Separation of effects of main dish and subreflector at the holography measurements.

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Abstract

The holography measurements give the phase error distribution at the antenna aperture. This distribution is a result of both main dish and subreflector errors.The separation of these two source of errors could help to address directly to the source of errors. A rotation of the subreflector may help at this separation. The four possible ways to use the rotation are analyzed. They are:

1.Least square fitting using more than two rotations.
2.Fitting polynoms to the difference rotated-non rotated subreflector.
3.Integration of the difference.
4.Fourier analysis

The contribution of the main dish and subreflector to errors at the dish aperture can be separated using the holography measurement with rotation of the subreflector *only having had additional appriori information about the searching functions.* This information is the value of the searching function along a given radius (method of integration of the difference measurement) or the mean value of the searching function along the circles of all radiuses (method of the Fourier analysis).
Using more than one rotation of the subreflector can not exclude the requirement of the appriori information.

1 The approach to the problem

The holography measurements of the phase error at the antenna aperture at polar coordinate system can be represented by the following equation:

$$\phi(r,\theta) = \varphi_{main}(r,\theta) + \varphi_{subr}(r,\theta + \Delta\theta_i) \tag{1}$$

where $\varphi_{main}(r,\theta)$ is the main dish contribution to the phase error;
$\varphi_{subr}(r,\theta)$ is the subreflector contribution to the phase error;
$\Delta\theta_i$ is the rotation value of the subreflector;
i = 1,2,3...N; N is number of rotations.

The problem which we need to solve is:
Having the measurement described by the equation 1 restore the functions $\varphi_{main}(r,\theta)$ and $\varphi_{subr}(r,\theta)$. The number and values of the subreflector rotations can be chosen arbitrary.

Having subtracted the pair of equations 1 with different i we can come to the following equations:

$$\phi(r,\theta;i=k)-\phi(r,\theta;i=l)=\varphi_{subr}(r,\theta+\Delta\theta_k)-\varphi_{subr}(r,\theta+\Delta\theta_l) \tag{2}$$

$$\phi(r,\theta-\Delta\theta_k;i=k)-\phi(r,\theta-\Delta\theta_l;i=l)=\varphi_{main}(r,\theta-\Delta\theta_k)-\varphi_{main}(r,\theta-\Delta\theta_l) \tag{3}$$

The measurements (the left part of the equation 2) at the equation(2) represent the subreflector contribution.
The measurements (the left part of the equation 3) at the equation 3 represent the main dish contribution.

In principle the problem of the separation is solved using equations 2 and/or equations 3. But actually there are problems on the way.

2 The least square method (as I understand it).

Let's consider the least square method of separation. This method was offered by Craig Walker. Divide each circle at the polar coordinate system by **n** identical sector. Then we have **2n** unknown values on the circle at the equation 1 - **n** for $\varphi_{main}(r,\theta)$ and **n** for $\varphi_{subr}(r,\theta)$. If we have the measurements (equations 1) for three different subreflector rotations then we have **3n** equations for the **2n** unknown values. So it looks like we have enough equations to find the solution for both $\varphi_{main}(r,\theta)$ and for $\varphi_{subr}(r,\theta)$. But the question is whether the all **3n** equations are independent.
To answer on this question let's consider equation (2) or equation (3) instead of equation (1). Then we can say that there are **2n** (the third difference is linear combination of the first two) equations for the **n** unknown values. Again it looks like there are more equations than number of variables. But it is clear that the value at the initial angle can not be found because any addition to its value will be subtracted. **Therefore the number of independent equations is less than n and least square method can not be used**.

3 Fitting polynoms to the difference measurement

Now let's consider another modification of application of the least square method. This method was offered by Barry Clark. Lets represent the function $\varphi_{subr}(r,\theta)$ by the polynom at the Cartesian coordinate system **X,Y**:

$$\varphi(r,\theta)=\sum_{i+k\leq N} a_{ik}X^iY^k \tag{4}$$

Let's use the polynom representation of the function $\varphi_{subr}(r,\theta)$ for the special rotation at $\Delta\theta_k=0$ and $\Delta\theta_l=180$degree. Rotation by 180 degrees is equivalent to changing the sign near X and Y. So if we use the polynomial representation then the coefficients a_{ik} for $i+k=even$ will be eliminated at the equation (2). That means that the coefficients a_{ik} with $i+k=even$ are not available from the measurements and can not be derived. Such coefficients are for instance: $a_{00},a_{11},a_{02},a_{20}...$

If we use 90 degrees rotation instead of 180 degrees then the coefficients with even first index will be eliminated but coefficients with odd first index will be not. In particular the coefficient a_{11} will be available but the coefficients a_{02},a_{20} still not. It is simple to prove that the coefficients a_{02},a_{20} are not available using any value of the sub-reflector rotation. Probably (definitely) there are many coefficients which will be eliminated using any rotation.
But because the coefficients a_{00},a_{02},a_{20} are not available from the difference measurement the method of polynom fitting can not be used for restoration of the main dish and subreflecor contribution to the holography measurement.

4 Using the Fourier analysis.

Let's rewrite the equation 2 marking the left side of the equation as $DIFF(r,\theta)$ the result of measurement, and simplifying the right side:

$$DIFF(r,\theta) = \varphi(r,\theta) - \varphi(r,\theta+\Delta\theta) \tag{5}$$

where $\Delta\theta$ is the the rotation of the subreflecor at the second measurement relatively the first one;
r is the radius of the given circle.

The both functions $DIFF(r,\theta)$ and $\varphi(r,\theta)$ are periodic by θ with period 2π. Therefore the both functions can be represented by the Fourier series.

$$\varphi(r,\theta) = \sum_{n=0}^{\infty} C_n(r)\exp(jn\theta)$$
$$C_n(r) = \frac{1}{2\pi}\int_0^{2\pi} \varphi(r,\theta)\exp(-jn\theta)\,d\theta \tag{6}$$

$$DIFF(r,\theta) = \sum_{n=0}^{\infty} D_n(r)\exp(jn\theta)$$
$$D_n(r) = \frac{1}{2\pi}\int_0^{2\pi} DIFF(r,\theta)\exp(-jn\theta)\,d\theta \tag{7}$$

Substituting (6) and (7) into (5) and comparing the relevant coefficients we can find the Fourier coefficients of the searching function $\varphi(r,\theta)$ through the Fourier coefficients of measured function $DIFF(r,\theta)$:

$$C_n(r) = \frac{D_n(r)}{1-\exp(jn\Delta\theta)} \tag{8}$$

The equation (8) can be used to find coefficients $C_n(r)$ for any n except $n = 0$ because $D_n(r) = 0$ for any r and the denominator at the equation (8) is equal zero also if $n = 0$. So the equation (6) can be rewritten as:

$$\varphi(r,\theta) = C_0(r) + \sum_{n=1}^{\infty} C_n(r)\exp(jn\theta)$$
$$C_n(r) = \frac{1}{2\pi}\int_0^{2\pi} \varphi(r,\theta)\exp(-jn\theta)\,d\theta; n \geq 1 \tag{9}$$

Thus we can reconstruct the function $\varphi(r,\theta)$ from the measurement of the difference function with accuracy of constant (on each circle) $C_0(r)$. The constant $C_0(r)$ can not be found from the given measurement of the difference function $DIFF(r,\theta)$. The constant $C_0(r)$ is the mean value of the searching function $\varphi(r,\theta)$ along the circle of radius r.
If we can derive the constant $C_0(r)$ from somewhere we can restore the searching function $\varphi(r,\theta)$ from the measurements of the difference function $DIFF(r,\theta)$. If not, the searching function $\varphi(r,\theta)$ can not be restored even if we have the measurements of the difference function $DIFF(r,\theta)$ for more than one rotations.

5 Integration of the difference measurement.

Looking at the equation of the difference measurement (eqn # 2) we can say that if we have even only one rotation measurement and know the initial value of the unknown function $\varphi_{subr}(r,\theta)$ along

any radius, we can restore the all function integrating the difference. Indeed, if we know the values of $\varphi_{subr}(r, \theta = 0)$ and all differences with the step $\Delta\theta$, then $\varphi_{subr}(r, \theta = \Delta\theta)$ can be found as the sum of the initial value and the measured first difference; $\varphi_{subr}(r, \theta = 2\Delta\theta)$ can be found as the sum of the previously found value and the measured second difference and so on.
If we know the initial values of $\varphi_{subr}(r, \theta = 0)$ along a radius and all differences with the step $\Delta\theta$, then the function $\varphi_{subr}(r, \theta)$ can be found with the step of angle $\Delta\theta$. It is clear that the only rotation of the subreflector should be small to have enough resolution at the edge of the aperture and therefore the number of steps covered the whole circle will be big. For example the number of steps will be 360 for the step value 1degree. Because the current value of the function is found as the previous value plus the measured difference the noise of the solution will be accumulated as $\sqrt{number\ of\ steps}$. In particular the last value of the restored function for step=1degree will have the rms of noise $\sqrt{360} \sim 20$ times bigger than the first value.
The noise problem can be partially solved if we carry out several rotation of the subreflector say 180,90,45, 22,11,6,3,2,1. In this case the noise will be increased at the worst case by $\sqrt{9} = 3$ times.

6 Conclusion

The contribution of the main dish and subreflector to errors at the dish aperture can be separated using the holography measurement with rotation of the subreflector *only having had additional appriori information about the searching functions.* This information is the value of the searching function along a given radius (method of integration of the difference measurement) or the mean value of the searching function along the circles of all radiuses (method of the Fourier analysis).
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