Interferometry Basics

Andrea Isella
Caltech

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Outline

1. Interferometry principles:
   - *uv* plane and visibilities.
   - angular resolution.

2. Aperture Synthesis
   - synthesized beam.
   - characteristic angular scales.

3. Data calibration , i.e., correcting for the atmospheric and instrumental terms in the visibility (introduction to Lazio's and Perez's talks)

4. From visibilities to images (introduction to Carpenter's talk):
   - weighting function
   - deconvolution

From Sky Brightness to Visibility

1. An Interferometer measures the interference pattern produced by two apertures.
2. The interference pattern is directly related to the source brightness. In particular, for small fields of view the complex visibility, V(u,v), is the 2D Fourier transform of the brightness on the sky, T(x,y)

(van Cittert-Zernike theorem)

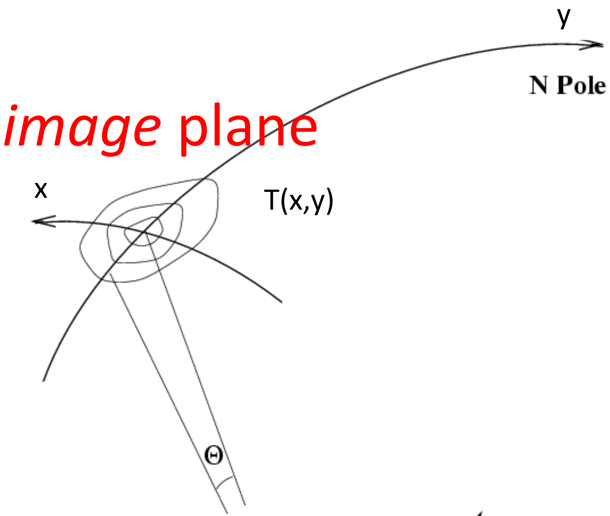
Fourier space/domain

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dxdy$$

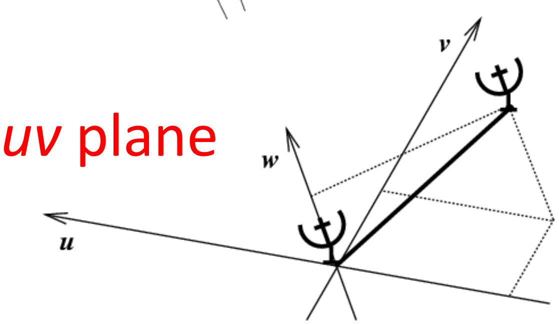
$$T(x,y) = \int\int V(u,v) e^{-2\pi i(ux+vy)} dudv$$

Image space/domain

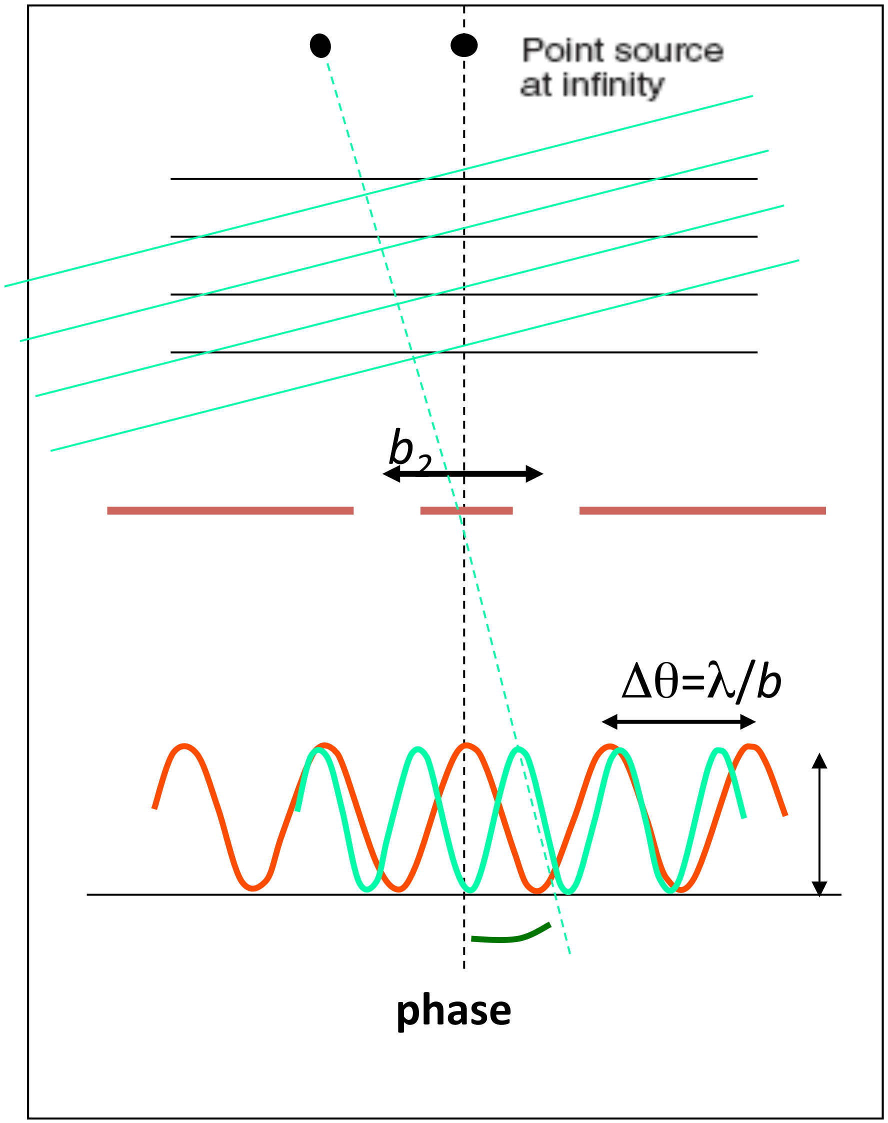
image plane

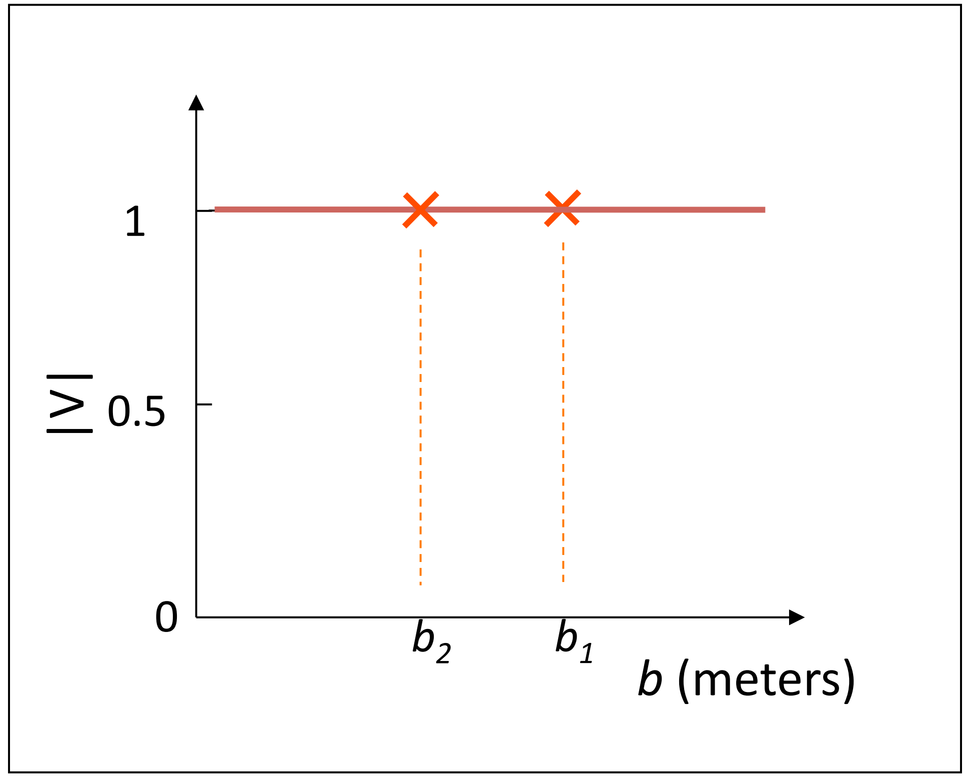


uv plane



Visibility and Sky Brightness



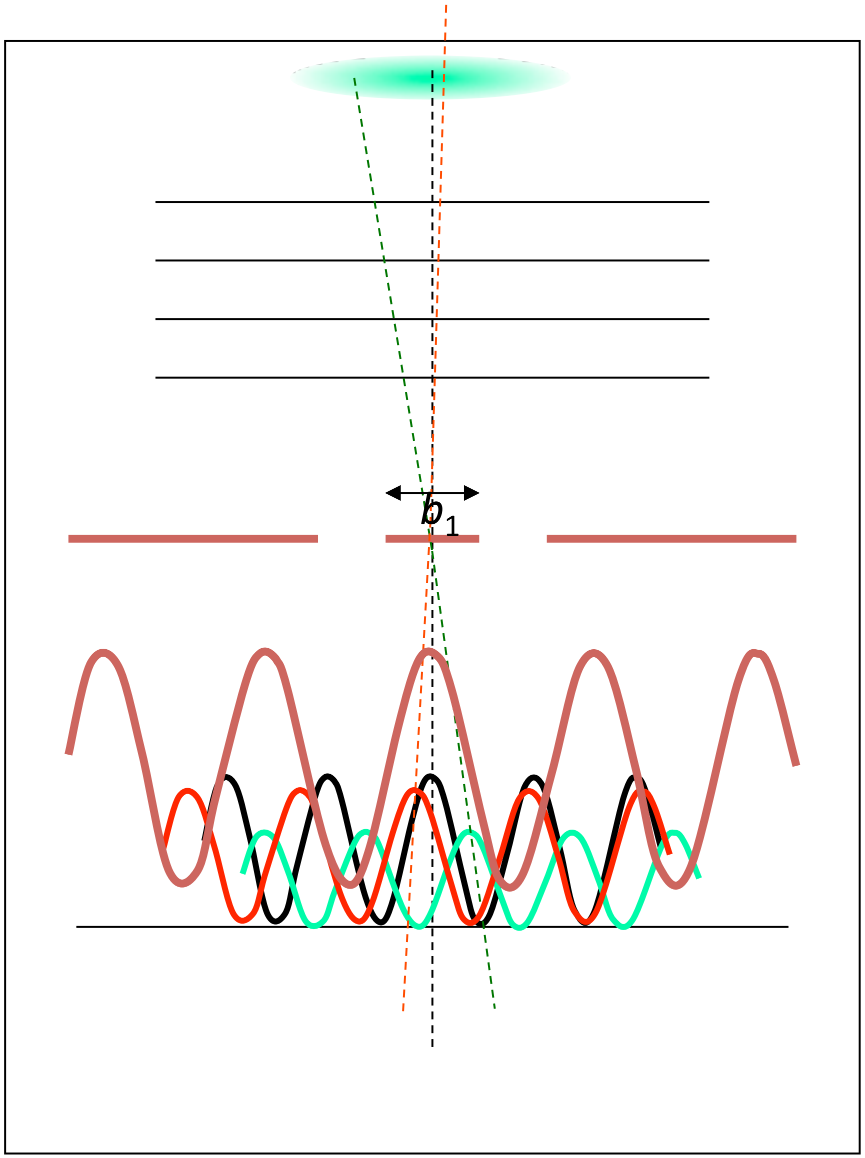


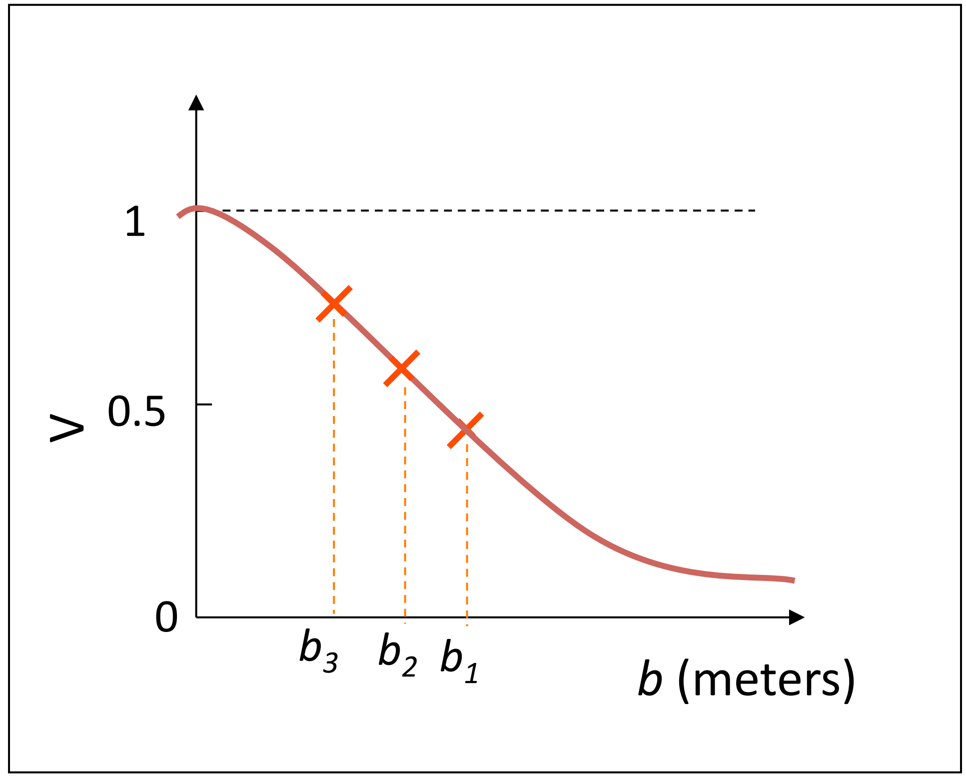
$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

The visibility is a complex quantity:

- amplitude tells "how much" of a certain frequency component
- phase tells "where" this component is located

Visibility and Sky Brightness



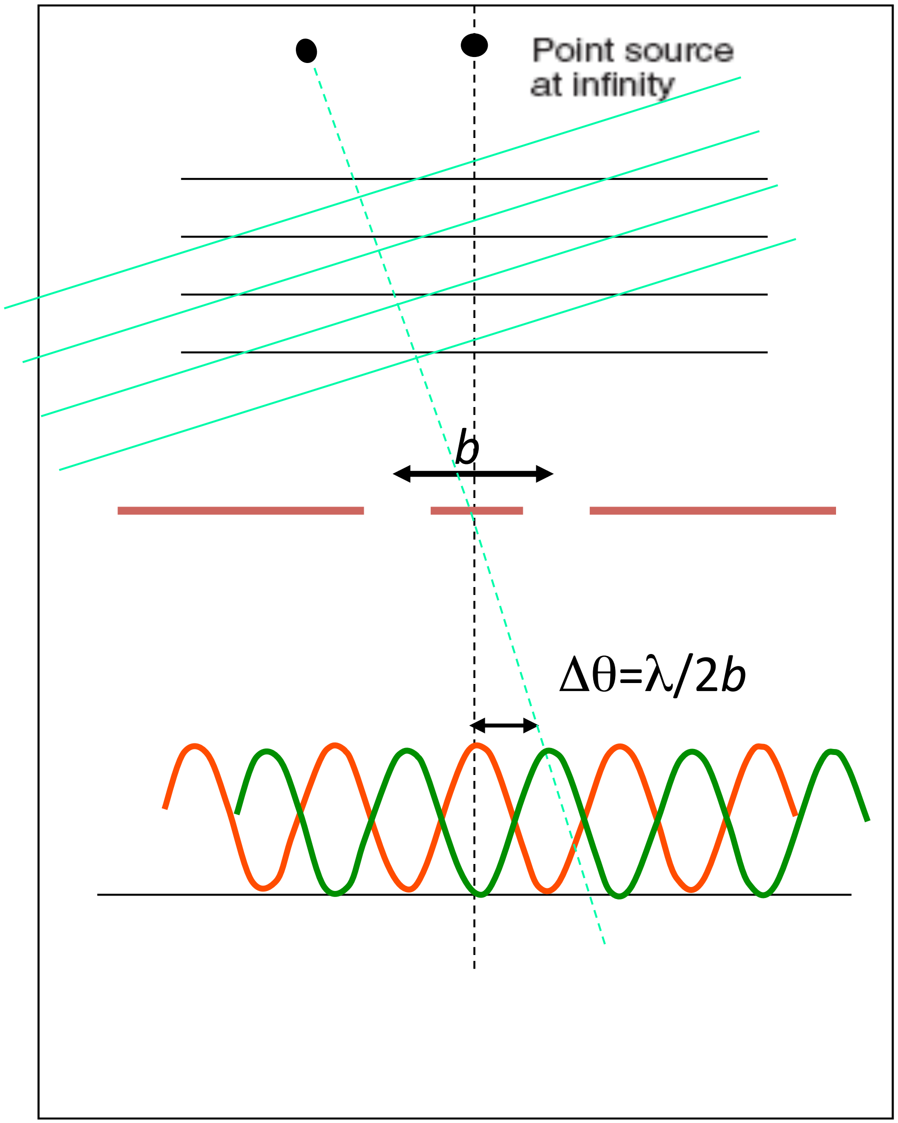


$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{Fringe\ Amplitude}{Average\ Intensity}$$

The visibility is a complex quantity:

- amplitude tells "how much" of a certain frequency component
- phase tells "where" this component is located

Angular resolution of an interferometer



A source is resolved when the visibility goes to zero on the longest baseline.

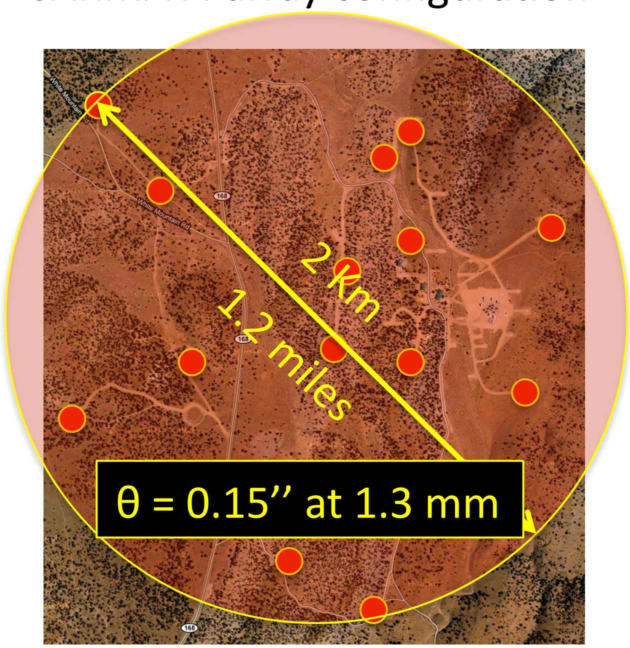
For example, in the case of a binary system, this corresponds to the angular separation of $\theta = \lambda/2B$.

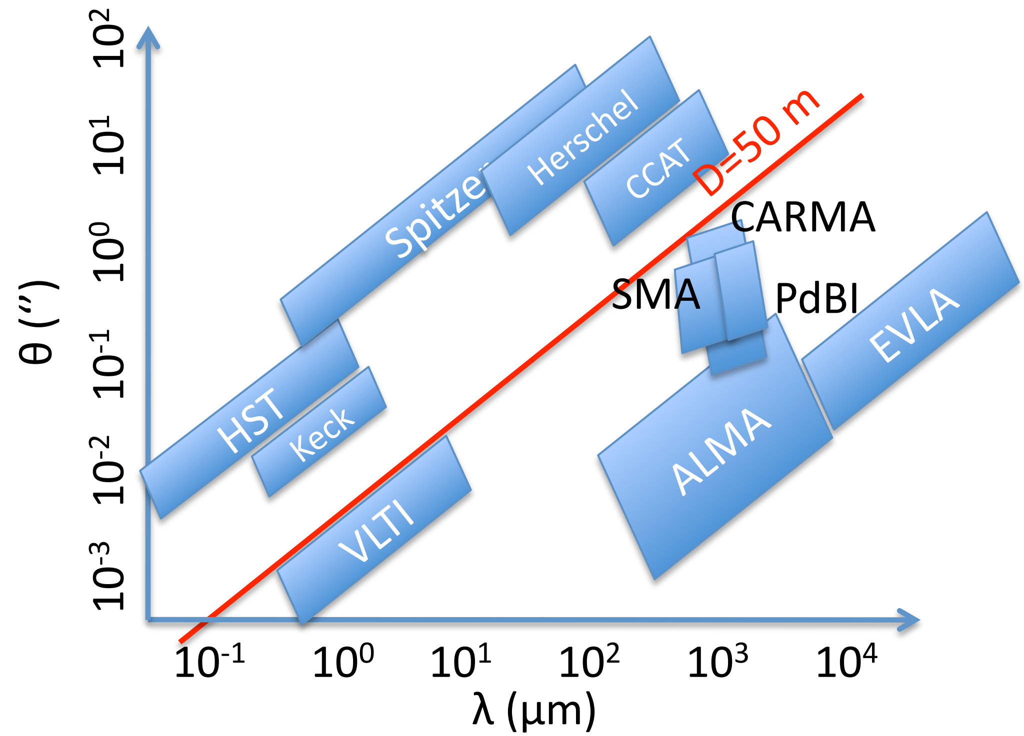
$$|V| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = 0$$

Need for high angular resolution

	Single telescope	Array of N telescopes
Angular resolution, θ	~ λ/D	$\sim \lambda/D_{max}$
Collecting area	$\sim D^2$	$\sim ND^2$

CARMA A-array configuration

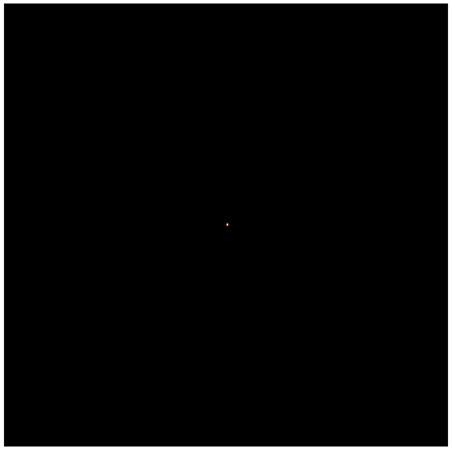




2D Fourier Transform Pairs

T(x,y) | |V(u,v)|

δ Function



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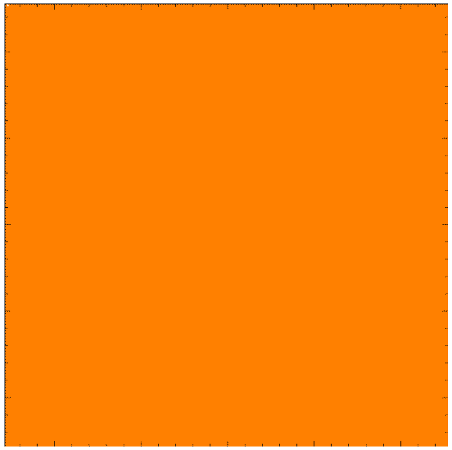
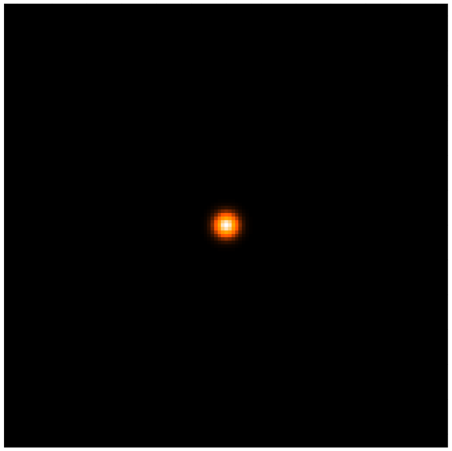


Constant

Gaussian



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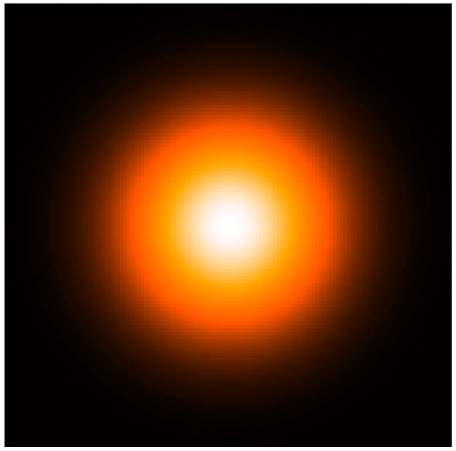


Gaussian

2D Fourier Transform Pairs

T(x,y)

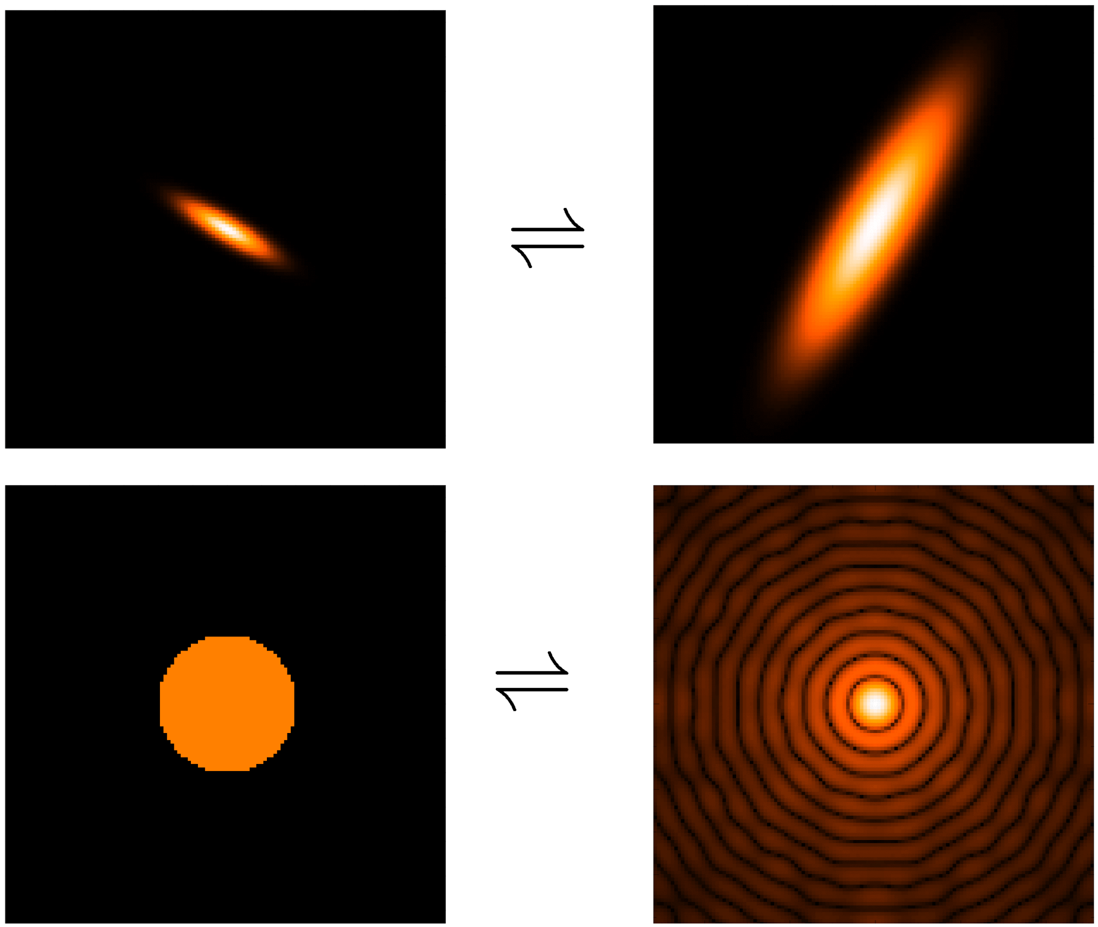
|V(u,v)|

elliptical Gaussian

elliptical Gaussian

Disk

Bessel

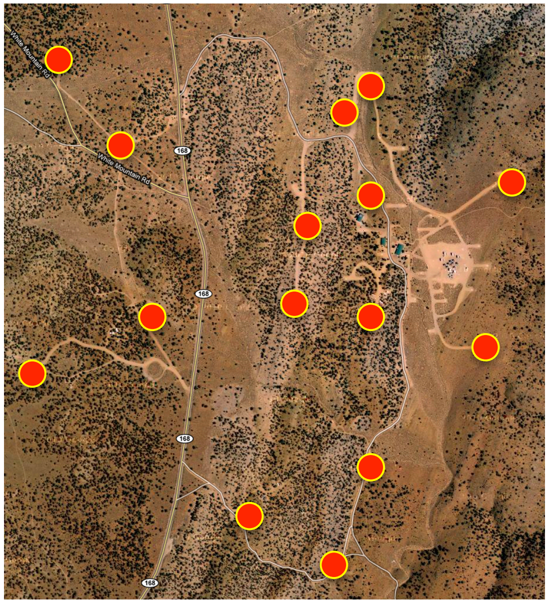


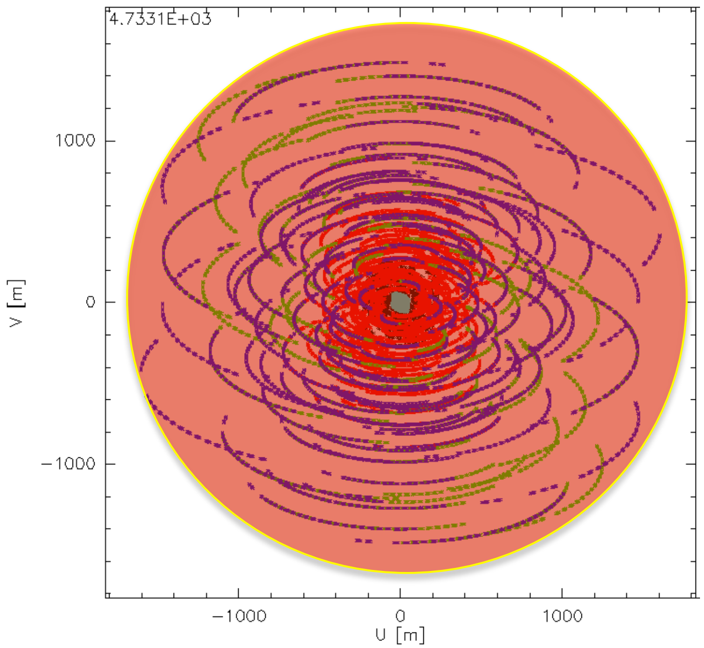
sharp edges result in many high spatial frequencies

Aperture Synthesis

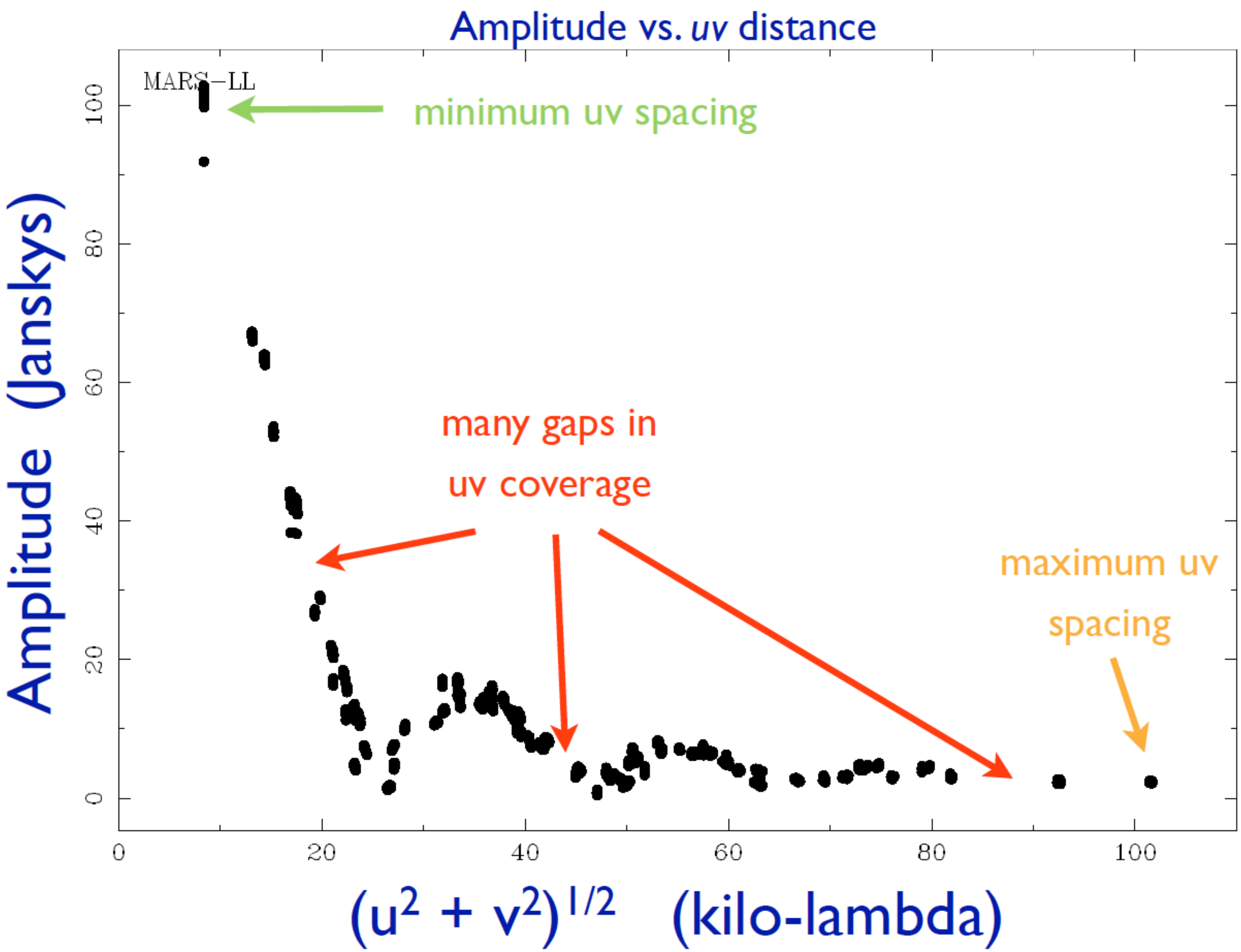
$$T(x,y) = \int\int V(u,v)e^{-2\pi i(ux+vy)}dudv$$

V(u,v) can be measured on a discrete number of points. A good image quality requires a good coverage of the *uv* plane. We can use the earth rotation to increase the *uv* coverage





CARMA Observations of Mars



Synthesized beam

Discrete sampling: $$T'(x,y) = \iint W(u,v)V(u,v)e^{2\pi i(uv+vy)}dudv$$

The **weighting function W(u,v)** is 0 where V is not sampled

T'(x,y) is FT of the product of W and V, which is the convolution of the FT of V and W:

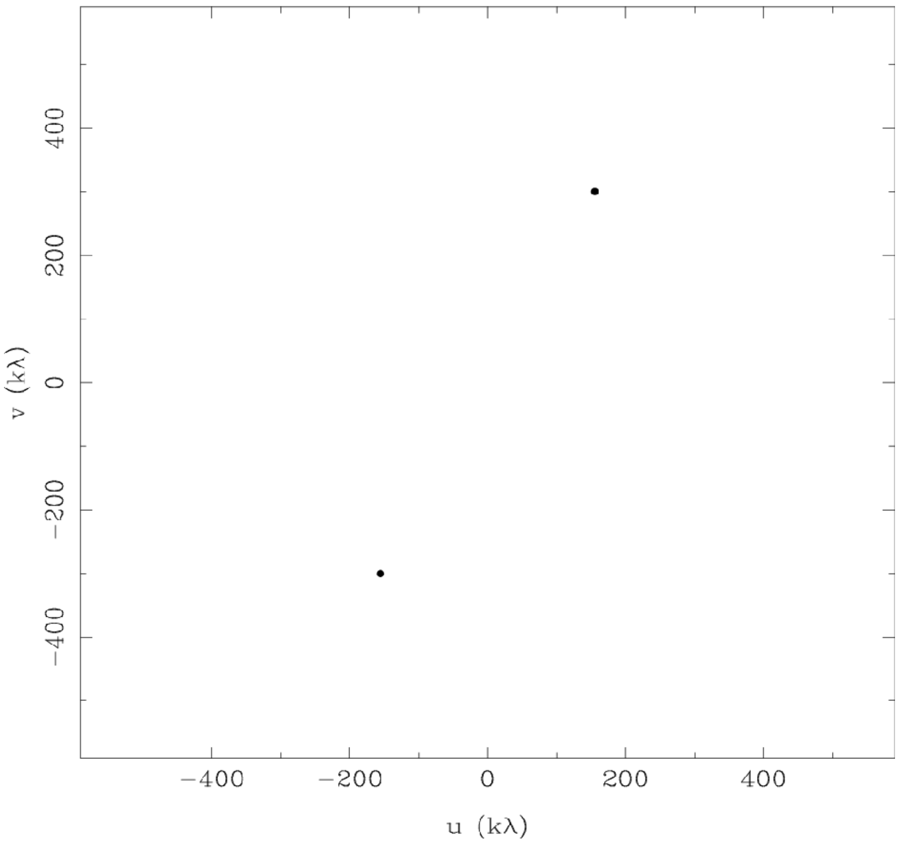
$$T'(x,y) = B(x,y) \otimes T(x,y)$$

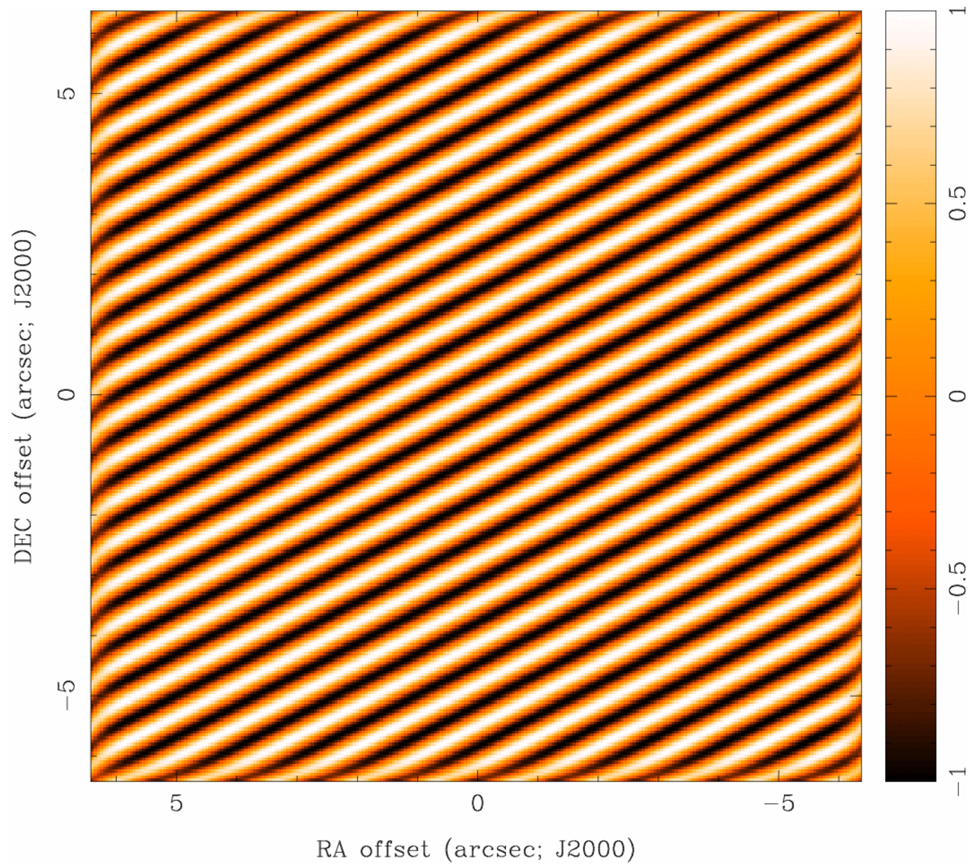
$$B(x,y) = \iint W(u,v)e^{2\pi i(uv+vy)}dudv$$

B(x,y) is the synthesized beam, analogous of the point-spread function in an optical telescope.

Effects of a sparse *uv* coverage

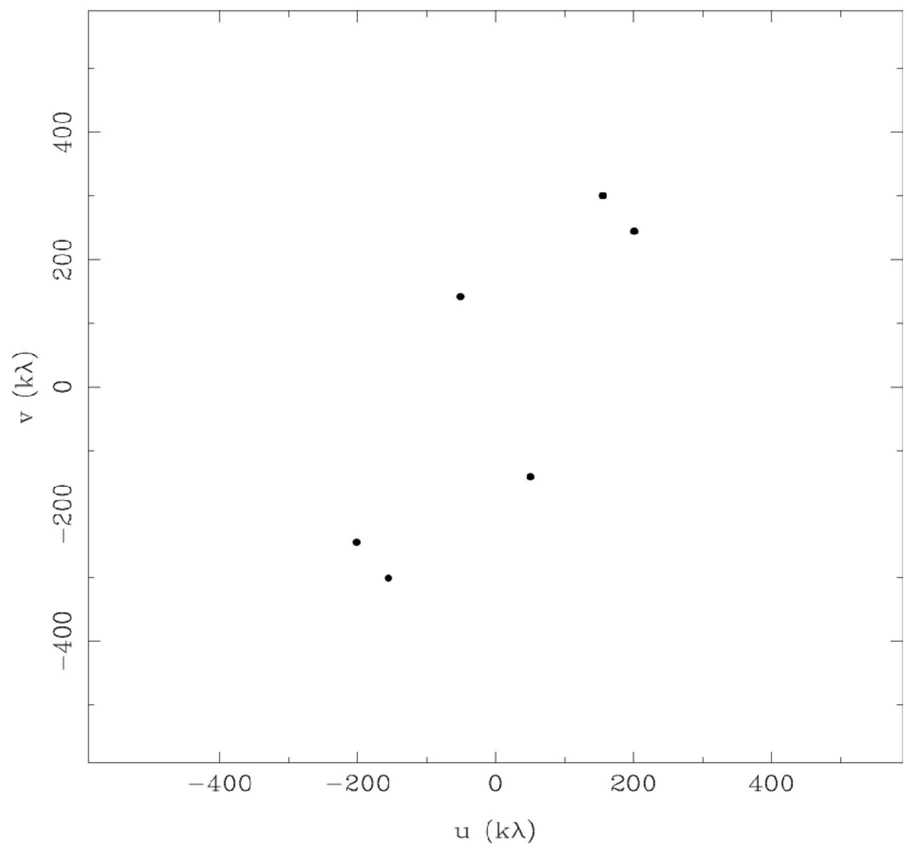
Synthesized Beam (i.e.,PSF) for 2 Antennas

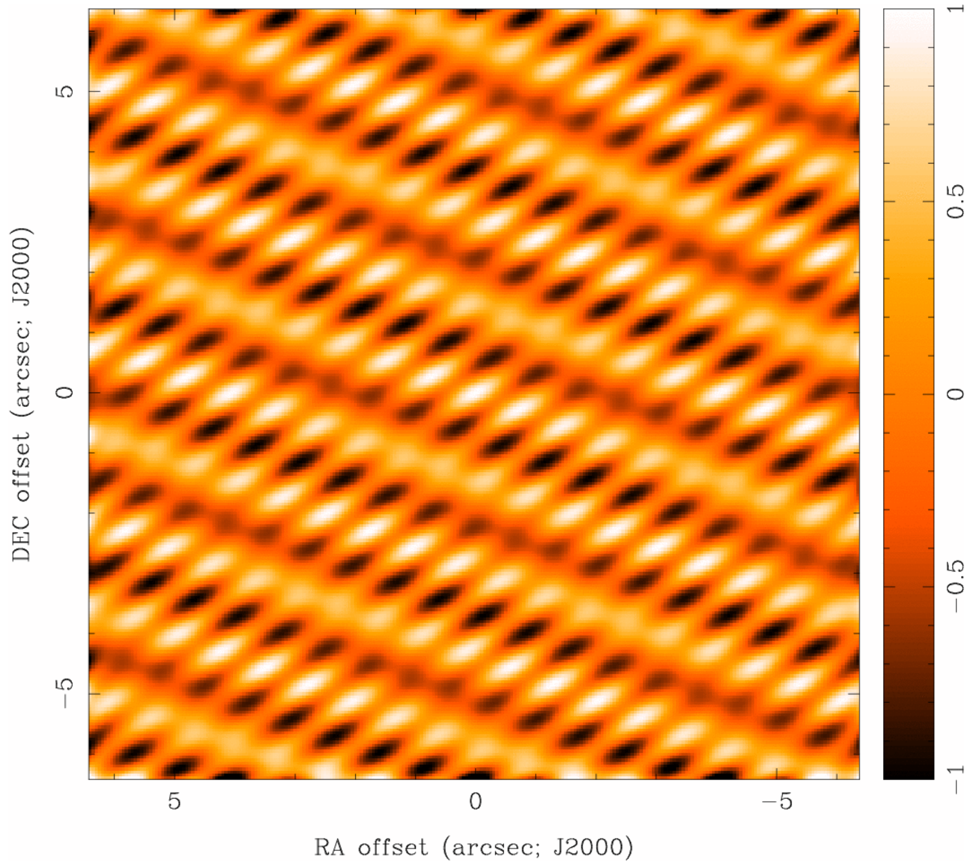




Effects of a sparse *uv* coverage

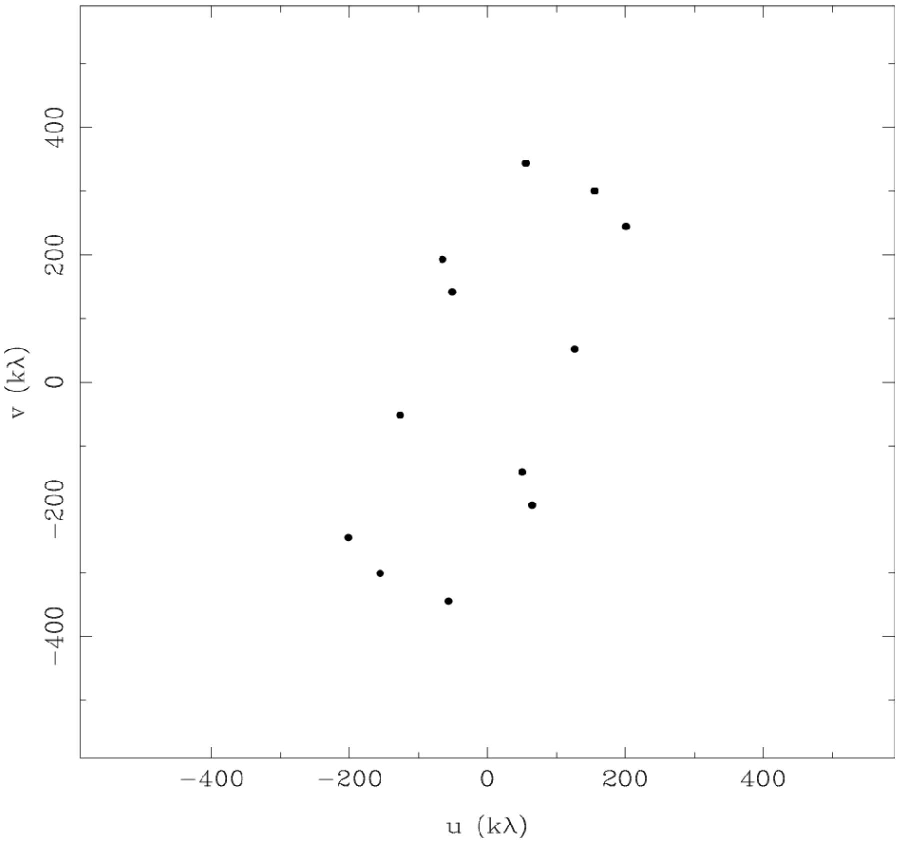
3 Antennas

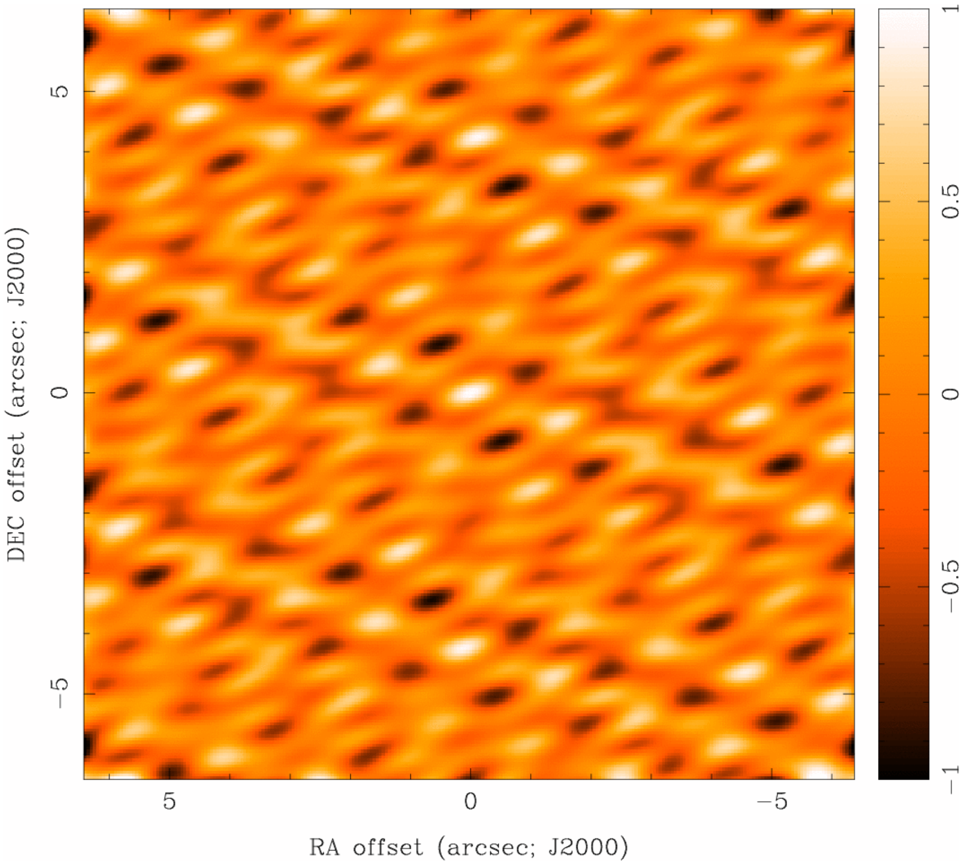




Effects of a sparse *uv* coverage

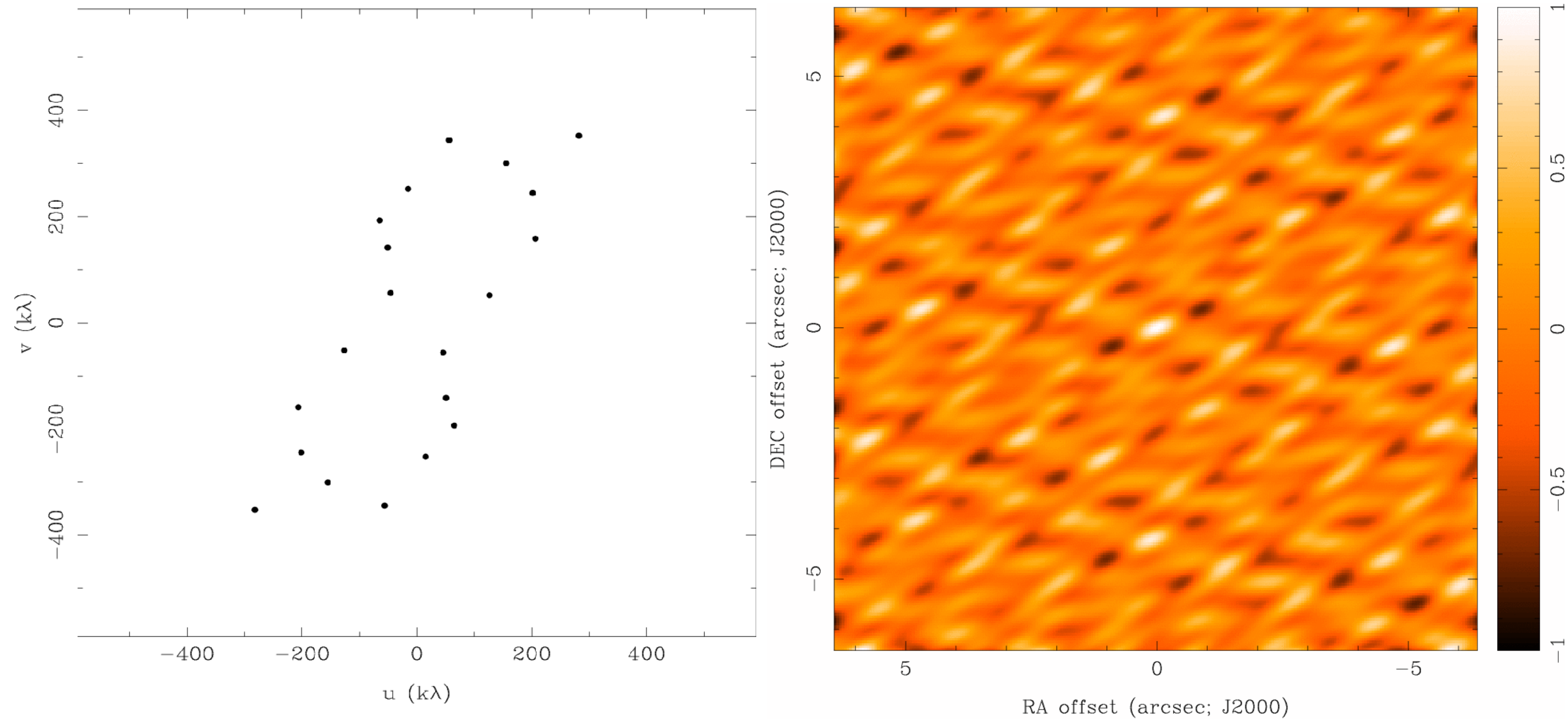
4 Antennas





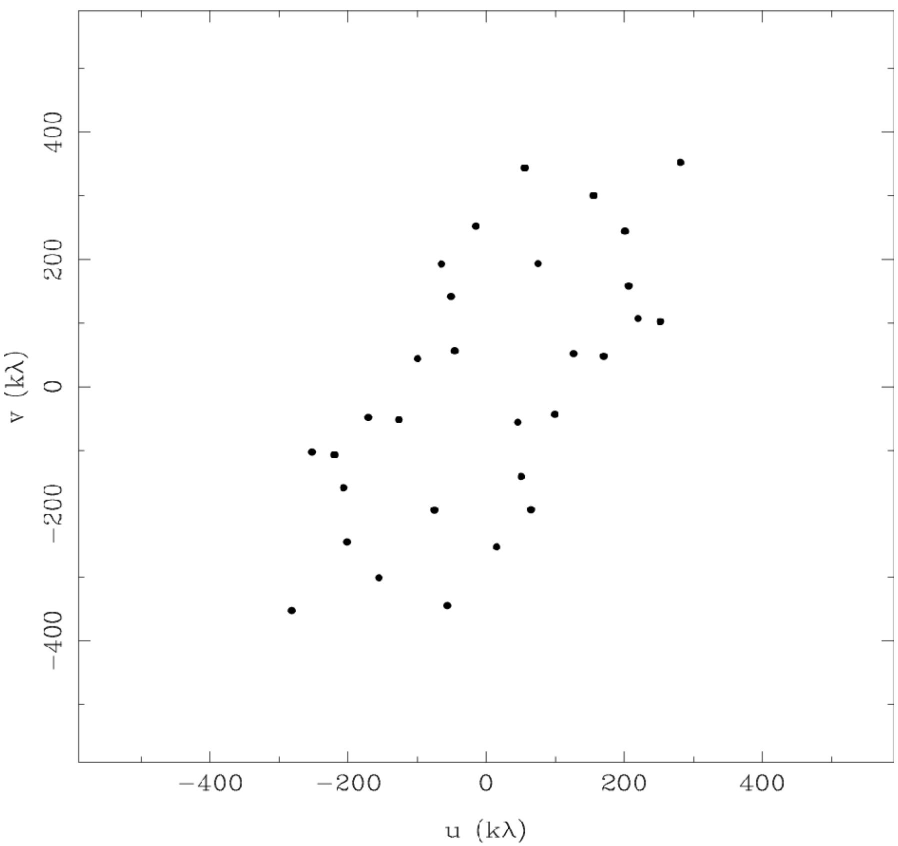
Effects of a sparse *uv* coverage

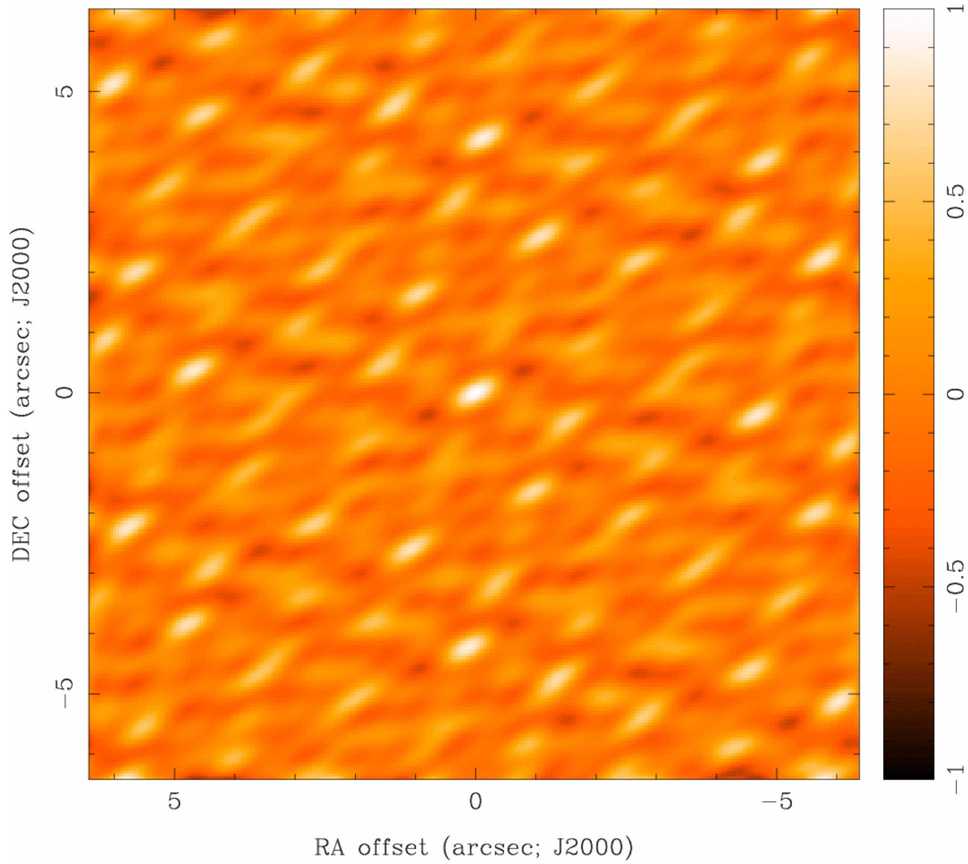
5 Antennas



Effects of a sparse *uv* coverage

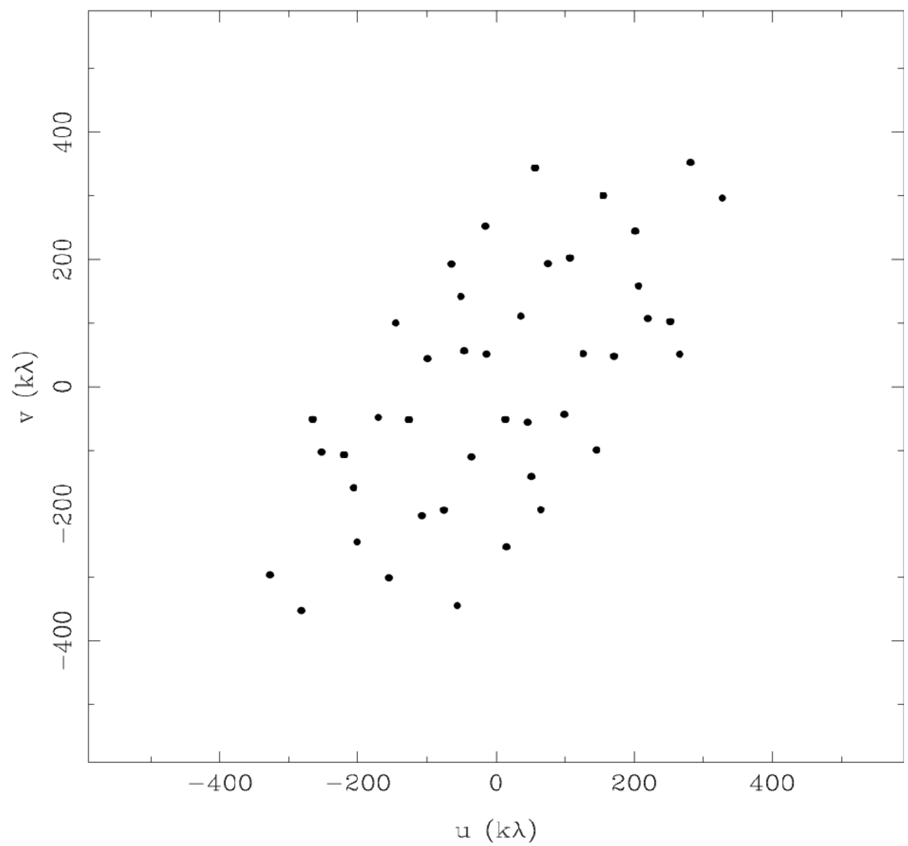
6 Antennas

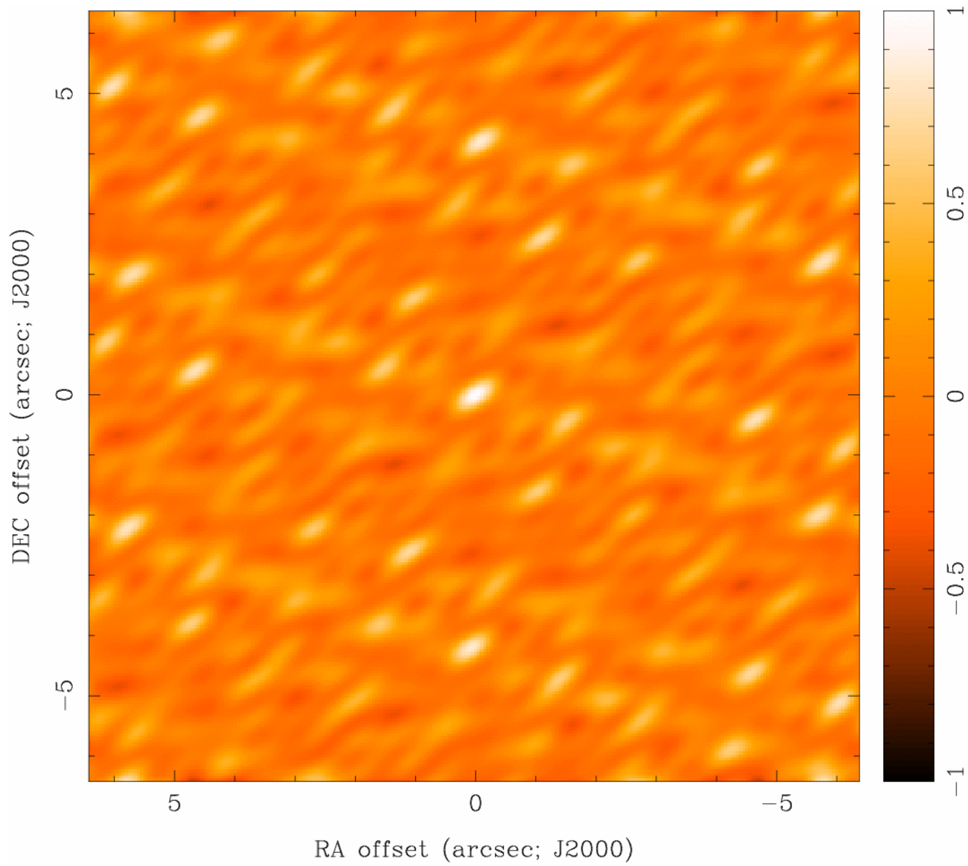




Effects of a sparse *uv* coverage

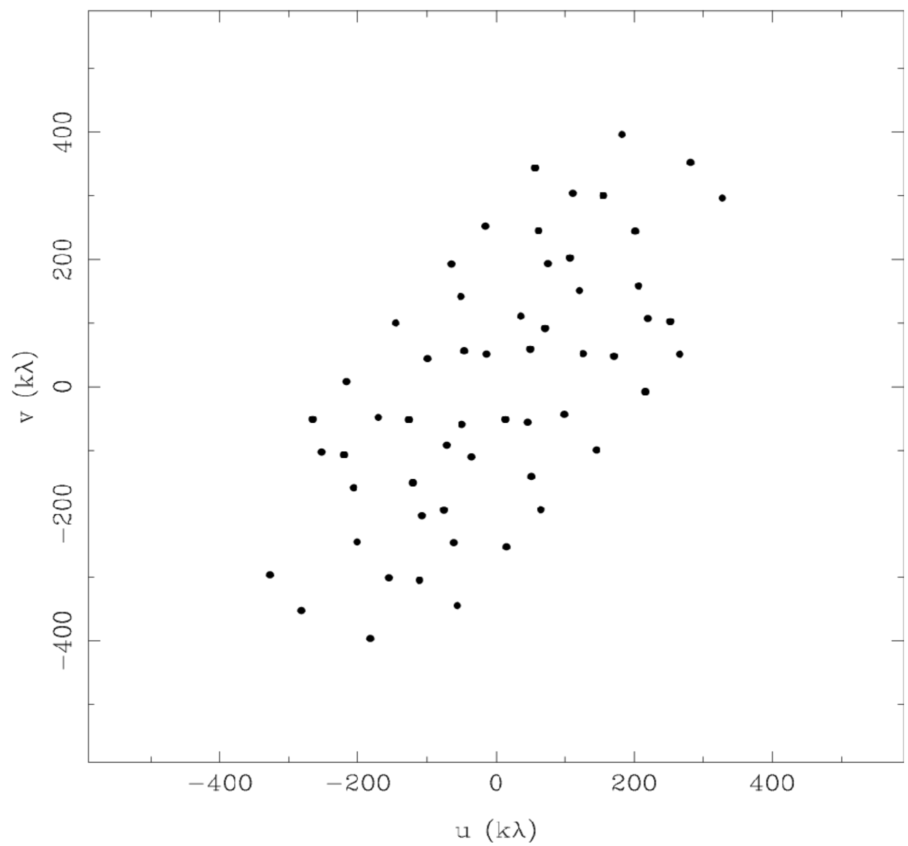
7 Antennas

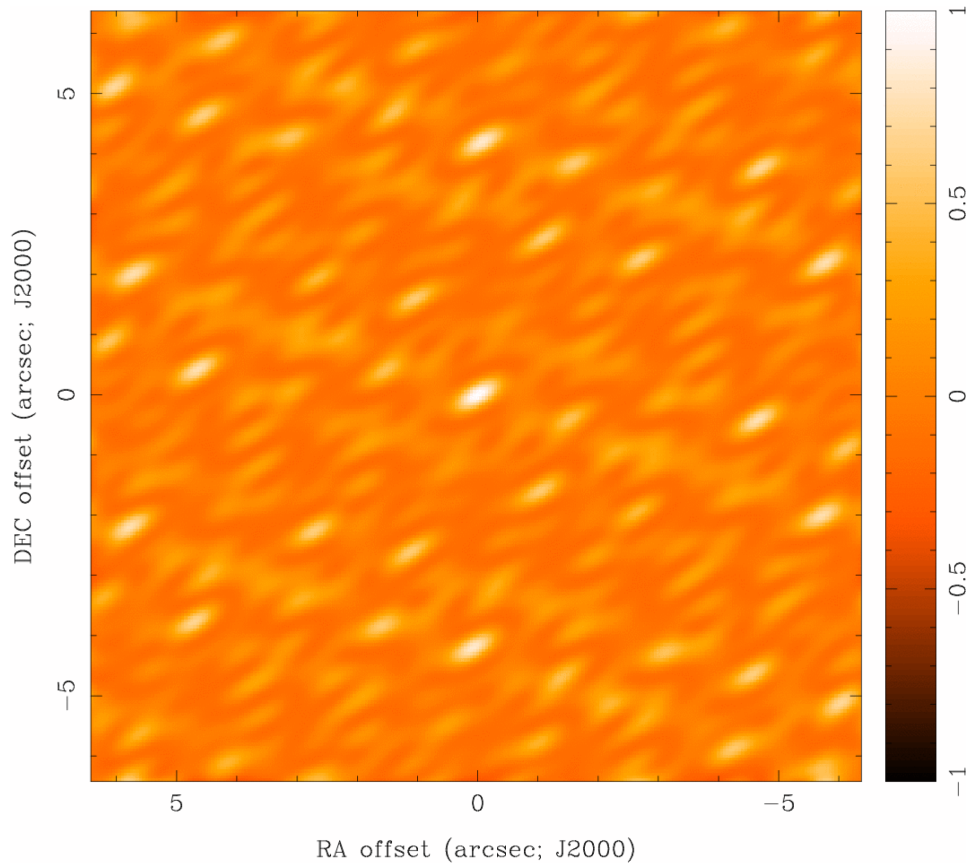




Effects of a sparse *uv* coverage

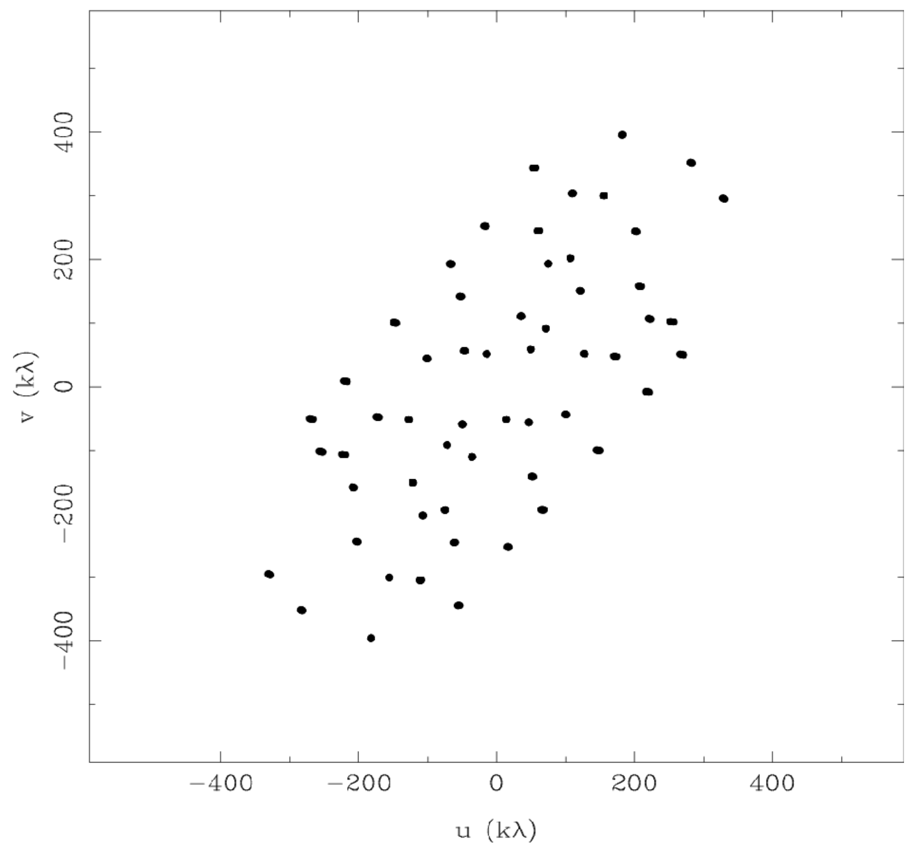
8 Antennas

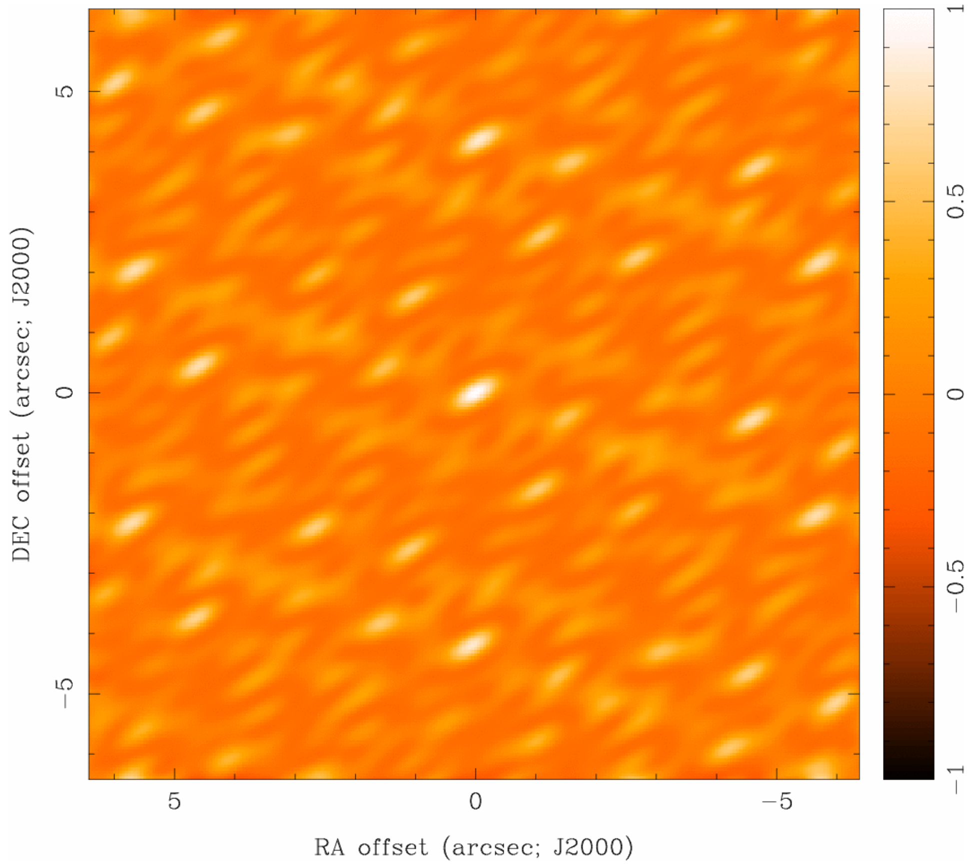




Effects of a sparse *uv* coverage

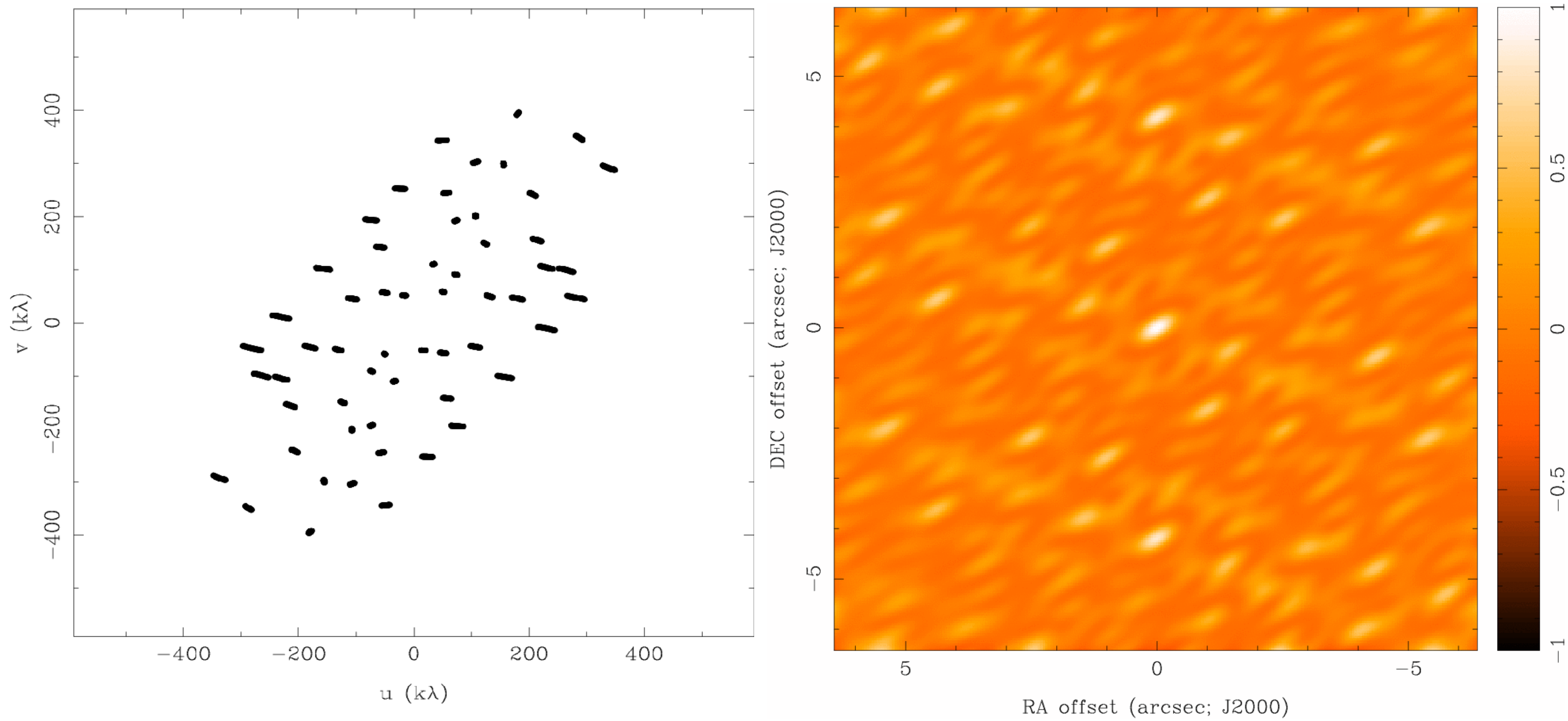
8 Antennas x 6 Samples





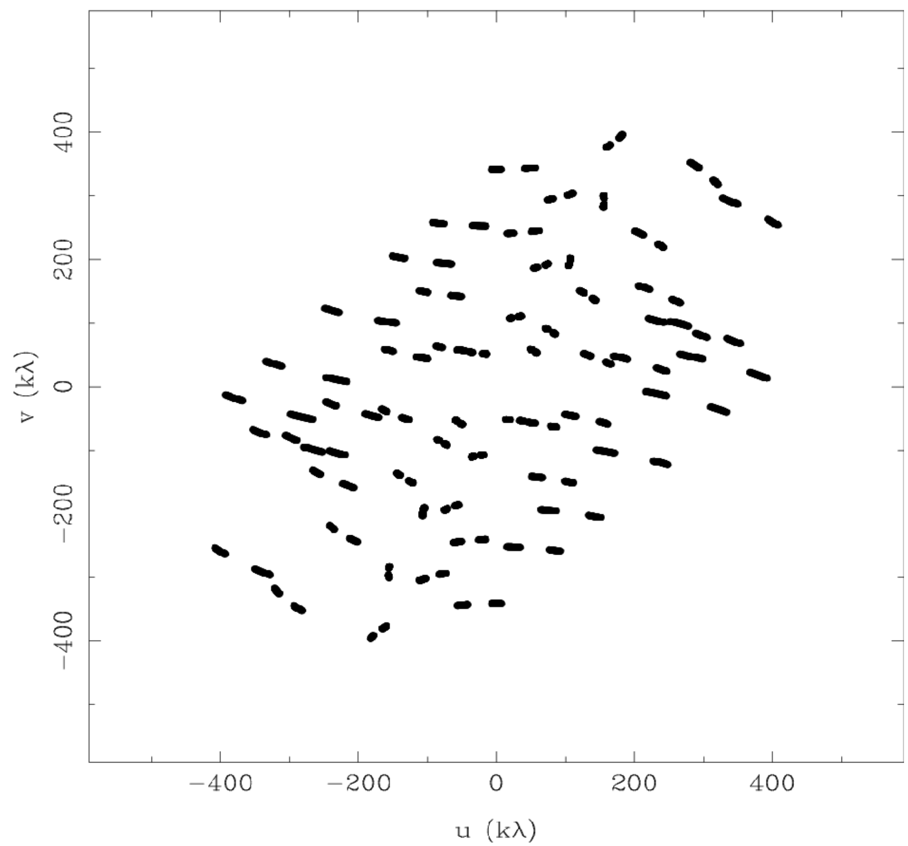
Effects of a sparse *uv* coverage

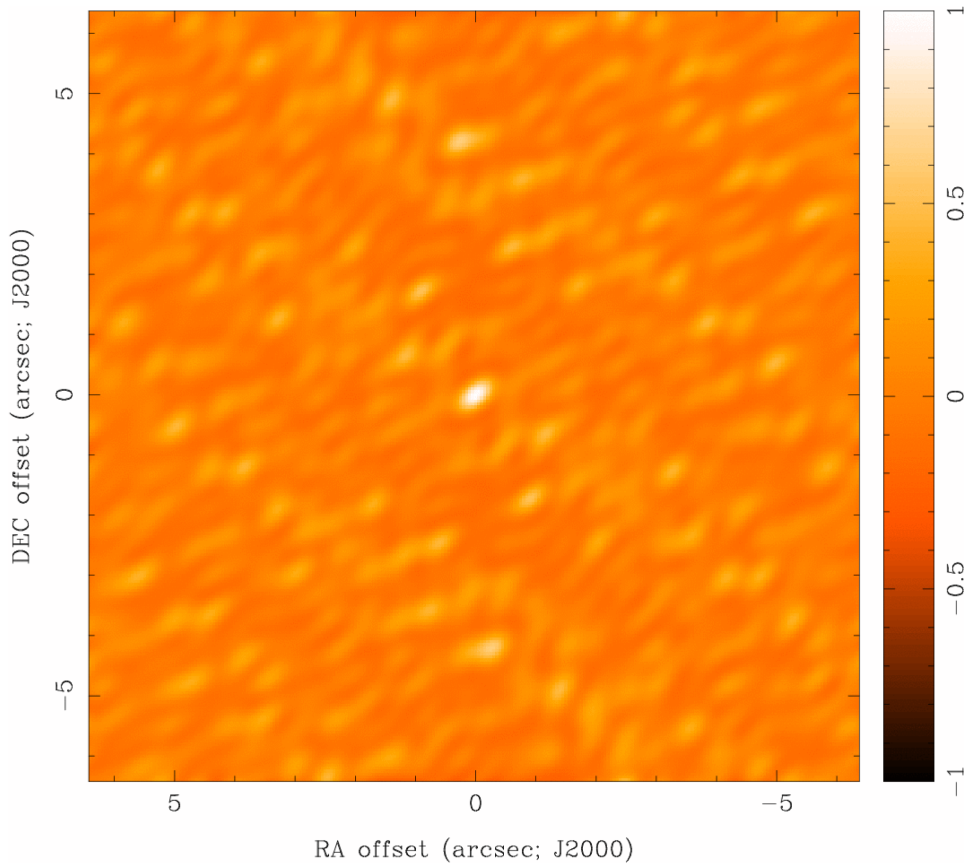
8 Antennas x 30 Samples



Effects of a sparse *uv* coverage

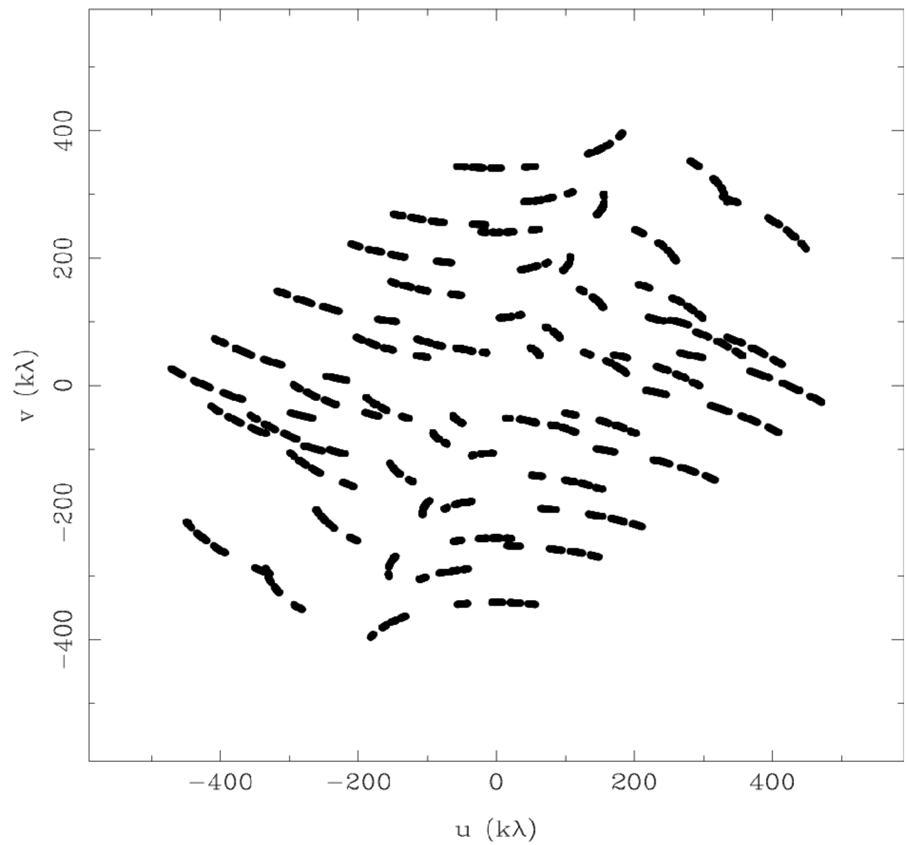
8 Antennas x 60 Samples

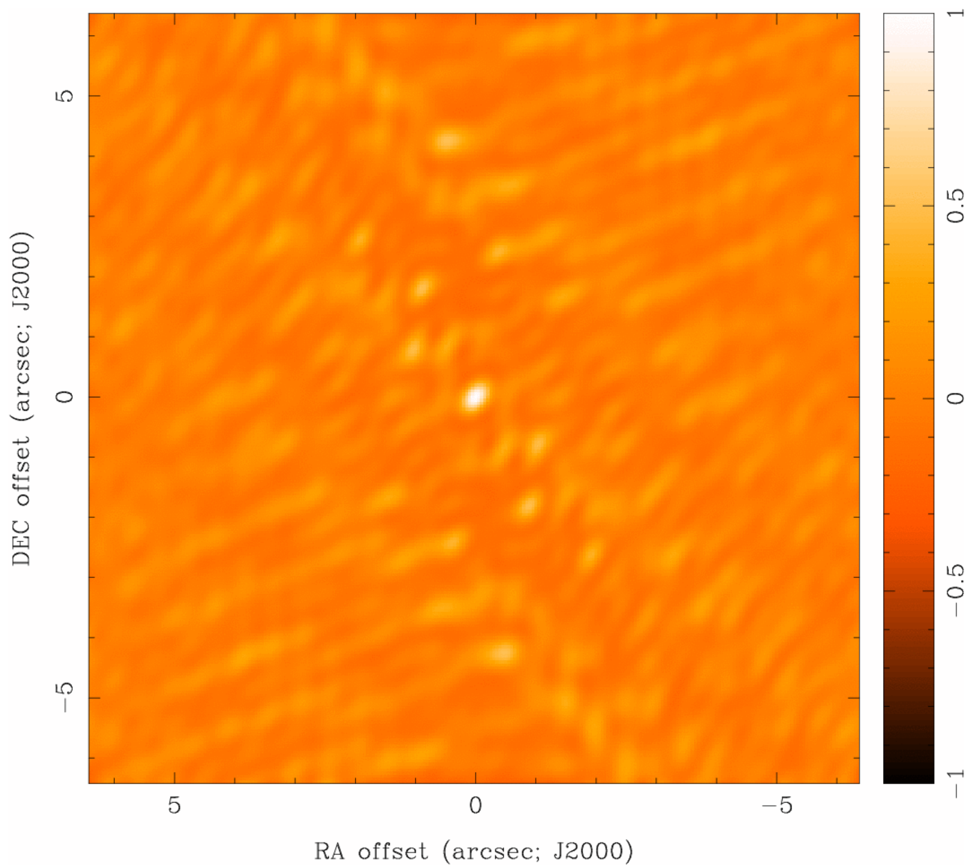




Effects of a sparse *uv* coverage

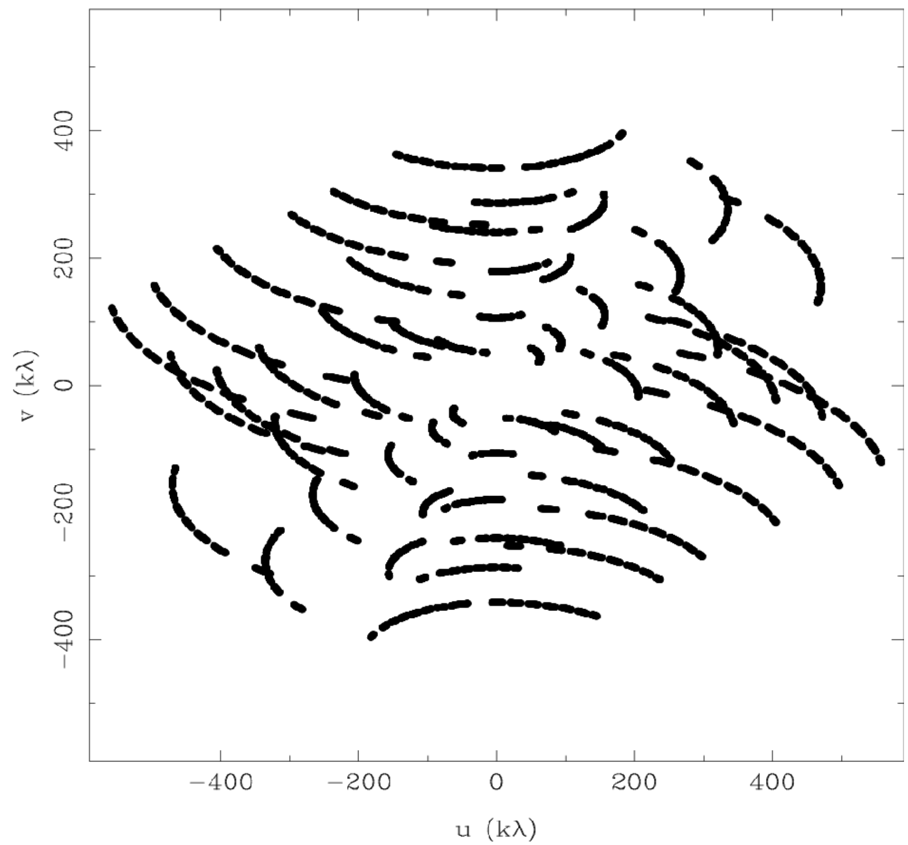
8 Antennas x 120 Samples

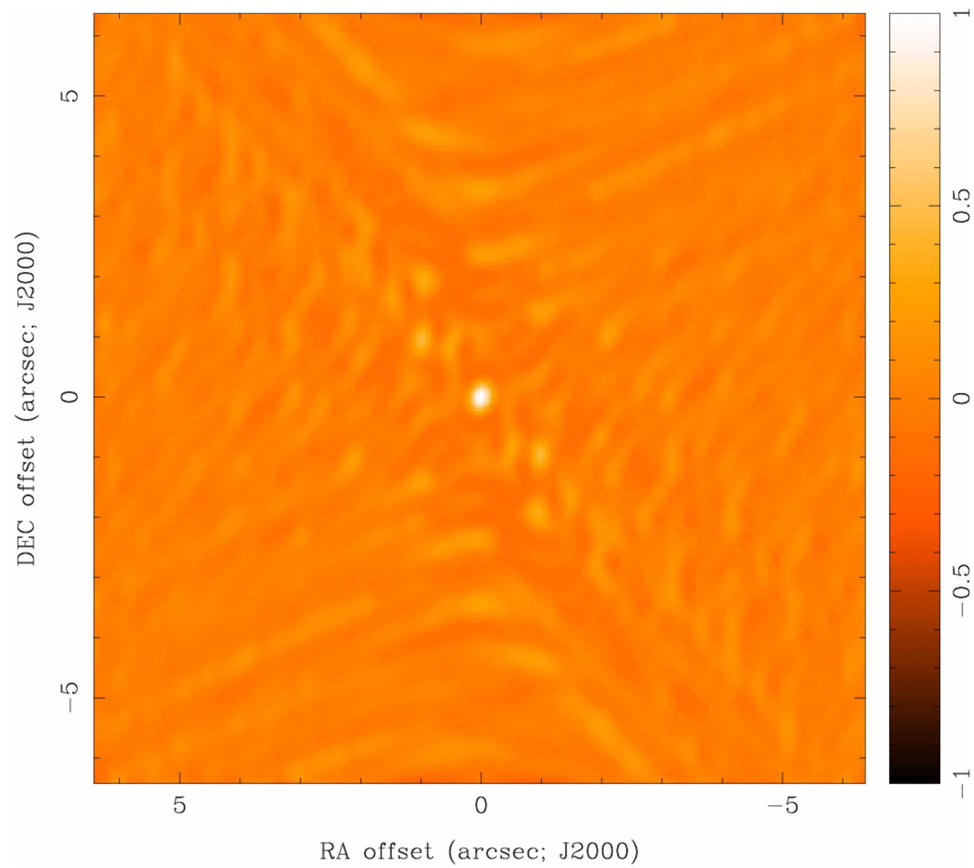




Effects of a sparse *uv* coverage

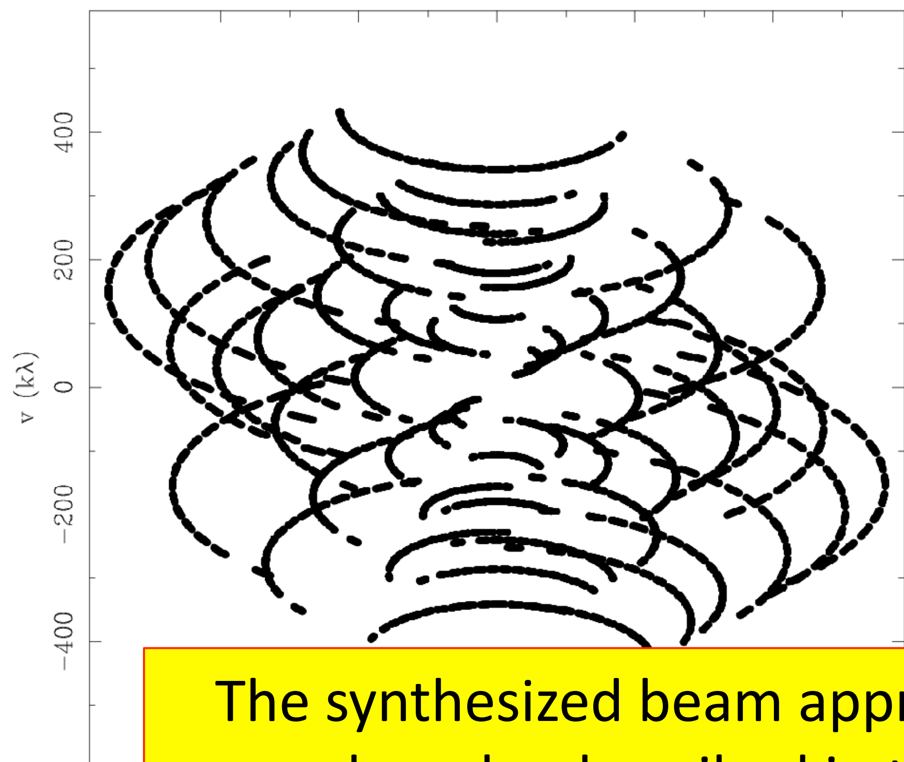
8 Antennas x 240 Samples

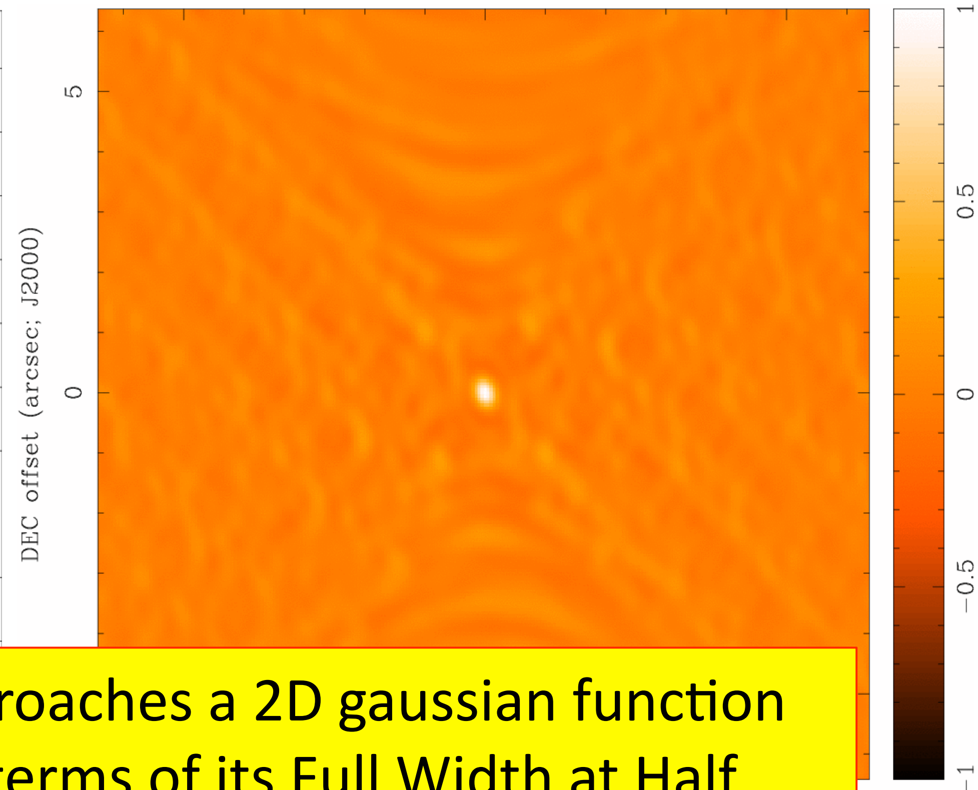




Effects of a sparse *uv* coverage

8 Antennas x 480 Samples





The synthesized beam approaches a 2D gaussian function and can be described in terms of its Full Width at Half Maximum (FWHM) and Position Angle (PA)

Characteristic angular scales

1. An interferometer has at least THREE important characteristic angular scales:
   - angular resolution: ~ λ/D_{max} , where D_{max} is the maximum separation between the apertures.
   - shortest spacing problem: the source is resolved if $\theta>\lambda/D_{min}$, where D_{min} is the minimum separation between apertures.

   An interferometer is sensitive to a range of angular sizes, λ/D_{max} - λ/D_{min}

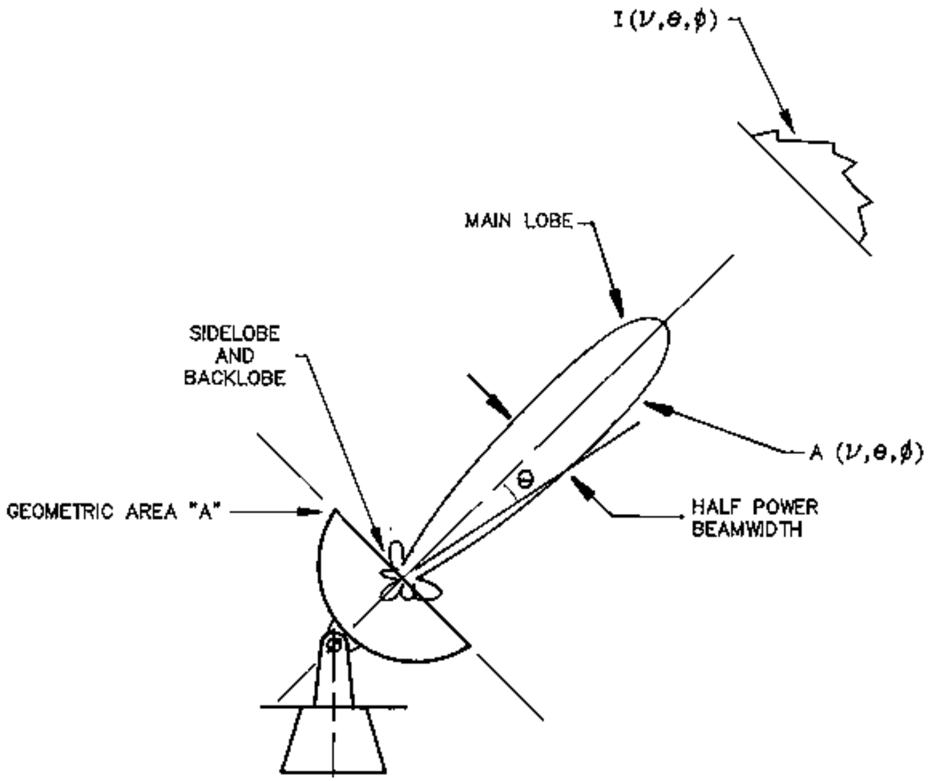
   and

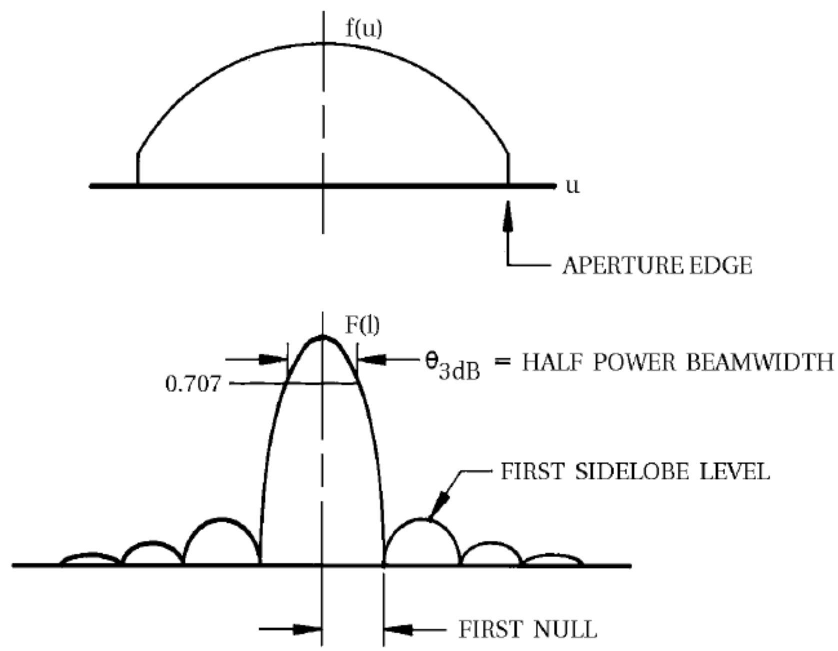
   since D_{min}> Aperture diameter, an interferometer is not sensitive to the large angular scales and cannot recover the total flux of resolved sources (you need a single dish, e.g., CSO, APEX, IRAM 30 m, ALMA total power array, CCAT).

2. Field of view of the single aperture ~ λ/D, where D is the diameter of the telescope. Source more extended than the field of view can be observed using multiple pointing centers in a mosaic.

Primary beam and Field of View

A telescope does not have uniform response across the entire sky
- main lobe approximately Gaussian, fwhm ~1.2λ/D = "primary beam"
- limited field of view ~ λ/D





Characteristic angular scales for the EVLA

Configuration	A	B	C	D
B_{max} (km[1])	36.4	11.1	3.4	1.03
B_{min} (km[1])	0.68	0.21	0.035[5]	0.035
	Synthesized Beamwidth θ_{HPBW}(arcsec)[1,2,3]			
74 MHz (4 band)	24	80	260	850
1.5 GHz (L)	1.3	4.3	14	46
3.0 GHz (S)[6]	0.65	2.1	7.0	23
6.0 GHz (C)	0.33	1.0	3.5	12
8.5 GHz (X)[7]	0.23	0.73	2.5	8.1
15 GHz (Ku)[6]	0.13	0.42	1.4	4.6
22 GHz (K)	0.089	0.28	0.95	3.1
33 GHz (Ka)	0.059	0.19	0.63	2.1
45 GHz (Q)	0.043	0.14	0.47	1.5

Configuration	A	B	C	D
B_{max} (km[1])	36.4	11.1	3.4	1.03
B_{min} (km[1])	0.68	0.21	0.035[5]	0.035
	Largest Angular Scale θ_{LAS}(arcsec)[1,4]			
74 MHz (4 band)	800	2200	20000	20000
1.5 GHz (L)	36	120	970	970
3.0 GHz (S)[6]	18	58	490	490
6.0 GHz (C)	8.9	29	240	240
8.5 GHz (X)[7]	6.3	20	170	170
15 GHz (Ku)[6]	3.6	12	97	97
22 GHz (K)	2.4	7.9	66	66
33 GHz (Ka)	1.6	5.3	44	44
45 GHz (Q)	1.2	3.9	32	32

Characteristic angular scales during ALMA early science

Dmax = 400 m, Dmin = 30 m, D = 12 m

Band	λ	λ/D_{max}	λ/D_{min}	FOV
3	3 mm	1.5''	10.5''	50''
6	1.3 mm	0.6''	4.5''	22''
7	0.9 mm	0.4''	3.0''	15''
9	0.45 mm	0.2''	1.5''	8''

1. How complex is the source? Is good (u,v) coverage needed? This may set additional constraints on the integration time if the source has a complex morphology.

2. How large is the source? If it is comparable to the primary beam (λ/D), you should mosaic several fields

Measuring the visibility

$$T'_\nu(x,y) = \iint W_\nu(u,v) V_\nu(u,v) e^{2\pi i(uv+vy)} du dv$$

Calibration formula:

$$\tilde{V}_\nu(u,v) = G_\nu(u,v) V_\nu(u,v) + \varepsilon_\nu(u,v) + \eta_\nu(u,v)$$

G(u,v) = complex gain, i.e., amplitude and phase gains.
Due to the atmosphere, i.e., different optical path and atmospheric opacity, and to the instrumental response to the astronomical signal.
ε(u,v) = complex offset, i.e., amplitude and phase offset. Engineers work hard to make this term close to zero
η(u,v) = random complex noise

Data Calibration

Basic idea: observe sources with known flux, shape and spectrum to derive the response of the instrument.

Bandpass calibration:
what does it do? It compensates for the change of gain with frequency.
How does it work? A strong source with a flat spectrum (i.e., bandpass calibrator) is observed (usually once during the track). Bandpass calibration is generally stable across the track.

Phase calibration:
What does it do? It compensates for relative temporal variation of the phase of the correlated signal on different antennas (or baselines).
How does it work?
- using Water Vapor Radiometers (see Laura's talk)
-a strong source with known shape (preferably a point source) is observed every few minutes. If the gain calibrator is a point source, than the gain are derived assuming that the intrinsic phase is 0.

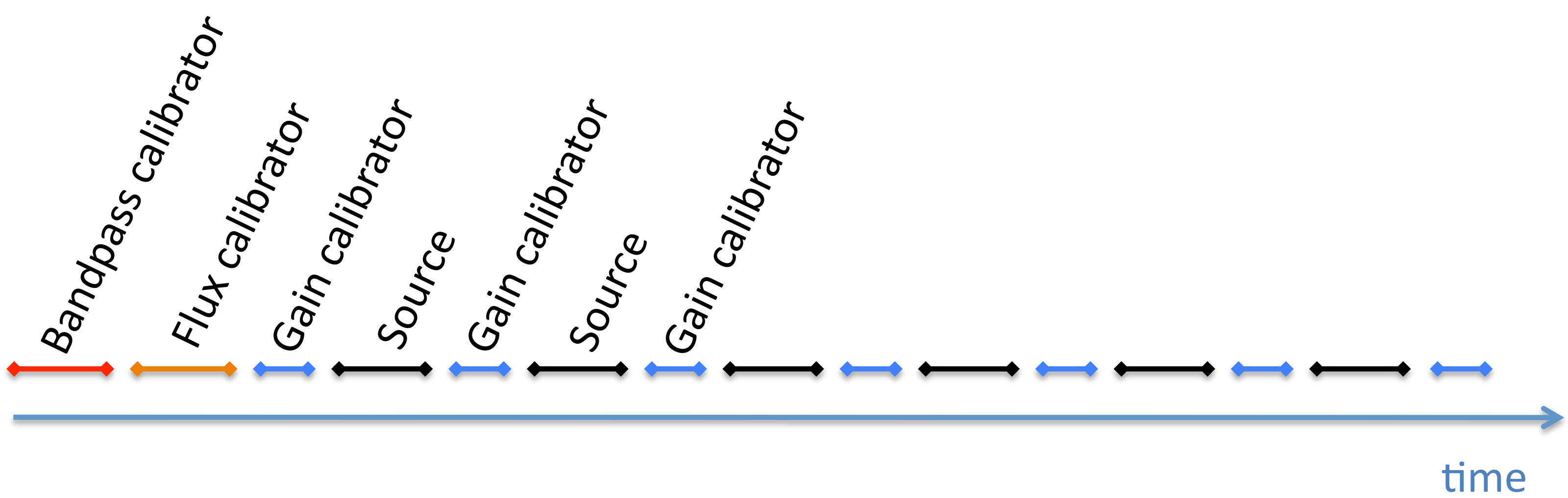
Data Calibration

Basic idea: observe sources with known flux, shape and spectrum to derive the response of the instrument.

Amplitude calibration:

What does it do? It compensates for atmospheric opacity and loss of signal within the interferometer (e.g., pointing errors).

How does it work? A source with known flux is observed. This can be a planet for which accurate models exists, or a *stable* radio source, which has been independently calibrated using, e.g., a planet. Accuracies < 15% in the absolute flux are challenging and might required specific observing strategies.



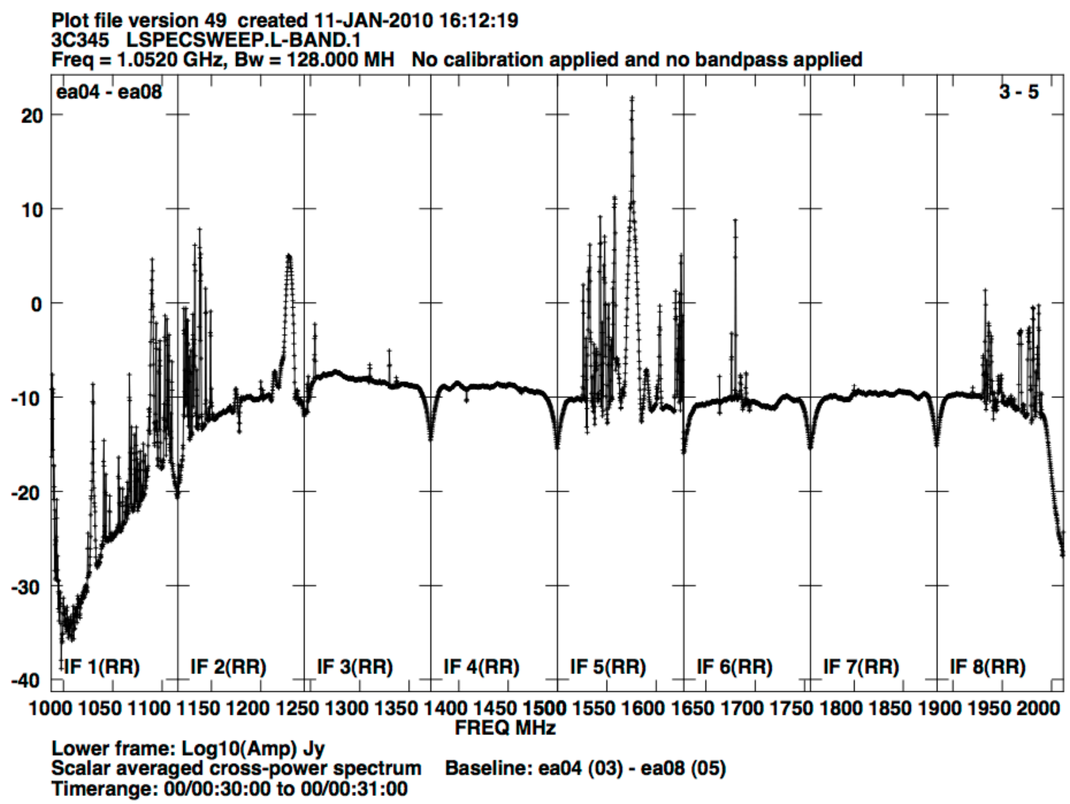
Radio-Frequency Interference (RFI)

RFI mainly affect low frequency observations

Table 10: Identified VLA RFI Between 1 and 4 GHz

Frequency (MHz)	Source	Comments
1025-1150	Aircraft navigation	Very strong
1200.0	VLA modem	
1217-1237	GPS L2	Very strong
1243-1251	GLONASS L2	
1254	Aeronautical radar	
1263	Aeronautical radar	
1268	COMPASS E6	
1310	Aeronautical radar	
1317	Aeronautical radar	
1330	Aeronautical radar	
1337	Aeronautical radar	
1376-1386	GPS L3	Intermittent
1525-1564	INMARSAT satellites	
1564-1584	GPS L1	Very strong
1598-1609	GLONASS L1	
1618-1627	IRIDIUM satellites	
1642	2nd harmonic VLA radios	Sporadic
1683-1687	GOES weather satellite	
1689-1693	GOES weather satellite	
1700-1702	NOAA weather satellite	
1705-1709	NOAA weather satellite	
1930-1990	PCS cell phone base stations	
2178-2195	???	
2320-2350	Satellite radio	

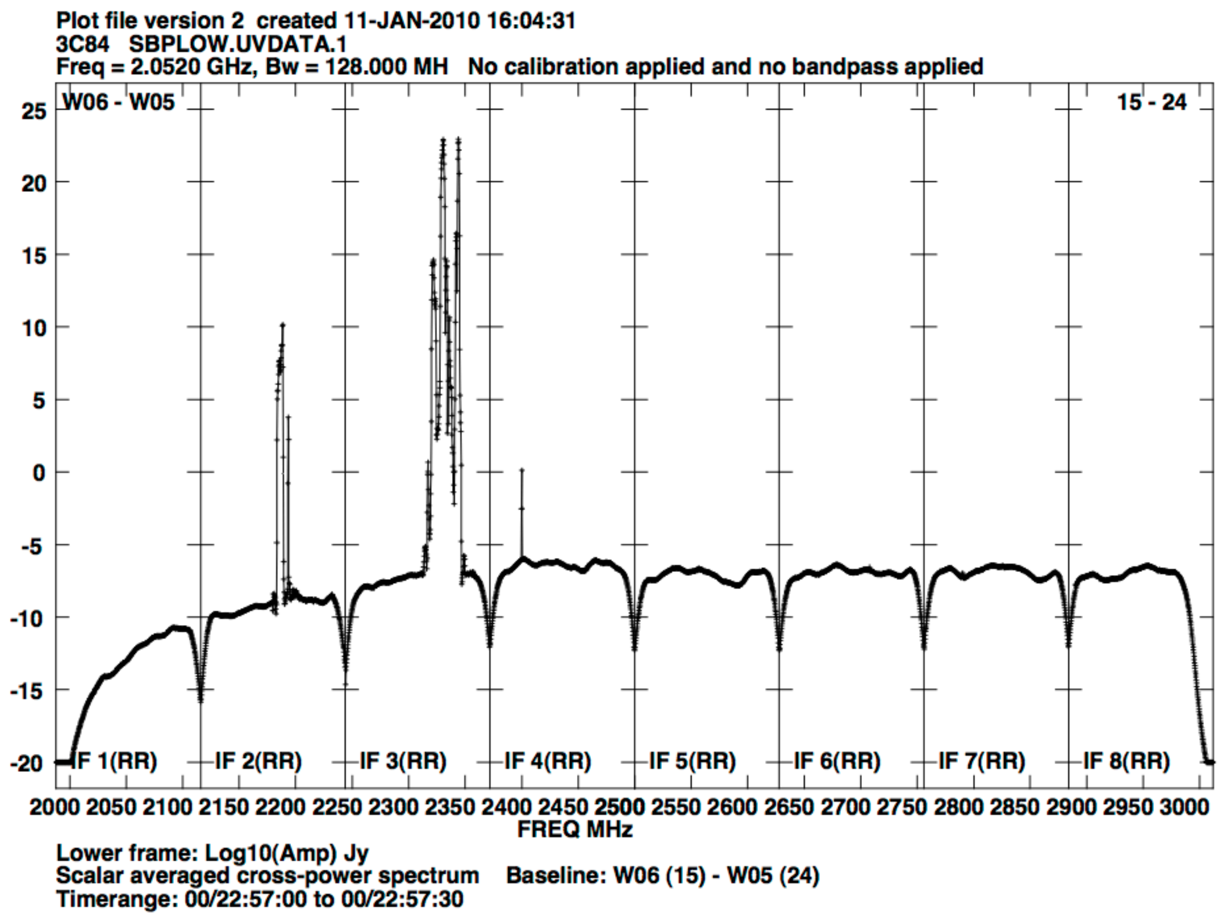
RFI in L-band from 1 to 2 GHz



http://evlaguides.nrao.edu/index.php?title=Observational_Status_Summary

Radio-Frequency Interference (RFI)

RFI in S-band from 2 to 3 GHz



http://evlaguides.nrao.edu/index.php?title=Observational_Status_Summary

Radio-Frequency Interference (RFI)

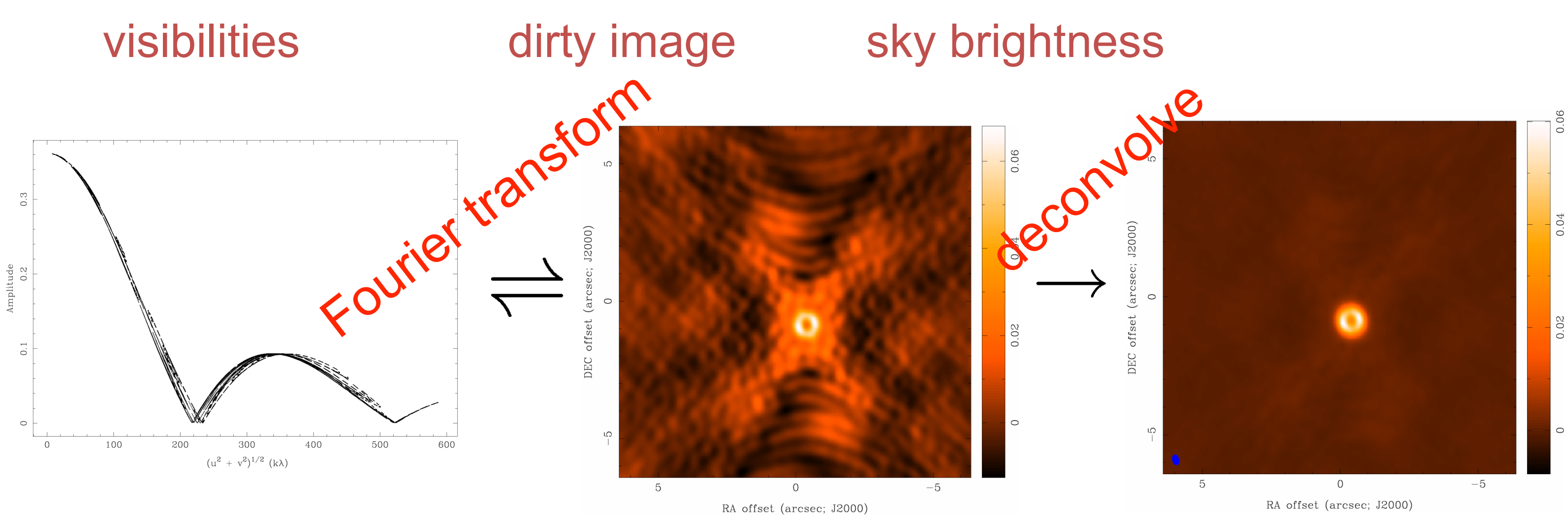
1. Avoid the frequencies affected by RFI.
Check http://evlaguides.nrao.edu/index.php?title=Observational_Status_Summary
For a complete list of known RFI

2. If you cannot avoid RFI, try to observe in the extended configurations, where RFI are less severe

3. Apply spectral smoothing to the data, but in general you will not be able to get Useful data within a few MHz from the RFI frequency,

From visibilities to images (see also John's talk tomorrow)

- uv plane analysis
  - best for "simple" sources, e.g. point sources, disks
- image plane analysis
  - Fourier transform V(u,v) samples to image plane, get $T'(x,y)$
  - but difficult to do science on dirty image
  - deconvolve b(x,y) from $T'(x,y)$ to determine (model of) T(x,y)



Weighting function

Measured flux: $$T'(x,y) = B(x,y) \otimes T(x,y)$$

Synthesized beam: $$B(x,y) = \iint W(u,v) e^{2\pi i(uv+vy)} du dv$$

You can change the angular resolution and sensitivity of the final image by changing the weighting function W(u,v)

Weighting function

• **"Natural"** weighting: W(u,v) = $1/\sigma^2(u,v)$, where $\sigma^2(u,v)$ is the noise variance of the (u,v) sample:

Advantage: gives the lowest noise in the final image, highlight extended structures.

Disadvantage: generally gives more weights to the short baseline (where there are more measurements of V) degrading the resolution

• **"Uniform"** weighting: W(u,v) is inversely proportional to the local density of (u,v) points. It generally gives more weights to the long baseline therefore leading to higher angular resolution.

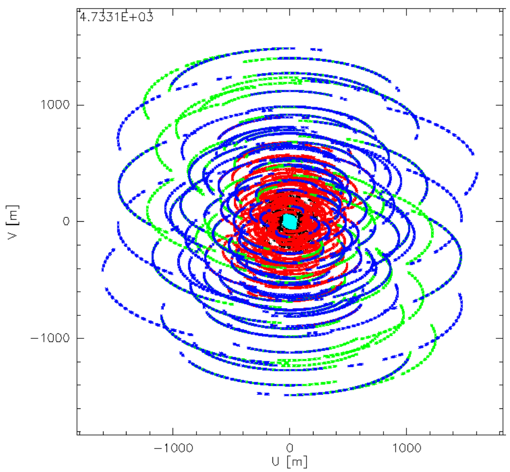
Advantage: better resolution and lower sidelobes

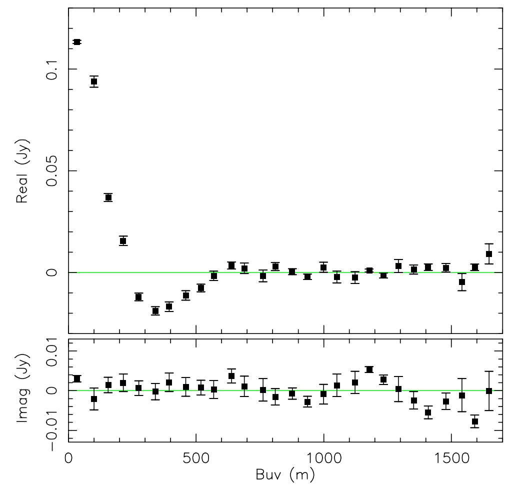
Disadvantage: higher noise in the final map

• **"Robust"** (Briggs) weighting: W(u,v) depends on a given threshold value S, so that a large S gives natural weighting and a small S gives uniform weighting.

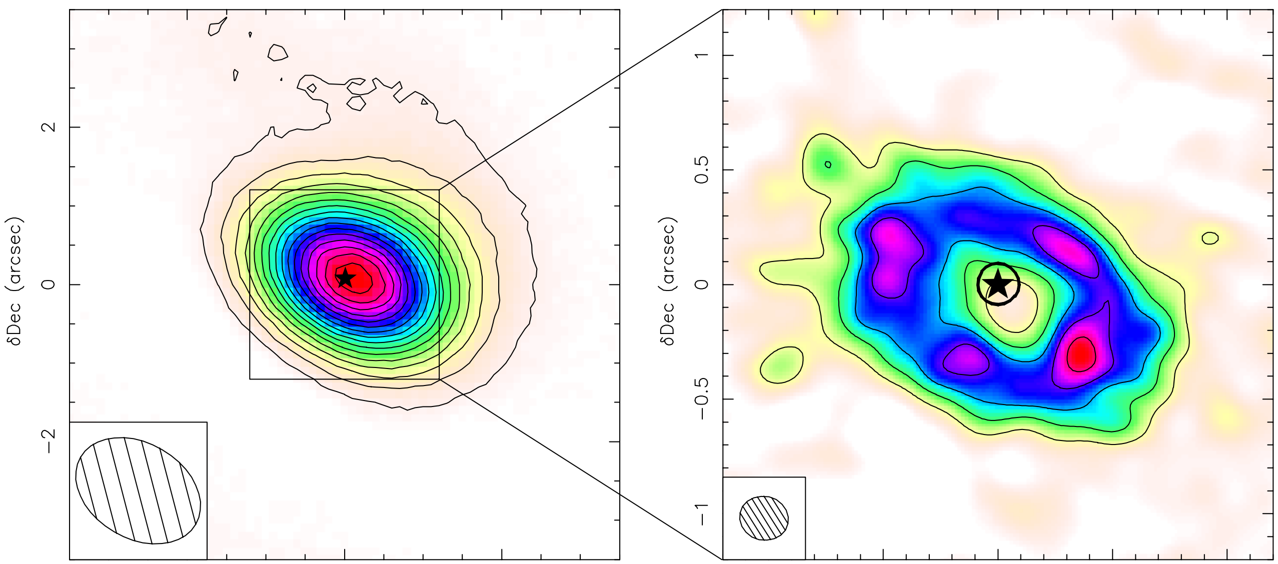
Advantage: continuous variation of the angular resolution.

Weighting function





NATURAL WEIGHTING



UNIFROM WEIGHTING

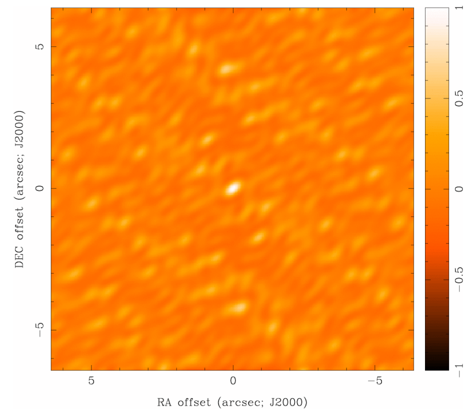
FWHM beam size = 1.7’’ x 1.2’’ 0.21’’ x 0.19’’

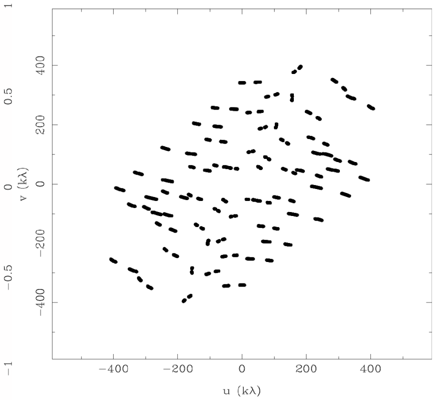
Deconvolution: Dirty Vs Clean image

A non complete coverage of the *uv* plane gives a synthesized beam with a lot of sidelobes, i.e. a 'dirty' beam. And since

$$T'(x,y) = B(x,y) \otimes T(x,y),$$

T'(x,y) is also characterized by sidelobes, i.e., 'dirty' image.

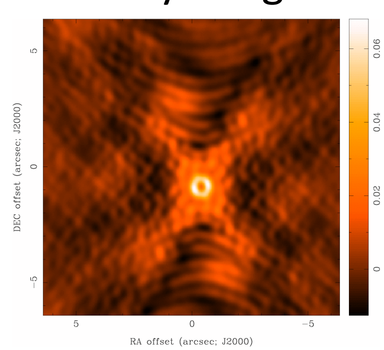




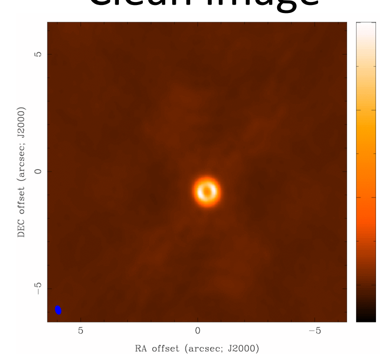
The deconvolution process consists in giving reasonable values to the visibility in the unmeasured (u,v) areas in order to get a nice gaussian beam without sidelobes. The most successful deconvolution procedure is the algorithm CLEAN (Hogbom 1974).

Deconvolution or Cleaning

Dirty image



Clean image



Concluding remarks. I

- Interferometry samples visibilities that are related to a sky brightness image by the Fourier transform.

  Keep in mind these three important scales:
  - angular resolution ~ λ/maximum baseline ,
  - largest mapped spatial scale ~ λ/minimum baseline
  - field of view ~ λ/antenna diameter

  Also note that to have "good" images you need a "good" sampling of the uv plane

- The measured visibilities contains wavelength-dependent atmospheric and instrumental terms. The "data reduction" consists in deriving these terms from the observations of sources with known spectrum, shape and flux.

  Re-reducing the same dataset 10 times is fine!

Concluding remarks. II

- Once you have calibrated your data, keep in mind that there is an infinite number of images compatible with the sampled visibilities. Play around with the weighting functions while using CASA

- Deconvolution (e.g. cleaning) try to correct for the incomplete sampling of the *uv*-plane. Play around with 'clean' while using CASA

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  - CARMA SUMMER school, July 2011, ask J. Carpenter for information