \mathbf{(16)}$$

Having combined (15) and (16) we receive the signal to noise ratio for the both cases:

$$\left(\frac{S}{N}\right)_{24} = \sqrt{\frac{2}{\pi}\left(\frac{S}{N}\right)_{44}} = \frac{2}{\pi}\frac{(n-1)\exp\left(-\frac{v^2}{2}\right)+1}{\sqrt{n^2-(n^2-1)erf\left(\frac{v}{\sqrt{2}}\right)}} \qquad \textbf{(17)}$$

It is clear from (17) that *if* $\mathbf{v} = 0.981599$ *and* $\mathbf{n} = 3.335875$ *are optimal values for the case FOUR TO FOUR (Swab, 1986) then the same values are optimal for the case TWO TO FOUR.* The signal/noise ratios for both cases versus threshold are shown at the Fig. 3. The maximum signal/noise ratio comparatively with analog case is equal 0.7495 for TWO TO FOUR correlator. This SNR drops less than 5% when the threshold differs from the optimal one (0.9816) as large as 0.4 at the left side and 1.6 at the right side.

3. THE RESULTS

We have analyzed here the normalized outputs of the digital correlator $\mathbf{r_n=r/r_{max}}$. The expression for $\mathbf{r_{max}}$ are given by (12) and (13) for the FOUR TO FOUR and TWO TO FOUR cases respectively. If the values of $\mathbf{v}$ and $\mathbf{n}$ are equal to the optimal ones then $\mathbf{r_{m24}}$= 5.8784 and $\mathbf{r_{m44}}$=4.3048. The expression of the derivatives $\mathbf{dr_n/d\rho}$ at these particular cases can be obtained from (8) having substituted the corresponded values of $\mathbf{v_1}$ and $\mathbf{v_2}$ ($\mathbf{v_1} = \mathbf{v_2} = \mathbf{v}$ for the FOUR TO FOUR case and $\mathbf{v_1} = 0$; $\mathbf{v_2} = \mathbf{v}$ for the TWO TO FOUR case):

$$\frac{dr_n}{d\rho} = \frac{1}{r_{m44}}\frac{1}{\pi}\frac{1}{\sqrt{1-\rho^2}}\,[\,(n-1)^2\left[\exp\left(-\frac{v^2}{1+\rho}\right)+\exp\left(-\frac{v^2}{1-\rho}\right)\right] + 4\,(n-1)\exp\left(-\frac{v^2}{2(1-\rho^2)}\right) + 2\,], \qquad \textit{FOUR - FOUR} \qquad \textbf{(18)}$$

$$\frac{dr_n}{d\rho} = \frac{1}{r_{m24}}\frac{2}{\pi}\frac{1}{\sqrt{1-\rho^2}}\,[(n-1)^2\exp\left(-\frac{v^2}{2(1-\rho^2)}\right) + (n-1)\left[\exp\left(-\frac{v^2}{2(1-\rho^2)}\right)+1\right]+1\,], \qquad \textit{TWO - FOUR} \qquad \textbf{(19)}$$

Using (12), (13), (18) and (19) we can receive the relation between $\mathbf{r_n}$ and ρ for the small correlation coefficient - the most typical case at the radio astronomy.

$$r_n = \frac{2}{\pi}\frac{\left[(n-1)\exp\left(-\frac{v^2}{2}\right)+1\right]^2}{n^2-(n^2-1)erf\left(\frac{v}{\sqrt{2}}\right)}\rho\,, \qquad \textit{FOUR - FOUR} \qquad \textbf{(20)}$$

$$r_n = \frac{2}{\pi} \frac{n\left[(n-1)\exp\left(-\frac{v^2}{2}\right) + 1\right]}{n^2 - (n^2 - n)\,erf\left(\frac{v}{\sqrt{2}}\right)}\rho\,, \qquad TWO - FOUR \qquad \textbf{(21)}$$

It is possible to prove that equation (20) and (21) are becoming the identity when **v** and **n** are equal to the optimal values. So the relation between $\mathbf{r_n}$ and ρ for the small correlation coefficient is the same for both TWO TO FOUR and FOUR TO FOUR correlator if **v** and **n** are equal to the optimal values.

$$r_n = 0.882518\,\rho\,, \qquad \rho = 1.133121\,r_n \qquad \textbf{(22)}$$

We have carried out the digital integrating of the derivative $\mathbf{dr_n/d\rho}$ for the FOUR TO FOUR and TWO TO FOUR cases and several values of number of points at an integrating sum. The result is represented at table 1 and table 2, and Fig. 4. It is seen from the tables that increasing of the number of points at an integrating sum more than 5000 does not change the result more than 2.10^{-5} for all possible ρ and for both cases. The linear equation (21) approximate the real relation with accuracy better than 10^{-3} if $\boldsymbol{\rho<0.1}$ and better than 10^{-2} if $\boldsymbol{\rho<0.2}$.

III. THE EFFECT OF FRINGE STOPPING PROCEDURE.

The correlation coefficient of the original signals is distorted usually by digitizing of the signals. This distortion is possible to recover because the function related measured correlation coefficient of the digitizing signals and original analog signals is known for the whole domain of the input correlation coefficients - from 0 till 1. But at VLBI case the estimation of the correlation coefficient is done after fringe stop procedure - multiplication on the complex exponent of the expecting fringe rate and following averaging of the product. This procedure brings additional distortion at the correlation coefficient estimation especially for the high correlated signals. So despite of the fact that we know the converted function for the whole range ρ (0,1) at the VLBI we can apply it correctly only for linear part of this function when the value of the correlation coefficient is very small. Definitely the case of small correlation coefficients is the most typical one at the radio astronomy. But the rather high correlation coefficients (even close to 1) have a place occasionally. For example the brightness of the maser water vapor source at the Orion nebula corresponded to 1.10^6 ja at 80th years. So VLBI observations of this source at Mark II system (the bandwidth less than 2MHz) provided the correlation coefficient close to 1.

At this section the estimation of whole distortion of the correlation coefficient (including digitizing and fringe stopping) has been done. The signal at the output of the fringe stop unit is determined by the next equation:

$$r_{out} = \frac{1}{\Delta t} \int_{t-\Delta t}^{t+\Delta t} F(\rho)\, expj(\omega t + \varphi)\, dt \tag{23}$$

where **F(ρ)** is the function related correlation coefficient of original analog signals ρ and digitized signals. This function estimates the signal at the input of the fringe stop unit just after the multiplication of the digitized signals. So **F(ρ)** is periodic function and can be represented by Fourie series:

$$F(\rho) = \sum_{k=1}^{\infty} a_k \cos(k\omega_{fr} t) \tag{24}$$

where ω_{fr} is the fringe rate. The Fourie coefficients can be calculated by standard formulas:

$$a_k = \frac{\omega}{\pi} \int_{-\pi/\omega}^{\pi/\omega} F(\rho) \cos(k\omega_{fr} t)\, dt = 2 \int_{-1/2}^{1/2} F(\rho) \cos(k 2\pi x)\, dx \tag{25}$$

Having substituted (24) at (23) and considering $\omega = \omega_{fr} + \Delta\omega$; $\Delta\omega\Delta t \ll 1$ we have:

$$r_{out} = \exp j(\Delta\omega t + \varphi) \left[\frac{1}{2} a_1 + O\left(\frac{a_1}{2\omega\Delta t}, \frac{a_2}{\omega\Delta t} \right) \right] \tag{26}$$

The fringe rate is usually of order unit of kHz and the time of average at a fringe stopper of order units of seconds. So magnitude of O() at (26) is usually less than 10^{-3} and we neglect it as well as it is neglected elsewhere when the fringe stopper output is analyzed. Having substituted (25) at (26) we are receiving the next formula for absolute value of the signal at the output of the fringe stopper.

$$|r_{out}| = \frac{1}{2} a_1 = \int_{-1/2}^{1/2} F(\rho \cos(2\pi x)) \cos(2\pi x)\, dx \tag{27}$$

Having changed the variables at the integral **x = z/(2π)+1/4** we can simplify the equation (27):

$$|r_{out}| = \frac{2}{\pi} \int_{0}^{\pi/2} F(\rho \sin z) \sin z\, dz \tag{28}$$

or after substitution **t = sin(x)**

$$|r_{out}| = \frac{2}{\pi}\int_0^1 F(\rho\, t)\,\frac{t}{\sqrt{(1-t^2)}}\,dt \tag{29}$$

This equation connects amplitude of fringe lobes ρ with a measured absolute value of the signal at the output of the fringe stopper $\mathbf{r}_{out}$. Now let's apply method of integration by parts to the equation (29):

$$|r_{out}| = \frac{2}{\pi}\left(-F(\rho\, t)\sqrt{1-t^2}\,\Big|_0^1 + \int_0^1 \frac{dF(\rho\, t)}{dt}\sqrt{1-t^2}\,dt\right) \tag{30}$$

The first adding of (30) is equal 0 because **F(0)=0** and $\sqrt{1-t^2} = 0$ if **t=1**. Finally we receive the next formula expressed $\mathbf{r}_{out}$:

$$F_1(\rho) = |r_{out}| = \frac{2}{\pi}\,\rho\int_0^1 \frac{dF(\rho\, t)}{d(\rho\, t)}\sqrt{1-t^2}\,dt \tag{31}$$

The expression for **dF(x)/dx** is known at the elementary functions for all three combinations of the digitizers level TWO-TWO, TWO-FOUR, FOUR-FOUR (see formulas 1, 17, 18).

At the TWO-TWO case (two levels digitizers are at the both site) the function **F(ρt)** is represented by well known Van Vleck relation **F(ρt)=(2/π)arcsin(ρt)** and the expression for $\mathbf{r}_{out}$ can be rewritten at this case:

$$F_1(\rho) = |r_{out}| = \left(\frac{2}{\pi}\right)^2\rho\int_0^1 \frac{\sqrt{1-t^2}}{\sqrt{1-(\rho\, t)^2}}\,dt = \left(\frac{2}{\pi}\right)^2\rho\, B(\rho) \tag{32}$$

where **B(ρ)** is the linear combination of the complete elliptic integrals of the first and second kind. The tables of the function **B(ρ)** can be found elsewhere for example at Emde & Yanke. The numerical calculation of the integral (30) can be applied at the cases TWO-FOUR and FOUR-FOUR. The plots of the graph $\mathbf{2r}_{out}$ versus amplitude of original correlation are shown at the Fig. 5. The tables 3,4,5 represent the values of the function $\mathbf{F}_1(\rho)$ for three possible combination of digitizers at VLBI (TWO-TWO, TWO-FOUR, FOUR-FOUR). The analogous tables of function **F(ρ)** are represented by tables 12, 13, 14. The function $\mathbf{F}_1(\rho)$ relates the output of the fringe stopping procedure with the input correlation ρ and is applied at a cross correlation case. The function **F(ρ)** relates the output of the correlator with the input correlation ρ and is applied at a auto correlation case. The conversion function **F(ρ)** is linear for small value of ρ and has a rise at the region of large values of ρ. The fringe stopper procedure on the contrary suppresses the large amplitudes of input correlation. So jointly they have to linearise the relation between input correlation amplitude and output of the fringe stopper procedure. The evidence of this is clear from Fig. 6-8 where relative error of linear approximation is shown for the three combinations of digitizers levels at the interferometers sites. The optimum value of the line slope has been found using criteria of minimum of integral of square of deviation of $\mathbf{F_1(\rho)}$ from the linear law. The optimal values of the slope were calculated separately for the three combinations of the digitizer levels and for three region of input correlation amplitudes: (0, 1), (0, 0.5) and (0, 0.25). The optimal values of the slope and corresponding deviation from the linear laws are shown at the table 6.

TABLE 6.

Range of r	TWO-TWO		TWO-FOUR		FOUR-FOUR	
	kopt	Δr/r,%	kopt	Δr/r,%	kopt	Δr/r,%
(0, 1)	0.710	10	0.909	2.6	0.897	2.5
(0, 0.5)	0.649	1.3	0.890	0.7	0.886	0.3
(0, 0.25)	0.640	0.3	0.884	0.1	0.883	0.08

It seen from the table that the function $\mathbf{F_1(\rho)}$ is linear at the whole range of ρ (0,1) with accuracy better than 2.5% for the cases TWO-FOUR and FOUR-FOUR. For the limit range of ρ the linearity is more better. So if we are satisfied such degree of linearity we can use corresponded factor to restore the correlation amplitude from measured output of the fringe stopper procedure.

IV. THE EFFECT OF FRACTIONAL BIT SHIFT CORRECTION

The delay introduced to align the bit stream is quantized at the sampling rate which is equal to or more than the Nyquist rate for the original analog signal with limited spectra at the range **0-Δf**. Thus there is a periodic sawtooth delay error with a peak-to-peak amplitude equal to the sampling period. This effect known as a fractional bit shift error can be compensated by the corresponded phase shift of the cross power spectrum. The cross correlation function and cross power spectrum are the Fourie transform pair. So we take the Fourie transform of the sampled cross correlation function receiving the cross power spectrum, multiply its components by complex exponent with phase proportional to the frequency and delay which we desire to compensate and which is known for us in advance for every time. Then we take inverse Fourie transform and receive restored crosscorrelation function. This process is equivalent to evaluating of the cross correlation function at the arbitrary value of its argument using the known values of this function at the sampled time moments. Definitely we are interested to find this value for argument (delay) which is equal zero. It is well known that the value of the function at arbitrary value of its argument can be restored completely from the known sampled values if the spectrum of the function (its Fourie transform) is limited and if the sampling frequency is at least two times more than maximum frequency of the spectrum. These two conditions are satisfied for the cross correlation function of the analog original signals but they are not satisfied for cross correlation function of the digitized signals because the spectrum of such signals continue to infinitive. So the procedure of fractional bit shift correction does not restore the maximum value of correlation if it is applied to sampled digitized signals. We have fulfilled some numerical calculation to estimate the signal loosing due to described above reason. It is possible to show that the procedure of fractional bit shift correction can be expressed by the next equation if we use the FFT for Fourie transformation.

$$r_{-}(x) = \sum_{i=-\frac{N}{2}}^{\frac{N}{2}-1} F_1[r(i)]\,\frac{\sin\pi\,(i-x)}{N\sin\pi\,\frac{(i-x)}{N}}\cos\pi\,\frac{(i-x)}{N} \qquad \textbf{(33)}$$

where **r(i)** is the sampled value of correlation,

r_(x) is restored value of correlation at the point **x**,

N is the even number of points at the FFTs.

$\mathbf{F_1[r]}$ is the conversion function from correlation of analog signals to the estimation of it at the fringe stop output.

We have used the expression (33) for the case of continuum spectrum from **0** to **Δf** and for the all three above described combination of digitizers. The corresponded conversion functions $\mathbf{F_1[r]}$ were taken from tables 3,4,5. We were installing the delay shift at the original sampled data and then calculated **r_(x)** at the point equal to this shift using (33). The result is shown at the tables 7,8,9 for the cases TWO-TWO, TWO-FOUR and FOUR-FOUR. Each table contents the calculation result for five values (0.2, 0.4, 0.6, 0.8 and 1.0) of maximum input correlation **r**. The first line is the delay shift at parts of sample period; the second line corresponds to the analog case ($\mathbf{F_1[r]=r}$) and demonstrates the expected complete restore of the maximum value; the third line is restored relative value of the maxima; the fourth line is restored absolute value. We see from the tables 7,8,9 that the loosing of the signal due to unideal restoration of the correlation maxima is negligible if the correlation is less than 0.6. For the maximum value of the correlation (**r=1**) the average loosing of the signal is less than 3% for the case TWO-TWO and 1% for the cases TWO-FOUR and FOUR-FOUR. This loosing of the signal is improving a little bit the linearity of the conversion function.

V. FX CORRELATOR

Signal at the output of a FX correlator can be represented by the next equation (D'Addario, 1989):

$$\zeta(\Omega) = \sum_{t}\sum_{l=-K}^{K} Tr(l)\,F(R(t,\tau))\,\exp(-j\Omega\,(l+\Delta l)\,\delta\tau)\,\exp(-j\omega\,t) \qquad \textbf{(34)}$$

where **K** is the number of points at the FFT; **Ω** is the frequency at the video bandwidth; **ω** is the fringe rate; **Tr(l)** is triangular form function determined by the next equation:

$$Tr(l) = \begin{cases} K - |l|, & if\ |l| < K \\ 0, & otherwise. \end{cases} \qquad (35)$$

Δl(t) is the fraction of a bit delay introduced to provide fractional bit shift correction; **F(R)** is the conversion function related original correlation of the analog signals and correlation of the real digitized signals; **R(t,τ)** is the crosscorrelation function connected with the spectrum of the signal by the next equation:

$$\begin{aligned} R(t,\tau) &= \rho \cdot |b((l+\Delta l)\delta\tau)| \cdot \cos(\omega t + arg(b) + \varphi) \\ b(\tau) &= \int_{\Omega} B(\Omega)\exp(j\Omega\tau)\,d\Omega \end{aligned} \qquad (36)$$

where ρ is the maximum correlation of the input signals; **b(0)=1**; The noncompensated delay **Δl(t)** and the phase of fringe lobes **ωt** depend upon time periodically. The ratio of their periods determined by the next equation $\mathbf{T_d/T_{fr} = f/(2\Delta f)}$ is very large. So we can calculate the sums by **t** using the time intervals which are rather large comparatively with $\mathbf{T_{fr}}$ and small comparatively with $\mathbf{T_d}$ (Fig. 9). The function **F(R)** is an odd one and effects on the output of the first stage of the time averaging by the suppressing the amplitude of the first harmonic of the fringe rate. This suppressing is described by the function $\mathbf{F_1(x)}$ analyzed at the section III for different combinations of digitizers levels. So as a result of the first stage of the time averaging (fringe stopping procedure) we have:

$$\zeta(\Omega) = \sum_{l=-K}^{K} Tr(l) \sum_{t} \frac{F_1(\rho|b((l+\Delta l)\delta\tau)|)}{\rho|b((l+\Delta l)\delta\tau)|} \rho b((l+\Delta l)\delta\tau)\exp(-j\Omega(l+\Delta l)\delta\tau) \qquad (37)$$

We calculated the expression (37) taking the time interval of final summarizing 10 times grater than period $\mathbf{T_d}$ and we were taking ten points for this period. So the whole number of terms at the internal sum is equal 100. The calculation was carried out for two spectrum of the input signals: the continuum and two spectral features located at the most low and upper part of the bandwidth. The upper feature was selected to have 2 times less bandwidth than lower one. The result as graphs of the function $|\varsigma(\Omega)|$ for different maximum value of input correlation - 0.4, 0.6, 0.8, 1.0 is shown at the figures 10-16 .

The case of continuum spectra.

To analyze this case we added amplitudes at each frequency channels to estimate the correlation corresponded to the whole bandwidth. The dependence of this sum upon the input correlation is represented by graphs at Fig. 17 for three combinations of digitizers TWO-TWO, TWO-FOUR and FOUR-FOUR. The graphs corresponded to the cases TWO-FOUR and FOUR-FOUR are very linear. Nonlinearity is less than 1% for the FOUR-FOUR case and 1.5% for the TWO-FOUR for the whole range of input correlation from 0 till 1. Nonlinearity of the graph corresponded to the case TWO-TWO is definitely more and equal $\pm 5\%$. So we can use just multiplication on the constant coefficient to estimate the input correlation from measured output of FX correlator.

The case of a spectrum with spectral features.

The digitizing does not change the flat spectrum dramatically but it does change the ratio of intensities of spectral features. It is demonstrated by figures 14-16. The relative change of the ratio of the two components intensity is shown at the table 10.

Table 10.
Relative error of ratio of two components for different combinations of digitizers and different correlations.

	r = 0.4	r = 0.6	r = 0.8	r = 1.0
TWO-TWO, %	1.66	3.95	7.85	15.45
TWO-FOUR, %	0.67	1.49	2.48	3.42
FOUR-FOUR,%	0.29	0.66	1.21	2.45

It is seen from the table 10 that distortion of the components ratio is less than 2.5% and 3.5% for the of FOUR-FOUR and TWO-FOUR respectively for the whole range of possible input correlation from 0 till 1. The distortion of the component ratio has to depend upon this ratio itself. To investigate this dependence we repeated our calculation for different ratio of the features intensity. The result of these calculations is shown at the table 11. Relative error of the component intensity ratio is indicated at percents.

Table 11.
Error of determining of the ratio of components intensity (at %) versus the input correlation and the ratio itself. FOUR-FOUR case.

	500	100	20	10	1	1/10	1/20	1/100	1/500
r = 0.4	0.37	0.053	0.048	0.046	0.026	0.29	0.38	0.44	0.51
r = 0.6	0.52	0.18	0.14	0.11	0.066	0.66	0.85	0.95	1.16
r = 0.8	0.85	0.39	0.25	0.21	0.115	1.21	1.57	1.869	1.96
r = 1.0	19.6	6.8	2.31	1.12	0.18	2.45	3.91	9.25	14.1

It is seen from the table that the smallest error in ratio of the component has a place for the equal intensity of the components. When the ratio of the component intensity is raising to 500 the error of its ratio is increased till 19% for the limit correlation. The smaller error corresponds to smaller upper (narrow) component. But in any case the error of ratio of the components intensity is less than 2% if the ratio is less 500 and input correlation less than 0.8 at the case FOUR-FOUR digitizer. The sharp increasing of error of intensity component ratio at the case **ρ>0.8** is connected with nonlinearity of the function **F_1(x)**(see Fig. 5) for this values of ρ. *So, we have to multiply outputs of FX correlator by the constant coefficient to restore the original crosscorrelation spectrum if we are satisfied by accuracy shown at the tables 10 and 11.* This is true for the selected two features components spectrum. And it is probably true for any other spectrum. If not then the measured spectrum has to be Fourie transformed to the delays domain where the known conversion function has to be applied and then inverse Fourie transform gives us the restored spectrum. It is definitely necessary for the autocorrelation spectrum when he correlation is equal 1 for zero delay.

VI. CONCLUSION

The provided analysis gives the conversion functions from original correlation coefficient to the point of its estimation for the three combinations of digitizers at the VLBI sites and for the whole range of input correlation. At XF correlator different conversion functions have to be applied to receive auto and cross correlation function. The function **F(ρ)**(Fig.4) need to apply for the auto correlation and **F_1[ρ]** (Fig.5) for cross correlation. The function **F_1[ρ]** is more linear than **F(ρ)** due to inverse nonlinearity of fringe stopping procedure. Definitely it has a sense only at a very special case of high correlation. The linear part of the function **F_1[ρ]** can be successfully used for the most typical case at radio astronomy - small correlation. The linear conversion function can be used to restore original correlation of continuum spectra source from measured correlation at the output of FX correlator (with accuracy better than 1% for FOUR-FOUR and 1.5% for TWO-FOUR) for the whole range of correlation. The ratio of amplitudes of two component spectra (which we have chosen as an exemple) is restored by FX correlator with accuracy better than 2% if the ratio itself is less than 500 and correlation less 0.8 for the FOUR-FOUR digitizer .

I acknowledge Phil Diamond for involving me at this interesting investigation and supervising the work.

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APPENDIX

The bivariate normal integral is equal:

$$L(k,h,\rho) = \int_k^\infty\int_h^\infty P(x,y,\rho)\,dx\,d\rho \tag{1}$$

where:

$$P(x,y,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}\exp\left[-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}\right] \tag{2}$$

Having carried out the change of the integrating variable

$$x = u\sqrt{1-\rho^2}+\rho y \tag{3}$$

we are receiving another expression for L(k,h,ρ)

$$L(k,h,\rho) = \frac{1}{2\pi}\int_k^\infty \exp\left(-\frac{y^2}{2}\right)\left[\int_{\frac{h-\rho y}{\sqrt{1-\rho^2}}}^\infty \exp\left(-\frac{u^2}{2}\right)du\right]dy \tag{4}$$

The derivative of the equation (4) is equal:

$$\frac{dL(k,h,\rho)}{d\rho} = \frac{1}{2\pi}\int_k^\infty \exp\left[-\frac{y^2-2h\rho y+h^2}{2(1-\rho^2)}\right]\frac{y-h\rho}{1-\rho^2}\,\frac{1}{\sqrt{1-\rho^2}}\,dy \tag{5}$$

Having paid attention that fraction after exponent at (5) is exactly equal to the derivative by y of the exponents power we can take the integral and receive the final expression for the derivative dL/dρ.

$$\frac{dL(k,h,\rho)}{d\rho} = \frac{1}{2\pi}\,\frac{1}{\sqrt{1-\rho^2}}\exp\left(-\frac{k^2-2kh\rho+h^2}{2(1-\rho^2)}\right) = P(k,h,\rho) \tag{6}$$

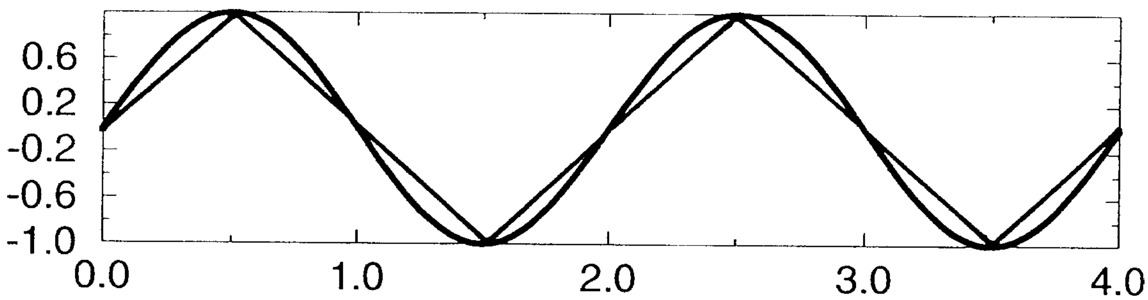

Fig. 1. Conversion of sine wave correlation to the triangle one at the case of one bit digitizers.

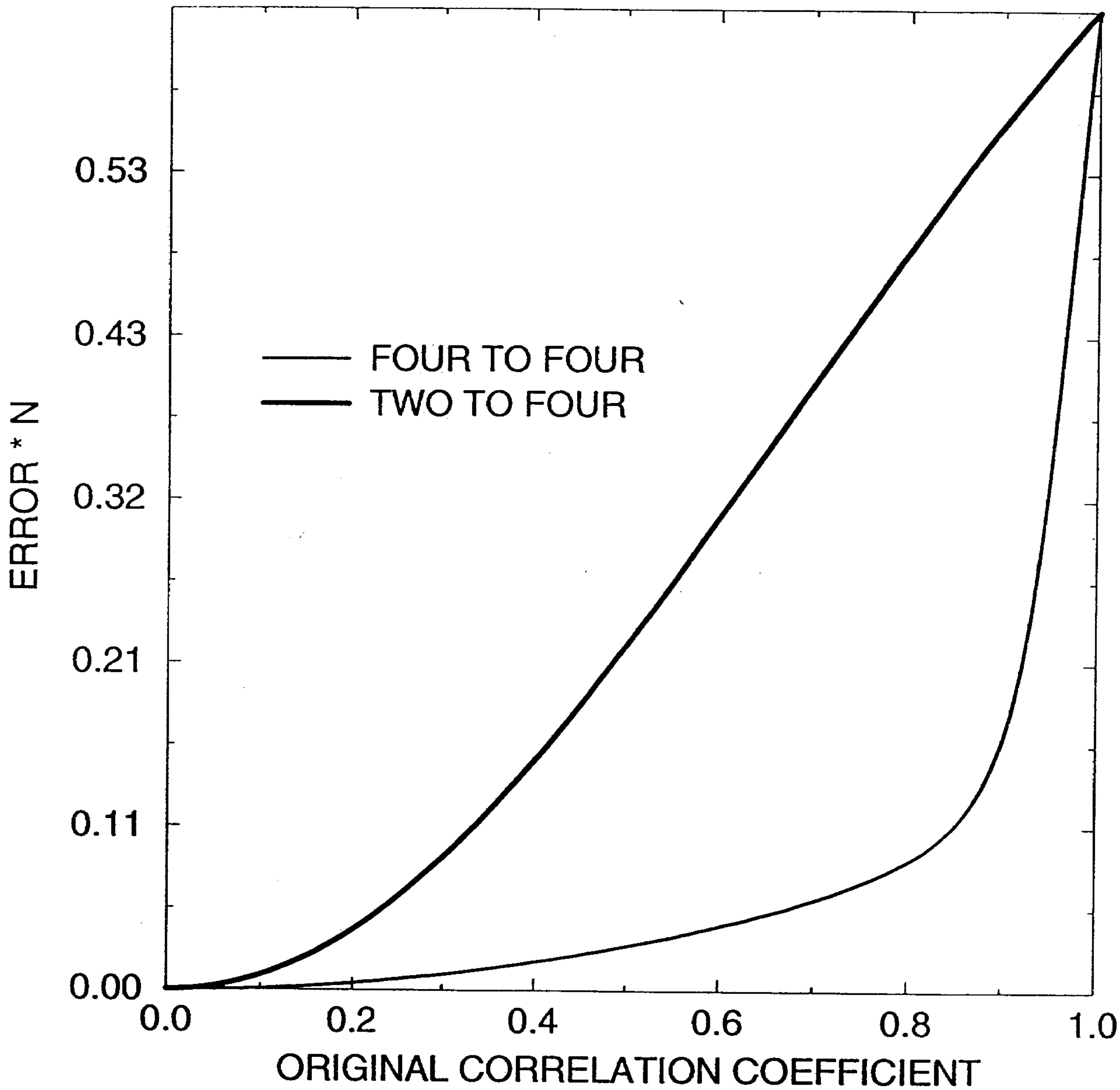

Fig. 2. Error of correlation coefficient estimation

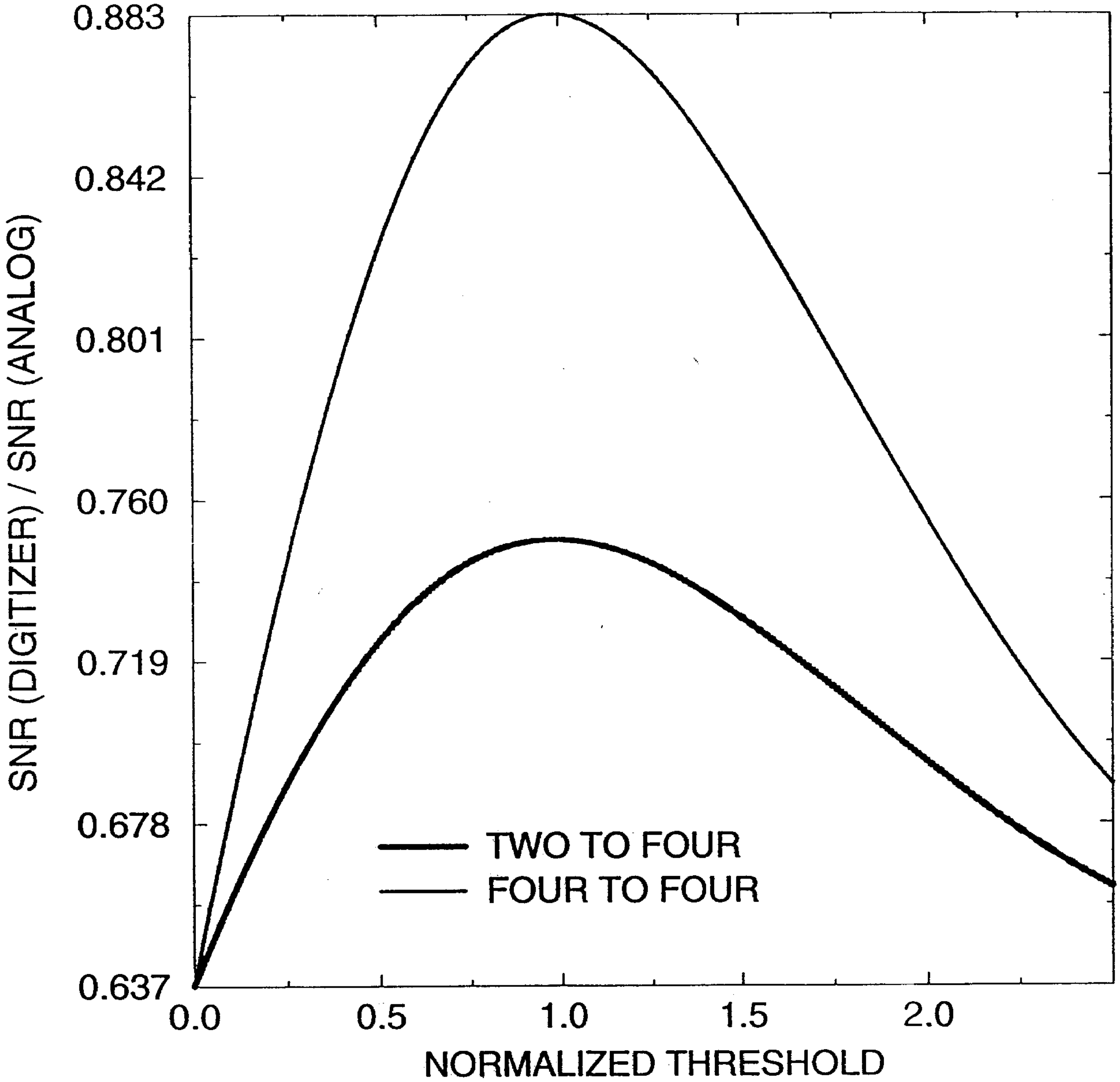

Fig. 3. Signal/noise ratio versus normalized threshold

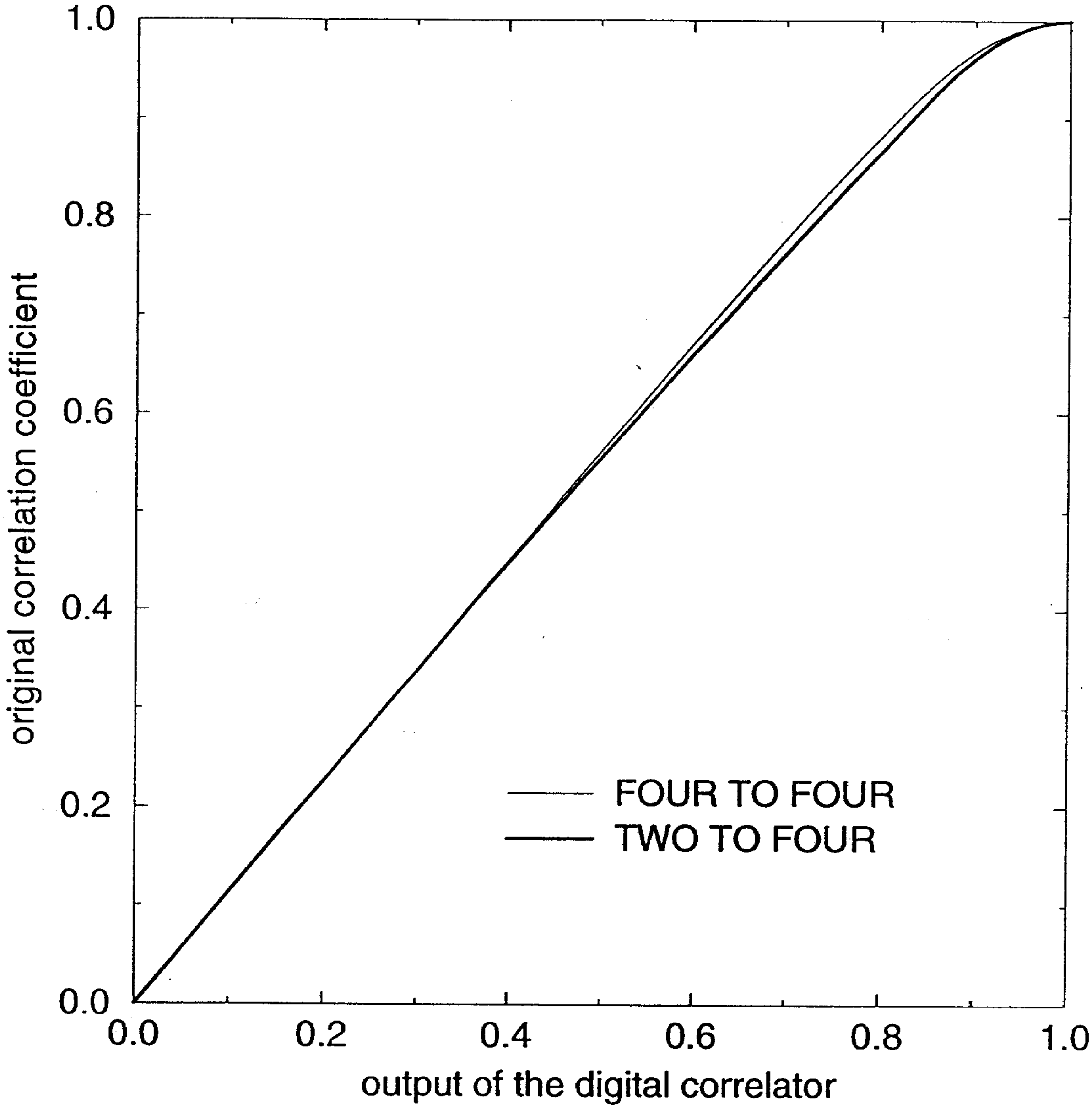

Fig. 4. Original correlation coefficient versus a measured one

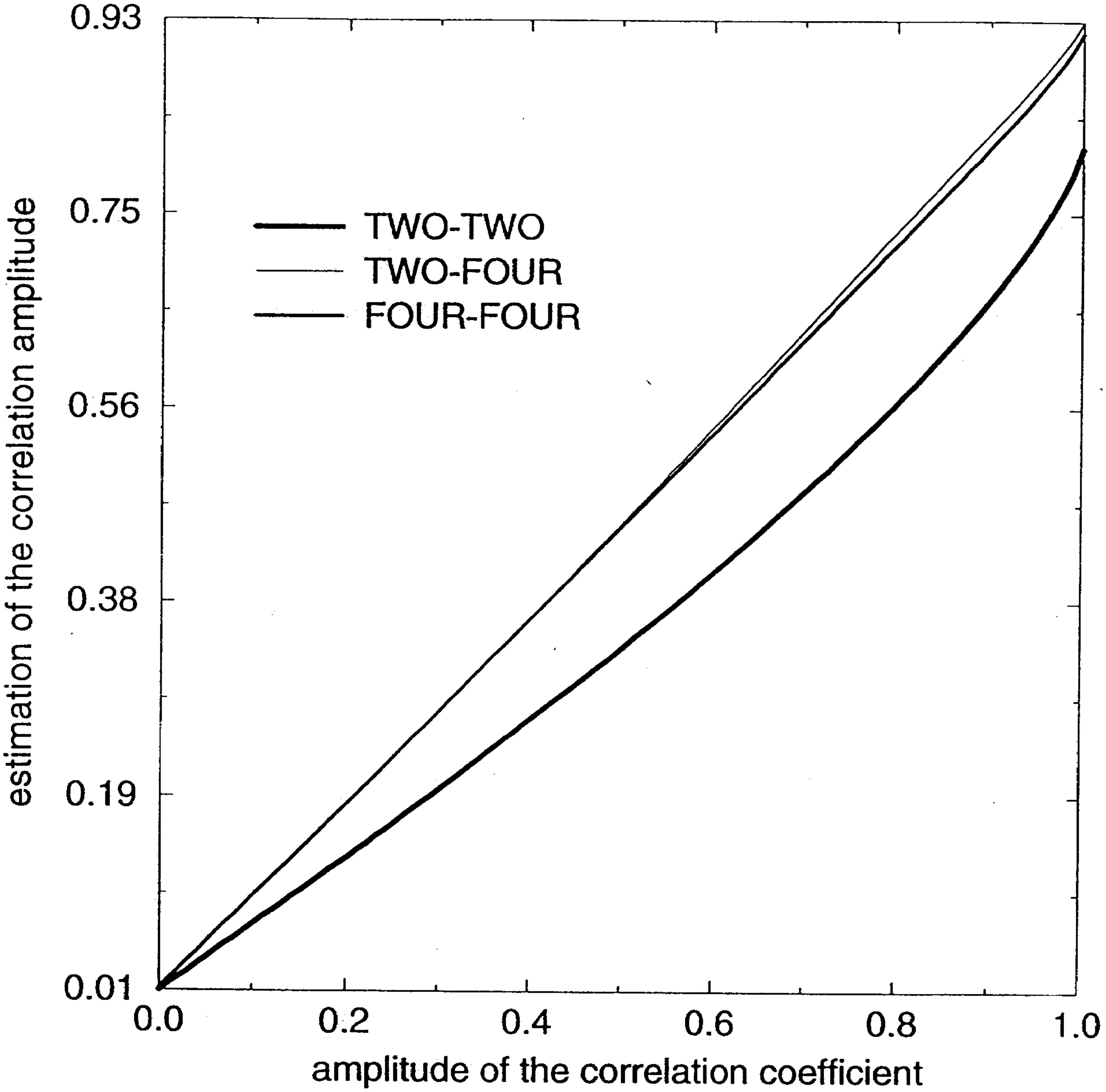

Fig. **5** Estimation of the correlation amplitude at the output of the fringe stopping unit

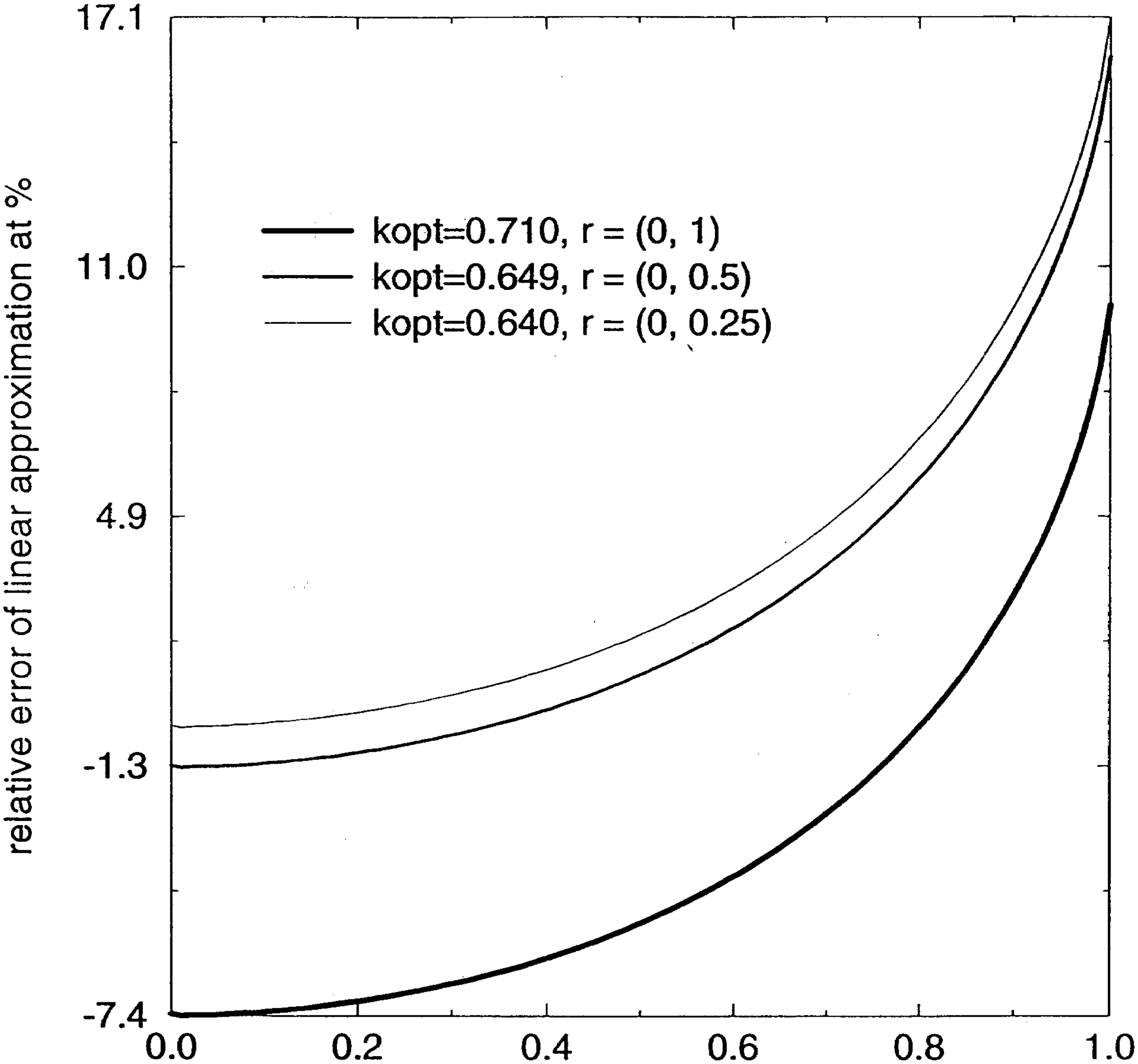

Fig. 6 Relative error of linear approximation with optimal value of the slope for different correlation range. The case TWO-TWO.

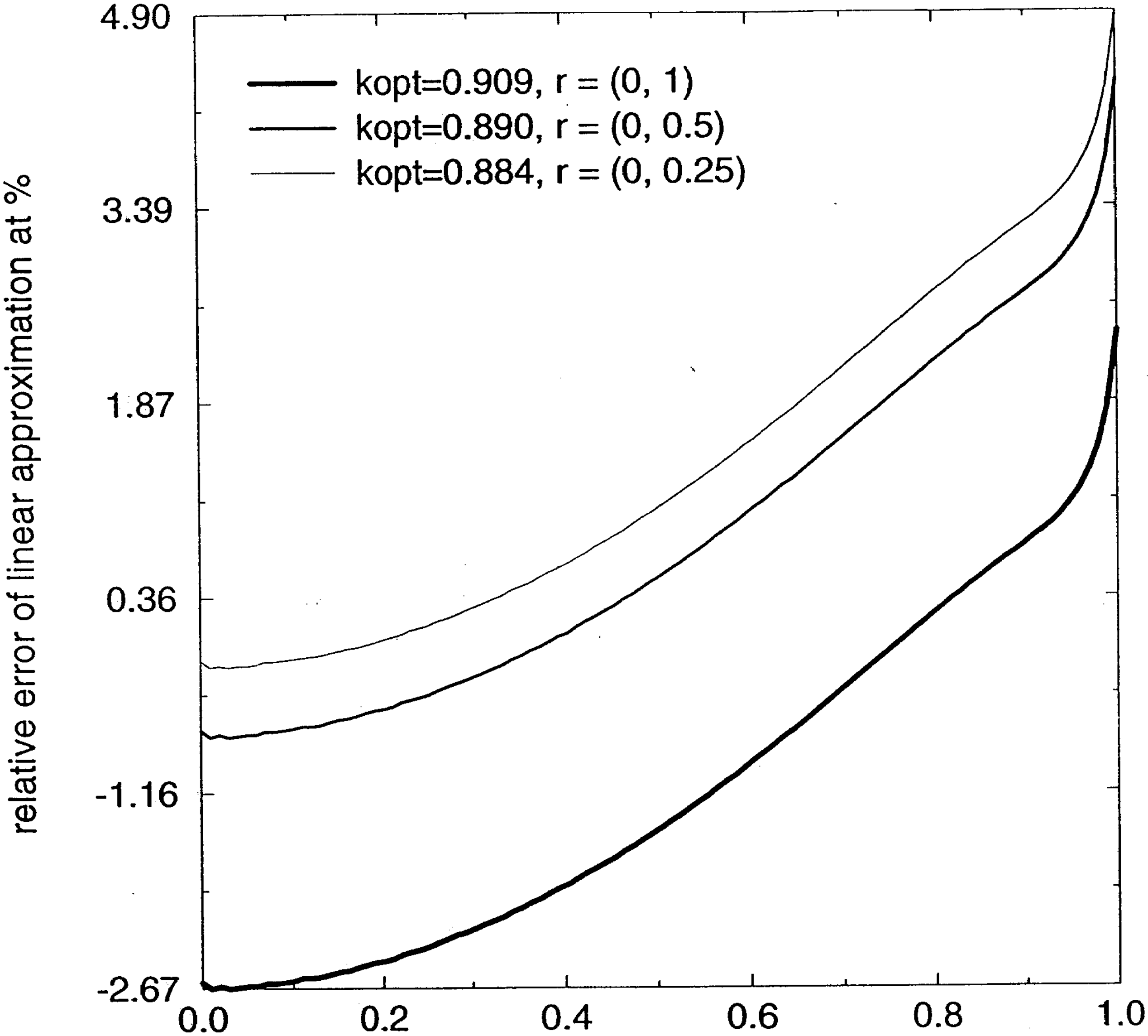

Fig. 7 Relative error of linear approximation with optimal value of the slope for different correlation ranges. The case TWO-FOUR.

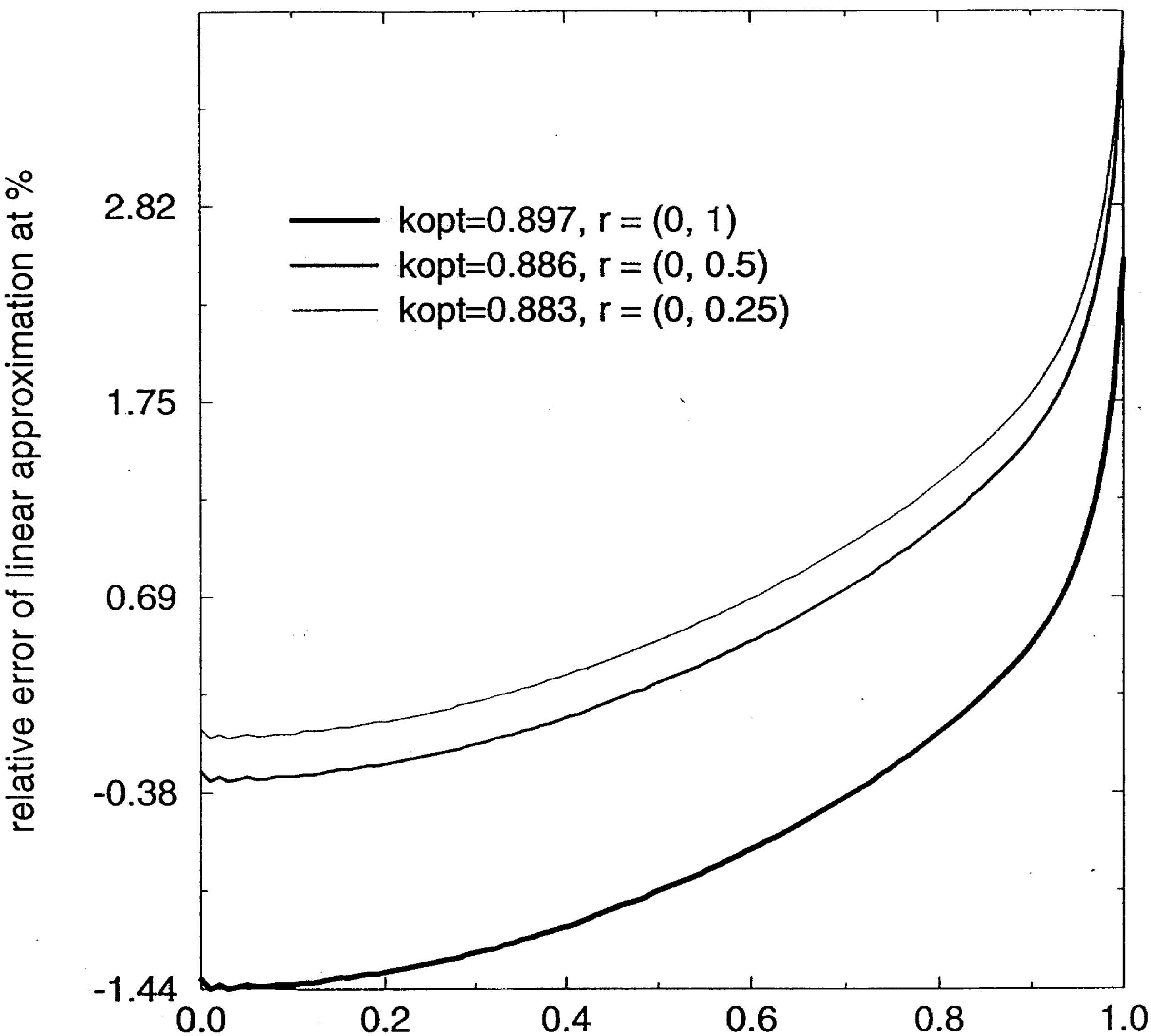

Fig. 8Relative error of linear approximation with optimal value of the slope for different correlation ranges. The case FOUR-FOUR.

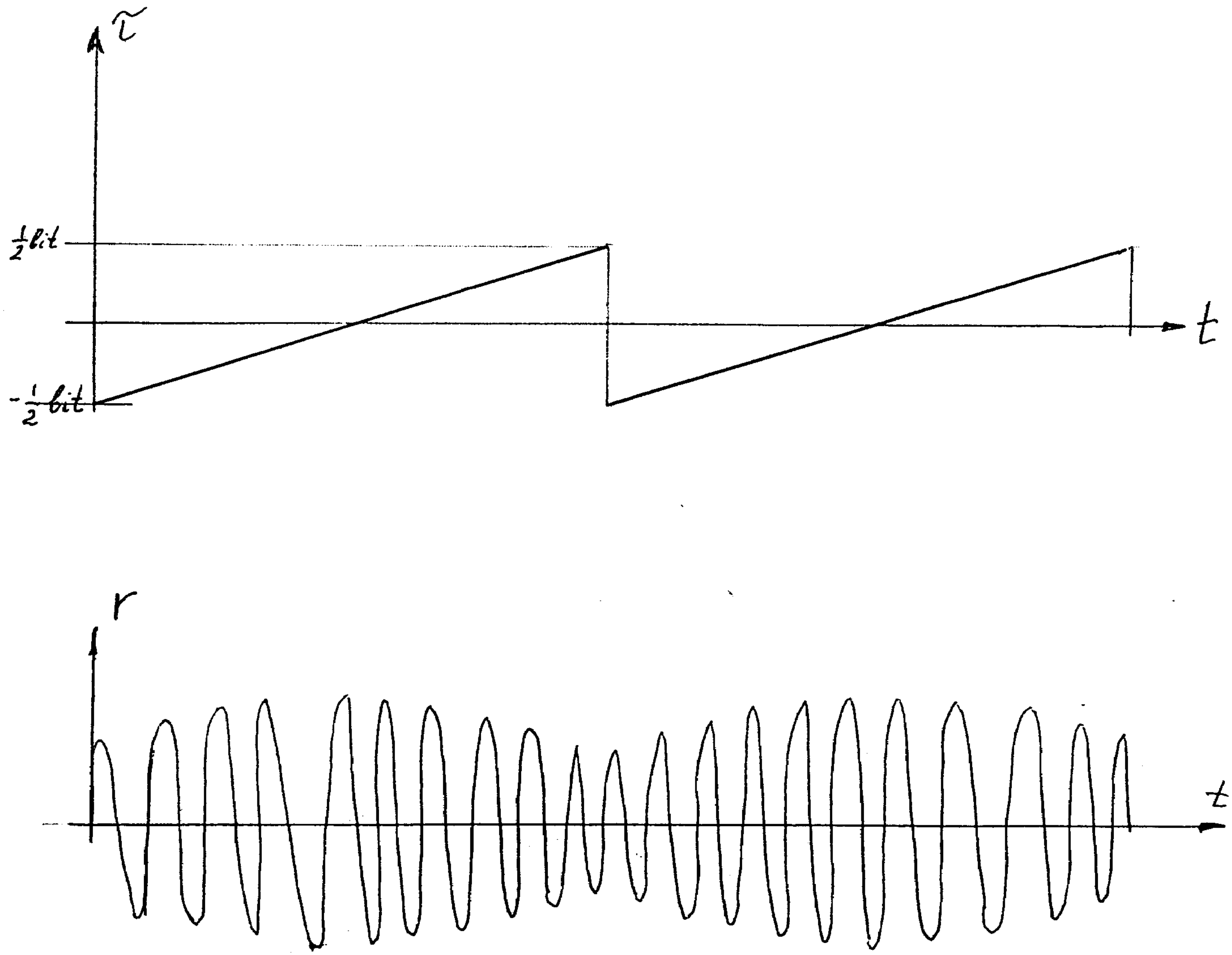

Fig. 9. Changing of a correlation corresponding to the uncompensated delay.

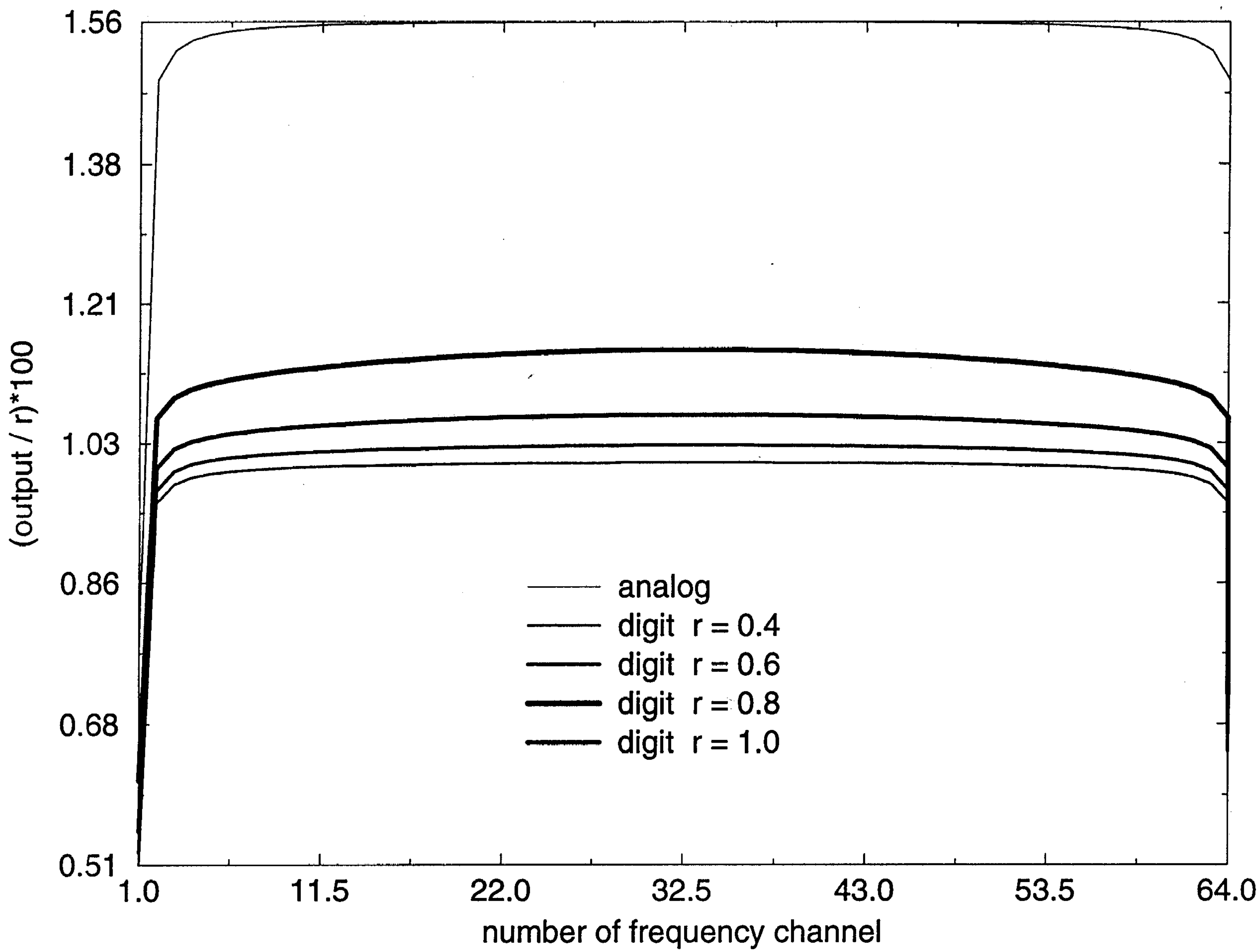

Fig.10 Response of FX correlator. Continuum spectrum. TWO-TWO.

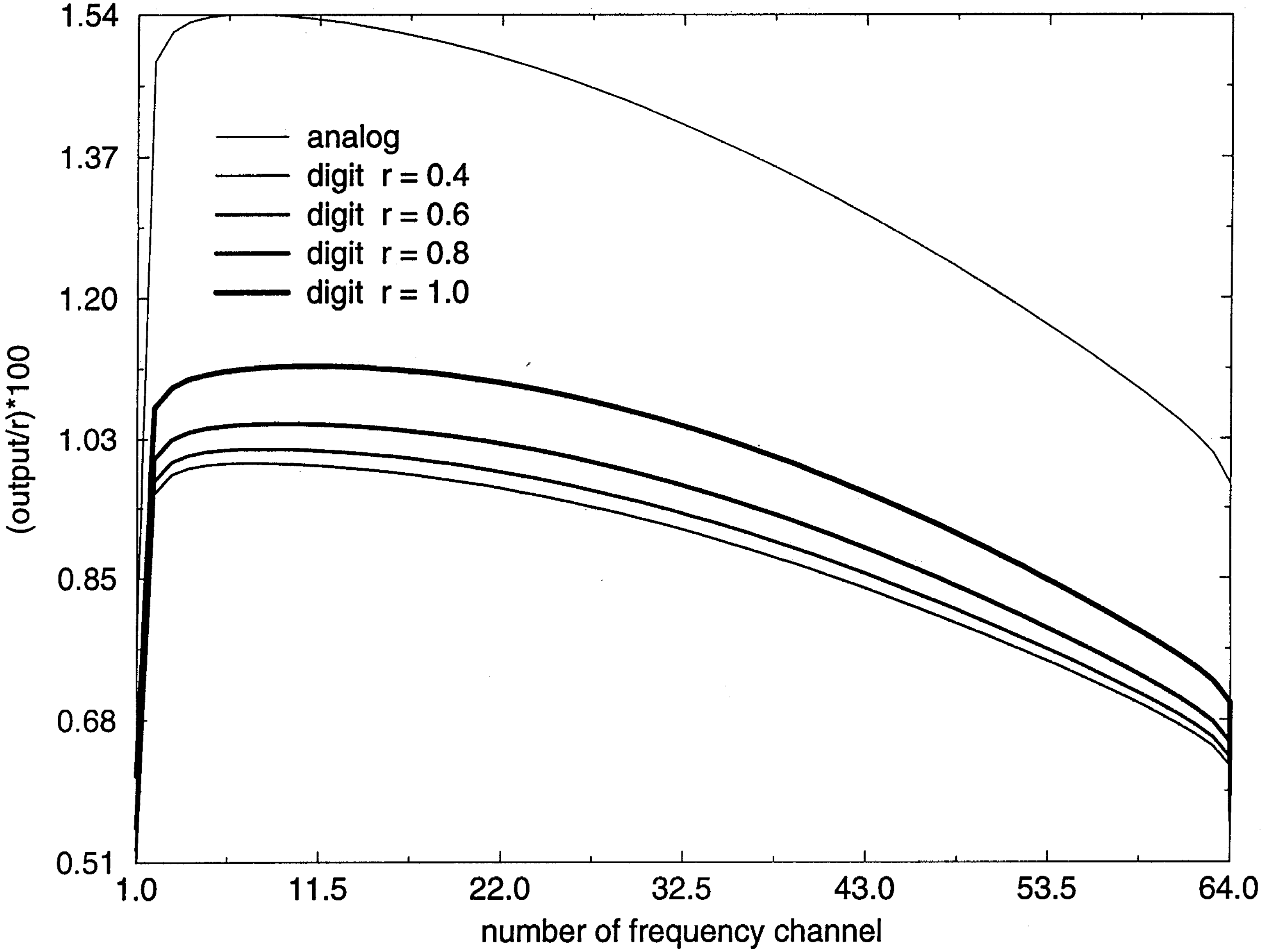

Fig.11 Response of FX correlator. Continuum. One bit shift correction is not applied. TWO-TWO.

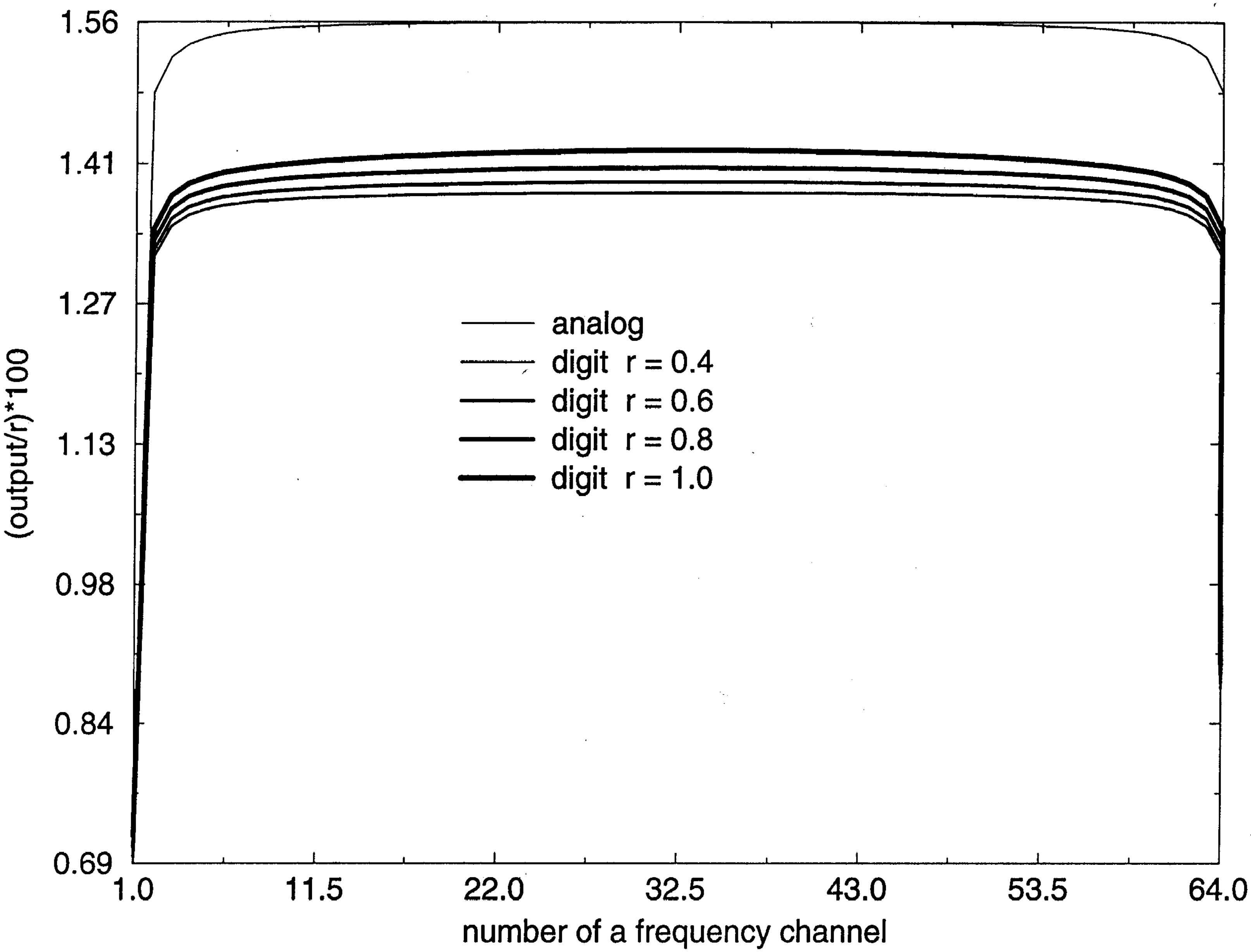

Fig. 12 Response of FX correlator. Continuum spectrum. TWO-FOUR.

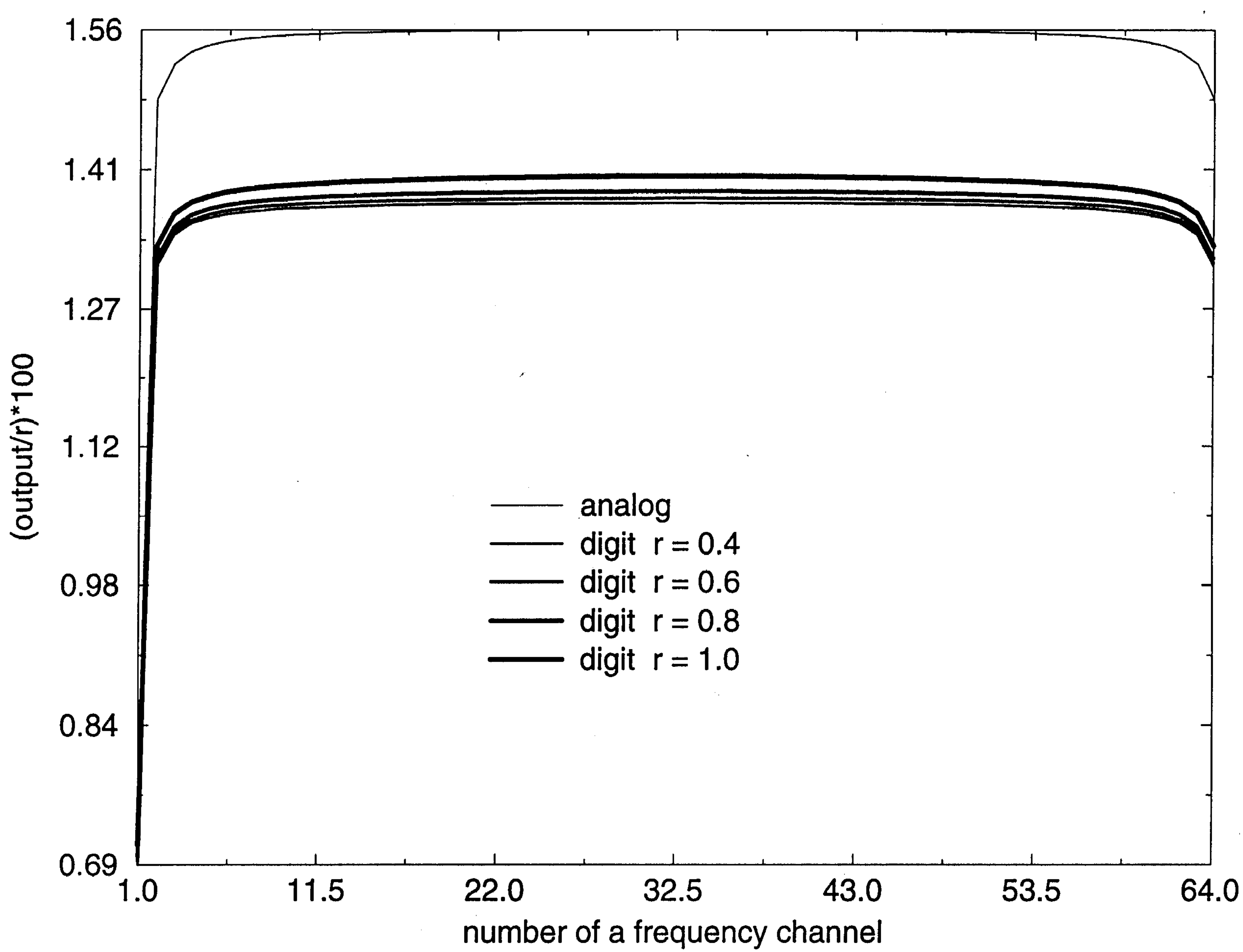

Fig. 13 Response of FX correlator. Continuum spectrum. FOUR-FOUR.

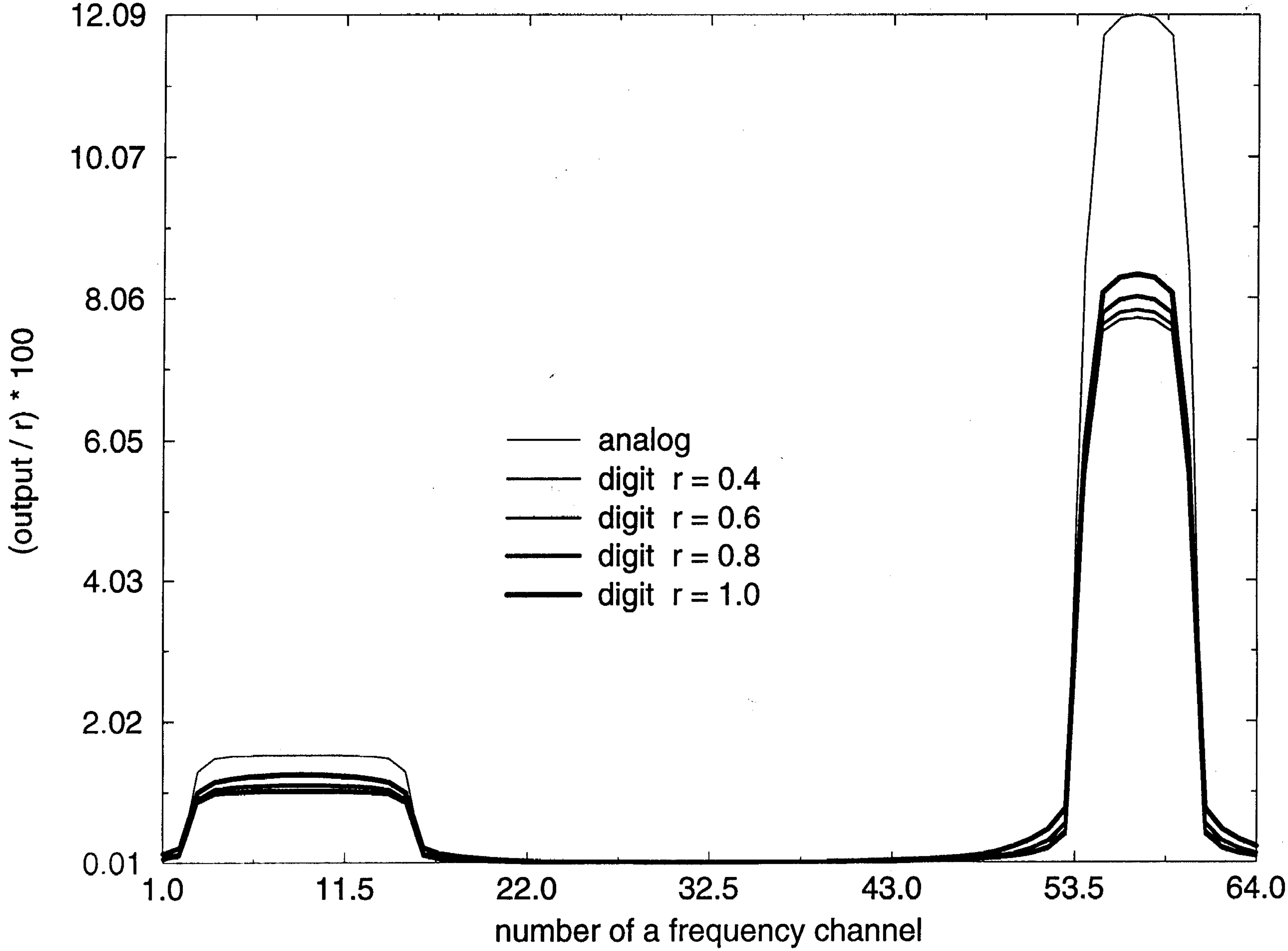

Fig. 14 Response of FX correlator. Two features spectrum. TWO-TWO.

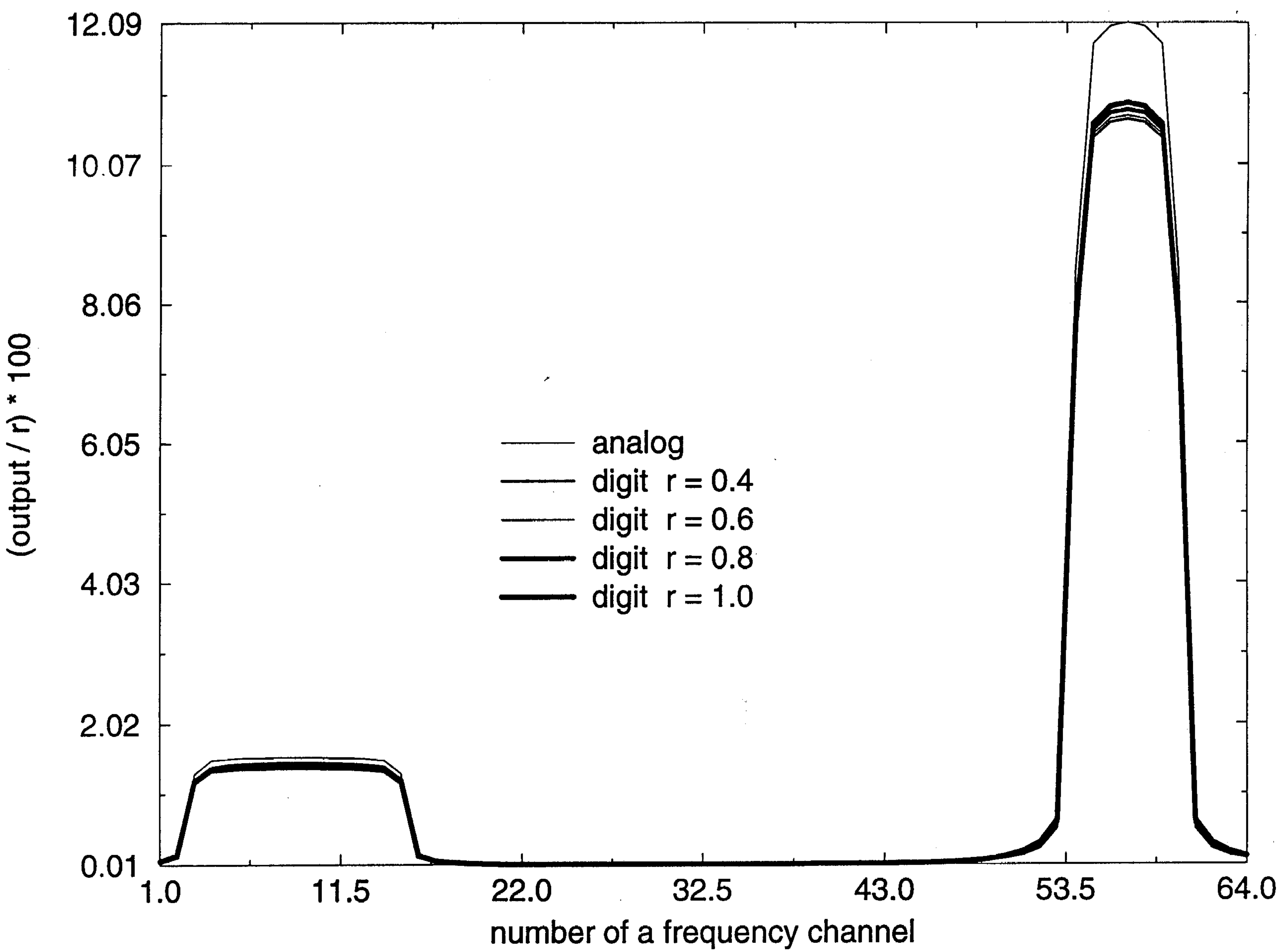

Fig.15 Response of FX correlator. Two features spectrum. TWO-FOUR.

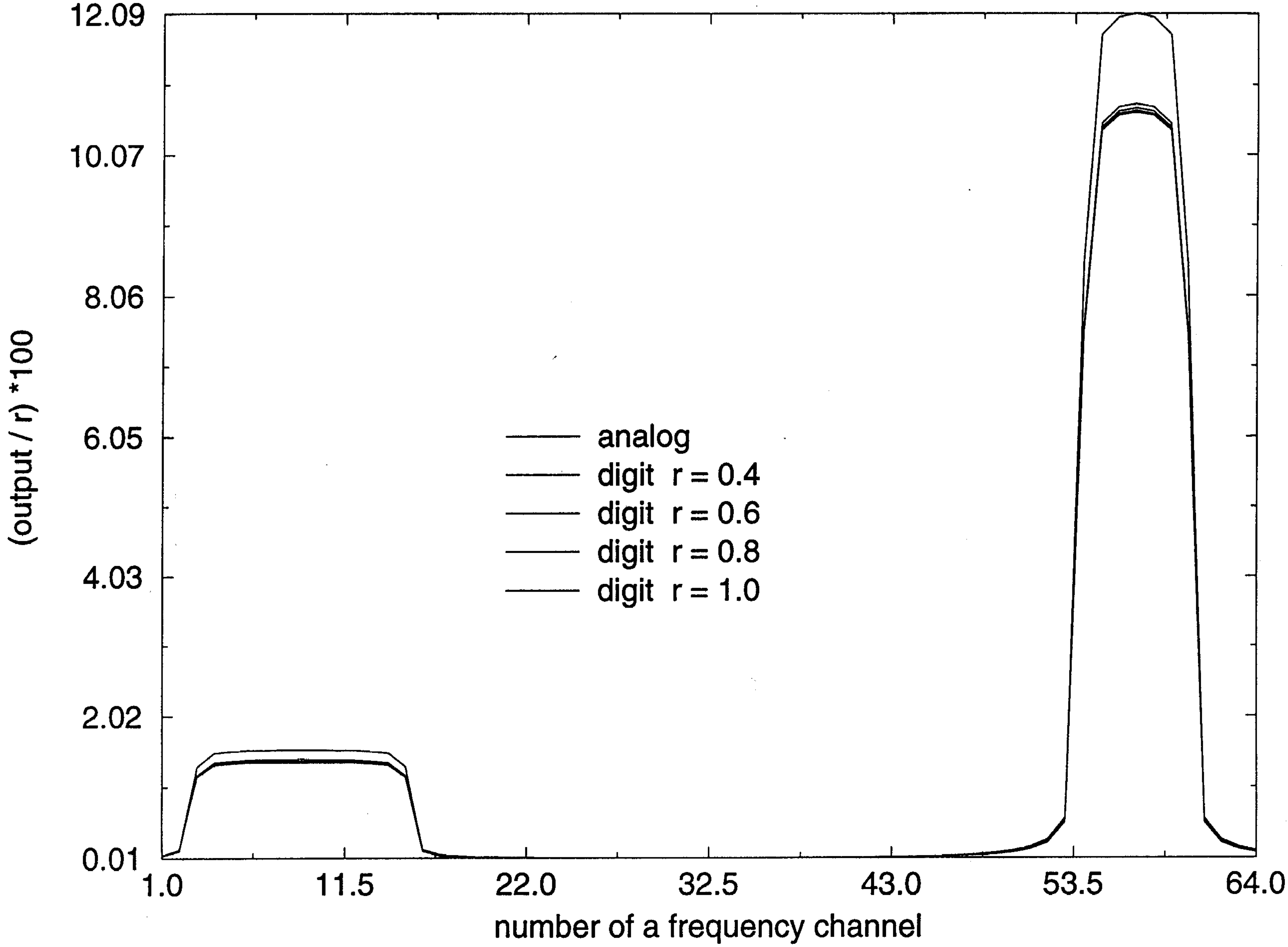

Fig. 16 Response of FX correlator. Two features spectrum. FOUR-FOUR.

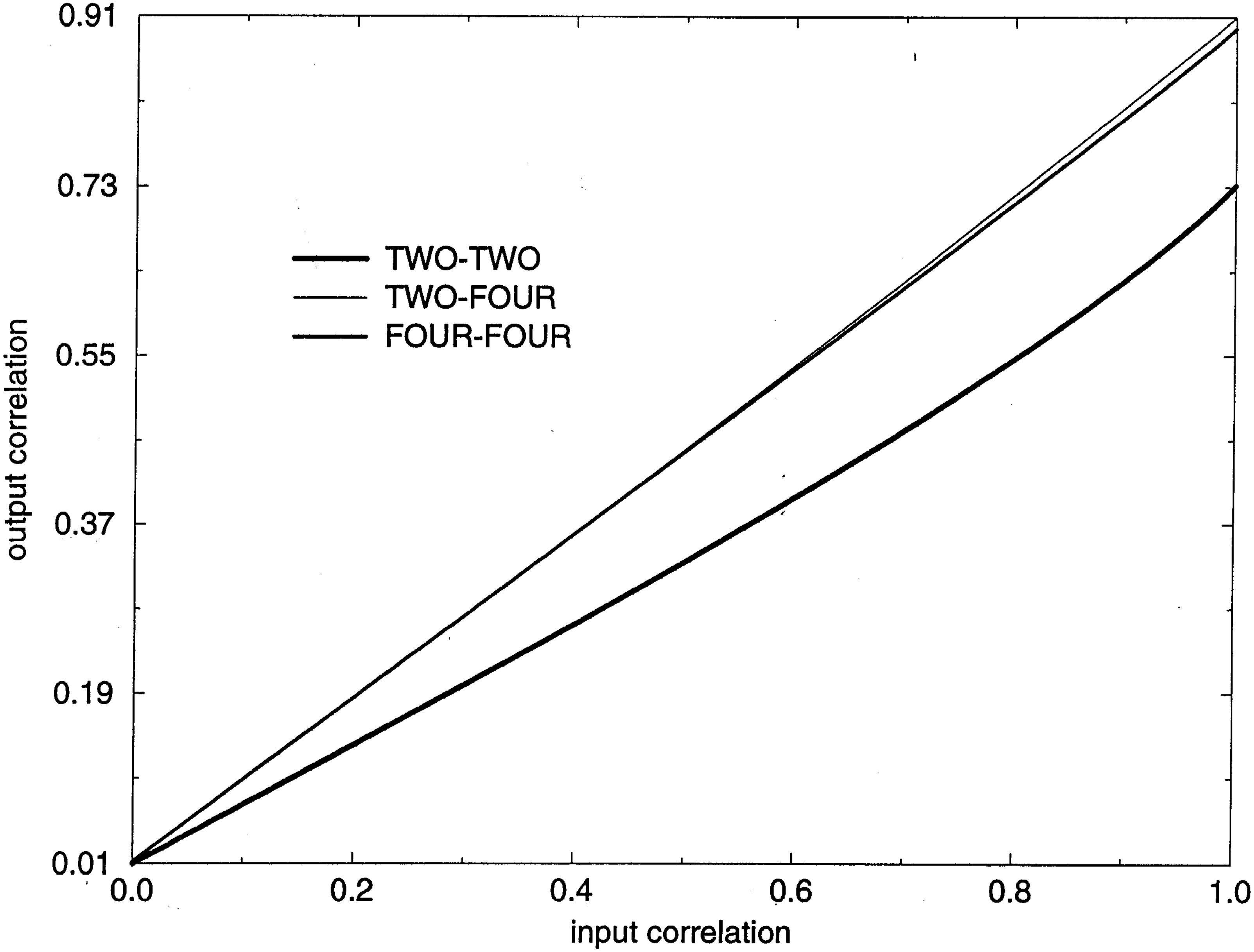

Fig. 17. Correlation at the output of FX correlator versus input correlation. Specrum is continuum

TABLE 1.

The original correlation coefficients calculated from FOUR-FOUR levels correlator using different number of points at the integrating sums.

Ratio of the levels is equal optimal value 3.336.

A = 0.9816 B = 0.9816

N	R	500	1000	5000	10000	20000
1	0.02000	0.02266	0.02266	0.02266	0.02266	0.02266
2	0.04000	0.04532	0.04532	0.04532	0.04532	0.04532
3	0.06000	0.06798	0.06798	0.06798	0.06798	0.06798
4	0.08000	0.09063	0.09063	0.09063	0.09063	0.09063
5	0.10000	0.11327	0.11327	0.11327	0.11327	0.11327
6	0.12000	0.13590	0.13590	0.13590	0.13590	0.13590
7	0.14000	0.15852	0.15851	0.15851	0.15851	0.15851
8	0.16000	0.18112	0.18112	0.18111	0.18111	0.18111
9	0.18000	0.20370	0.20370	0.20370	0.20370	0.20370
10	0.20000	0.22627	0.22627	0.22626	0.22626	0.22626
11	0.22000	0.24881	0.24881	0.24881	0.24881	0.24881
12	0.24000	0.27134	0.27133	0.27133	0.27133	0.27133
13	0.26000	0.29383	0.29383	0.29382	0.29382	0.29382
14	0.28000	0.31630	0.31630	0.31629	0.31629	0.31629
15	0.30000	0.33874	0.33874	0.33873	0.33873	0.33873
16	0.32000	0.36115	0.36114	0.36114	0.36114	0.36114
17	0.34000	0.38352	0.38352	0.38351	0.38351	0.38351
18	0.36000	0.40586	0.40586	0.40585	0.40585	0.40585
19	0.38000	0.42817	0.42816	0.42815	0.42815	0.42815
20	0.40000	0.45043	0.45042	0.45041	0.45041	0.45041
21	0.42000	0.47265	0.47264	0.47263	0.47263	0.47262
22	0.44000	0.49482	0.49481	0.49480	0.49480	0.49480
23	0.46000	0.51695	0.51694	0.51693	0.51693	0.51693
24	0.48000	0.53903	0.53902	0.53901	0.53901	0.53900
25	0.50000	0.56107	0.56105	0.56104	0.56103	0.56103
26	0.52000	0.58304	0.58302	0.58301	0.58301	0.58301
27	0.54000	0.60496	0.60494	0.60493	0.60493	0.60492
28	0.56000	0.62682	0.62680	0.62678	0.62678	0.62678
29	0.58000	0.64862	0.64860	0.64858	0.64858	0.64857
30	0.60000	0.67035	0.67032	0.67030	0.67030	0.67030
31	0.62000	0.69200	0.69198	0.69196	0.69196	0.69195
32	0.64000	0.71358	0.71356	0.71353	0.71353	0.71353
33	0.66000	0.73508	0.73505	0.73502	0.73502	0.73502
34	0.68000	0.75648	0.75645	0.75642	0.75642	0.75642
35	0.70000	0.77778	0.77775	0.77772	0.77771	0.77771
36	0.72000	0.79896	0.79893	0.79890	0.79889	0.79889
37	0.74000	0.82001	0.81997	0.81993	0.81993	0.81993
38	0.76000	0.84088	0.84084	0.84080	0.84080	0.84080
39	0.78000	0.86154	0.86149	0.86145	0.86144	0.86144
40	0.80000	0.88188	0.88182	0.88177	0.88176	0.88176
41	0.82000	0.90173	0.90166	0.90160	0.90160	0.90159
42	0.84000	0.92083	0.92073	0.92066	0.92065	0.92064
43	0.86000	0.93871	0.93859	0.93849	0.93848	0.93847
44	0.88000	0.95484	0.95468	0.95456	0.95454	0.95453
45	0.90000	0.96873	0.96854	0.96839	0.96838	0.96837
46	0.92000	0.98014	0.97994	0.97977	0.97975	0.97974
47	0.94000	0.98901	0.98880	0.98863	0.98861	0.98860
48	0.96000	0.99531	0.99512	0.99496	0.99495	0.99494
49	0.98000	0.99901	0.99887	0.99876	0.99874	0.99874
50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

TABLE 2.

The original correlation coefficients calculated from TWO-FOUR levels correlator using different number of points at the integrating sums.

Ratio of the levels is equal optimal value 3.336.

A = 0.9816 B = 0.0000

N	R	500	1000	5000	10000	20000
	0.0	0.0	0.0	0.0	0.0	0.0
1	0.02000	0.02266	0.02266	0.02266	0.02266	0.02266
2	0.04000	0.04532	0.04532	0.04532	0.04532	0.04532
3	0.06000	0.06797	0.06796	0.06796	0.06796	0.06796
4	0.08000	0.09060	0.09060	0.09060	0.09060	0.09060
5	0.10000	0.11321	0.11321	0.11321	0.11321	0.11321
6	0.12000	0.13580	0.13580	0.13579	0.13579	0.13579
7	0.14000	0.15836	0.15835	0.15835	0.15835	0.15835
8	0.16000	0.18088	0.18088	0.18087	0.18087	0.18087
9	0.18000	0.20337	0.20336	0.20336	0.20336	0.20336
10	0.20000	0.22581	0.22580	0.22580	0.22579	0.22579
11	0.22000	0.24820	0.24819	0.24819	0.24818	0.24818
12	0.24000	0.27054	0.27053	0.27052	0.27052	0.27052
13	0.26000	0.29282	0.29281	0.29280	0.29280	0.29280
14	0.28000	0.31504	0.31503	0.31502	0.31502	0.31502
15	0.30000	0.33720	0.33718	0.33717	0.33717	0.33717
16	0.32000	0.35928	0.35927	0.35926	0.35926	0.35925
17	0.34000	0.38130	0.38128	0.38127	0.38126	0.38126
18	0.36000	0.40323	0.40321	0.40320	0.40320	0.40320
19	0.38000	0.42509	0.42507	0.42505	0.42505	0.42505
20	0.40000	0.44686	0.44684	0.44682	0.44682	0.44682
21	0.42000	0.46855	0.46853	0.46851	0.46850	0.46850
22	0.44000	0.49015	0.49013	0.49010	0.49010	0.49010
23	0.46000	0.51166	0.51164	0.51161	0.51161	0.51161
24	0.48000	0.53309	0.53305	0.53303	0.53303	0.53303
25	0.50000	0.55442	0.55438	0.55436	0.55435	0.55435
26	0.52000	0.57565	0.57562	0.57559	0.57559	0.57559
27	0.54000	0.59680	0.59677	0.59674	0.59673	0.59673
28	0.56000	0.61786	0.61782	0.61779	0.61779	0.61779
29	0.58000	0.63884	0.63880	0.63876	0.63876	0.63876
30	0.60000	0.65973	0.65969	0.65965	0.65965	0.65965
31	0.62000	0.68055	0.68050	0.68047	0.68046	0.68046
32	0.64000	0.70130	0.70125	0.70121	0.70121	0.70121
33	0.66000	0.72199	0.72194	0.72190	0.72190	0.72190
34	0.68000	0.74263	0.74258	0.74254	0.74254	0.74253
35	0.70000	0.76324	0.76319	0.76315	0.76315	0.76314
36	0.72000	0.78383	0.78378	0.78374	0.78374	0.78374
37	0.74000	0.80442	0.80438	0.80434	0.80433	0.80434
38	0.76000	0.82504	0.82499	0.82495	0.82494	0.82495
39	0.78000	0.84568	0.84563	0.84559	0.84558	0.84559
40	0.80000	0.86634	0.86629	0.86625	0.86624	0.86626
41	0.82000	0.88696	0.88691	0.88687	0.88686	0.88688
42	0.84000	0.90738	0.90732	0.90727	0.90726	0.90728
43	0.86000	0.92721	0.92714	0.92707	0.92706	0.92708
44	0.88000	0.94579	0.94568	0.94559	0.94558	0.94559
45	0.90000	0.96226	0.96210	0.96198	0.96196	0.96197
46	0.92000	0.97596	0.97577	0.97562	0.97560	0.97560
47	0.94000	0.98666	0.98645	0.98628	0.98626	0.98626
48	0.96000	0.99428	0.99408	0.99392	0.99390	0.99389
49	0.98000	0.99877	0.99862	0.99850	0.99848	0.99848
50	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

TABLE 3.

Table of the converted function F(ro) taking in acount fringe stopping procedure. The first line is the step of the input correlation

TWO - TWO

0.00100									
0.00064	0.00127	0.00191	0.00255	0.00318	0.00382	0.00446	0.00509	0.00573	0.00637
0.00700	0.00764	0.00828	0.00891	0.00955	0.01019	0.01082	0.01146	0.01210	0.01273
0.01337	0.01401	0.01464	0.01528	0.01592	0.01655	0.01719	0.01783	0.01846	0.01910
0.01974	0.02037	0.02101	0.02165	0.02229	0.02292	0.02356	0.02420	0.02483	0.02547
0.02611	0.02674	0.02738	0.02802	0.02866	0.02929	0.02993	0.03057	0.03120	0.03184
0.03248	0.03312	0.03375	0.03439	0.03503	0.03566	0.03630	0.03694	0.03758	0.03821
0.03885	0.03949	0.04013	0.04076	0.04140	0.04204	0.04268	0.04332	0.04395	0.04459
0.04523	0.04587	0.04650	0.04714	0.04778	0.04842	0.04906	0.04969	0.05033	0.05097
0.05161	0.05225	0.05289	0.05352	0.05416	0.05480	0.05544	0.05608	0.05672	0.05735
0.05799	0.05863	0.05927	0.05991	0.06055	0.06119	0.06183	0.06246	0.06310	0.06374
0.06438	0.06502	0.06566	0.06630	0.06694	0.06758	0.06822	0.06886	0.06950	0.07013
0.07077	0.07141	0.07205	0.07269	0.07333	0.07397	0.07461	0.07525	0.07589	0.07653
0.07717	0.07781	0.07845	0.07909	0.07973	0.08037	0.08101	0.08166	0.08230	0.08294
0.08358	0.08422	0.08486	0.08550	0.08614	0.08678	0.08742	0.08806	0.08871	0.08935
0.08999	0.09063	0.09127	0.09191	0.09255	0.09320	0.09384	0.09448	0.09512	0.09576
0.09641	0.09705	0.09769	0.09833	0.09898	0.09962	0.10026	0.10090	0.10155	0.10219
0.10283	0.10347	0.10412	0.10476	0.10540	0.10605	0.10669	0.10733	0.10798	0.10862
0.10926	0.10991	0.11055	0.11120	0.11184	0.11248	0.11313	0.11377	0.11442	0.11506
0.11571	0.11635	0.11700	0.11764	0.11829	0.11893	0.11958	0.12022	0.12087	0.12151
0.12216	0.12280	0.12345	0.12409	0.12474	0.12539	0.12603	0.12668	0.12732	0.12797
0.12862	0.12926	0.12991	0.13056	0.13120	0.13185	0.13250	0.13315	0.13379	0.13444
0.13509	0.13574	0.13638	0.13703	0.13768	0.13833	0.13897	0.13962	0.14027	0.14092
0.14157	0.14222	0.14287	0.14352	0.14416	0.14481	0.14546	0.14611	0.14676	0.14741
0.14806	0.14871	0.14936	0.15001	0.15066	0.15131	0.15196	0.15261	0.15326	0.15391
0.15456	0.15522	0.15587	0.15652	0.15717	0.15782	0.15847	0.15912	0.15978	0.16043
0.16108	0.16173	0.16239	0.16304	0.16369	0.16434	0.16500	0.16565	0.16630	0.16696
0.16761	0.16826	0.16892	0.16957	0.17023	0.17088	0.17153	0.17219	0.17284	0.17350
0.17415	0.17481	0.17546	0.17612	0.17677	0.17743	0.17809	0.17874	0.17940	0.18005
0.18071	0.18137	0.18202	0.18268	0.18334	0.18400	0.18465	0.18531	0.18597	0.18663
0.18728	0.18794	0.18860	0.18926	0.18992	0.19058	0.19123	0.19189	0.19255	0.19321
0.19387	0.19453	0.19519	0.19585	0.19651	0.19717	0.19783	0.19849	0.19915	0.19981
0.20047	0.20114	0.20180	0.20246	0.20312	0.20378	0.20445	0.20511	0.20577	0.20643
0.20710	0.20776	0.20842	0.20908	0.20975	0.21041	0.21108	0.21174	0.21240	0.21307
0.21373	0.21440	0.21506	0.21573	0.21639	0.21706	0.21773	0.21839	0.21906	0.21972
0.22039	0.22106	0.22172	0.22239	0.22306	0.22373	0.22439	0.22506	0.22573	0.22640
0.22707	0.22773	0.22840	0.22907	0.22974	0.23041	0.23108	0.23175	0.23242	0.23309
0.23376	0.23443	0.23510	0.23577	0.23644	0.23712	0.23779	0.23846	0.23913	0.23980
0.24048	0.24115	0.24182	0.24250	0.24317	0.24384	0.24452	0.24519	0.24586	0.24654
0.24721	0.24789	0.24856	0.24924	0.24991	0.25059	0.25127	0.25194	0.25262	0.25330
0.25397	0.25465	0.25533	0.25601	0.25668	0.25736	0.25804	0.25872	0.25940	0.26008
0.26075	0.26143	0.26211	0.26279	0.26347	0.26415	0.26484	0.26552	0.26620	0.26688
0.26756	0.26824	0.26892	0.26961	0.27029	0.27097	0.27166	0.27234	0.27302	0.27371
0.27439	0.27508	0.27576	0.27645	0.27713	0.27782	0.27850	0.27919	0.27987	0.28056
0.28125	0.28193	0.28262	0.28331	0.28400	0.28469	0.28537	0.28606	0.28675	0.28744
0.28813	0.28882	0.28951	0.29020	0.29089	0.29158	0.29227	0.29296	0.29366	0.29435
0.29504	0.29573	0.29643	0.29712	0.29781	0.29851	0.29920	0.29990	0.30059	0.30128
0.30198	0.30268	0.30337	0.30407	0.30476	0.30546	0.30616	0.30685	0.30755	0.30825
0.30895	0.30965	0.31035	0.31104	0.31174	0.31244	0.31314	0.31384	0.31455	0.31525
0.31595	0.31665	0.31735	0.31805	0.31876	0.31946	0.32016	0.32087	0.32157	0.32227
0.32298	0.32368	0.32439	0.32509	0.32580	0.32651	0.32721	0.32792	0.32863	0.32933
0.33004	0.33075	0.33146	0.33217	0.33288	0.33359	0.33430	0.33501	0.33572	0.33643
0.33714	0.33785	0.33856	0.33928	0.33999	0.34070	0.34142	0.34213	0.34285	0.34356
0.34427	0.34499	0.34571	0.34642	0.34714	0.34786	0.34857	0.34929	0.35001	0.35073
0.35145	0.35216	0.35288	0.35360	0.35432	0.35504	0.35577	0.35649	0.35721	0.35793
0.35865	0.35938	0.36010	0.36082	0.36155	0.36227	0.36300	0.36372	0.36445	0.36518
0.36590	0.36663	0.36736	0.36809	0.36881	0.36954	0.37027	0.37100	0.37173	0.37246
0.37319	0.37392	0.37466	0.37539	0.37612	0.37685	0.37759	0.37832	0.37905	0.37979
0.38052	0.38126	0.38200	0.38273	0.38347	0.38421	0.38495	0.38568	0.38642	0.38716
0.38790	0.38864	0.38938	0.39012	0.39086	0.39161	0.39235	0.39309	0.39384	0.39458
0.39532	0.39607	0.39681	0.39756	0.39831	0.39905	0.39980	0.40055	0.40130	0.40205
0.40279	0.40354	0.40429	0.40505	0.40580	0.40655	0.40730	0.40805	0.40881	0.40956
0.41032	0.41107	0.41183	0.41258	0.41334	0.41409	0.41485	0.41561	0.41637	0.41713
0.41789	0.41865	0.41941	0.42017	0.42093	0.42169	0.42246	0.42322	0.42398	0.42475
0.42551	0.42628	0.42705	0.42781	0.42858	0.42935	0.43012	0.43089	0.43166	0.43243
0.43320	0.43397	0.43474	0.43551	0.43629	0.43706	0.43783	0.43861	0.43939	0.44016
0.44094	0.44172	0.44249	0.44327	0.44405	0.44483	0.44561	0.44639	0.44718	0.44796
0.44874	0.44953	0.45031	0.45109	0.45188	0.45267	0.45345	0.45424	0.45503	0.45582

0.45661	0.45740	0.45819	0.45898	0.45977	0.46057	0.46136	0.46215	0.46295	0.46374
0.46454	0.46534	0.46614	0.46693	0.46773	0.46853	0.46933	0.47014	0.47094	0.47174
0.47254	0.47335	0.47415	0.47496	0.47577	0.47657	0.47738	0.47819	0.47900	0.47981
0.48062	0.48143	0.48225	0.48306	0.48387	0.48469	0.48550	0.48632	0.48714	0.48795
0.48877	0.48959	0.49041	0.49123	0.49206	0.49288	0.49370	0.49453	0.49535	0.49618
0.49701	0.49783	0.49866	0.49949	0.50032	0.50115	0.50199	0.50282	0.50365	0.50449
0.50532	0.50616	0.50700	0.50784	0.50868	0.50952	0.51036	0.51120	0.51204	0.51289
0.51373	0.51458	0.51542	0.51627	0.51712	0.51797	0.51882	0.51967	0.52052	0.52138
0.52223	0.52309	0.52394	0.52480	0.52566	0.52652	0.52738	0.52824	0.52910	0.52996
0.53083	0.53169	0.53256	0.53343	0.53430	0.53517	0.53604	0.53691	0.53778	0.53866
0.53953	0.54041	0.54128	0.54216	0.54304	0.54392	0.54481	0.54569	0.54657	0.54746
0.54834	0.54923	0.55012	0.55101	0.55190	0.55279	0.55369	0.55458	0.55548	0.55638
0.55727	0.55817	0.55907	0.55998	0.56088	0.56178	0.56269	0.56360	0.56451	0.56542
0.56633	0.56724	0.56815	0.56907	0.56999	0.57090	0.57182	0.57274	0.57367	0.57459
0.57551	0.57644	0.57737	0.57830	0.57923	0.58016	0.58109	0.58203	0.58297	0.58390
0.58484	0.58578	0.58673	0.58767	0.58862	0.58956	0.59051	0.59146	0.59241	0.59337
0.59432	0.59528	0.59624	0.59720	0.59816	0.59912	0.60009	0.60106	0.60202	0.60299
0.60397	0.60494	0.60592	0.60689	0.60787	0.60885	0.60984	0.61082	0.61181	0.61280
0.61379	0.61478	0.61577	0.61677	0.61777	0.61877	0.61977	0.62077	0.62178	0.62279
0.62380	0.62481	0.62582	0.62684	0.62786	0.62888	0.62990	0.63093	0.63195	0.63298
0.63401	0.63505	0.63608	0.63712	0.63816	0.63921	0.64025	0.64130	0.64235	0.64341
0.64446	0.64552	0.64658	0.64764	0.64871	0.64978	0.65085	0.65192	0.65300	0.65408
0.65516	0.65624	0.65733	0.65842	0.65951	0.66061	0.66171	0.66281	0.66392	0.66503
0.66614	0.66725	0.66837	0.66949	0.67061	0.67174	0.67287	0.67400	0.67514	0.67628
0.67743	0.67857	0.67973	0.68088	0.68204	0.68320	0.68437	0.68554	0.68671	0.68789
0.68907	0.69026	0.69145	0.69264	0.69384	0.69504	0.69625	0.69746	0.69868	0.69990
0.70113	0.70236	0.70359	0.70483	0.70608	0.70733	0.70858	0.70984	0.71111	0.71238
0.71366	0.71494	0.71623	0.71752	0.71882	0.72013	0.72144	0.72276	0.72409	0.72542
0.72676	0.72811	0.72946	0.73082	0.73219	0.73356	0.73495	0.73634	0.73774	0.73915
0.74057	0.74199	0.74343	0.74487	0.74633	0.74779	0.74927	0.75076	0.75226	0.75376
0.75529	0.75682	0.75837	0.75993	0.76150	0.76309	0.76469	0.76631	0.76795	0.76961
0.77128	0.77297	0.77469	0.77642	0.77818	0.77996	0.78178	0.78362	0.78549	0.78740
0.78934	0.79133	0.79337	0.79546	0.79762	0.79986	0.80220	0.80468	0.80737	0.81057

TABLE 4.

Table of the converted function $F_{i}(ro)$ taking in acount fringe stopping procedure. The first line is the step of the input correlation

TWO - FOUR

0.00100									
0.00088	0.00177	0.00265	0.00353	0.00441	0.00530	0.00618	0.00706	0.00794	0.00883
0.00971	0.01059	0.01147	0.01236	0.01324	0.01412	0.01500	0.01589	0.01677	0.01765
0.01853	0.01942	0.02030	0.02118	0.02206	0.02295	0.02383	0.02471	0.02559	0.02648
0.02736	0.02824	0.02913	0.03001	0.03089	0.03177	0.03266	0.03354	0.03442	0.03530
0.03619	0.03707	0.03795	0.03884	0.03972	0.04060	0.04148	0.04237	0.04325	0.04413
0.04502	0.04590	0.04678	0.04766	0.04855	0.04943	0.05031	0.05120	0.05208	0.05296
0.05384	0.05473	0.05561	0.05649	0.05738	0.05826	0.05914	0.06003	0.06091	0.06179
0.06268	0.06356	0.06444	0.06533	0.06621	0.06709	0.06798	0.06886	0.06974	0.07063
0.07151	0.07239	0.07328	0.07416	0.07504	0.07593	0.07681	0.07769	0.07858	0.07946
0.08035	0.08123	0.08211	0.08300	0.08388	0.08476	0.08565	0.08653	0.08742	0.08830
0.08918	0.09007	0.09095	0.09184	0.09272	0.09360	0.09449	0.09537	0.09626	0.09714
0.09803	0.09891	0.09979	0.10068	0.10156	0.10245	0.10333	0.10422	0.10510	0.10599
0.10687	0.10775	0.10864	0.10952	0.11041	0.11129	0.11218	0.11306	0.11395	0.11483
0.11572	0.11660	0.11749	0.11837	0.11926	0.12014	0.12103	0.12191	0.12280	0.12368
0.12457	0.12546	0.12634	0.12723	0.12811	0.12900	0.12988	0.13077	0.13165	0.13254
0.13343	0.13431	0.13520	0.13608	0.13697	0.13785	0.13874	0.13963	0.14051	0.14140
0.14229	0.14317	0.14406	0.14494	0.14583	0.14672	0.14760	0.14849	0.14938	0.15026
0.15115	0.15204	0.15292	0.15381	0.15470	0.15558	0.15647	0.15736	0.15825	0.15913
0.16002	0.16091	0.16179	0.16268	0.16357	0.16446	0.16534	0.16623	0.16712	0.16801
0.16889	0.16978	0.17067	0.17156	0.17245	0.17333	0.17422	0.17511	0.17600	0.17689
0.17777	0.17866	0.17955	0.18044	0.18133	0.18222	0.18311	0.18399	0.18488	0.18577
0.18666	0.18755	0.18844	0.18933	0.19022	0.19111	0.19199	0.19288	0.19377	0.19466
0.19555	0.19644	0.19733	0.19822	0.19911	0.20000	0.20089	0.20178	0.20267	0.20356
0.20445	0.20534	0.20623	0.20712	0.20801	0.20890	0.20979	0.21068	0.21157	0.21247
0.21336	0.21425	0.21514	0.21603	0.21692	0.21781	0.21870	0.21959	0.22048	0.22138
0.22227	0.22316	0.22405	0.22494	0.22583	0.22673	0.22762	0.22851	0.22940	0.23029
0.23119	0.23208	0.23297	0.23386	0.23476	0.23565	0.23654	0.23743	0.23833	0.23922
0.24011	0.24101	0.24190	0.24279	0.24369	0.24458	0.24547	0.24637	0.24726	0.24815
0.24905	0.24994	0.25084	0.25173	0.25262	0.25352	0.25441	0.25531	0.25620	0.25710
0.25799	0.25888	0.25978	0.26067	0.26157	0.26246	0.26336	0.26425	0.26515	0.26605
0.26694	0.26784	0.26873	0.26963	0.27052	0.27142	0.27232	0.27321	0.27411	0.27500
0.27590	0.27680	0.27769	0.27859	0.27949	0.28038	0.28128	0.28218	0.28307	0.28397
0.28487	0.28577	0.28666	0.28756	0.28846	0.28936	0.29025	0.29115	0.29205	0.29295
0.29385	0.29474	0.29564	0.29654	0.29744	0.29834	0.29924	0.30014	0.30103	0.30193
0.30283	0.30373	0.30463	0.30553	0.30643	0.30733	0.30823	0.30913	0.31003	0.31093
0.31183	0.31273	0.31363	0.31453	0.31543	0.31633	0.31723	0.31813	0.31903	0.31994
0.32084	0.32174	0.32264	0.32354	0.32444	0.32534	0.32625	0.32715	0.32805	0.32895
0.32985	0.33076	0.33166	0.33256	0.33346	0.33437	0.33527	0.33617	0.33707	0.33798
0.33888	0.33978	0.34069	0.34159	0.34250	0.34340	0.34430	0.34521	0.34611	0.34702
0.34792	0.34882	0.34973	0.35063	0.35154	0.35244	0.35335	0.35425	0.35516	0.35606
0.35697	0.35788	0.35878	0.35969	0.36059	0.36150	0.36240	0.36331	0.36422	0.36512
0.36603	0.36694	0.36784	0.36875	0.36966	0.37057	0.37147	0.37238	0.37329	0.37420
0.37510	0.37601	0.37692	0.37783	0.37874	0.37964	0.38055	0.38146	0.38237	0.38328
0.38419	0.38510	0.38601	0.38692	0.38783	0.38874	0.38965	0.39056	0.39147	0.39238
0.39329	0.39420	0.39511	0.39602	0.39693	0.39784	0.39875	0.39966	0.40057	0.40148
0.40240	0.40331	0.40422	0.40513	0.40604	0.40695	0.40787	0.40878	0.40969	0.41060
0.41152	0.41243	0.41334	0.41426	0.41517	0.41608	0.41700	0.41791	0.41882	0.41974
0.42065	0.42157	0.42248	0.42340	0.42431	0.42523	0.42614	0.42706	0.42797	0.42889
0.42980	0.43072	0.43163	0.43255	0.43346	0.43438	0.43530	0.43621	0.43713	0.43805
0.43896	0.43988	0.44080	0.44171	0.44263	0.44355	0.44447	0.44538	0.44630	0.44722
0.44814	0.44906	0.44998	0.45089	0.45181	0.45273	0.45365	0.45457	0.45549	0.45641
0.45733	0.45825	0.45917	0.46009	0.46101	0.46193	0.46285	0.46377	0.46469	0.46561
0.46653	0.46745	0.46837	0.46930	0.47022	0.47114	0.47206	0.47298	0.47390	0.47483
0.47575	0.47667	0.47759	0.47852	0.47944	0.48036	0.48129	0.48221	0.48313	0.48406
0.48498	0.48591	0.48683	0.48775	0.48868	0.48960	0.49053	0.49145	0.49238	0.49330
0.49423	0.49515	0.49608	0.49701	0.49793	0.49886	0.49978	0.50071	0.50164	0.50256
0.50349	0.50442	0.50534	0.50627	0.50720	0.50813	0.50905	0.50998	0.51091	0.51184
0.51277	0.51370	0.51462	0.51555	0.51648	0.51741	0.51834	0.51927	0.52020	0.52113
0.52206	0.52299	0.52392	0.52485	0.52578	0.52671	0.52764	0.52857	0.52950	0.53043
0.53137	0.53230	0.53323	0.53416	0.53509	0.53602	0.53696	0.53789	0.53882	0.53975
0.54069	0.54162	0.54255	0.54349	0.54442	0.54535	0.54629	0.54722	0.54816	0.54909
0.55002	0.55096	0.55189	0.55283	0.55376	0.55470	0.55563	0.55657	0.55750	0.55844
0.55938	0.56031	0.56125	0.56218	0.56312	0.56406	0.56499	0.56593	0.56687	0.56781
0.56874	0.56968	0.57062	0.57156	0.57249	0.57343	0.57437	0.57531	0.57625	0.57719
0.57813	0.57907	0.58000	0.58094	0.58188	0.58282	0.58376	0.58470	0.58564	0.58658
0.58752	0.58846	0.58941	0.59035	0.59129	0.59223	0.59317	0.59411	0.59505	0.59599

0.59694	0.59788	0.59882	0.59976	0.60071	0.60165	0.60259	0.60353	0.60448	0.60542
0.60636	0.60731	0.60825	0.60920	0.61014	0.61108	0.61203	0.61297	0.61392	0.61486
0.61581	0.61675	0.61770	0.61864	0.61959	0.62053	0.62148	0.62242	0.62337	0.62432
0.62526	0.62621	0.62716	0.62810	0.62905	0.63000	0.63094	0.63189	0.63284	0.63379
0.63473	0.63568	0.63663	0.63758	0.63853	0.63947	0.64042	0.64137	0.64232	0.64327
0.64422	0.64517	0.64612	0.64707	0.64801	0.64896	0.64991	0.65086	0.65181	0.65276
0.65371	0.65467	0.65562	0.65657	0.65752	0.65847	0.65942	0.66037	0.66132	0.66227
0.66322	0.66418	0.66513	0.66608	0.66703	0.66798	0.66894	0.66989	0.67084	0.67179
0.67275	0.67370	0.67465	0.67561	0.67656	0.67751	0.67847	0.67942	0.68037	0.68133
0.68228	0.68323	0.68419	0.68514	0.68610	0.68705	0.68801	0.68896	0.68991	0.69087
0.69182	0.69278	0.69373	0.69469	0.69564	0.69660	0.69755	0.69851	0.69947	0.70042
0.70138	0.70233	0.70329	0.70425	0.70520	0.70616	0.70711	0.70807	0.70903	0.70998
0.71094	0.71190	0.71285	0.71381	0.71477	0.71572	0.71668	0.71764	0.71860	0.71955
0.72051	0.72147	0.72243	0.72338	0.72434	0.72530	0.72626	0.72721	0.72817	0.72913
0.73009	0.73105	0.73200	0.73296	0.73392	0.73488	0.73584	0.73680	0.73775	0.73871
0.73967	0.74063	0.74159	0.74255	0.74351	0.74446	0.74542	0.74638	0.74734	0.74830
0.74926	0.75022	0.75118	0.75214	0.75309	0.75405	0.75501	0.75597	0.75693	0.75789
0.75885	0.75981	0.76077	0.76173	0.76269	0.76365	0.76461	0.76557	0.76653	0.76749
0.76844	0.76940	0.77036	0.77132	0.77228	0.77324	0.77420	0.77516	0.77612	0.77708
0.77804	0.77900	0.77996	0.78092	0.78188	0.78284	0.78380	0.78476	0.78572	0.78668
0.78764	0.78860	0.78956	0.79052	0.79148	0.79244	0.79340	0.79436	0.79532	0.79628
0.79724	0.79820	0.79916	0.80012	0.80108	0.80205	0.80301	0.80397	0.80493	0.80589
0.80685	0.80781	0.80877	0.80973	0.81069	0.81165	0.81262	0.81358	0.81454	0.81550
0.81646	0.81743	0.81839	0.81935	0.82031	0.82128	0.82224	0.82320	0.82416	0.82513
0.82609	0.82706	0.82802	0.82898	0.82995	0.83091	0.83188	0.83285	0.83381	0.83478
0.83574	0.83671	0.83768	0.83865	0.83962	0.84059	0.84155	0.84252	0.84349	0.84447
0.84544	0.84641	0.84738	0.84836	0.84933	0.85031	0.85128	0.85226	0.85324	0.85421
0.85519	0.85617	0.85716	0.85814	0.85912	0.86011	0.86109	0.86208	0.86307	0.86406
0.86505	0.86605	0.86704	0.86804	0.86904	0.87004	0.87104	0.87204	0.87305	0.87406
0.87507	0.87608	0.87710	0.87811	0.87914	0.88016	0.88119	0.88222	0.88325	0.88429
0.88533	0.88637	0.88742	0.88847	0.88953	0.89059	0.89165	0.89272	0.89380	0.89488
0.89596	0.89705	0.89815	0.89926	0.90037	0.90149	0.90261	0.90375	0.90489	0.90604
0.90720	0.90837	0.90955	0.91074	0.91195	0.91317	0.91440	0.91565	0.91691	0.91820
0.91950	0.92083	0.92219	0.92357	0.92500	0.92647	0.92799	0.92959	0.93131	0.93332

TABLE 5.

Table of the converted function F(ro) taking in acount fringe stopping procedure. The first line is the step of the input correlation

FOUR - FOUR

0.00100									
0.00088	0.00177	0.00265	0.00353	0.00441	0.00530	0.00618	0.00706	0.00794	0.00883
0.00971	0.01059	0.01147	0.01236	0.01324	0.01412	0.01500	0.01589	0.01677	0.01765
0.01853	0.01942	0.02030	0.02118	0.02206	0.02295	0.02383	0.02471	0.02559	0.02648
0.02736	0.02824	0.02912	0.03001	0.03089	0.03177	0.03265	0.03354	0.03442	0.03530
0.03618	0.03707	0.03795	0.03883	0.03971	0.04060	0.04148	0.04236	0.04325	0.04413
0.04501	0.04589	0.04678	0.04766	0.04854	0.04942	0.05031	0.05119	0.05207	0.05296
0.05384	0.05472	0.05560	0.05649	0.05737	0.05825	0.05913	0.06002	0.06090	0.06178
0.06267	0.06355	0.06443	0.06531	0.06620	0.06708	0.06796	0.06885	0.06973	0.07061
0.07149	0.07238	0.07326	0.07414	0.07503	0.07591	0.07679	0.07768	0.07856	0.07944
0.08032	0.08121	0.08209	0.08297	0.08386	0.08474	0.08562	0.08651	0.08739	0.08827
0.08915	0.09004	0.09092	0.09180	0.09269	0.09357	0.09445	0.09534	0.09622	0.09710
0.09799	0.09887	0.09975	0.10064	0.10152	0.10240	0.10329	0.10417	0.10505	0.10594
0.10682	0.10770	0.10859	0.10947	0.11035	0.11124	0.11212	0.11300	0.11389	0.11477
0.11566	0.11654	0.11742	0.11831	0.11919	0.12007	0.12096	0.12184	0.12272	0.12361
0.12449	0.12538	0.12626	0.12714	0.12803	0.12891	0.12979	0.13068	0.13156	0.13245
0.13333	0.13421	0.13510	0.13598	0.13687	0.13775	0.13863	0.13952	0.14040	0.14129
0.14217	0.14305	0.14394	0.14482	0.14571	0.14659	0.14747	0.14836	0.14924	0.15013
0.15101	0.15190	0.15278	0.15367	0.15455	0.15543	0.15632	0.15720	0.15809	0.15897
0.15986	0.16074	0.16163	0.16251	0.16339	0.16428	0.16516	0.16605	0.16693	0.16782
0.16870	0.16959	0.17047	0.17136	0.17224	0.17313	0.17401	0.17490	0.17578	0.17667
0.17755	0.17844	0.17932	0.18021	0.18109	0.18198	0.18286	0.18375	0.18463	0.18552
0.18640	0.18729	0.18817	0.18906	0.18994	0.19083	0.19171	0.19260	0.19349	0.19437
0.19526	0.19614	0.19703	0.19791	0.19880	0.19968	0.20057	0.20146	0.20234	0.20323
0.20411	0.20500	0.20588	0.20677	0.20766	0.20854	0.20943	0.21031	0.21120	0.21209
0.21297	0.21386	0.21474	0.21563	0.21652	0.21740	0.21829	0.21918	0.22006	0.22095
0.22183	0.22272	0.22361	0.22449	0.22538	0.22627	0.22715	0.22804	0.22893	0.22981
0.23070	0.23159	0.23247	0.23336	0.23425	0.23513	0.23602	0.23691	0.23779	0.23868
0.23957	0.24046	0.24134	0.24223	0.24312	0.24400	0.24489	0.24578	0.24667	0.24755
0.24844	0.24933	0.25022	0.25110	0.25199	0.25288	0.25377	0.25465	0.25554	0.25643
0.25732	0.25820	0.25909	0.25998	0.26087	0.26176	0.26264	0.26353	0.26442	0.26531
0.26620	0.26708	0.26797	0.26886	0.26975	0.27064	0.27152	0.27241	0.27330	0.27419
0.27508	0.27597	0.27685	0.27774	0.27863	0.27952	0.28041	0.28130	0.28219	0.28308
0.28396	0.28485	0.28574	0.28663	0.28752	0.28841	0.28930	0.29019	0.29108	0.29197
0.29286	0.29374	0.29463	0.29552	0.29641	0.29730	0.29819	0.29908	0.29997	0.30086
0.30175	0.30264	0.30353	0.30442	0.30531	0.30620	0.30709	0.30798	0.30887	0.30976
0.31065	0.31154	0.31243	0.31332	0.31421	0.31510	0.31599	0.31688	0.31777	0.31866
0.31955	0.32044	0.32133	0.32222	0.32311	0.32401	0.32490	0.32579	0.32668	0.32757
0.32846	0.32935	0.33024	0.33113	0.33202	0.33292	0.33381	0.33470	0.33559	0.33648
0.33737	0.33826	0.33915	0.34005	0.34094	0.34183	0.34272	0.34361	0.34451	0.34540
0.34629	0.34718	0.34807	0.34896	0.34986	0.35075	0.35164	0.35253	0.35343	0.35432
0.35521	0.35610	0.35700	0.35789	0.35878	0.35967	0.36057	0.36146	0.36235	0.36324
0.36414	0.36503	0.36592	0.36682	0.36771	0.36860	0.36950	0.37039	0.37128	0.37218
0.37307	0.37396	0.37486	0.37575	0.37664	0.37754	0.37843	0.37933	0.38022	0.38111
0.38201	0.38290	0.38379	0.38469	0.38558	0.38648	0.38737	0.38827	0.38916	0.39006
0.39095	0.39184	0.39274	0.39363	0.39453	0.39542	0.39632	0.39721	0.39811	0.39900
0.39990	0.40079	0.40169	0.40258	0.40348	0.40437	0.40527	0.40616	0.40706	0.40796
0.40885	0.40975	0.41064	0.41154	0.41243	0.41333	0.41423	0.41512	0.41602	0.41692
0.41781	0.41871	0.41960	0.42050	0.42140	0.42229	0.42319	0.42409	0.42498	0.42588
0.42678	0.42767	0.42857	0.42947	0.43036	0.43126	0.43216	0.43306	0.43395	0.43485
0.43575	0.43665	0.43754	0.43844	0.43934	0.44024	0.44113	0.44203	0.44293	0.44383
0.44473	0.44562	0.44652	0.44742	0.44832	0.44922	0.45012	0.45101	0.45191	0.45281
0.45371	0.45461	0.45551	0.45641	0.45731	0.45821	0.45910	0.46000	0.46090	0.46180
0.46270	0.46360	0.46450	0.46540	0.46630	0.46720	0.46810	0.46900	0.46990	0.47080
0.47170	0.47260	0.47350	0.47440	0.47530	0.47620	0.47710	0.47800	0.47890	0.47980
0.48070	0.48160	0.48251	0.48341	0.48431	0.48521	0.48611	0.48701	0.48791	0.48881
0.48972	0.49062	0.49152	0.49242	0.49332	0.49422	0.49513	0.49603	0.49693	0.49783
0.49873	0.49964	0.50054	0.50144	0.50234	0.50325	0.50415	0.50505	0.50595	0.50686
0.50776	0.50866	0.50957	0.51047	0.51137	0.51228	0.51318	0.51408	0.51499	0.51589
0.51679	0.51770	0.51860	0.51950	0.52041	0.52131	0.52222	0.52312	0.52403	0.52493
0.52583	0.52674	0.52764	0.52855	0.52945	0.53036	0.53126	0.53217	0.53307	0.53398
0.53488	0.53579	0.53669	0.53760	0.53851	0.53941	0.54032	0.54122	0.54213	0.54304
0.54394	0.54485	0.54575	0.54666	0.54757	0.54847	0.54938	0.55029	0.55119	0.55210
0.55301	0.55391	0.55482	0.55573	0.55664	0.55754	0.55845	0.55936	0.56027	0.56117
0.56208	0.56299	0.56390	0.56481	0.56571	0.56662	0.56753	0.56844	0.56935	0.57026
0.57116	0.57207	0.57298	0.57389	0.57480	0.57571	0.57662	0.57753	0.57844	0.57935

0.58026	0.58117	0.58208	0.58299	0.58390	0.58481	0.58572	0.58663	0.58754	0.58845
0.58936	0.59027	0.59118	0.59209	0.59300	0.59391	0.59482	0.59573	0.59665	0.59756
0.59847	0.59938	0.60029	0.60120	0.60212	0.60303	0.60394	0.60485	0.60576	0.60668
0.60759	0.60850	0.60941	0.61033	0.61124	0.61215	0.61307	0.61398	0.61489	0.61581
0.61672	0.61763	0.61855	0.61946	0.62037	0.62129	0.62220	0.62312	0.62403	0.62495
0.62586	0.62678	0.62769	0.62861	0.62952	0.63044	0.63135	0.63227	0.63318	0.63410
0.63501	0.63593	0.63684	0.63776	0.63868	0.63959	0.64051	0.64143	0.64234	0.64326
0.64418	0.64509	0.64601	0.64693	0.64784	0.64876	0.64968	0.65060	0.65151	0.65243
0.65335	0.65427	0.65519	0.65611	0.65702	0.65794	0.65886	0.65978	0.66070	0.66162
0.66254	0.66346	0.66438	0.66530	0.66622	0.66714	0.66806	0.66898	0.66990	0.67082
0.67174	0.67266	0.67358	0.67450	0.67542	0.67634	0.67726	0.67818	0.67911	0.68003
0.68095	0.68187	0.68279	0.68372	0.68464	0.68556	0.68648	0.68741	0.68833	0.68925
0.69018	0.69110	0.69202	0.69295	0.69387	0.69479	0.69572	0.69664	0.69757	0.69849
0.69942	0.70034	0.70127	0.70219	0.70312	0.70404	0.70497	0.70589	0.70682	0.70774
0.70867	0.70960	0.71052	0.71145	0.71238	0.71330	0.71423	0.71516	0.71609	0.71701
0.71794	0.71887	0.71980	0.72073	0.72166	0.72258	0.72351	0.72444	0.72537	0.72630
0.72723	0.72816	0.72909	0.73002	0.73095	0.73188	0.73281	0.73374	0.73467	0.73561
0.73654	0.73747	0.73840	0.73933	0.74027	0.74120	0.74213	0.74306	0.74400	0.74493
0.74586	0.74680	0.74773	0.74867	0.74960	0.75053	0.75147	0.75240	0.75334	0.75428
0.75521	0.75615	0.75708	0.75802	0.75896	0.75989	0.76083	0.76177	0.76271	0.76365
0.76458	0.76552	0.76646	0.76740	0.76834	0.76928	0.77022	0.77116	0.77210	0.77304
0.77398	0.77492	0.77587	0.77681	0.77775	0.77869	0.77964	0.78058	0.78152	0.78247
0.78341	0.78436	0.78530	0.78625	0.78719	0.78814	0.78909	0.79004	0.79098	0.79193
0.79288	0.79383	0.79478	0.79573	0.79668	0.79763	0.79858	0.79953	0.80048	0.80144
0.80239	0.80334	0.80430	0.80525	0.80621	0.80716	0.80812	0.80908	0.81004	0.81099
0.81195	0.81291	0.81387	0.81484	0.81580	0.81676	0.81772	0.81869	0.81965	0.82062
0.82159	0.82255	0.82352	0.82449	0.82546	0.82643	0.82741	0.82838	0.82936	0.83033
0.83131	0.83229	0.83326	0.83425	0.83523	0.83621	0.83719	0.83818	0.83917	0.84016
0.84115	0.84214	0.84313	0.84413	0.84512	0.84612	0.84712	0.84812	0.84913	0.85014
0.85114	0.85215	0.85317	0.85418	0.85520	0.85622	0.85724	0.85827	0.85930	0.86033
0.86136	0.86240	0.86344	0.86448	0.86553	0.86658	0.86764	0.86869	0.86976	0.87082
0.87189	0.87297	0.87405	0.87513	0.87622	0.87732	0.87842	0.87953	0.88064	0.88176
0.88288	0.88401	0.88515	0.88630	0.88745	0.88862	0.88979	0.89097	0.89216	0.89336
0.89457	0.89579	0.89702	0.89827	0.89953	0.90081	0.90210	0.90341	0.90474	0.90609
0.90746	0.90886	0.91029	0.91175	0.91326	0.91481	0.91642	0.91812	0.91995	0.92210

TABLE 7.

TWO - TWO

Two levels digitizers are at both sites.

The restoration of maximum value of correlation coefficient for different shifts of correlation set and for different value of maximum correlation
Correlation set corresponds to the video flat spectrum.
Shift is given at sample periods;

RAN is line of restored correlations at the analog signals case
RDIGN is a line of restored correlations at the digitized signal case (relative to the expected maximum)
RDIG is a line of restored correlations at the digitized signal case (absolute value)

Maximum correlation = 0.2

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9997	0.9998	0.9998	0.9996	0.9996	0.9996	0.9998	0.9998	0.9997
RDIG	I	0.1280	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279	0.1279

Maximum correlation = 0.4

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9998	0.9993	0.9987	0.9982	0.9980	0.9982	0.9987	0.9993	0.9998
RDIG	I	0.2601	0.2600	0.2599	0.2597	0.2596	0.2596	0.2596	0.2597	0.2599	0.2600

Maximum correlation = 0.6

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9994	0.9981	0.9963	0.9949	0.9944	0.9949	0.9963	0.9981	0.9994
RDIG	I	0.4021	0.4018	0.4013	0.4006	0.4000	0.3998	0.4000	0.4006	0.4013	0.4018

Maximum correlation = 0.8

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9985	0.9948	0.9902	0.9865	0.9851	0.9865	0.9902	0.9948	0.9985
RDIG	I	0.5654	0.5646	0.5625	0.5599	0.5578	0.5570	0.5578	0.5599	0.5625	0.5646

Maximum correlation = 1.0

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9890	0.9689	0.9487	0.9342	0.9290	0.9342	0.9487	0.9689	0.9890
RDIG	I	0.8106	0.8017	0.7854	0.7690	0.7572	0.7530	0.7572	0.7690	0.7854	0.8017

TABLE 8.

TWO - FOUR

Two levels digitizer at one site site meets four level one at the other site.

The restoration of maximum value of correlation coefficient for different shifts of correlation set and for different value of maximum correlation
Correlation set corresponds to the video flat spectrum.
Shift is given at sample periods;
RAN is line of restored correlations at the analog signals case
RDIGN is a line of restored correlations at the digitized signal case (relative to the expected maximum)
RDIG is a line of restored correlations at the digitized signal case (absolute value)

Maximum correlation = 0.2

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9998	0.9999	0.9999	0.9998	0.9998	0.9998	0.9999	0.9999	0.9998
RDIG	I	0.1769	0.1768	0.1769	0.1769	0.1769	0.1769	0.1769	0.1769	0.1769	0.1768

Maximum correlation = 0.4

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9999	0.9998	0.9995	0.9994	0.9993	0.9994	0.9995	0.9998	0.9999
RDIG	I	0.3561	0.3560	0.3560	0.3559	0.3558	0.3558	0.3558	0.3559	0.3560	0.3560

Maximum correlation = 0.6

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9998	0.9995	0.9990	0.9986	0.9985	0.9986	0.9990	0.9995	0.9998
RDIG	I	0.5397	0.5397	0.5395	0.5392	0.5390	0.5389	0.5390	0.5392	0.5395	0.5397

Maximum correlation = 0.8

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9998	0.9994	0.9988	0.9984	0.9982	0.9984	0.9988	0.9994	0.9998
RDIG	I	0.7291	0.7290	0.7287	0.7283	0.7279	0.7278	0.7279	0.7283	0.7287	0.7290

Maximum correlation = 1.0

SHIFT	I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN	I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN	I	1.0000	0.9965	0.9918	0.9887	0.9873	0.9871	0.9873	0.9887	0.9918	0.9965
RDIG	I	0.9333	0.9301	0.9257	0.9228	0.9215	0.9212	0.9215	0.9228	0.9257	0.9301

TABLE 9.

FOUR - FOUR

Four levels digitizer at one site site meets four level one at the other site.

The restoration of maximum value of correlation coefficient for different shifts of correlation set and for different value of maximum correlation
Correlation set corresponds to the video flat spectrum.
Shift is given at sample periods;
RAN is line of restored correlations at the analog signals case
RDIGN is a line of restored correlations at the digitized signal case (relative to the expected maximum)
RDIG is a line of restored correlations at the digitized signal case (absolute value)

Maximum correlation = 0.2

SHIFT I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN I	1.0000	0.9997	0.9999	1.0000	0.9999	0.9999	0.9999	1.0000	0.9999	0.9997
RDIG I	0.1767	0.1766	0.1767	0.1767	0.1767	0.1767	0.1767	0.1767	0.1767	0.1766

Maximum correlation = 0.4

SHIFT I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN I	1.0000	0.9999	0.9999	0.9998	0.9997	0.9997	0.9997	0.9998	0.9999	0.9999
RDIG I	0.3543	0.3543	0.3543	0.3542	0.3542	0.3542	0.3542	0.3542	0.3543	0.3543

Maximum correlation = 0.6

SHIFT I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN I	1.0000	0.9999	0.9997	0.9995	0.9993	0.9993	0.9993	0.9995	0.9997	0.9999
RDIG I	0.5340	0.5339	0.5338	0.5337	0.5336	0.5336	0.5336	0.5337	0.5338	0.5339

Maximum correlation = 0.8

SHIFT I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN I	1.0000	0.9998	0.9995	0.9990	0.9986	0.9984	0.9986	0.9990	0.9995	0.9998
RDIG I	0.7170	0.7169	0.7166	0.7163	0.7160	0.7159	0.7160	0.7163	0.7166	0.7169

Maximum correlation = 1.0

SHIFT I	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
RAN I	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RDIGN I	1.0000	0.9957	0.9891	0.9838	0.9808	0.9799	0.9808	0.9838	0.9891	0.9957
RDIG I	0.9221	0.9181	0.9121	0.9072	0.9044	0.9036	0.9044	0.9072	0.9121	0.9181

TABLE 12.

Table of the converted function F(ro)
The first line is the step of the input correlation

TWO - TWO

0.00100

0.00064	0.00127	0.00191	0.00255	0.00318	0.00382	0.00446	0.00509	0.00573	0.00637
0.00700	0.00764	0.00828	0.00891	0.00955	0.01019	0.01082	0.01146	0.01210	0.01273
0.01337	0.01401	0.01464	0.01528	0.01592	0.01655	0.01719	0.01783	0.01846	0.01910
0.01974	0.02038	0.02101	0.02165	0.02229	0.02292	0.02356	0.02420	0.02483	0.02547
0.02611	0.02675	0.02738	0.02802	0.02866	0.02929	0.02993	0.03057	0.03121	0.03184
0.03248	0.03312	0.03376	0.03439	0.03503	0.03567	0.03631	0.03694	0.03758	0.03822
0.03886	0.03950	0.04013	0.04077	0.04141	0.04205	0.04269	0.04332	0.04396	0.04460
0.04524	0.04588	0.04651	0.04715	0.04779	0.04843	0.04907	0.04971	0.05035	0.05098
0.05162	0.05226	0.05290	0.05354	0.05418	0.05482	0.05546	0.05610	0.05673	0.05737
0.05801	0.05865	0.05929	0.05993	0.06057	0.06121	0.06185	0.06249	0.06313	0.06377
0.06441	0.06505	0.06569	0.06633	0.06697	0.06761	0.06825	0.06889	0.06953	0.07017
0.07081	0.07145	0.07209	0.07273	0.07337	0.07401	0.07466	0.07530	0.07594	0.07658
0.07722	0.07786	0.07850	0.07914	0.07979	0.08043	0.08107	0.08171	0.08235	0.08300
0.08364	0.08428	0.08492	0.08556	0.08621	0.08685	0.08749	0.08813	0.08878	0.08942
0.09006	0.09071	0.09135	0.09199	0.09264	0.09328	0.09392	0.09457	0.09521	0.09585
0.09650	0.09714	0.09779	0.09843	0.09908	0.09972	0.10036	0.10101	0.10165	0.10230
0.10294	0.10359	0.10423	0.10488	0.10552	0.10617	0.10682	0.10746	0.10811	0.10875
0.10940	0.11005	0.11069	0.11134	0.11199	0.11263	0.11328	0.11393	0.11457	0.11522
0.11587	0.11651	0.11716	0.11781	0.11846	0.11910	0.11975	0.12040	0.12105	0.12170
0.12235	0.12299	0.12364	0.12429	0.12494	0.12559	0.12624	0.12689	0.12754	0.12819
0.12884	0.12949	0.13014	0.13079	0.13144	0.13209	0.13274	0.13339	0.13404	0.13469
0.13534	0.13600	0.13665	0.13730	0.13795	0.13860	0.13925	0.13991	0.14056	0.14121
0.14186	0.14252	0.14317	0.14382	0.14448	0.14513	0.14578	0.14644	0.14709	0.14775
0.14840	0.14905	0.14971	0.15036	0.15102	0.15167	0.15233	0.15298	0.15364	0.15429
0.15495	0.15561	0.15626	0.15692	0.15758	0.15823	0.15889	0.15955	0.16020	0.16086
0.16152	0.16218	0.16283	0.16349	0.16415	0.16481	0.16547	0.16613	0.16679	0.16745
0.16810	0.16876	0.16942	0.17008	0.17074	0.17140	0.17206	0.17273	0.17339	0.17405
0.17471	0.17537	0.17603	0.17669	0.17736	0.17802	0.17868	0.17934	0.18001	0.18067
0.18133	0.18200	0.18266	0.18332	0.18399	0.18465	0.18532	0.18598	0.18665	0.18731
0.18798	0.18864	0.18931	0.18997	0.19064	0.19131	0.19197	0.19264	0.19331	0.19397
0.19464	0.19531	0.19598	0.19664	0.19731	0.19798	0.19865	0.19932	0.19999	0.20066
0.20133	0.20200	0.20267	0.20334	0.20401	0.20468	0.20535	0.20602	0.20669	0.20737
0.20804	0.20871	0.20938	0.21006	0.21073	0.21140	0.21208	0.21275	0.21342	0.21410
0.21477	0.21545	0.21612	0.21680	0.21747	0.21815	0.21882	0.21950	0.22018	0.22085
0.22153	0.22221	0.22289	0.22356	0.22424	0.22492	0.22560	0.22628	0.22696	0.22764
0.22832	0.22900	0.22968	0.23036	0.23104	0.23172	0.23240	0.23308	0.23376	0.23445
0.23513	0.23581	0.23650	0.23718	0.23786	0.23855	0.23923	0.23991	0.24060	0.24128
0.24197	0.24266	0.24334	0.24403	0.24471	0.24540	0.24609	0.24678	0.24746	0.24815
0.24884	0.24953	0.25022	0.25091	0.25160	0.25229	0.25298	0.25367	0.25436	0.25505
0.25574	0.25643	0.25713	0.25782	0.25851	0.25920	0.25990	0.26059	0.26129	0.26198
0.26267	0.26337	0.26407	0.26476	0.26546	0.26615	0.26685	0.26755	0.26824	0.26894
0.26964	0.27034	0.27104	0.27174	0.27244	0.27314	0.27384	0.27454	0.27524	0.27594
0.27664	0.27734	0.27805	0.27875	0.27945	0.28016	0.28086	0.28156	0.28227	0.28297
0.28368	0.28438	0.28509	0.28580	0.28650	0.28721	0.28792	0.28863	0.28933	0.29004
0.29075	0.29146	0.29217	0.29288	0.29359	0.29430	0.29502	0.29573	0.29644	0.29715
0.29787	0.29858	0.29929	0.30001	0.30072	0.30144	0.30215	0.30287	0.30358	0.30430
0.30502	0.30574	0.30645	0.30717	0.30789	0.30861	0.30933	0.31005	0.31077	0.31149
0.31221	0.31294	0.31366	0.31438	0.31510	0.31583	0.31655	0.31728	0.31800	0.31873
0.31945	0.32018	0.32091	0.32163	0.32236	0.32309	0.32382	0.32455	0.32528	0.32601
0.32674	0.32747	0.32820	0.32893	0.32966	0.33040	0.33113	0.33186	0.33260	0.33333
0.33407	0.33480	0.33554	0.33628	0.33702	0.33775	0.33849	0.33923	0.33997	0.34071
0.34145	0.34219	0.34293	0.34367	0.34442	0.34516	0.34590	0.34665	0.34739	0.34814
0.34888	0.34963	0.35037	0.35112	0.35187	0.35262	0.35337	0.35412	0.35487	0.35562
0.35637	0.35712	0.35787	0.35862	0.35938	0.36013	0.36088	0.36164	0.36240	0.36315
0.36391	0.36467	0.36542	0.36618	0.36694	0.36770	0.36846	0.36922	0.36998	0.37074
0.37151	0.37227	0.37303	0.37380	0.37456	0.37533	0.37610	0.37686	0.37763	0.37840
0.37917	0.37994	0.38071	0.38148	0.38225	0.38302	0.38379	0.38457	0.38534	0.38611
0.38689	0.38766	0.38844	0.38922	0.39000	0.39077	0.39155	0.39233	0.39311	0.39389
0.39468	0.39546	0.39624	0.39703	0.39781	0.39860	0.39938	0.40017	0.40096	0.40174
0.40253	0.40332	0.40411	0.40490	0.40570	0.40649	0.40728	0.40808	0.40887	0.40967
0.41046	0.41126	0.41206	0.41285	0.41365	0.41445	0.41525	0.41606	0.41686	0.41766
0.41846	0.41927	0.42007	0.42088	0.42169	0.42250	0.42330	0.42411	0.42492	0.42573
0.42655	0.42736	0.42817	0.42899	0.42980	0.43062	0.43143	0.43225	0.43307	0.43389
0.43471	0.43553	0.43635	0.43718	0.43800	0.43882	0.43965	0.44048	0.44130	0.44213
0.44296	0.44379	0.44462	0.44545	0.44629	0.44712	0.44795	0.44879	0.44962	0.45046
0.45130	0.45214	0.45298	0.45382	0.45466	0.45551	0.45635	0.45719	0.45804	0.45889
0.45974	0.46058	0.46143	0.46229	0.46314	0.46399	0.46484	0.46570	0.46655	0.46741

0.46827	0.46913	0.46999	0.47085	0.47171	0.47258	0.47344	0.47431	0.47517	0.47604
0.47691	0.47778	0.47865	0.47952	0.48040	0.48127	0.48215	0.48302	0.48390	0.48478
0.48566	0.48654	0.48742	0.48831	0.48919	0.49008	0.49096	0.49185	0.49274	0.49363
0.49453	0.49542	0.49631	0.49721	0.49811	0.49900	0.49990	0.50080	0.50171	0.50261
0.50351	0.50442	0.50533	0.50624	0.50715	0.50806	0.50897	0.50988	0.51080	0.51172
0.51263	0.51355	0.51447	0.51540	0.51632	0.51725	0.51817	0.51910	0.52003	0.52096
0.52189	0.52283	0.52376	0.52470	0.52564	0.52658	0.52752	0.52846	0.52940	0.53035
0.53130	0.53225	0.53320	0.53415	0.53510	0.53606	0.53701	0.53797	0.53893	0.53989
0.54086	0.54182	0.54279	0.54376	0.54473	0.54570	0.54667	0.54765	0.54862	0.54960
0.55058	0.55156	0.55255	0.55353	0.55452	0.55551	0.55650	0.55750	0.55849	0.55949
0.56049	0.56149	0.56249	0.56349	0.56450	0.56551	0.56652	0.56753	0.56855	0.56956
0.57058	0.57160	0.57262	0.57365	0.57467	0.57570	0.57673	0.57777	0.57880	0.57984
0.58088	0.58192	0.58296	0.58401	0.58506	0.58611	0.58716	0.58822	0.58927	0.59033
0.59140	0.59246	0.59353	0.59460	0.59567	0.59674	0.59782	0.59890	0.59998	0.60107
0.60215	0.60324	0.60433	0.60543	0.60653	0.60763	0.60873	0.60983	0.61094	0.61205
0.61317	0.61428	0.61540	0.61652	0.61765	0.61878	0.61991	0.62104	0.62218	0.62332
0.62446	0.62561	0.62676	0.62791	0.62906	0.63022	0.63139	0.63255	0.63372	0.63489
0.63607	0.63724	0.63843	0.63961	0.64080	0.64199	0.64319	0.64439	0.64559	0.64680
0.64801	0.64922	0.65044	0.65166	0.65289	0.65412	0.65535	0.65659	0.65783	0.65907
0.66032	0.66158	0.66283	0.66410	0.66536	0.66663	0.66791	0.66919	0.67047	0.67176
0.67306	0.67435	0.67566	0.67696	0.67828	0.67959	0.68092	0.68224	0.68358	0.68492
0.68626	0.68761	0.68896	0.69032	0.69168	0.69305	0.69443	0.69581	0.69720	0.69859
0.69999	0.70140	0.70281	0.70423	0.70565	0.70708	0.70852	0.70996	0.71141	0.71287
0.71433	0.71580	0.71728	0.71877	0.72026	0.72176	0.72327	0.72478	0.72631	0.72784
0.72938	0.73092	0.73248	0.73405	0.73562	0.73720	0.73879	0.74039	0.74200	0.74362
0.74525	0.74689	0.74854	0.75020	0.75187	0.75355	0.75524	0.75695	0.75866	0.76039
0.76212	0.76388	0.76564	0.76741	0.76920	0.77100	0.77282	0.77465	0.77649	0.77835
0.78022	0.78211	0.78402	0.78594	0.78788	0.78983	0.79181	0.79380	0.79581	0.79783
0.79988	0.80195	0.80404	0.80616	0.80829	0.81045	0.81263	0.81484	0.81707	0.81933
0.82162	0.82394	0.82628	0.82866	0.83107	0.83352	0.83600	0.83851	0.84107	0.84367
0.84631	0.84899	0.85173	0.85451	0.85735	0.86024	0.86320	0.86622	0.86930	0.87246
0.87570	0.87903	0.88245	0.88597	0.88960	0.89335	0.89724	0.90128	0.90549	0.90989
0.91452	0.91942	0.92463	0.93023	0.93631	0.94304	0.95068	0.95973	0.97153	0.99973

TABLE 13.

Table of the converted function F(ro)
The first line is the step of the input correlation

TWO - FOUR

0.00100

0.00088	0.00177	0.00265	0.00353	0.00441	0.00530	0.00618	0.00706	0.00794	0.00883
0.00971	0.01059	0.01147	0.01236	0.01324	0.01412	0.01500	0.01589	0.01677	0.01765
0.01853	0.01942	0.02030	0.02118	0.02206	0.02295	0.02383	0.02471	0.02559	0.02648
0.02736	0.02824	0.02913	0.03001	0.03089	0.03177	0.03266	0.03354	0.03442	0.03530
0.03619	0.03707	0.03795	0.03884	0.03972	0.04060	0.04148	0.04237	0.04325	0.04413
0.04502	0.04590	0.04678	0.04767	0.04855	0.04943	0.05032	0.05120	0.05208	0.05296
0.05385	0.05473	0.05561	0.05650	0.05738	0.05826	0.05915	0.06003	0.06091	0.06180
0.06268	0.06356	0.06445	0.06533	0.06622	0.06710	0.06798	0.06887	0.06975	0.07063
0.07152	0.07240	0.07329	0.07417	0.07505	0.07594	0.07682	0.07770	0.07859	0.07947
0.08036	0.08124	0.08213	0.08301	0.08389	0.08478	0.08566	0.08655	0.08743	0.08832
0.08920	0.09008	0.09097	0.09185	0.09274	0.09362	0.09451	0.09539	0.09628	0.09716
0.09805	0.09893	0.09982	0.10070	0.10159	0.10247	0.10336	0.10424	0.10513	0.10601
0.10690	0.10778	0.10867	0.10955	0.11044	0.11132	0.11221	0.11310	0.11398	0.11487
0.11575	0.11664	0.11752	0.11841	0.11930	0.12018	0.12107	0.12195	0.12284	0.12373
0.12461	0.12550	0.12639	0.12727	0.12816	0.12905	0.12993	0.13082	0.13171	0.13259
0.13348	0.13437	0.13525	0.13614	0.13703	0.13791	0.13880	0.13969	0.14058	0.14146
0.14235	0.14324	0.14413	0.14501	0.14590	0.14679	0.14768	0.14856	0.14945	0.15034
0.15123	0.15212	0.15300	0.15389	0.15478	0.15567	0.15656	0.15745	0.15833	0.15922
0.16011	0.16100	0.16189	0.16278	0.16367	0.16456	0.16545	0.16634	0.16722	0.16811
0.16900	0.16989	0.17078	0.17167	0.17256	0.17345	0.17434	0.17523	0.17612	0.17701
0.17790	0.17879	0.17968	0.18057	0.18146	0.18235	0.18324	0.18414	0.18503	0.18592
0.18681	0.18770	0.18859	0.18948	0.19037	0.19126	0.19216	0.19305	0.19394	0.19483
0.19572	0.19661	0.19751	0.19840	0.19929	0.20018	0.20107	0.20197	0.20286	0.20375
0.20464	0.20554	0.20643	0.20732	0.20822	0.20911	0.21000	0.21090	0.21179	0.21268
0.21358	0.21447	0.21536	0.21626	0.21715	0.21805	0.21894	0.21983	0.22073	0.22162
0.22252	0.22341	0.22431	0.22520	0.22610	0.22699	0.22789	0.22878	0.22968	0.23057
0.23147	0.23236	0.23326	0.23415	0.23505	0.23595	0.23684	0.23774	0.23863	0.23953
0.24043	0.24132	0.24222	0.24312	0.24401	0.24491	0.24581	0.24671	0.24760	0.24850
0.24940	0.25030	0.25119	0.25209	0.25299	0.25389	0.25479	0.25568	0.25658	0.25748
0.25838	0.25928	0.26018	0.26108	0.26197	0.26287	0.26377	0.26467	0.26557	0.26647
0.26737	0.26827	0.26917	0.27007	0.27097	0.27187	0.27277	0.27367	0.27457	0.27547
0.27638	0.27728	0.27818	0.27908	0.27998	0.28088	0.28178	0.28269	0.28359	0.28449
0.28539	0.28629	0.28720	0.28810	0.28900	0.28990	0.29081	0.29171	0.29261	0.29352
0.29442	0.29532	0.29623	0.29713	0.29803	0.29894	0.29984	0.30075	0.30165	0.30256
0.30346	0.30437	0.30527	0.30618	0.30708	0.30799	0.30889	0.30980	0.31070	0.31161
0.31251	0.31342	0.31433	0.31523	0.31614	0.31705	0.31795	0.31886	0.31977	0.32067
0.32158	0.32249	0.32340	0.32430	0.32521	0.32612	0.32703	0.32794	0.32885	0.32975
0.33066	0.33157	0.33248	0.33339	0.33430	0.33521	0.33612	0.33703	0.33794	0.33885
0.33976	0.34067	0.34158	0.34249	0.34340	0.34431	0.34522	0.34613	0.34704	0.34796
0.34887	0.34978	0.35069	0.35160	0.35252	0.35343	0.35434	0.35525	0.35617	0.35708
0.35799	0.35891	0.35982	0.36073	0.36165	0.36256	0.36347	0.36439	0.36530	0.36622
0.36713	0.36805	0.36896	0.36988	0.37079	0.37171	0.37262	0.37354	0.37445	0.37537
0.37629	0.37720	0.37812	0.37904	0.37995	0.38087	0.38179	0.38270	0.38362	0.38454
0.38546	0.38638	0.38729	0.38821	0.38913	0.39005	0.39097	0.39189	0.39281	0.39372
0.39464	0.39556	0.39648	0.39740	0.39832	0.39924	0.40016	0.40109	0.40201	0.40293
0.40385	0.40477	0.40569	0.40661	0.40753	0.40846	0.40938	0.41030	0.41122	0.41215
0.41307	0.41399	0.41491	0.41584	0.41676	0.41768	0.41861	0.41953	0.42046	0.42138
0.42231	0.42323	0.42416	0.42508	0.42601	0.42693	0.42786	0.42878	0.42971	0.43063
0.43156	0.43249	0.43341	0.43434	0.43527	0.43619	0.43712	0.43805	0.43898	0.43990
0.44083	0.44176	0.44269	0.44362	0.44455	0.44548	0.44640	0.44733	0.44826	0.44919
0.45012	0.45105	0.45198	0.45291	0.45384	0.45478	0.45571	0.45664	0.45757	0.45850
0.45943	0.46036	0.46130	0.46223	0.46316	0.46409	0.46503	0.46596	0.46689	0.46782
0.46876	0.46969	0.47063	0.47156	0.47249	0.47343	0.47436	0.47530	0.47623	0.47717
0.47810	0.47904	0.47997	0.48091	0.48185	0.48278	0.48372	0.48466	0.48559	0.48653
0.48747	0.48840	0.48934	0.49028	0.49122	0.49216	0.49309	0.49403	0.49497	0.49591
0.49685	0.49779	0.49873	0.49967	0.50061	0.50155	0.50249	0.50343	0.50437	0.50531
0.50625	0.50719	0.50813	0.50908	0.51002	0.51096	0.51190	0.51284	0.51379	0.51473
0.51567	0.51661	0.51756	0.51850	0.51945	0.52039	0.52133	0.52228	0.52322	0.52417
0.52511	0.52606	0.52700	0.52795	0.52889	0.52984	0.53078	0.53173	0.53268	0.53362
0.53457	0.53552	0.53646	0.53741	0.53836	0.53931	0.54025	0.54120	0.54215	0.54310
0.54405	0.54500	0.54594	0.54689	0.54784	0.54879	0.54974	0.55069	0.55164	0.55259
0.55354	0.55449	0.55544	0.55640	0.55735	0.55830	0.55925	0.56020	0.56115	0.56211
0.56306	0.56401	0.56496	0.56592	0.56687	0.56782	0.56878	0.56973	0.57068	0.57164
0.57259	0.57354	0.57450	0.57545	0.57641	0.57736	0.57832	0.57927	0.58023	0.58119
0.58214	0.58310	0.58405	0.58501	0.58597	0.58692	0.58788	0.58884	0.58980	0.59075
0.59171	0.59267	0.59363	0.59458	0.59554	0.59650	0.59746	0.59842	0.59938	0.60034
0.60130	0.60226	0.60322	0.60418	0.60514	0.60610	0.60706	0.60802	0.60898	0.60994

0.61090	0.61186	0.61282	0.61378	0.61474	0.61571	0.61667	0.61763	0.61859	0.61955
0.62052	0.62148	0.62244	0.62341	0.62437	0.62533	0.62630	0.62726	0.62822	0.62919
0.63015	0.63111	0.63208	0.63304	0.63401	0.63497	0.63594	0.63690	0.63787	0.63883
0.63980	0.64076	0.64173	0.64270	0.64366	0.64463	0.64559	0.64656	0.64753	0.64849
0.64946	0.65043	0.65139	0.65236	0.65333	0.65429	0.65526	0.65623	0.65720	0.65816
0.65913	0.66010	0.66107	0.66204	0.66300	0.66397	0.66494	0.66591	0.66688	0.66785
0.66882	0.66978	0.67075	0.67172	0.67269	0.67366	0.67463	0.67560	0.67657	0.67754
0.67851	0.67948	0.68045	0.68142	0.68239	0.68336	0.68433	0.68530	0.68627	0.68724
0.68821	0.68918	0.69015	0.69112	0.69209	0.69306	0.69403	0.69500	0.69598	0.69695
0.69792	0.69889	0.69986	0.70083	0.70180	0.70277	0.70374	0.70471	0.70569	0.70666
0.70763	0.70860	0.70957	0.71054	0.71151	0.71248	0.71346	0.71443	0.71540	0.71637
0.71734	0.71831	0.71928	0.72026	0.72123	0.72220	0.72317	0.72414	0.72511	0.72608
0.72705	0.72803	0.72900	0.72997	0.73094	0.73191	0.73288	0.73385	0.73482	0.73580
0.73677	0.73774	0.73871	0.73968	0.74065	0.74162	0.74259	0.74356	0.74453	0.74550
0.74647	0.74744	0.74842	0.74939	0.75036	0.75133	0.75230	0.75327	0.75424	0.75521
0.75618	0.75715	0.75812	0.75909	0.76006	0.76103	0.76199	0.76296	0.76393	0.76490
0.76587	0.76684	0.76781	0.76878	0.76975	0.77072	0.77169	0.77265	0.77362	0.77459
0.77556	0.77653	0.77750	0.77847	0.77943	0.78040	0.78137	0.78234	0.78331	0.78428
0.78524	0.78621	0.78718	0.78815	0.78912	0.79008	0.79105	0.79202	0.79299	0.79395
0.79492	0.79589	0.79686	0.79783	0.79879	0.79976	0.80073	0.80170	0.80267	0.80364
0.80461	0.80557	0.80654	0.80751	0.80848	0.80945	0.81042	0.81139	0.81236	0.81333
0.81430	0.81527	0.81624	0.81722	0.81819	0.81916	0.82013	0.82111	0.82208	0.82306
0.82403	0.82501	0.82598	0.82696	0.82794	0.82891	0.82989	0.83087	0.83185	0.83284
0.83382	0.83480	0.83579	0.83677	0.83776	0.83875	0.83974	0.84073	0.84172	0.84272
0.84371	0.84471	0.84571	0.84671	0.84771	0.84872	0.84972	0.85073	0.85174	0.85276
0.85377	0.85479	0.85582	0.85684	0.85787	0.85890	0.85994	0.86097	0.86202	0.86306
0.86411	0.86517	0.86622	0.86729	0.86835	0.86943	0.87051	0.87159	0.87268	0.87377
0.87488	0.87598	0.87710	0.87822	0.87935	0.88049	0.88163	0.88278	0.88395	0.88512
0.88630	0.88749	0.88869	0.88991	0.89113	0.89237	0.89361	0.89488	0.89615	0.89744
0.89874	0.90006	0.90140	0.90275	0.90412	0.90551	0.90692	0.90835	0.90981	0.91128
0.91278	0.91431	0.91586	0.91744	0.91905	0.92069	0.92237	0.92408	0.92583	0.92763
0.92946	0.93135	0.93329	0.93529	0.93735	0.93948	0.94168	0.94398	0.94637	0.94887
0.95149	0.95427	0.95723	0.96041	0.96386	0.96768	0.97201	0.97715	0.98384	0.99985

TABLE 14.

Table of the converted function F(ro)
The first line is the step of the input correlation

FOUR - FOUR

0.00100									
0.00088	0.00177	0.00265	0.00353	0.00441	0.00530	0.00618	0.00706	0.00794	0.00883
0.00971	0.01059	0.01147	0.01236	0.01324	0.01412	0.01500	0.01589	0.01677	0.01765
0.01853	0.01942	0.02030	0.02118	0.02206	0.02295	0.02383	0.02471	0.02559	0.02648
0.02736	0.02824	0.02912	0.03001	0.03089	0.03177	0.03265	0.03354	0.03442	0.03530
0.03619	0.03707	0.03795	0.03883	0.03972	0.04060	0.04148	0.04236	0.04325	0.04413
0.04501	0.04589	0.04678	0.04766	0.04854	0.04943	0.05031	0.05119	0.05207	0.05296
0.05384	0.05472	0.05561	0.05649	0.05737	0.05825	0.05914	0.06002	0.06090	0.06179
0.06267	0.06355	0.06443	0.06532	0.06620	0.06708	0.06797	0.06885	0.06973	0.07062
0.07150	0.07238	0.07326	0.07415	0.07503	0.07591	0.07680	0.07768	0.07856	0.07945
0.08033	0.08121	0.08210	0.08298	0.08386	0.08475	0.08563	0.08651	0.08740	0.08828
0.08916	0.09005	0.09093	0.09181	0.09270	0.09358	0.09446	0.09535	0.09623	0.09711
0.09800	0.09888	0.09976	0.10065	0.10153	0.10241	0.10330	0.10418	0.10507	0.10595
0.10683	0.10772	0.10860	0.10948	0.11037	0.11125	0.11214	0.11302	0.11390	0.11479
0.11567	0.11656	0.11744	0.11832	0.11921	0.12009	0.12098	0.12186	0.12274	0.12363
0.12451	0.12540	0.12628	0.12716	0.12805	0.12893	0.12982	0.13070	0.13159	0.13247
0.13335	0.13424	0.13512	0.13601	0.13689	0.13778	0.13866	0.13955	0.14043	0.14131
0.14220	0.14308	0.14397	0.14485	0.14574	0.14662	0.14751	0.14839	0.14928	0.15016
0.15105	0.15193	0.15282	0.15370	0.15459	0.15547	0.15636	0.15724	0.15813	0.15901
0.15990	0.16078	0.16167	0.16255	0.16344	0.16432	0.16521	0.16609	0.16698	0.16787
0.16875	0.16964	0.17052	0.17141	0.17229	0.17318	0.17406	0.17495	0.17584	0.17672
0.17761	0.17849	0.17938	0.18027	0.18115	0.18204	0.18292	0.18381	0.18470	0.18558
0.18647	0.18735	0.18824	0.18913	0.19001	0.19090	0.19179	0.19267	0.19356	0.19444
0.19533	0.19622	0.19710	0.19799	0.19888	0.19976	0.20065	0.20154	0.20242	0.20331
0.20420	0.20509	0.20597	0.20686	0.20775	0.20863	0.20952	0.21041	0.21129	0.21218
0.21307	0.21396	0.21484	0.21573	0.21662	0.21751	0.21839	0.21928	0.22017	0.22106
0.22194	0.22283	0.22372	0.22461	0.22550	0.22638	0.22727	0.22816	0.22905	0.22993
0.23082	0.23171	0.23260	0.23349	0.23438	0.23526	0.23615	0.23704	0.23793	0.23882
0.23971	0.24059	0.24148	0.24237	0.24326	0.24415	0.24504	0.24593	0.24682	0.24771
0.24859	0.24948	0.25037	0.25126	0.25215	0.25304	0.25393	0.25482	0.25571	0.25660
0.25749	0.25838	0.25927	0.26015	0.26104	0.26193	0.26282	0.26371	0.26460	0.26549
0.26638	0.26727	0.26816	0.26905	0.26994	0.27083	0.27172	0.27261	0.27350	0.27440
0.27529	0.27618	0.27707	0.27796	0.27885	0.27974	0.28063	0.28152	0.28241	0.28330
0.28419	0.28508	0.28597	0.28687	0.28776	0.28865	0.28954	0.29043	0.29132	0.29221
0.29311	0.29400	0.29489	0.29578	0.29667	0.29756	0.29846	0.29935	0.30024	0.30113
0.30202	0.30292	0.30381	0.30470	0.30559	0.30648	0.30738	0.30827	0.30916	0.31005
0.31095	0.31184	0.31273	0.31362	0.31452	0.31541	0.31630	0.31720	0.31809	0.31898
0.31988	0.32077	0.32166	0.32256	0.32345	0.32434	0.32524	0.32613	0.32702	0.32792
0.32881	0.32971	0.33060	0.33149	0.33239	0.33328	0.33418	0.33507	0.33596	0.33686
0.33775	0.33865	0.33954	0.34044	0.34133	0.34223	0.34312	0.34402	0.34491	0.34581
0.34670	0.34760	0.34849	0.34939	0.35028	0.35118	0.35207	0.35297	0.35386	0.35476
0.35566	0.35655	0.35745	0.35834	0.35924	0.36014	0.36103	0.36193	0.36282	0.36372
0.36462	0.36551	0.36641	0.36731	0.36820	0.36910	0.37000	0.37089	0.37179	0.37269
0.37358	0.37448	0.37538	0.37628	0.37717	0.37807	0.37897	0.37987	0.38076	0.38166
0.38256	0.38346	0.38436	0.38525	0.38615	0.38705	0.38795	0.38885	0.38974	0.39064
0.39154	0.39244	0.39334	0.39424	0.39514	0.39604	0.39693	0.39783	0.39873	0.39963
0.40053	0.40143	0.40233	0.40323	0.40413	0.40503	0.40593	0.40683	0.40773	0.40863
0.40953	0.41043	0.41133	0.41223	0.41313	0.41403	0.41493	0.41583	0.41673	0.41763
0.41853	0.41943	0.42034	0.42124	0.42214	0.42304	0.42394	0.42484	0.42574	0.42665
0.42755	0.42845	0.42935	0.43025	0.43115	0.43206	0.43296	0.43386	0.43476	0.43567
0.43657	0.43747	0.43837	0.43928	0.44018	0.44108	0.44199	0.44289	0.44379	0.44470
0.44560	0.44650	0.44741	0.44831	0.44921	0.45012	0.45102	0.45192	0.45283	0.45373
0.45464	0.45554	0.45645	0.45735	0.45825	0.45916	0.46006	0.46097	0.46187	0.46278
0.46368	0.46459	0.46549	0.46640	0.46731	0.46821	0.46912	0.47002	0.47093	0.47183
0.47274	0.47365	0.47455	0.47546	0.47637	0.47727	0.47818	0.47909	0.47999	0.48090
0.48181	0.48271	0.48362	0.48453	0.48544	0.48634	0.48725	0.48816	0.48907	0.48997
0.49088	0.49179	0.49270	0.49361	0.49452	0.49542	0.49633	0.49724	0.49815	0.49906
0.49997	0.50088	0.50179	0.50269	0.50360	0.50451	0.50542	0.50633	0.50724	0.50815
0.50906	0.50997	0.51088	0.51179	0.51270	0.51361	0.51453	0.51544	0.51635	0.51726
0.51817	0.51908	0.51999	0.52090	0.52181	0.52273	0.52364	0.52455	0.52546	0.52637
0.52729	0.52820	0.52911	0.53002	0.53093	0.53185	0.53276	0.53367	0.53459	0.53550
0.53641	0.53733	0.53824	0.53915	0.54007	0.54098	0.54189	0.54281	0.54372	0.54464
0.54555	0.54647	0.54738	0.54830	0.54921	0.55013	0.55104	0.55196	0.55287	0.55379
0.55470	0.55562	0.55653	0.55745	0.55837	0.55928	0.56020	0.56111	0.56203	0.56295
0.56386	0.56478	0.56570	0.56662	0.56753	0.56845	0.56937	0.57029	0.57120	0.57212
0.57304	0.57396	0.57488	0.57579	0.57671	0.57763	0.57855	0.57947	0.58039	0.58131
0.58223	0.58315	0.58407	0.58499	0.58591	0.58683	0.58775	0.58867	0.58959	0.59051
0.59143	0.59235	0.59327	0.59419	0.59511	0.59603	0.59695	0.59788	0.59880	0.59972

0.60064	0.60156	0.60249	0.60341	0.60433	0.60525	0.60618	0.60710	0.60802	0.60895
0.60987	0.61079	0.61172	0.61264	0.61357	0.61449	0.61542	0.61634	0.61726	0.61819
0.61911	0.62004	0.62097	0.62189	0.62282	0.62374	0.62467	0.62559	0.62652	0.62745
0.62837	0.62930	0.63023	0.63115	0.63208	0.63301	0.63394	0.63486	0.63579	0.63672
0.63765	0.63858	0.63951	0.64043	0.64136	0.64229	0.64322	0.64415	0.64508	0.64601
0.64694	0.64787	0.64880	0.64973	0.65066	0.65159	0.65252	0.65345	0.65439	0.65532
0.65625	0.65718	0.65811	0.65905	0.65998	0.66091	0.66184	0.66278	0.66371	0.66464
0.66558	0.66651	0.66745	0.66838	0.66931	0.67025	0.67118	0.67212	0.67305	0.67399
0.67492	0.67586	0.67680	0.67773	0.67867	0.67961	0.68054	0.68148	0.68242	0.68335
0.68429	0.68523	0.68617	0.68711	0.68805	0.68898	0.68992	0.69086	0.69180	0.69274
0.69368	0.69462	0.69556	0.69650	0.69744	0.69838	0.69933	0.70027	0.70121	0.70215
0.70309	0.70404	0.70498	0.70592	0.70687	0.70781	0.70875	0.70970	0.71064	0.71159
0.71253	0.71348	0.71442	0.71537	0.71631	0.71726	0.71821	0.71915	0.72010	0.72105
0.72200	0.72295	0.72389	0.72484	0.72579	0.72674	0.72769	0.72864	0.72959	0.73054
0.73149	0.73244	0.73340	0.73435	0.73530	0.73625	0.73720	0.73816	0.73911	0.74007
0.74102	0.74198	0.74293	0.74389	0.74484	0.74580	0.74676	0.74771	0.74867	0.74963
0.75059	0.75154	0.75250	0.75346	0.75442	0.75538	0.75635	0.75731	0.75827	0.75923
0.76019	0.76116	0.76212	0.76309	0.76405	0.76502	0.76598	0.76695	0.76792	0.76888
0.76985	0.77082	0.77179	0.77276	0.77373	0.77470	0.77568	0.77665	0.77762	0.77860
0.77957	0.78055	0.78152	0.78250	0.78348	0.78446	0.78544	0.78642	0.78740	0.78838
0.78936	0.79035	0.79133	0.79232	0.79330	0.79429	0.79528	0.79627	0.79726	0.79825
0.79924	0.80024	0.80123	0.80223	0.80323	0.80423	0.80523	0.80623	0.80723	0.80824
0.80924	0.81025	0.81126	0.81227	0.81328	0.81429	0.81531	0.81633	0.81735	0.81837
0.81939	0.82042	0.82145	0.82247	0.82351	0.82454	0.82558	0.82662	0.82766	0.82870
0.82975	0.83080	0.83185	0.83291	0.83396	0.83503	0.83609	0.83716	0.83823	0.83931
0.84038	0.84147	0.84255	0.84365	0.84474	0.84584	0.84694	0.84805	0.84917	0.85029
0.85141	0.85254	0.85368	0.85482	0.85597	0.85712	0.85828	0.85945	0.86062	0.86180
0.86299	0.86419	0.86540	0.86661	0.86783	0.86906	0.87031	0.87156	0.87282	0.87409
0.87537	0.87667	0.87798	0.87930	0.88063	0.88197	0.88333	0.88471	0.88610	0.88751
0.88893	0.89037	0.89183	0.89331	0.89481	0.89632	0.89787	0.89943	0.90102	0.90263
0.90427	0.90594	0.90764	0.90937	0.91113	0.91293	0.91477	0.91664	0.91856	0.92053
0.92254	0.92461	0.92674	0.92893	0.93119	0.93352	0.93594	0.93846	0.94108	0.94382
0.94671	0.94976	0.95300	0.95649	0.96028	0.96448	0.96924	0.97488	0.98224	0.99983