Signal to Noise Ratio of an FX Correlator in Dependence on Weighting Function in FFT

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1 Introduction

It is clear that signal to noise ratio of an FX correlator depends on a weighting function in FFT. General equations have been deduced for estimation of the signal to noise ratio for any weighting function. The special calculation has been done for UNIFORM and HANNING weighting functions implemented in VLBA correlator. The signal to noise ratio has been estimated for three cases:
1.Individual spectral channel;
2.Averaging of spectral channels;
3.Averaging of overlapping FFTs.

2 Analysis of a signal to noise ratio in an individual spectral channel.

In the following analysis we suppose that fringe rate for a given pair of antennas is equal zero and correlated signals are not digitized. These suppositions simplify the mathematical analysys and at the same time do not limit the area of application of obtained results. So the signals from each antenna can be described by following equations:

$$\xi_1(k) = \sqrt{P_{s1}}\,\xi_s(k) + \sqrt{P_{n1}}\,n_1(k); \qquad \xi_2(k) = \sqrt{P_{s2}}\,\xi_s(k) + \sqrt{P_{n2}}\,n_2(k) \tag{1}$$

where P_{s1}, P_{s2} is the power of correlated signals in the antennas.
P_{n1}, P_{n2} is the power of independent noises in the antennas.
$\xi_s(k), n_1(k), n_2(k)$ are normalized random process' with the following properties:

$$\overline{\xi_s(k)} = \overline{n_1(k)} = \overline{n_2(k)} = 0; \qquad \overline{\xi_s^2(k)} = \overline{n_1^2(k)} = \overline{n_2^2(k)} = 1; \tag{2}$$

$$\overline{n_1(k_1)\cdot n_1(k_2)} = \overline{n_2(k_1)\cdot n_2(k_2)} = 0, \qquad if \quad k_1 \neq k_2 \tag{3}$$

$$\overline{n_1(k)\cdot n_2(k)} = \overline{n_1(k)\cdot \xi_s(k)} = \overline{n_2(k)\cdot \xi_s(k)} = 0 \qquad for \quad any \quad k \tag{4}$$

The property (3) is valid for a flat spectrum of the signals if a sampling frequency is equal double bandwidth of the spectrum. Spectrum of the noise is determined by the filters partern of receivers and usually close to the flat form. Signals at the FFT outputs are described by the next equations:

$$\eta_1(l) = \frac{1}{N}\sum_{k=1}^{N}\xi_1(k)H(k)\exp j2\pi\frac{k\,l}{N}; \qquad \eta_2(l) = \frac{1}{N}\sum_{k=1}^{N}\xi_2(k)H(k)\exp j2\pi\frac{k\,l}{N} \tag{5}$$

where H(k) is a weighting function.
The FFT output signals corresponded to antennas of a given interferometer are multiplyed (the second output is taken in conjagate form). As a result taking into account equations (5), we obtain the next expression for the output of an FX correlator from one FFT:

$$\zeta(l) = \eta_1(l)\,\eta_2^*(l) = \frac{1}{N^2}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N} H(k_1)H(k_2)\xi_1(k_1)\xi_2(k_2)\exp j2\pi\frac{(k_1-k_2)\,l}{N} \tag{6}$$

Now we'll calculate mathematical expectation and rms of random magnitude $\zeta(l)$ to estimate a signal to noise ratio.

Calculation of mathematical expectation $\overline{\zeta(l)}$ - signal

Starting with the mathematical expectation we can note that the complex exponent and mathematical expectation of $\xi_1(k_1)\xi_2(k_2)$ depend on the difference k_1-k_2. Therefore it is comfortable to swith the variables of the double sums from $k_1,\ k_2$ to $k_1, i = k_1 - k_2$. Following this and taking into account the pair independence of the random process' n_1, n_2, ξ_s (1, 4), we can deduce the following equation for the mathematical expectation of $\zeta(l)$:

$$\overline{\zeta(l)} = \frac{1}{N}\sqrt{P_{s1}P_{s2}}\sum_{i=-N}^{N} h(i)r(i)\exp j2\pi\frac{i\,l}{N} \tag{7}$$

where $r(i) = \overline{\xi_s(k)\,\xi_s(k+i)}$ is coefficient of auto correlation of the signal as a function of delay;
$h(i)$ is an auto convolution function of the weighting function:

$$h(i) = \frac{1}{N}\sum_{k=1}^{N} H(k)H(k+i) \tag{8}$$

The auto correlation function of a random process and its power spectrum are a related Fourier pair. That is why the equation (7) can be rewritten on the spectral language:

$$\overline{\zeta(l)} = \frac{1}{N}\sqrt{P_{s1}P_{s2}}\int_0^1 F(x)\,f\left(x-\frac{l}{\frac{N}{2}}\right)\,dx \tag{9}$$

where $F(x)$ is a spectrum of a given signal; $x = \frac{f}{\Delta f}$.
$f(z)$ is an equivalent filter corresponded to the given function $h(i)$:

$$f(z) = \sum_{i=-N}^{N} h(i)\cos(\pi i z) \tag{10}$$

Let's consider the width of the filter pattern $f(z)$ is much less of the features width in the spectrum $F(x)$. Then we can take out of the integral $F(l)$ and the equation (9) will be simplified:

$$\overline{\zeta(l)} = \frac{1}{N}\sqrt{P_{s1}P_{s2}}\,F(l)\int_0^1 f(x-\frac{l}{\frac{N}{2}})\,dx \tag{11}$$

But the integral in (11) is the area under the equivalent filter pattern $f(z)$. This area is equal $h(0)$ because $f(z)$ and $h(i)$ are a related Fourier pair (10). So finally we obtain the following expression for the mathematical expectation of $\zeta(l)$:

$$\overline{\zeta(l)} = \frac{1}{N}\sqrt{P_{s1}P_{s2}}\,F(l)\,h(0) \tag{12}$$

Calculation of rms $\zeta(l)$ - noise

Calculating the noise we'll consider the signal power to be negligible in comparison with a noise power. This supposition simplifyes calculations because different antennas signals are become independent. At the same time the case of strong signal is not actual for the signal to noise ratio analysis. Taking into account the independence of $\eta_1(l)$ and $\eta_2(l)$ in (6) we can write the following expression for a dispersion of $\zeta(l)$:

$$D\zeta = \overline{\eta_1\,\eta_1^*}\;\overline{\eta_2\,\eta_2^*} = \frac{1}{N^4}P_{n1}P_{n2}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N} H(k_1)H(k_2)\,\overline{n_1(k_1)\,n_1(k_2)}\,\exp j2\pi\frac{(k_1-k_2)\,l}{N}\;\cdot$$
$$\sum_{k_1=1}^{N}\sum_{k_2=1}^{N} H(k_1)H(k_2)\,\overline{n_2(k_1)\,n_2(k_2)}\,\exp j2\pi\frac{(k_1-k_2)\,l}{N} \qquad (13)$$

The random process' $n_1(k)$ and $n_2(k)$ are ortonormalized (look equations 2, 3). So the equation (13) can be simplifiyed:

$$D\zeta = P_{n1}P_{n2}\left[\frac{1}{N^2}\sum_{k=1}^{N} H^2(k)\right]^2 \qquad (14)$$

But $\frac{1}{N}\sum_{k=1}^{N} H^2(k) = h(0)$ (Look expression 8). Having substituted this expression into (14) we can deduce the following equation for *rms* of ζ:

$$\sigma\,\zeta = \frac{1}{N}\sqrt{P_{n1}P_{n2}}\,h(0) \qquad (15)$$

Having devided equation (12) over (15) we obtain the signal to noise ratio.

$$\frac{S}{N} = \frac{\overline{\zeta(l)}}{\sigma\,\zeta} = \sqrt{\frac{P_{s1}\,P_{s2}}{P_{n1}\,P_{n2}}}\,F(l) \qquad (16)$$

We see that $h(0)$ has been canceled in this division.

Signal to noise ratio in an individual spectral channel is identical for any weighting function in FFT

This conclusion is rather comprehensible physically. Hanning weighting does not use the all information on the time axis and for this reason hanning has to have worse signal to noise ratio in comparison with uniform weighting. But from the other hand the equivalent filter on the frequency axis is wider for hanning and for this reason hanning has to have better signal to noise ratio. These two effect compensate each other. Such a full compensation is valid in comparison of any weighting function with the uniform weighting function.

3 Analysis of a signal to noise ratio after averaging of spectral channels.

The equivalent filter of hanning weigting function is wider than of uniform one. As a result neighbour channels noises are more correlated in the hanning case. That means the uniform weighting function provides better averaging of the noises averaging the frequency channels. So the signal to noise ratio has to be better in the case of uniform weighting function. The answer on the question 'How much?' is given in the following analysis.
We are going to compare nathematical expectation (signal) and rms (noise) of the random magnitude γ:

$$\gamma = \frac{1}{L}\sum_{l=1}^{L}\zeta(l) \qquad (17)$$

where L is a number of averaging spectral channels.

Calculation of mathematical expectation $\overline{\gamma}$ - signal

Using equations (12) and (17) we can simply obtain the following expression for mathematical expectation of γ:

$$\overline{\gamma} = \frac{1}{N}\sqrt{P_{s1}P_{s2}}\,h(0)\,\overline{F} \tag{18}$$

where $\overline{F} = \frac{1}{L}\sum_{l=1}^{L} F(l)$ is a mean value of the spectrum in averaging channels.

Calculation of rms γ - noise

Calculating the noise we'll again consider the signal power to be negligible in comparison with a noise power. Taking into account the independence of $\eta_1(l)$ and $\eta_2(l)$ in (6) we can write the following expression for a dispersion of γ:

$$D\gamma = \frac{1}{L^2}\sum_{l_1=1}^{L}\sum_{l_2=1}^{L}\overline{\eta_1(l_1)\,\eta_1^*(l_2)}\;\overline{\eta_2^*(l_1)\,\eta_2(l_2)} \tag{19}$$

Having used an expression (5) and remembering that $\xi_1(k)$ is supposed to equal $n_1(k)$ we can obtain the next expression for $\eta_1(l_1)\,\eta_1^*(l_2)$:

$$\overline{\eta_1(l_1)\,\eta_1^*(l_2)} = \frac{1}{N^2}P_{n1}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}H(k_1)H(k_2)\,\overline{n_1(k_1)n_1(k_2)}\,\exp j2\pi\frac{(k_1l_1 - k_2l_2)}{N} \tag{20}$$

But $\overline{n_1(k_1)n_1(k_2)} = 0$ if $k_1 \neq k_2$ and $\overline{n_1(k_1)n_1(k_2)} = 1$ if $k_1 = k_2$ (See equation 2, 3). This property gives us opportunity to transform double sum expression (20) in single sum expression:

$$\overline{\eta_1(l_1)\,\eta_1^*(l_2)} = \frac{1}{N^2}P_{n1}\sum_{k=1}^{N}H^2(k)\,\exp j2\pi\frac{k(l_1 - l_2)}{N} \tag{21}$$

Analogously:

$$\overline{\eta_2^*(l_1)\,\eta_2(l_2)} = \frac{1}{N^2}P_{n2}\sum_{k=1}^{N}H^2(k)\,\exp -j2\pi\frac{k(l_1 - l_2)}{N} \tag{22}$$

Having substituted (21, 22) into (19) we can obtain the following expression for dispersion of γ:

$$D\gamma = \frac{1}{L^2}\sum_{l_1=1}^{L}\sum_{l_2=1}^{L}\frac{1}{N^4}P_{n1}P_{n2}\left|\sum_{k=1}^{N}H^2(k)\exp j2\pi\frac{k\,(l_1 - l_2)}{N}\right|^2 \tag{23}$$

Now let's analyse equation (23) separately for uniform and hanning weighting function.

UNIFORM

For uniform weighting function $H(k) = H^2(k) = 1$ and therefore:

$$\sum_{k=1}^{N}H^2(k)\exp j2\pi\frac{k\,(l_1 - l_2)}{N} = N\cdot\delta(l_1 - l_2) \tag{24}$$

where $\delta(i)$ is delta function:

$$\delta(i) = \begin{cases} 1 & \text{if } i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

Having substituted (24) into (23) we can obtain the following expression for dispersion of γ in the case of uniform weighting function:

$$D\gamma = \frac{1}{L^2\,N^2} P_{n1} P_{n2} \sum_{l_1=1}^{L} \sum_{l_2=1}^{L} \delta(l_1 - l_2) = \frac{P_{n1} P_{n2}}{N^2\,L} \tag{26}$$

An expression for rms can be simply obtained from (26)

$$\sigma\gamma = \frac{\sqrt{P_{n1} P_{n2}}}{N\,\sqrt{L}} \tag{27}$$

Signal to noise ratio is obtained by division (18) over (27) and substituting $h(0) = 1$

$$\left(\frac{S}{N}\right)_{uni} = \sqrt{\frac{P_{s1} P_{s2}}{P_{n1} P_{n2}}}\,\overline{F}\,\sqrt{L} \tag{28}$$

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For hanning type weighting function $H(k) = 0.5(1 - \cos 2\pi \frac{k}{N})$ the following expressions are valid:

$$\sum_{k=1}^{N} H^2(k) \exp j2\pi \frac{k\,(l_1 - l_2)}{N} = N \cdot$$
$$\left(\tfrac{3}{8} \cdot \delta(l_1 - l_2) - \tfrac{1}{4}\delta(l_1 - l_2 - 1) - \tfrac{1}{4}\delta(l_1 - l_2 + 1) + \tfrac{1}{16}\delta(l_1 - l_2 - 2) + \tfrac{1}{16}\delta(l_1 - l_2 + 2)\right) \tag{29}$$

$$\left|\sum_{k=1}^{N} H^2(k) \exp j2\pi \frac{k\,(l_1 - l_2)}{N}\right|^2 = \left(\frac{3}{8}N\right)^2 \cdot$$
$$\left(\delta(l_1 - l_2) + \tfrac{4}{9}\delta(l_1 - l_2 - 1) + \tfrac{4}{9}\delta(l_1 - l_2 + 1) + \tfrac{1}{36}\delta(l_1 - l_2 - 2) + \tfrac{1}{36}\delta(l_1 - l_2 + 2)\right) \tag{30}$$

Having substituted (30) into (23) we can obtain the following expression for dispersion of γ in the case of hanning type weighting function:

$$\begin{aligned} D\gamma &= \frac{1}{L^2\,N^2} P_{n1} P_{n2} \left(\frac{3}{8}\right)^2 \left(L + \frac{8}{9}(L-1) + \frac{1}{18}(L-2)\right) \\ &= \frac{1}{L\,N^2} P_{n1} P_{n2} \left(\frac{3}{8}\right)^2 \frac{35}{18} \left(1 - \frac{18}{35L}\right) \end{aligned} \tag{31}$$

An expression for rms can be simply obtained from (31)

$$\sigma\gamma = \frac{3}{8}\frac{1}{N}\sqrt{P_{n1} P_{n2}}\,\sqrt{\frac{35}{18}}\,\sqrt{1 - \frac{18}{35L}}\,\frac{1}{\sqrt{L}} \tag{32}$$

Signal to noise ratio is obtained by division (18) over (32) and substituting $h(0) = \frac{3}{8}$

$$\left(\frac{S}{N}\right)_{han} = \sqrt{\frac{P_{s1} P_{s2}}{P_{n1} P_{n2}}}\,\overline{F}\,\sqrt{L}\,\sqrt{\frac{18}{35}}\left(1 + \frac{9}{35\,L}\right) \tag{33}$$

Now we can compare signal to noise ratios for the uniform and hanning weighting functions, deviding (28) over (33)

$$\left(\frac{S}{N}\right)_{uni} \div \left(\frac{S}{N}\right)_{han} = \sqrt{\frac{35}{18}}\left(1 - \frac{9}{35\,L}\right) \simeq 1.4 \tag{34}$$

Having averaged spectral channels, signal to noise ratio for uniform weighting function is better than for hanning one in $\simeq 1.4$ times

4 Analysis of a signal to noise ratio in the case of overlapping FFT segments in time axis

We limit ourselfs by the case of one half segment overlapping. Signals at an outputs of FFT # m are described by the next equations:

$$\eta_1(l,m) = \frac{1}{N}\sum_{k=1}^{N}\xi_1(km)H(k)\exp j2\pi\frac{(km)\,l}{N}; \qquad \eta_2(l,m) = \frac{1}{N}\sum_{k=1}^{N}\xi_2(km)H(k)\exp j2\pi\frac{(km)\,l}{N} \tag{35}$$

where $km = k + (m-1)\,\frac{N}{2}$

The outputs of coresponded FFTs from antennas of a given interferometer are multiplyed forming the product $\zeta(m,l)$. Then the products are averaged through all M FFT's outputs. Generally speaking this averaging is provided in any case. But if there is no overlapping then all FFT's output are independent and effect of this averaging is easy comprehended. In the case of overlapping the FFT's outputs are correlated. The calculation of signal to noise ratio in the presence of this correlation is not so simple and is a subject of the following analysis.

We are going to compare nathematical expectation (signal) and rms (noise) of the random magnitude γ:

$$\gamma = \frac{1}{M}\sum_{m=1}^{M}\eta_1(l,m)\eta_2^*(l,m) \tag{36}$$

where M is a number of averaging FFTs.

Calculation of mathematical expectation $\overline{\gamma}$ - signal

Mathematical expectation of $(\eta_1(l,m)\eta_2(l,m))$ does not depend on m in accordance with equation (12) and we infer that mathematical expectation of γ is determined by the same equation:

$$\overline{\gamma} = \frac{1}{N}\sqrt{P_{s1}P_{s2}}\,F(l)\,h(0) \tag{37}$$

Calculation of rms γ - noise

Calculating the noise we'll again consider the signal power to be negligible in comparison with a noise power. Taking into account the independence of $\eta_1(l)$ and $\eta_2(l)$ in (6) we can write the following expression for a dispersion of γ:

$$D\gamma = \frac{1}{M^2}\sum_{m_1=1}^{N}\sum_{m_2=1}^{N}\overline{\eta_1(m_1)\,\eta_1^*(m_2)}\;\overline{\eta_2^*(m_1)\,\eta_2(m_2)} \tag{38}$$

Having used an expression (35) and remembering that $\xi_1(k)$ is supposed to equal $n_1(k)$ we can obtain the next expression for $\overline{\eta_1(m_1)\,\eta_1^*(m_2)}$:

$$\begin{aligned}\overline{\eta_1(m_1)\,\eta_1^*(m_2)} \;&=\; \frac{1}{N^2}P_{n1}\sum_{k_1=1}^{N}\sum_{k_2=1}^{N}H(k_1)H(k_2)\,\overline{n_1\left(k_1+(m_1-1)\frac{N}{2}\right)\,n_1\left(k_2+(m_2-1)\frac{N}{2}\right)} \\ &\cdot\;\exp j2\pi\frac{l((k_1-k_2)+(m_1-m_2)\frac{N}{2})}{N}\end{aligned} \tag{39}$$

But $\overline{n_1(k_1+(m_1-1)\frac{N}{2})\,n_1(k_2+(m_2-1)\frac{N}{2})} = 0$ if $k_1+(m_1-1)\frac{N}{2} \neq k_2+(m_2-1)\frac{N}{2}$ and $\overline{n_1(k_1+(m_1-1)\frac{N}{2})\,n_1(k_2+(m_2-1)\frac{N}{2})} = 1$ if $k_1+(m_1-1)\frac{N}{2} = k_2+(m_2-1)\frac{N}{2}$ (See equation 2, 3).

The last one can happen only in three cases:

1) $m_1 = m_2;\ k_1 = k_2$. From (39) we have:

$$\overline{\eta_1(m_1)\,\eta_1^*(m_2)} = \frac{1}{N^2} P_{n1} \sum_{k=1}^{N} H(k)^2 = \frac{1}{N} P_{n1} h(0) \tag{40}$$

2) $m_1 = m_2 + 1;\ k_1 = k_2 - \frac{N}{2}$. From (39) we have:

$$\overline{\eta_1(m_1)\,\eta_1^*(m_2)} = \frac{1}{N^2} P_{n1} \sum_{k=1}^{N/2} H(k_1) H\left(k_1 + \frac{N}{2}\right) = \frac{1}{N} P_{n1} h\left(\frac{N}{2}\right) \tag{41}$$

3) $m_1 = m_2 - 1;\ k_1 = k_2 + \frac{N}{2}$. From (39) we have:

$$\overline{\eta_1(m_1)\,\eta_1^*(m_2)} = \frac{1}{N^2} P_{n1} \sum_{k=1}^{N/2} H(k_1) H\left(k_1 - \frac{N}{2}\right) = \frac{1}{N} P_{n1} h\left(-\frac{N}{2}\right) = \frac{1}{N} P_{n1} h\left(\frac{N}{2}\right) \tag{42}$$

Combining (40)(41)(42) we can obtain the following expression for $\overline{\eta_1(m_1)\,\eta_1^*(m_2)}$:

$$\overline{\eta_1(m_1)\,\eta_1^*(m_2)} = \frac{1}{N} P_{n1} \begin{cases} h(0) & \text{if } m_1 = m_2 \\ h(\frac{N}{2}) & \text{if } m_1 = m_2 \pm 1 \\ 0 & \text{otherwise} \end{cases} = \frac{1}{N} P_{n1}\, D(m_1, m_2) \tag{43}$$

Using (43) and analogous expression for $\overline{\eta_2(m_1)\,\eta_2^*(m_2)}$ we can deduce from (38) the following expression for dispersion of γ:

$$D\gamma = \frac{1}{N^2}\,\frac{1}{M^2} P_{n1} P_{n2} \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} D^2(m_1, m_2) = \frac{1}{N^2}\,\frac{1}{M^2} P_{n1} P_{n2} \left[M\, h^2(0) + 2\,(M-1)\, h^2\left(\frac{N}{2}\right)\right] \tag{44}$$

Now let's analyse equation (44) separately for uniform and hanning type weighting function.

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For uniform weighting function $h(0) = 1;\quad h\left(\frac{N}{2}\right) = \frac{1}{2}$ and therefore following (44):

$$D\gamma = \frac{1}{N^2}\,\frac{1}{M^2} P_{n1} P_{n2} \left[M + \frac{1}{2}(M-1)\right] = \frac{1}{N^2}\, P_{n1} P_{n2} \frac{3}{2\,M} \left(1 - \frac{1}{3\,M}\right) \tag{45}$$

An expression for rms can be simply obtained from (45)

$$\sigma\gamma = \frac{1}{N}\, \sqrt{P_{n1} P_{n2}} \sqrt{\frac{3}{2\,M}} \left(1 - \frac{1}{6\,M}\right) \tag{46}$$

Signal to noise ratio is obtained by division (37) over (46) and substituting $h(0) = 1$

$$\left(\frac{S}{N}\right)_{uni} = \sqrt{\frac{P_{s1} P_{s2}}{P_{n1} P_{n2}}} F(l) \sqrt{\frac{2\,M}{3}} \left(1 + \frac{1}{6\,M}\right) \tag{47}$$

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For hanning type weighting function $h(0) = \frac{3}{8},\quad h\left(\frac{N}{2}\right) = \frac{1}{16}$ and therefore following (44):

$$D\gamma = \frac{1}{N^2}\,\frac{1}{M^2} P_{n1} P_{n2} \left[M \left(\frac{3}{8}\right)^2 + 2(M-1)\left(\frac{1}{16}\right)^2\right] = \frac{1}{N^2}\, P_{n1} P_{n2} \frac{9}{64\,M}\,\frac{19}{18} \left(1 - \frac{1}{19\,M}\right) \tag{48}$$

An expression for rms can be simply obtained from (48)

$$\sigma\gamma = \frac{1}{N}\sqrt{P_{n1}P_{n2}}\sqrt{\frac{1}{M}\frac{3}{8}}\sqrt{\frac{19}{18}}\left(1-\frac{1}{38\,M}\right) \tag{49}$$

Signal to noise ratio is obtained by division (37) over (49) and substituting $h(0) = \frac{3}{8}$

$$\left(\frac{S}{N}\right)_{han} = \sqrt{\frac{P_{s1}P_{s2}}{P_{n1}P_{n2}}}F(l)\sqrt{M}\sqrt{\frac{18}{19}}\left(1+\frac{1}{38\,M}\right) \tag{50}$$

Now we can compare signal to noise ratios for the uniform and hanning type weighting functions, deviding (47) over (50)

$$\left(\frac{S}{N}\right)_{uni} \div \left(\frac{S}{N}\right)_{han} = \frac{\sqrt{\frac{2}{3}}\,(1+\frac{1}{6\,M})}{\sqrt{\frac{18}{19}}\,(1+\frac{1}{38\,M})} \simeq 0.84 \tag{51}$$

Having averaged overlapped FFT's , signal to noise ratio for uniform weighting function is 0.84 **of for hanning one**

To compare the signal to noise ratios of hanning and uniform weighting functions with ovelapping on time axis and frequency channels averaging we have to multiply equations (34) and (51). In a result we can came to the following conclusion:

Having averaged frequency channels of overlapped FFT's , signal to noise ratio for uniform weighting function is better than for hanning one in $\simeq 1.17$ **times**

The final result is formed in table 1.

Table 1: Signal to Noise Ratios

		No overlapping	overlapping
Uniform	One channel	$1.0\sqrt{M}$	$\sqrt{\frac{2}{3}}\sqrt{2\,M} \simeq 0.82\sqrt{2\,M}$
	L channels	$1.0\sqrt{M\,L}$	$\sqrt{\frac{2}{3}}\sqrt{2\,M\,L} \simeq 0.82\sqrt{2\,M\,L}$
Hanning	One channel	$1.0\sqrt{M}$	$\sqrt{\frac{18}{19}}\sqrt{2\,M} \simeq 0.97\sqrt{2\,M}$
	L channels	$\sqrt{\frac{18}{35}}\sqrt{M\,L} \simeq 0.72\sqrt{M\,L}$	$\simeq 0.7\sqrt{2\,M\,L}$
$\frac{Uniform}{Hanning}$	One channel	1.0	0.84
	L channels	1.39	1.17

Comparing overlapping with no overlapping we consider a constant integration time. That is why there is a double number of averaging FFTs - $2\,M$ in the last column of the table.