VLBA TEST MEMO 61

Pointing Improvements Using Rail Height Information

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1. Introduction.

Examination of pointing residuals for some VLBA antennas, especially Los Alamos, shows the obvious presence of systematic variations with short periods in azimuth. The current pointing equation does now allow for any variations with azimuth faster than 2θ. Optical level rail height measurements show that the rails do indeed have variations that are faster than this and those measurements indicate that LA has larger than normal variations.

I first tried to approach this problem by simply fitting the pointing offsets to $cos(N\theta)$ and $sin(N\theta)$ terms, much as we have been doing for the 2θ terms. This met with some success. But the number of terms gets large fast. For each N, six terms are required: the $cos(N\theta)$ and $sin(N\theta)$) terms for the elevation offset, for azimuth offsets that vary with $cos(El)$, and for azimuth offsets that vary with $sin(El)$.

To reduce the number of terms required, and to relate the fit results to actual physical effects, I have implemented two ways of using the rail height in the pointing analysis program *PTANAL*. The first is to fit for fourier coefficients of the rail height. The second is to utilize actual measurements of the rail height made with an optical level. This memo describes the effect of rail height on the pointing and shows how the rail height information can be used to improve pointing. Initial results using rail height are presented. The results are sufficiently encouraging that I recommend that we implement use of rail height as part of the regular pointing equation. Some recommendations are made on how to do this to maintain maximum flexibility.

This work builds on an email of 1998 October 14 from Barry Clark in which he presented a useful parameterization of the effect of rail height on pointing. It also utilizes results of a computer analysis of the VLBA base done by Jon Thunborg in early 1996, which languished for too long on my desk.

2. Equations.

Consider the response of the pointing to lifting a wheel. To first order, the VLBA antenna structure can be thought of as two triangles, one on each side of the elevation axel. At the two bottom corners of the triangle are wheels. At the top corner is the elevation axel support. When a wheel is lifted, the triangle can be thought of as rotating about the other wheel. This has three effects important to pointing:

- The end of elevation axel will be lifted, which causes an offset in azimuth pointing that scales with $\sin(El)$. This is like an axis non-perpendicularity term. For a lift of a front wheel of h mm, parameterize the pointing offset as $\Delta Az \times \cos(El) = \text{azoff} = a \times h \times \sin(El)$

- The end of the elevation axis will be pushed away from the raised wheel as the support structure on that side of the antenna rotates. Parameterize this pointing offset for a front wheel as $\Delta Az \times \cos(El) = \text{azoff} = b \times h \times \cos(El)$

- The elevation encoder is mounted on one of the support triangles so when it rotates, the pointing changes because the servo trys to go to a particular indicated position. Parameterize this pointing offset as $\Delta El = \text{eloff} = c \times h$

In principle, we need a set of parameters for each wheel. However the side-to-side symmetry of the antenna allows us to use the same a and b for both front wheels or both back wheels, although the equations must contain a sign flip since lifting one wheel will push the pointing in the opposite direction from lifting the corresponding wheel on the other side. However the large (2.13m) axis offset gives a front-to-back asymmetry that forces use of different values for rear wheels than for front wheels. Adopt the convention that a and b are for front wheels and a corresponding d and e are for the rear wheels.

Correspondingly, assume that c is for front wheels and f is for rear wheels. However, the fact that the encoder is on only one side forces us to have separate parameter for each side. Call them c_r, c_l, f_r, and f_l.

The antenna base is square. The wheels are spaced at 90 degree intervals and the two front ones are offset 45 degrees on either side of the pointing direction.

The above can be used to specify the pointing offsets due to rail height variations:

$$
\begin{aligned}
\text{azoff} &= a \times \sin(El) \times [h(Az_{+45}) - h(Az_{-45})] + \\
&\quad d \times \sin(El) \times [h(Az_{+135}) - h(Az_{-135})] - \\
&\quad b \times \cos(El) \times [h(Az_{+45}) - h(Az_{-45})] + \\
&\quad e \times \cos(El) \times [h(Az_{+135}) - h(Az_{-135})] \qquad (1) \\
\text{eloff} &= f_r h(Az_{+135}) + f_l h(Az_{-135}) - c_r h(Az_{+45}) - c_l h(Az_{-45}) \qquad (2)
\end{aligned}
$$

It is possible to find relationships between many of the parameters based on geometric arguments. Some of these are based on requiring that simple situations, like an overall tilt, have the expected offsets.

- At the horizon, a tilt should give azoff = 0. At the horizon, $\sin El = 0$ and, for a pure tilt, $h(Az_{+45}) - h(Az_{-45}) = h(Az_{+135}) - h(Az_{-135})$. Therefore we require that $e = b$.

- The elevation encoder is on the right side of the antenna. To first order, this means that the elevation pointing will only be sensitive to motions of the right side wheels and $f_l = c_l = 0$. But Jon Thunborg's analysis does give small, but non-zero values for these terms.

- Lifting a front wheel will rotate the support structure about the corresponding rear wheel by the same amount that lifting a rear wheel by an equal amount will rotate it about the front wheel. It is just the rotation that matters for elevation pointing, so $c_r = f_r$ Also assume, to the extent that they are not zero, that $c_l = f_l$.

- For a pure tilt, the maximum azoff near the zenith must be equal to the maximum eloff, although those two maxima are reached at different azimuths. Using $h(Az_{+45}) - h(Az_{-45}) = h(Az_{+135}) - h(Az_{-135})$ for a tilt and then using the fact that the maximum difference in height between any two adjacent wheels is independent of wheel pair, we get $a + d = c_r$.

- If the pointing analysis program is to fit for both rail heights and the parameters, some more constraints are needed. It is possible to estimate the ratio of a and d based on the geometry of the antennas — specifically on the distance between the wheels on a side (10.78m) and the position of the elevation axis along the line between the wheels (2.13m from the center). Utilizing the previous constraint, we get $a = 0.70 \times c_r$ and $d = 0.30 \times c_r$.

- If fitting for the parameters and the rail heights, one must fix the apportionment of scale between rail height and the parameters. Calculating the angle by which the support structure on one side is rotated by lifting a wheel is relatively easy, giving $c_r = 0.001/10.78 = 0.32$ radians per mm.

3. Parameter estimates.

I have four sets of estimates of the coefficients relating pointing to rail height. One is from Barry, based on the simple description of the antenna as two triangles supporting the elevation axel. I have made estimates in a similar way, taking into account the constraints described above. The third set of estimates is from Jon Thunborg's computer analysis. Believing that the computer is always right, this is probably the best set. Seriously, Thunborg's analysis is a much more sophisticated analysis of the antenna than was performed by either Barry or myself. The final set is the result of attempts to treat some of the coefficients as free parameters in a pointing fit. Not all of Barry's values conform to all of the constraints described above. In particular, $a + d \neq c_r$. The analysis I have from Thunborg only includes the effects of lifting a front wheel, so the rear wheel terms must be deduced. The pointing fits depend on the quality of both the pointing data and the optical rail height measurements. But by using the measured rail height data, I can get good fits for a minimum set of parameters, which, when combined with the constraints derived earlier, specify the problem.

The values for the parameters relating pointing to rail height are given in Table 1.

There are some interesting facts to consider when dealing with the rail height in terms of fourier coefficients. The $4N\theta$ terms do not affect pointing because of the square antenna base. These terms simply move the antenna straight up and down. They would affect baseline measurements, but not pointing. The $\cos(El)$ azimuth terms in the pointing are the result of twisting of the antenna base when the the wheels do not remain on a plane. But all odd terms

TABLE 1
Rail Height — Pointing Coefficients

Coefficient	Clark Estimate	Walker Estimate	Thunborg Analysis	Fit Result
a	0.14	0.22	0.227	—
b	0.29	—	0.548	0.39
d	0.07	0.10	—	—
e	0.31	—	—	—
c_r	0.32	0.32	0.319	0.32
c_l	0.00	0.00	0.047	0.02
f_r	0.32	0.32	—	—
f_l	0.00	0.00	—	—

simply tilt the antenna and do not twist it. So the $\cos(El)$ azimuth terms are only sensitive to the $(4N-2)\theta$ terms.

4. Fitting methods and options.

I have implemented two rail height based fitting schemes in the pointing analysis program *PTANAL*. The first allows a fit for the parameters described above, plus fourier coefficients for the rail height. This required converting the program to use a non-linear fitting package, for which I ended up with *ODRPACK* from NIST. The other fitting scheme is based on external rail heights and just fits for the parameters described above. For the rail heights, I have used the results of optical level measurements provided by Bob Broilo. These measurements are made relative to bolt number — the 120 pairs of bolts that hold the rail have numbers written on them for this purpose. It was necessary to get the site techs to measure the azimuth of the antenna while a wheel was sitting over a known bolt in order to relate the bolt numbers to pointing azimuth.

Attempts to fit for the dependency parameters described above that relate rail height to pointing have met with mixed success. When combined with fitting for the fourier coefficients, there are 100% correlations between the some of the dependency parameters and some of the rail height fourier coefficients. Using the measured rail heights to fit for the coefficients was somewhat more productive. I got a "good" fit for the b, c_r, and c_l parameters as shown in the above table. The b parameter came out rather lower than Thunborg's estimate. The others are very close. By "good", I mean that the errors of the fit were small and that term was not highly correlated with others. Other parameters were related to the those three using the constraints described above. All of this is based on LA, which so far is the only antenna for which I know the azimuths of the rail height measurements, but which is also the antenna with the most significant, high N (as in $N\theta$) effects from rail irregularities.

5. Test results.

To test the use of the rail height data to improve pointing, a data set collected on 7 days over the months of March to May, 1998, was used. All hours of the day were represented except about

3 hours in mid morning. The pointing equation fit used both 1cm and 7mm data. The pointing fit utilized 324 measurements in RCP at 1cm, 314 in LCP at 1cm, 466 in RCP at 7mm and 480 in LCP at 7mm. This is one of the larger data sets that has been available.

To check the reasonableness of the fits of the fourier coefficients, the optical height data from LA were transformed to form the same coefficients. Actually, this was more than just an exercise to check consistency — it was important in checking my understanding of the information from the site on the relationship between bolt number and azimuth. Table 2 compares fourier coefficients for the rail height obtained from one of the pointing fits with coefficients derived from the optical level measurements. Both sources have measurement errors, but there is good agreement.

TABLE 2
Fitted and Measured Rail Height Fourier Coefficients

	Pointing Fit		Optical Data	
N as in $N\theta$	Amplitude (mm)	Phase	Amplitude (mm)	Phase
1	1.167	160.0	1.378	174.2
2	0.160	-108.9	0.207	-97.8
3	0.212	-107.5	0.232	-101.4
5	0.031	-74.2	0.054	-102.5
6	0.122	-179.2	0.132	-172.5
7	0.014	90.0	0.015	-12.6
9	0.038	116.8	0.036	114.9
10	0.066	-9.1	0.074	-26.1
11	0.081	159.5	0.054	137.8
13	0.054	-146.8	0.028	-138.8
14	0.047	-98.1	0.078	-98.4
15	0.035	-8.9	0.019	9.0
17	0.024	-164.6	0.026	-146.5
18	0.039	-99.8	0.063	-42.8
19	0.046	-62.8	0.043	-56.6

A variety of combinations of fit parameters were tried. First, for comparison, the results using the current pointing equation are shown in Figure 1, which shows the raw pointing data (effectively the offsets from the pointing equation assumed at the time of the observations) and the residuals from the best fit equation of the type currently in use. It is clear that Los Alamos has a problem with systematic variations in azimuth that cannot be described by the current equation.

The results of various variations on attempts to include rail height in the pointing are given in Table 3, which contains the RMS residuals after the various fits. Most fits were on LA using the March-May 1998 data. Only the 1cm and 7mm data were used in the fit. Only collimation offsets were determined for the other bands based on the residual from the newly fitted equation. As can be seen, any version of rail height fitting produces considerably better results than not using the rail height information. But the differences between the different schemes are not large. Plots of the residuals before and after the fit for one of the better solutions are shown in Figure 2.

Table 3 also gives results for some more recent data from September and October — this is

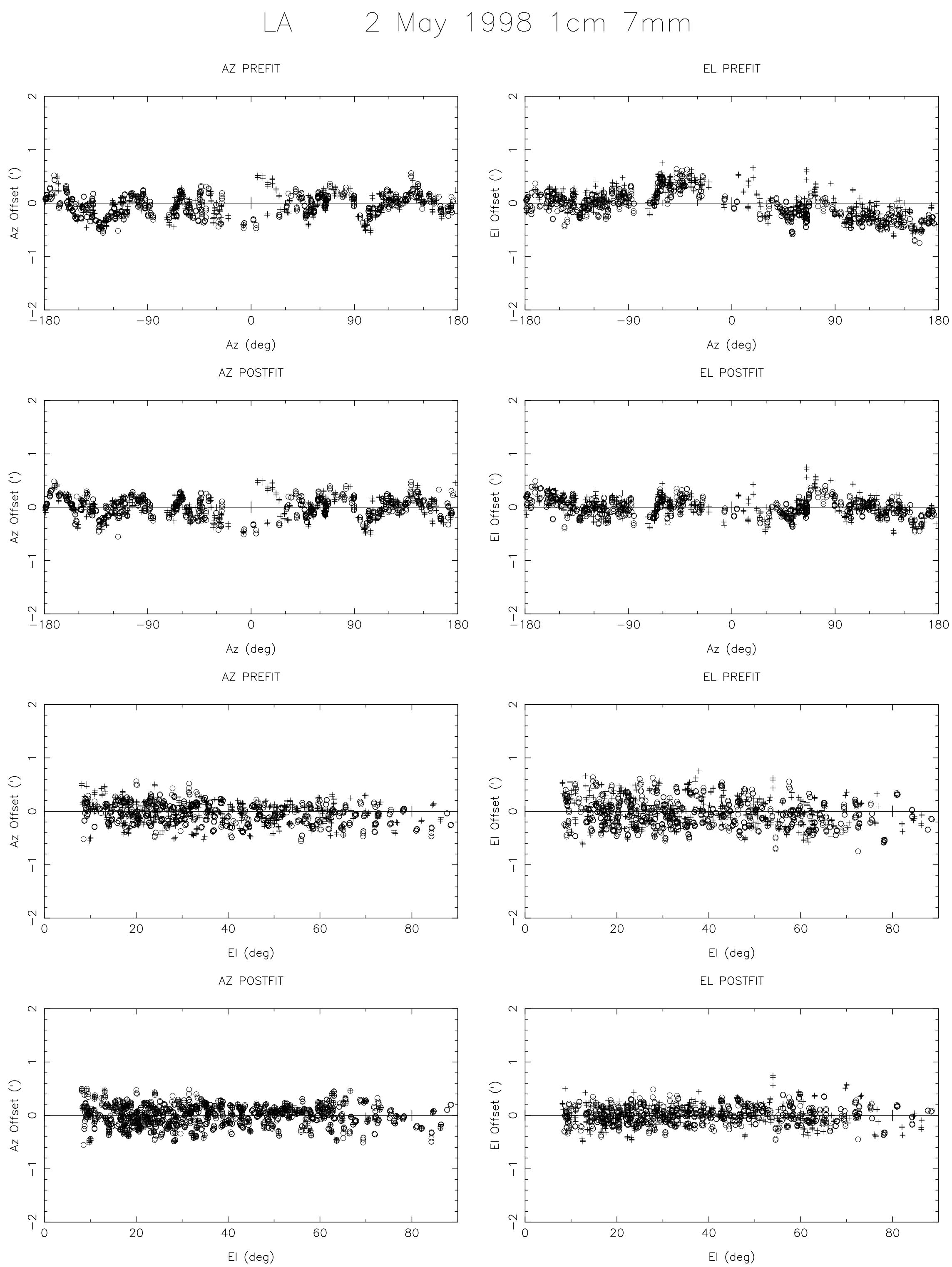

Fig. 1.— Pointing fit results for 1cm and 7mm data from March to May 1998 at the VLBA station at Los Alamos. The top and third rows of plots show the data as measured on the antenna. The second and bottom rows show the residuals after the fit for a new pointing equation. This figure shows results for the standard pointing equation currently in use.

TABLE 3
Los Alamos RMS Pointing Fit Residuals (Arcsec)

Fit Items		6cm	4cm	4cmsx	2cm	1cm	7mm
Old pointing equation (traditional fit).	Az:	15.0	14.7	12.8	9.6	12.1	10.1
	El:	19.7	15.5	18.1	7.5	11.1	9.2
Old pointing equation plus rail	Az:	15.2	12.1	8.5	8.7	7.8	8.4
measurements with fixed dependencies.	El:	17.2	12.1	17.8	5.6	8.1	6.5
Old equation without 2θ terms. Add	Az:	14.6	11.6	8.4	9.5	9.2	9.4
rail measurements with fixed dependencies.	El:	17.2	12.0	17.8	5.8	8.0	6.7
Old equation plus rail measurements and	Az:	15.0	11.6	9.3	8.7	7.7	7.2
fitted b.	El:	17.2	12.1	17.7	5.6	8.1	6.5
Old equation without tilt and 2θ.	Az:	13.4	10.4	9.6	8.0	7.8	7.0
Fit for rail height with fixed dependencies. $b = 0.55$.	El:	17.4	12.6	17.6	5.9	8.6	7.0
Old equation without tilt and 2θ.	Az:	13.7	10.7	9.9	8.3	7.9	6.9
Fit for rail height with fixed dependencies. $b = 0.39$.	El:	17.3	13.0	18.5	5.9	8.4	6.7
Old equation plus rail measurements with	Az:	14.9	11.4	9.1	8.6	7.7	7.2
fitted b, c_r, and c_l.	El:	17.1	12.2	17.9	5.7	8.0	6.5
Sept-Oct 1998 data. Old equation.	Az:	12.4	11.3	9.5	6.8	11.2	8.1
	El:	15.0	12.7	14.4	9.0	11.7	9.2
Sept-Oct 1998 data. Use equation from	Az:	10.7	6.8	7.5	5.6	8.8	5.6
March-May data, but fit for collimation offsets and b. Got $b = 0.41$.	El:	12.2	8.5	12.0	8.6	10.3	6.5
Sept-Oct 1998 data. Use equation from	Az:	9.8	5.6	6.5	5.7	8.1	5.3
March-May data, but fit for collimation offsets, tilt, and b. Got $b = 0.40$ Tilt: EW 0.07, NS -0.05.	El:	10.5	8.3	10.6	7.9	9.6	6.9
Sept-Oct 1998 data. Full fit for old	Az:	9.5	5.3	6.2	4.7	7.7	5.2
equation plus b, c_r, and c_l using rail measurements.	El:	10.3	8.4	10.6	7.3	9.4	6.7

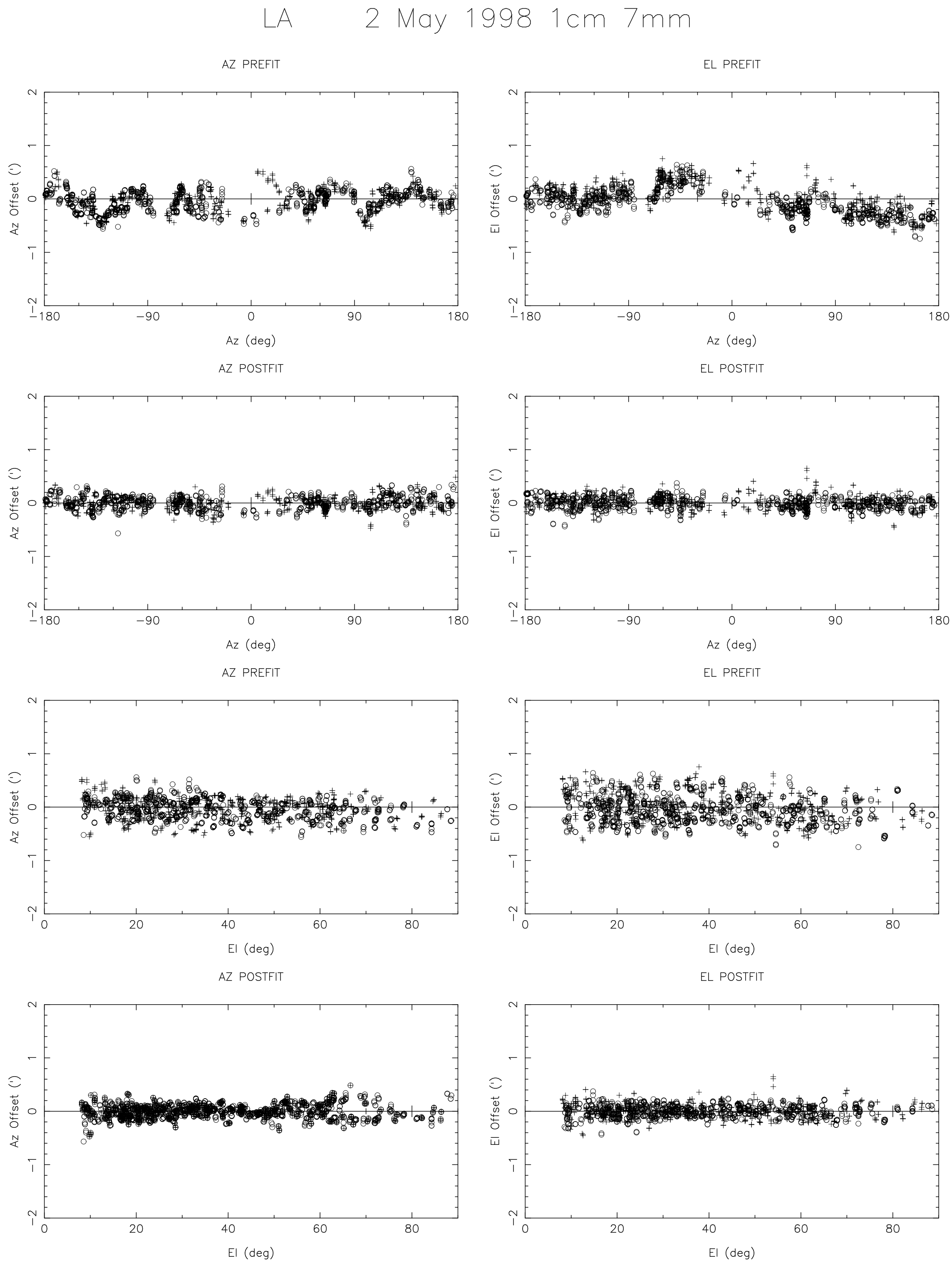

Fig. 2.— Pointing fit results for the same data as Figure 1, but including the measured rail height data. The parameters relating the pointing to rail height were free in the fit, as were additional tilt and 2θ terms on top of the rail height induced effects.

another multi-day data set of magnitude similar to the March-May data. This was an attempt to show the long term effectiveness of pointing based on rail height. For this data set, both use of the March-May pointing equation with rail heights and an entirely new fit gave roughly equivalent results that are slightly better, if anything, than the March-May results. At 1cm, the RMSs are a bit high. This seems to be the result of the inclusion of a day with higher than normal pointing noise — a day on which only 1cm was observed. But the conclusion is clear. The inclusion of rail height information improves pointing results over periods of at least months, and probably indefinitely.

The relationship between bolt number and azimuth is not yet available for antennas other than LA. To check the value of the rail height information, I ran an analysis of the Mar-May data for all antennas using a fit for the rail heights and for all normally-used terms in the old pointing equation. I did not attempt to fit for the dependencies of pointing on rail height and will await the azimuth information for the optical data to do so. I used the $b = 0.55$ theoretical value, although the LA fitted value might be better. The difference at LA was not large. In all cases, the postfit residuals improved. This is probably encouraging for the rail based pointing, but judgement should be reserved for when the optical measurements are available. The fit used here had 46 free parameters, compared to the usual 15, so it is not too surprising that the fit is a bit better. However even the site with the fewest data points, counting both polarizations and both frequencies used in the fit (1cm and 7mm), had nearly 1000 points and some sites had nearly 2000.

6. Recommendations.

The use of rail height information seems capable of providing significant improvements in the pointing model, especially on our worst antennas. I recommend that we implement such a capability at the stations. A reasonable way to do this, and the way that provides the most flexibility in how we use the capability, would be to add the parameters relating rail height to pointing offsets to the current list of pointing parameters and to provide for the use of a list of rail heights as a function of azimuth. In my opinion, the full list of parameters should be available. This allows for changes in our understanding of their relationships. It is much easier to set two parameters to be equal, for example, than to decide later that they shouldn't be equal and find that their equality is built into the system. Using the table of heights is the easiest way to use the data from the optical level measurements, and can be adapted to fitted results based on fourier coefficients simply by calculating such a table from the coefficients. A calculated table would not match a measured one because of the lack of $4N\theta$ terms, but it would work for pointing.

TABLE 4

Comparison of Old Equation and Rail Fit at All Antennas

Site and Item		6cm	4cm	4cmsx	2cm	1cm	7mm
SC Old Equation.	Az:	13.6	9.3	11.8	9.1	10.4	7.4
	El:	14.7	14.5	15.2	14.2	12.4	9.5
SC With Rail Fit	Az:	11.8	7.6	8.8	7.9	8.0	5.5
	El:	12.3	12.0	11.3	9.9	9.3	7.0
HN Old Equation.	Az:	10.7	8.9	6.3	6.5	8.2	7.2
	El:	12.7	10.4	20.0	8.5	8.4	7.9
HN With Rail Fit	Az:	8.6	5.7	4.2	5.1	5.3	5.1
	El:	11.0	8.4	18.3	6.4	6.7	6.3
NL Old Equation.	Az:	13.0	12.0	15.7	11.5	7.7	7.9
	El:	12.5	9.8	14.1	8.8	9.3	8.1
NL With Rail Fit	Az:	12.9	11.8	14.7	11.1	7.1	6.7
	El:	10.5	8.4	13.2	8.6	8.1	7.0
FD Old Equation.	Az:	15.3	10.6	10.6	5.9	7.8	6.7
	El:	10.0	7.4	30.6	6.5	9.1	7.0
FD With Rail Fit	Az:	14.7	11.4	10.1	5.7	7.2	5.8
	El:	8.9	6.1	28.0	4.2	7.6	4.8
LA Old Equation.	Az:	15.0	14.7	12.8	9.6	12.1	10.1
	El:	19.7	15.5	18.1	7.5	11.1	9.2
LA With Rail Fit	Az:	11.3	8.8	8.7	5.9	6.6	5.6
	El:	18.0	12.2	18.4	5.9	8.5	6.7
PT Old Equation.	Az:	13.3	10.6	11.5	9.8	9.7	8.9
	El:	17.1	12.9	15.1	16.8	11.8	9.7
PT With Rail Fit	Az:	12.4	8.7	9.8	8.3	7.9	7.5
	El:	17.9	11.7	15.0	13.7	10.3	8.4
KP Old Equation.	Az:	8.0	6.4	7.5	6.2	7.1	5.2
	El:	10.0	9.4	6.9	6.1	7.8	6.7
KP With Rail Fit	Az:	7.7	5.1	6.3	5.1	5.9	4.4
	El:	11.0	10.0	8.0	5.1	7.2	6.0
OV Old Equation.	Az:	11.8	9.3	7.2	7.2	7.6	6.9
	El:	10.5	8.5	7.9	6.5	8.5	9.3
OV With Rail Fit	Az:	11.1	7.9	5.8	6.3	6.9	6.5
	El:	9.2	7.9	7.2	5.5	8.0	8.6
BR Old Equation.	Az:	9.6	7.9	8.3	7.6	8.9	7.9
	El:	11.6	6.7	11.2	8.6	8.9	8.9
BR With Rail Fit	Az:	7.6	5.2	5.5	3.8	5.1	5.2
	El:	9.2	5.3	7.9	5.8	7.3	6.5
MK Old Equation.	Az:	11.0	7.6	7.9	6.7	6.5	6.3
	El:	7.6	7.8	10.2	5.4	6.9	7.8
MK With Rail Fit	Az:	9.7	5.5	7.7	6.5	5.0	5.1
	El:	8.6	6.7	9.0	5.7	6.2	6.9