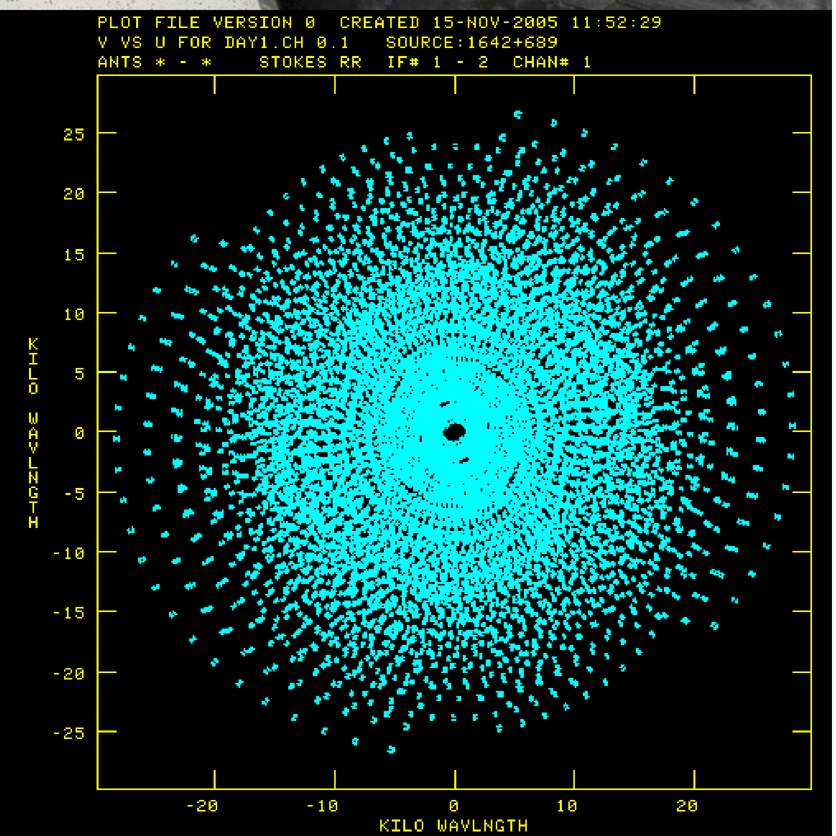


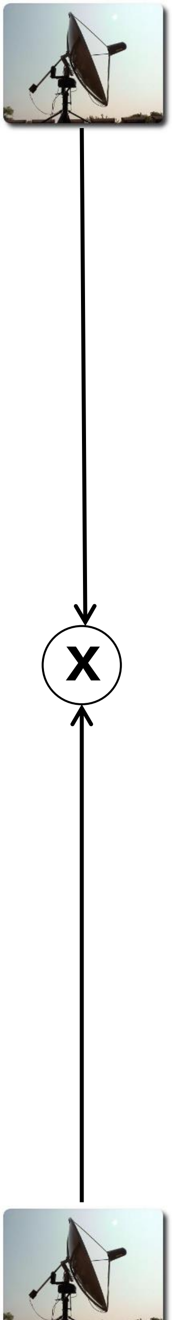
Cross correlators

for radio astronomy

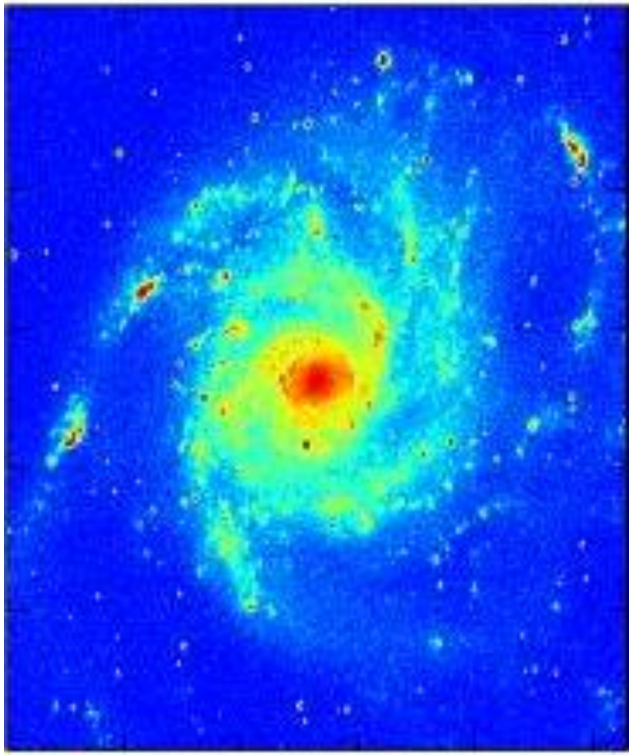
Adam Deller

May 12, 2014

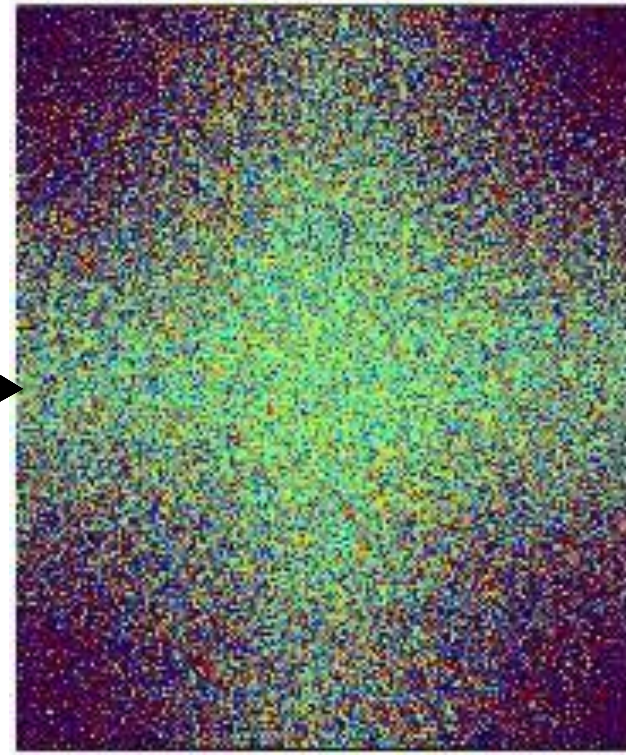
ASTRON



Correlators and Interferometry



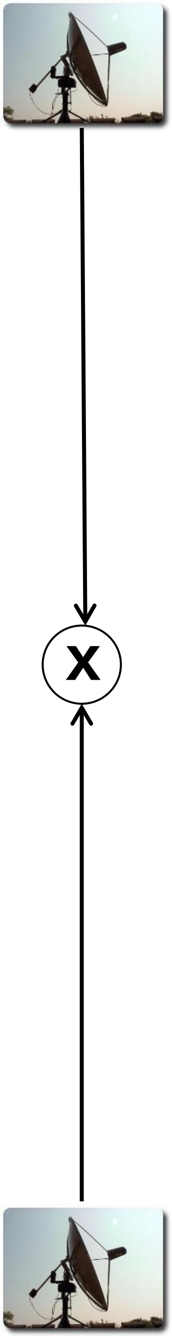
$\leftarrow \mathcal{F} \rightarrow$

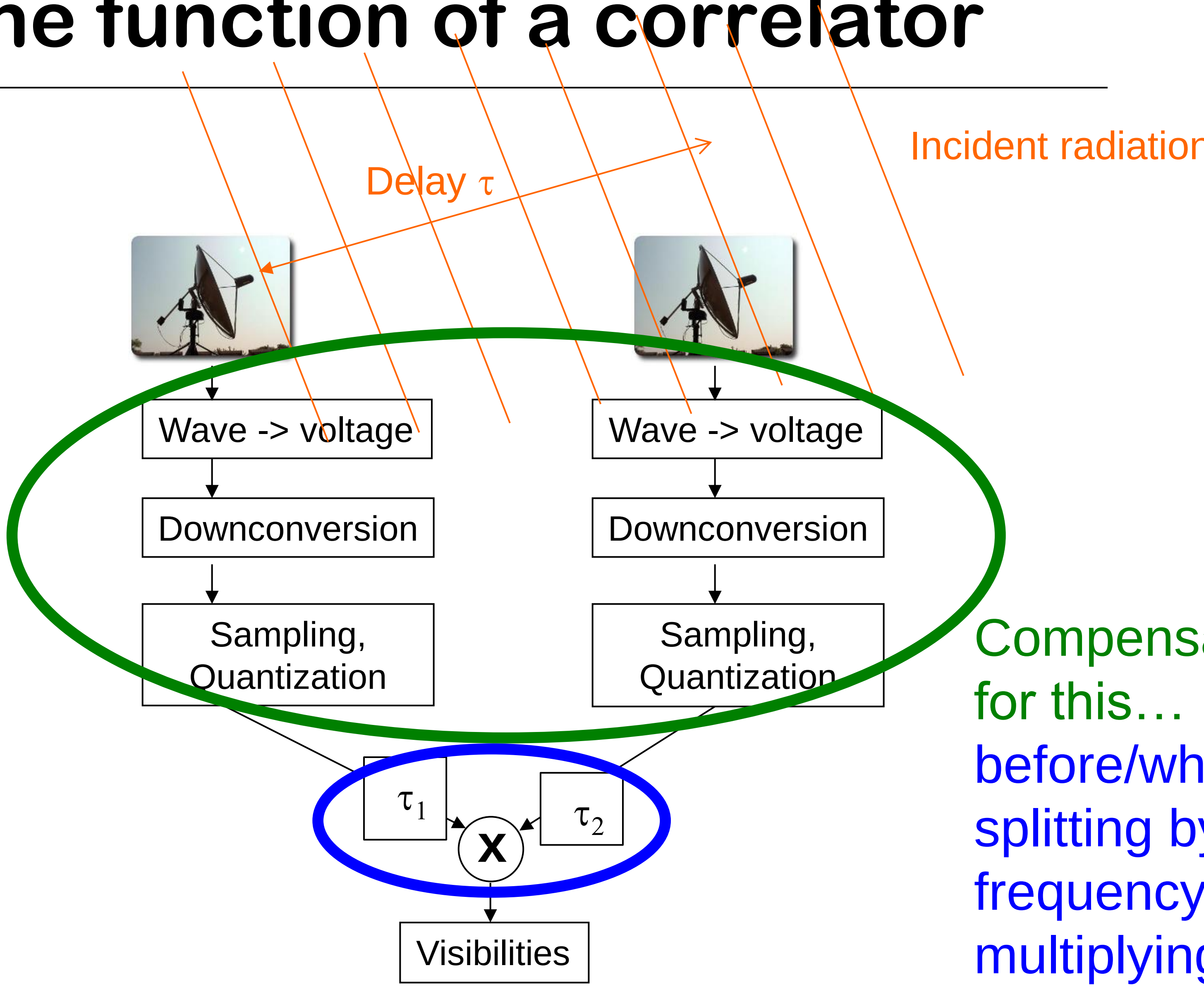


Sky brightness

Visibilities
(real component shown)

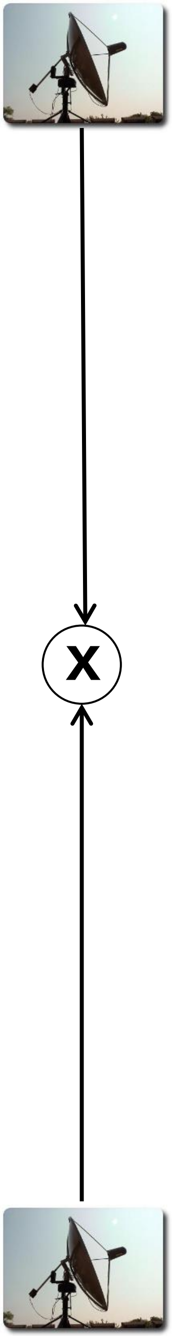
The function of a correlator





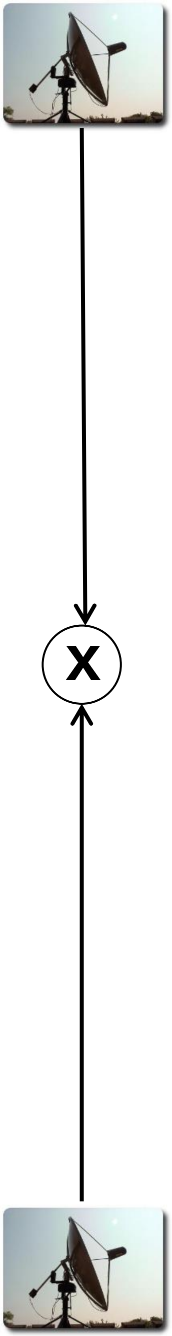
Compensate for this…

before/while splitting by frequency and multiplying



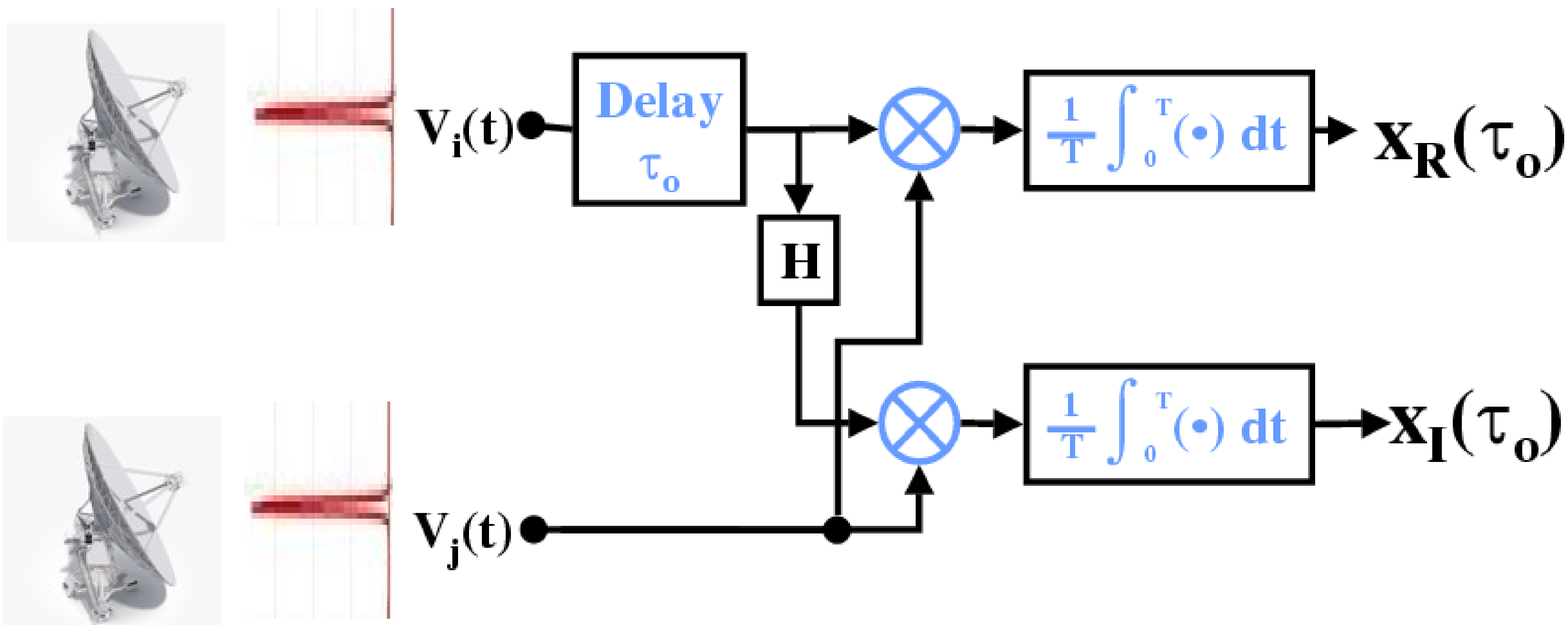
Why care about correlators?

1. One day you want to be a radio interferometry guru
2. To help you propose the right observations and identify problems in data or images



A “dumb” correlator

- Use many analog filters to make many narrow channels; correlate each one separately with a separate complex correlator



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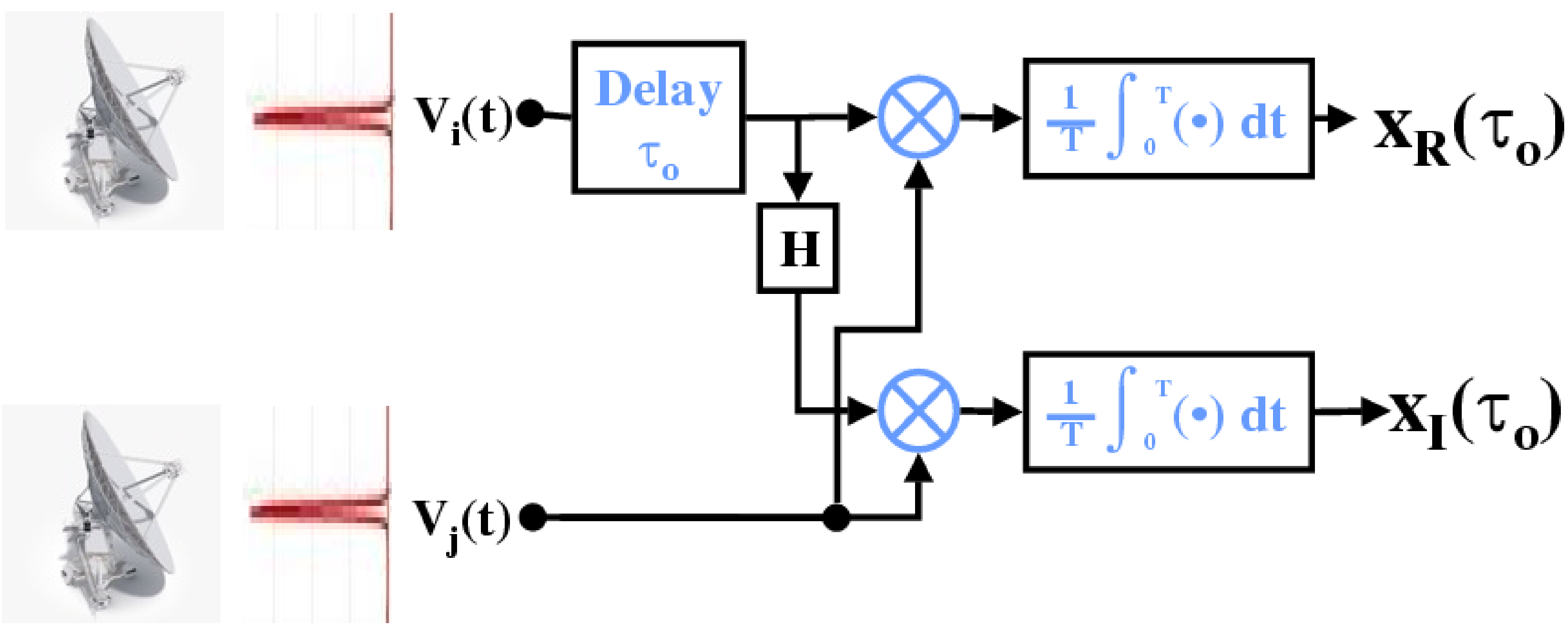
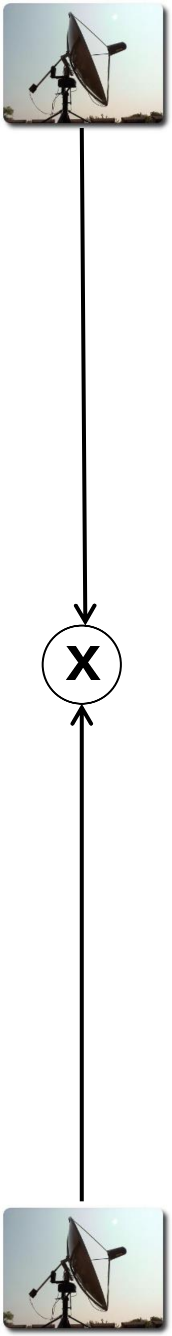
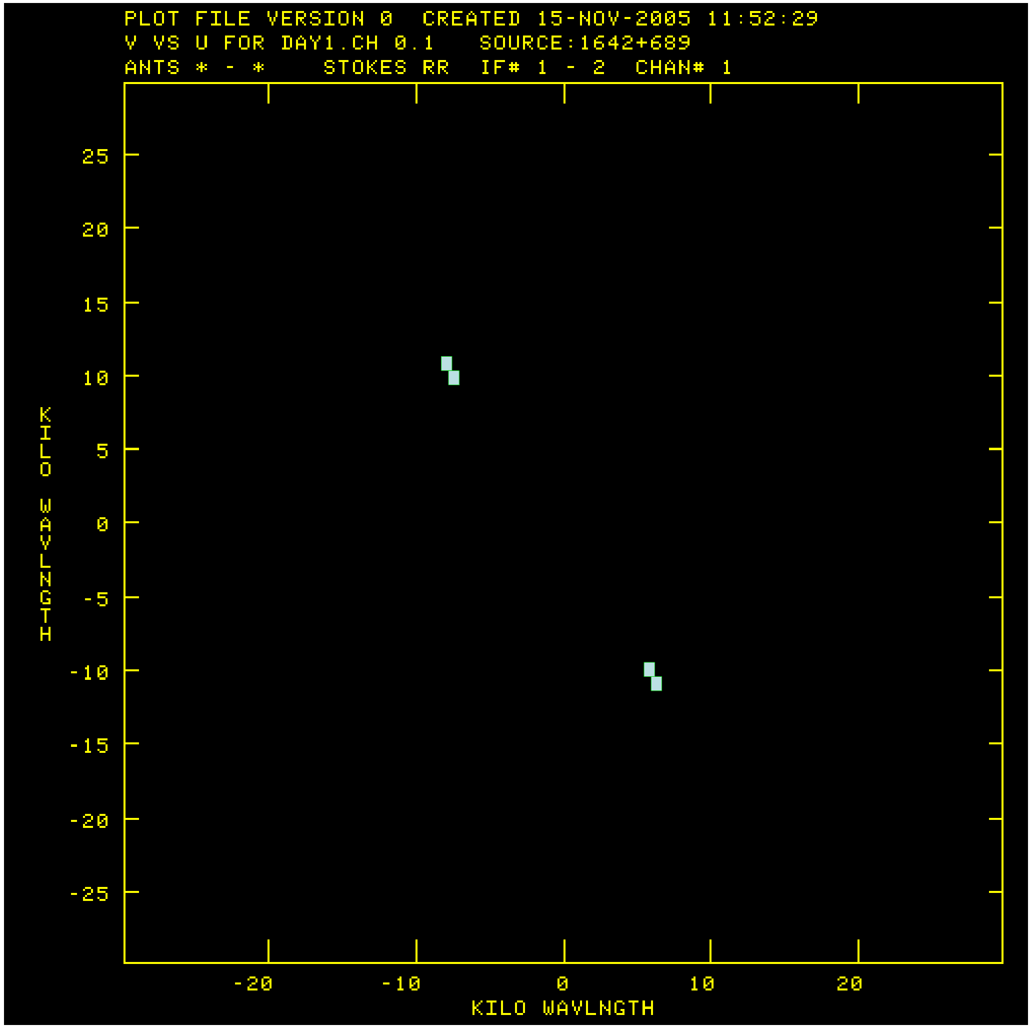
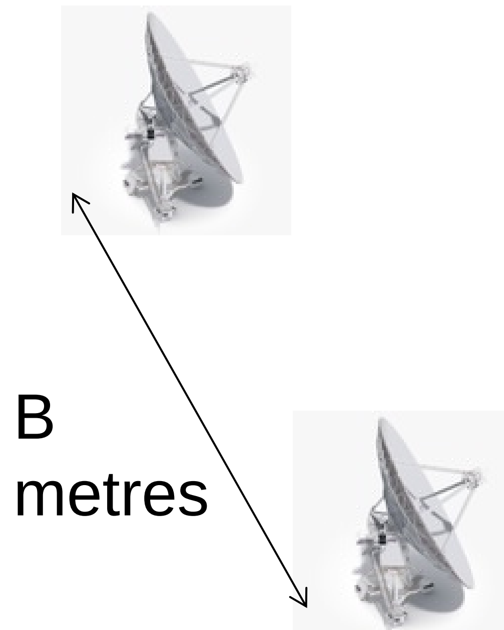


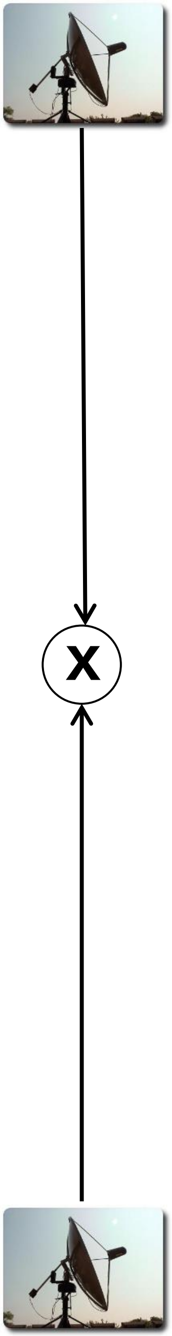
The output

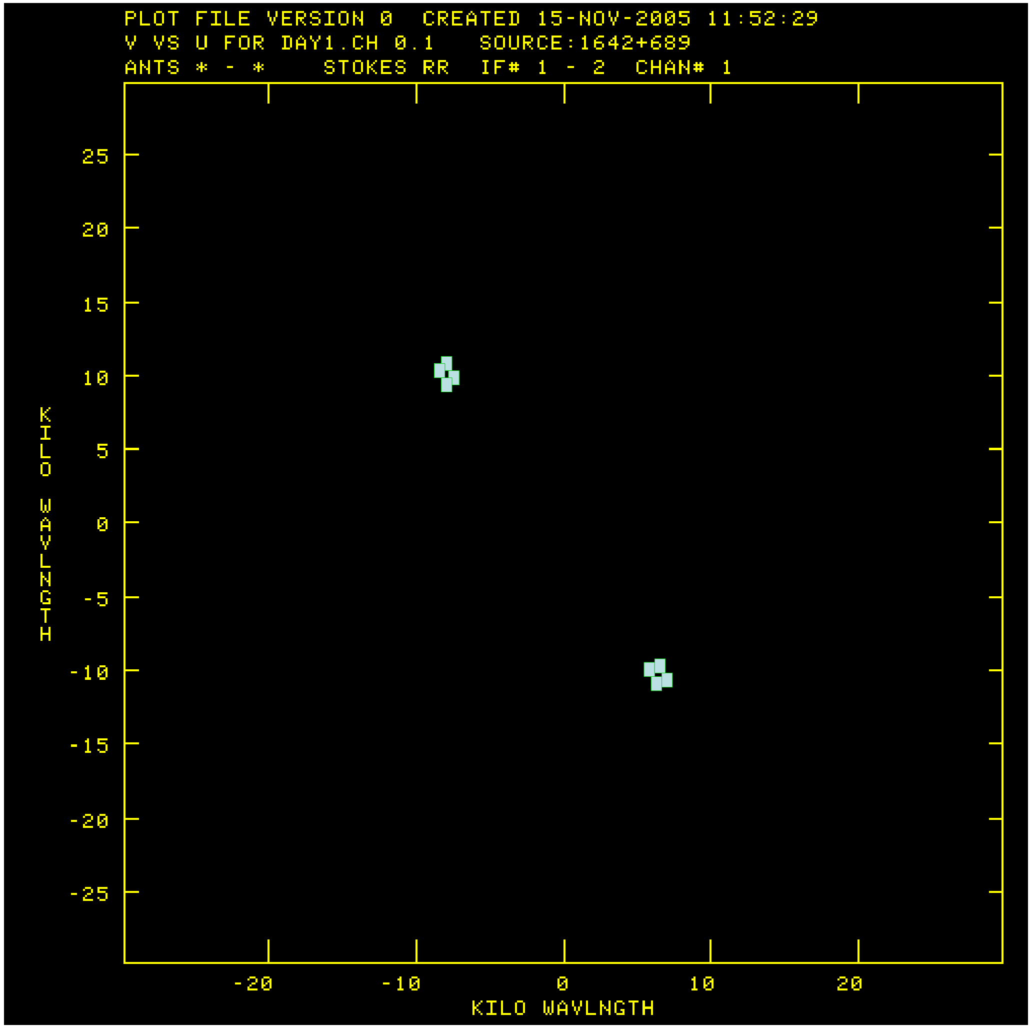

X


PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1   SOURCE:1642+689
ANTS * - *   STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
20


B
metres

The output


X


PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1 SOURCE:1642+689
ANTS * - * STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
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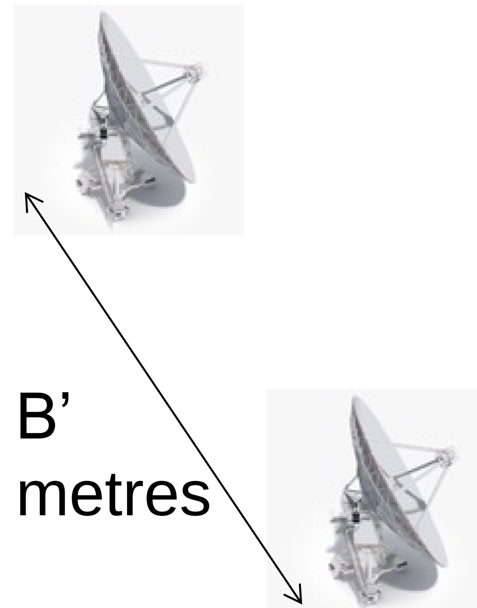
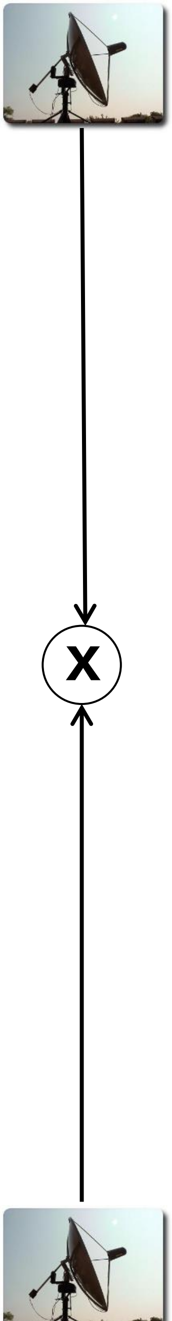
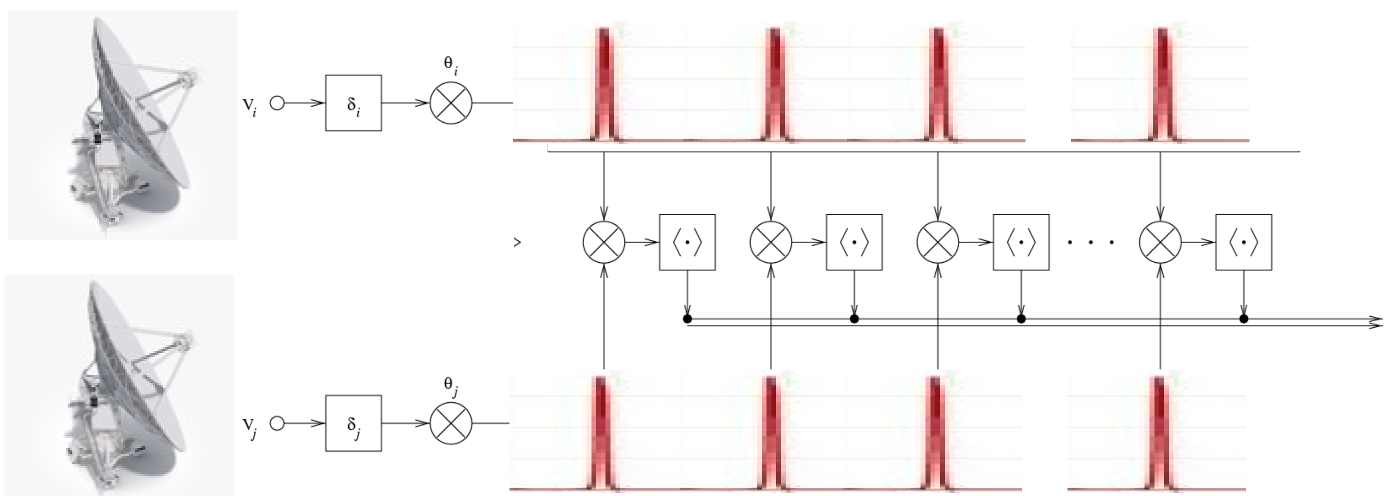
KILO WAVLNGTH

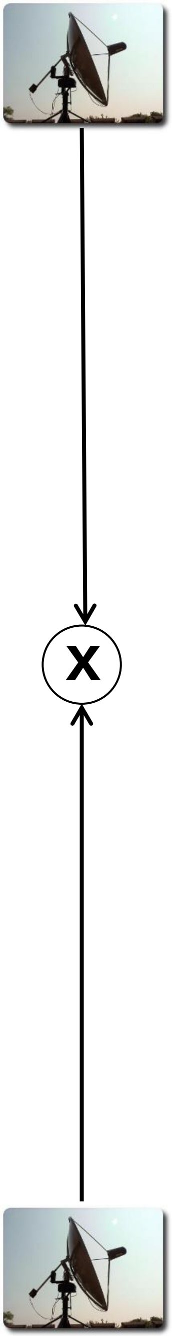

B'
metres



Making it feasible

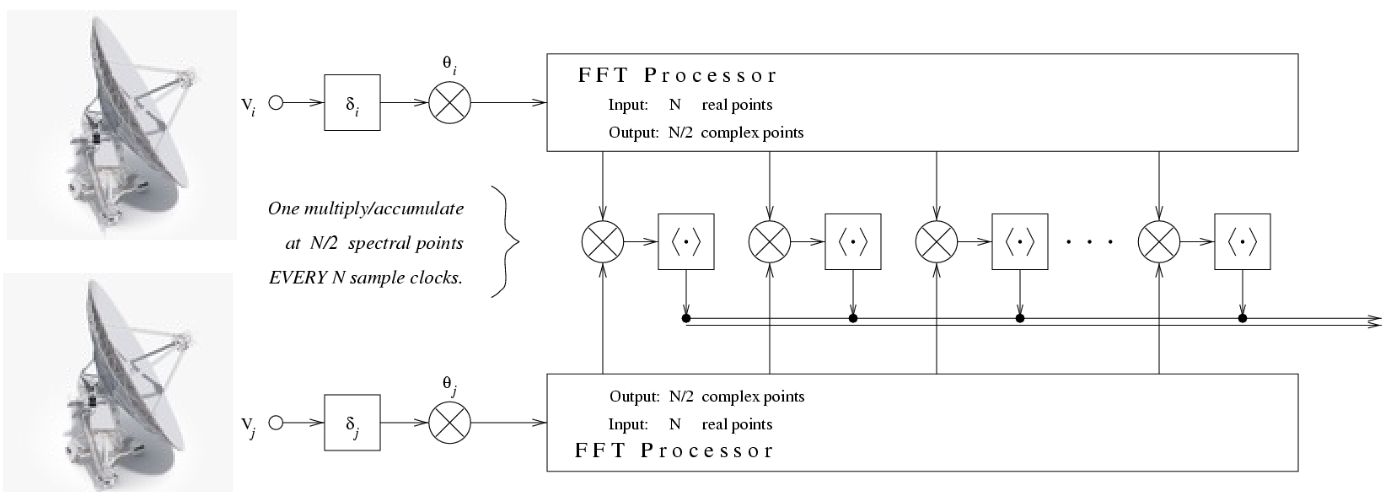
- Analog filters are costly & unstable; expensive and poor performance



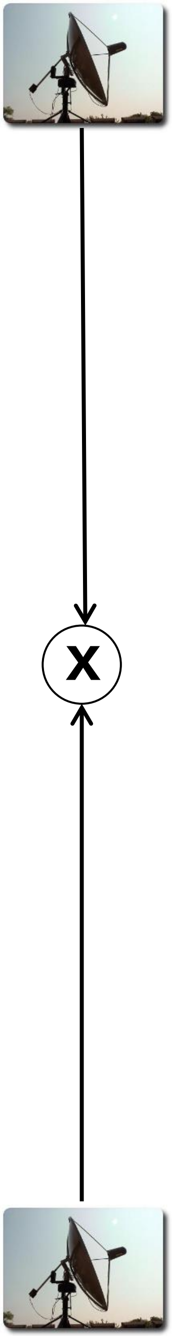


Making it feasible

- Analog filters are costly & unstable; expensive and poor performance
- Fortunately, we can (and do) digitize the signal – meaning we can use a digital substitute: **digital filterbank**

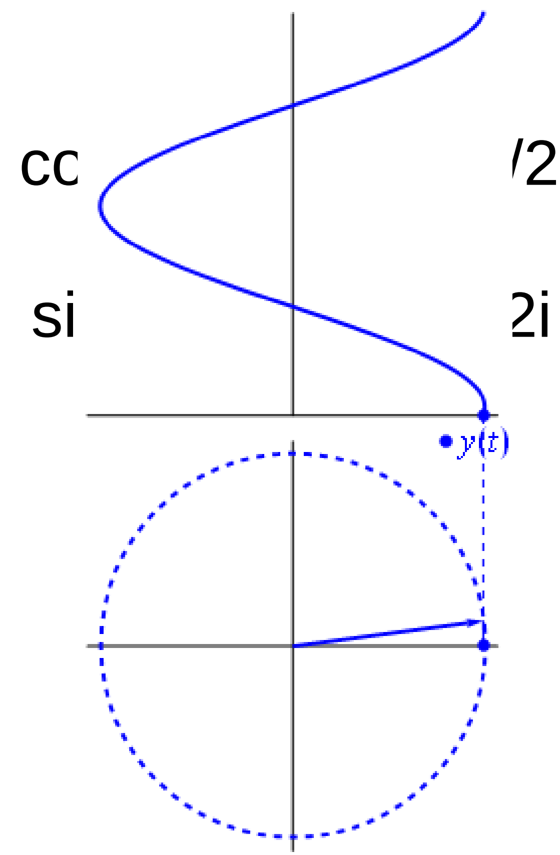


The advantage of going digital



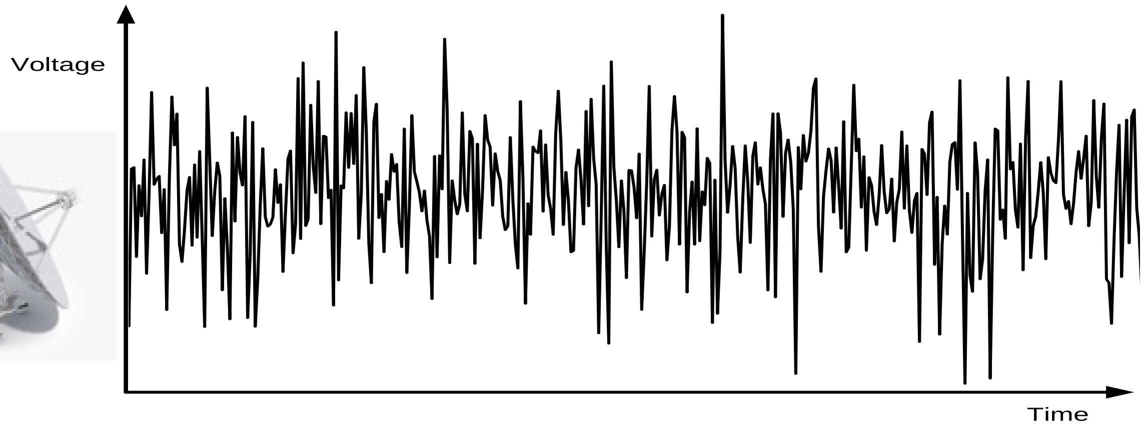
- Stable, cheap filters
- Produces complex output: use a 1 complex multiplier rather than 2 real multipliers and a phase shift

$e^{i\phi} = \cos\phi + i \sin\phi$



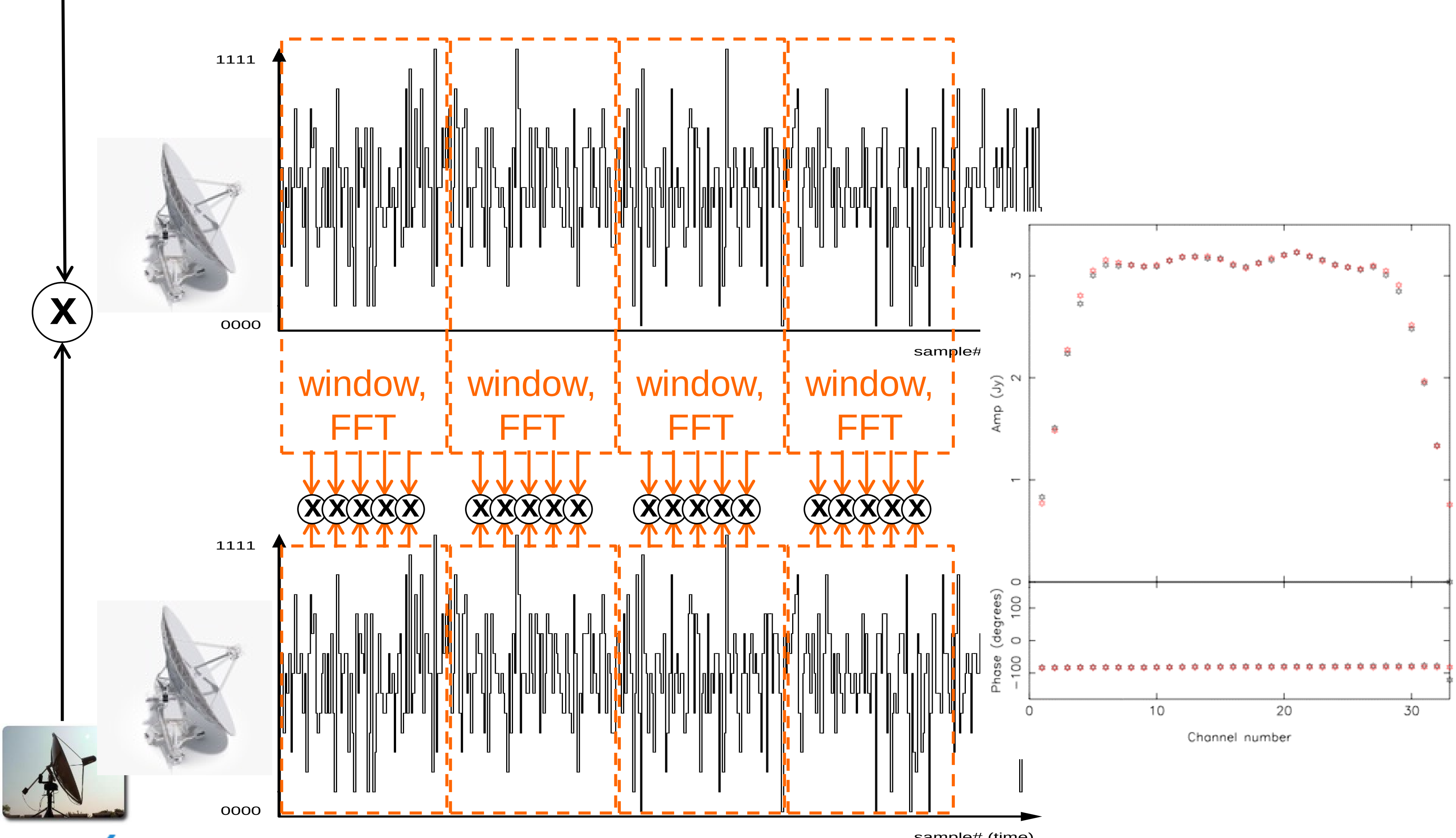
Animation from http://en.wikipedia.org/wiki/File:Unfasor.gif

The "FX" correlator



X

The “FX” correlator



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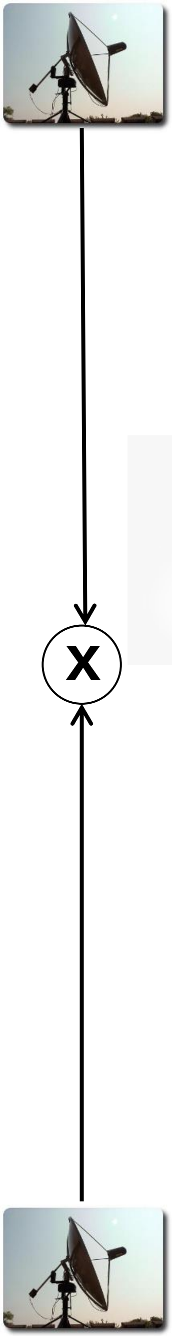
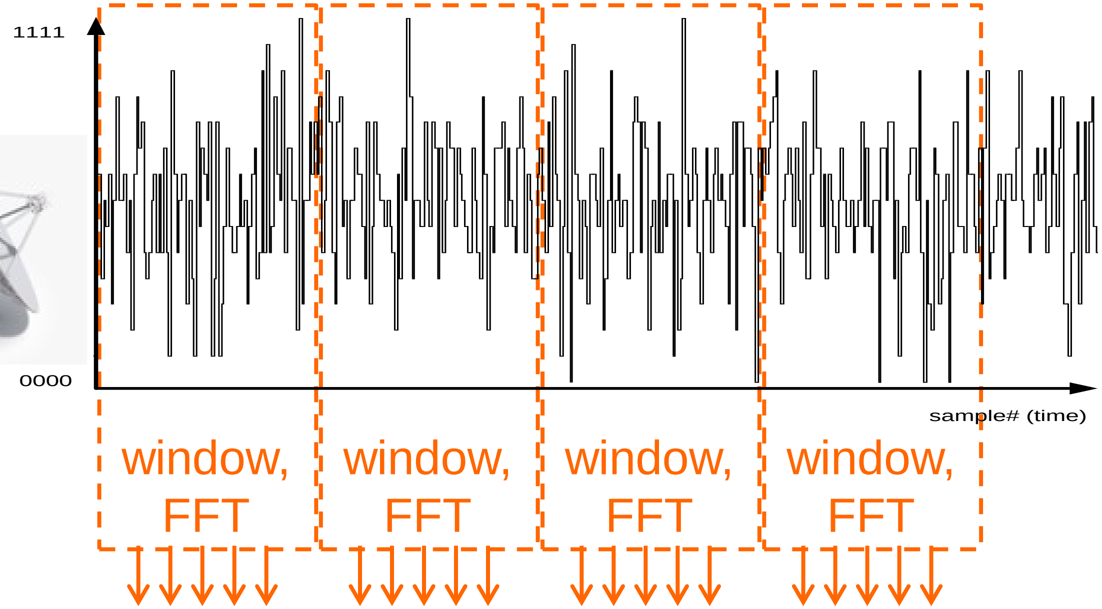
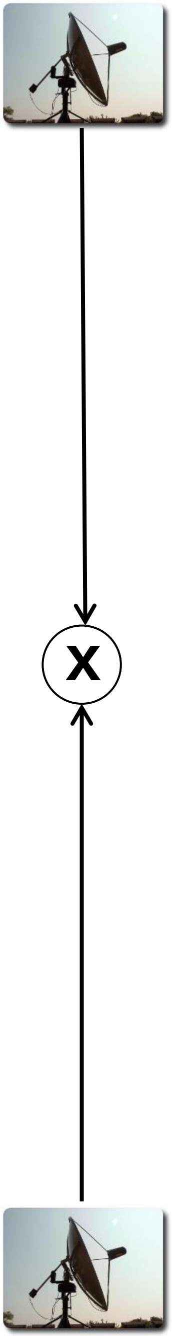
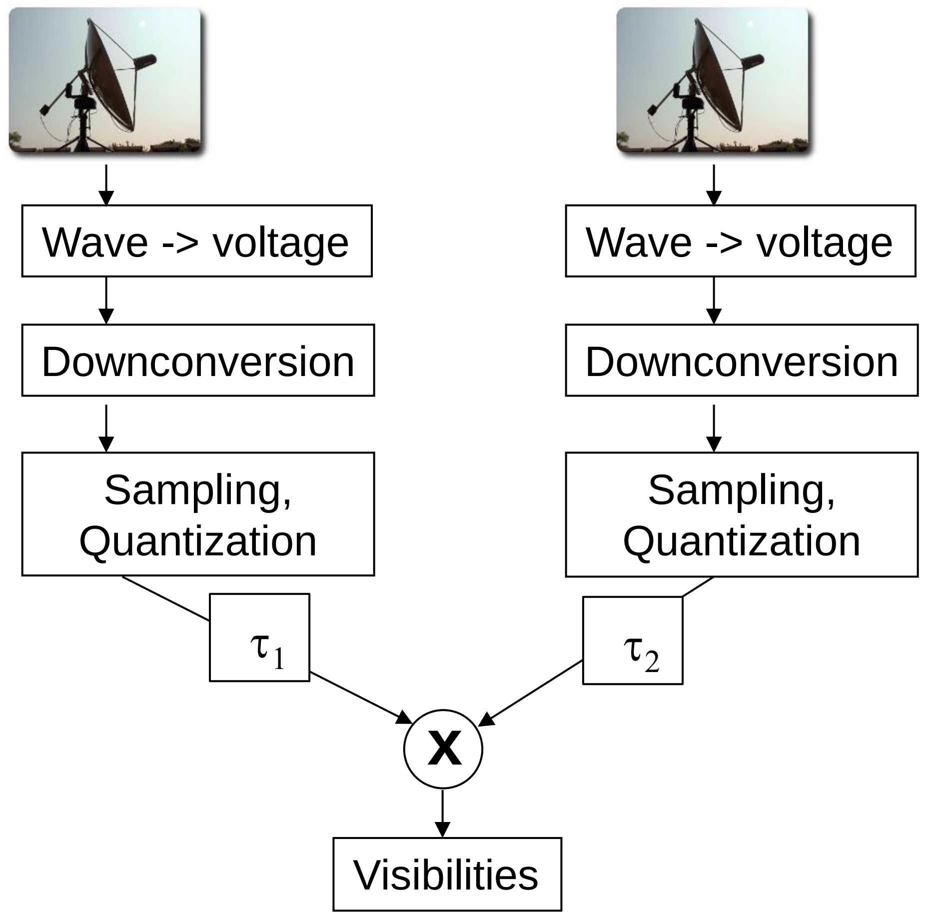


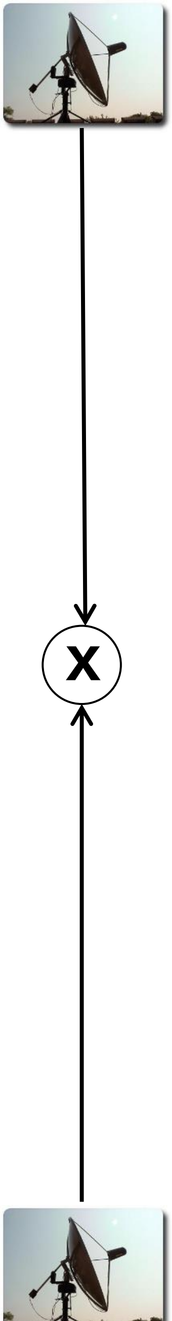


- Since this architecture consists of a Fourier transform (F) followed by cross-multiplication (X), we dub this the "FX" correlator

But first, we must compensate

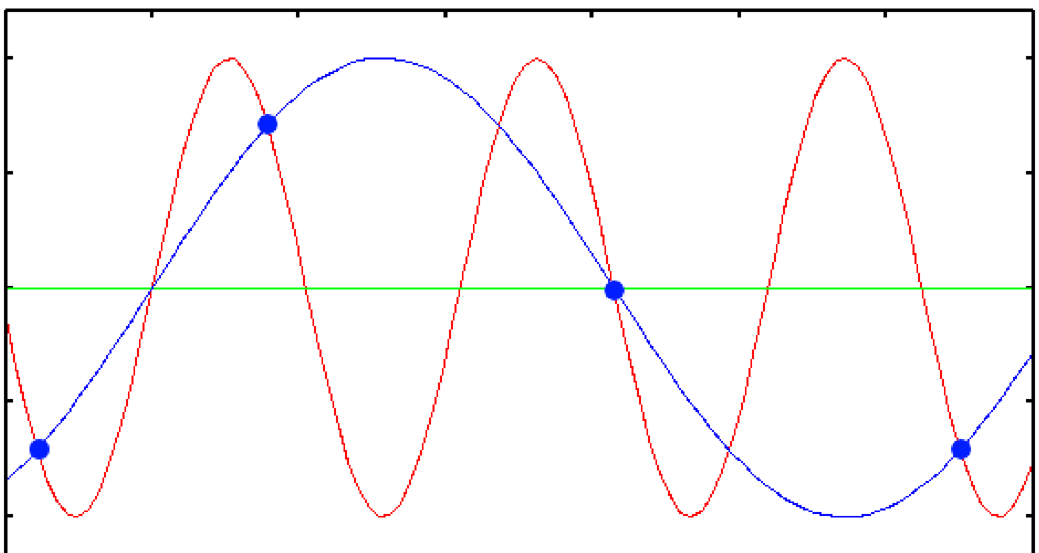






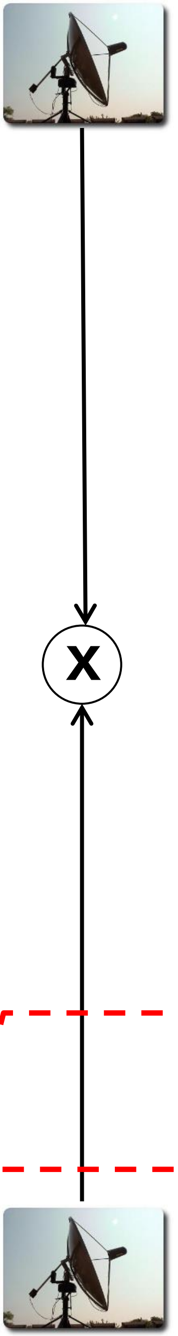
Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every Δt sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies ≥ 1 / (2 Δt) Hz (*band-limited*)



Adequately sampled

Undersampled, cannot be reconstructed



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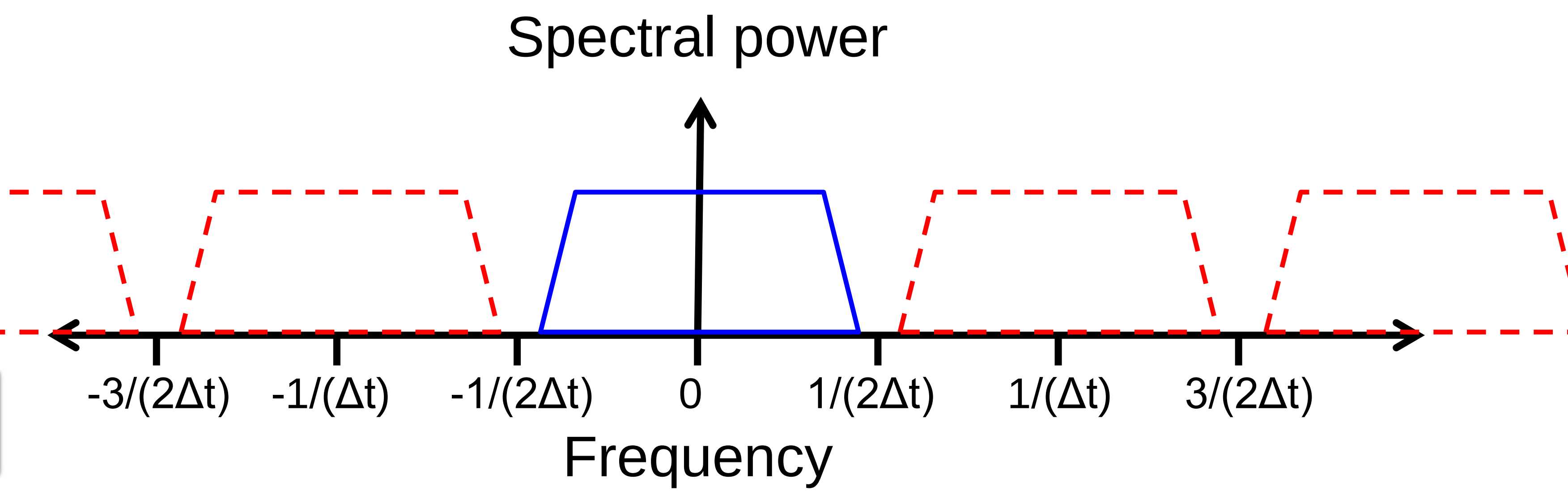

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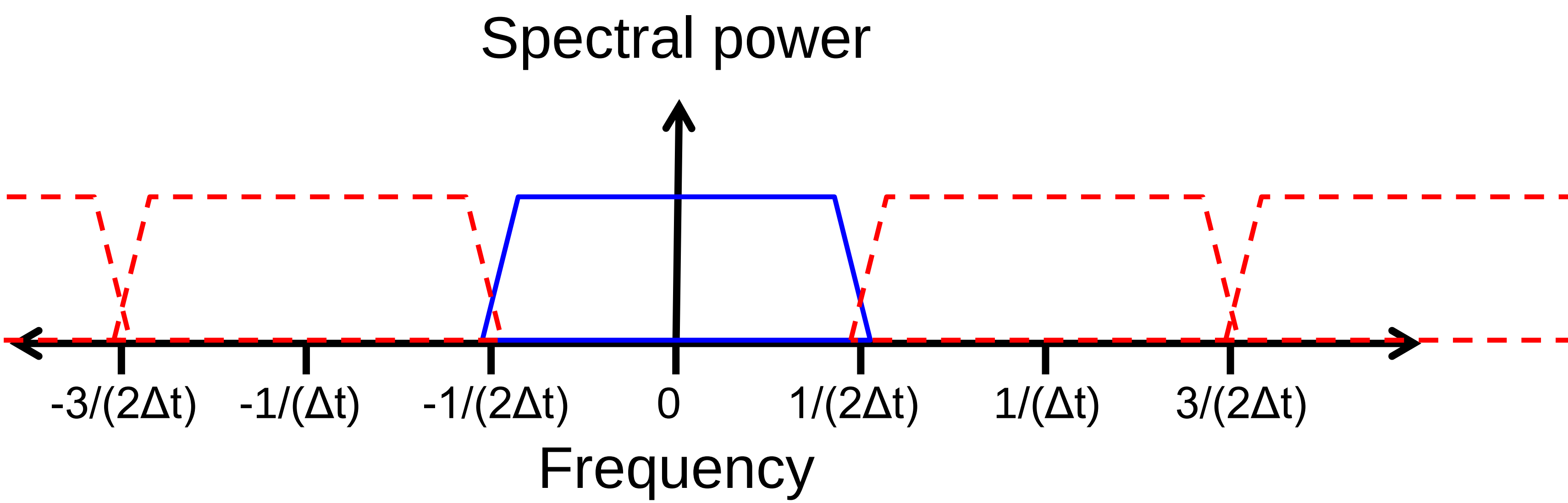
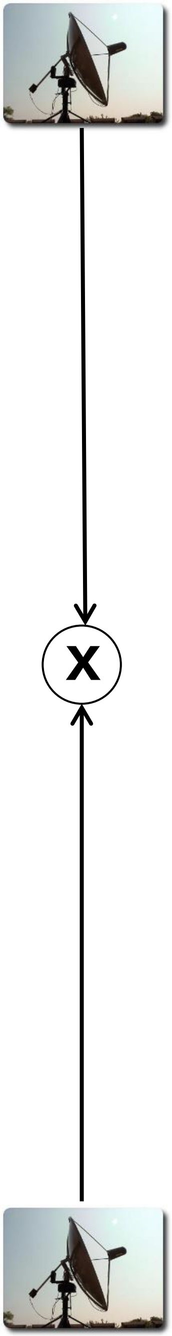
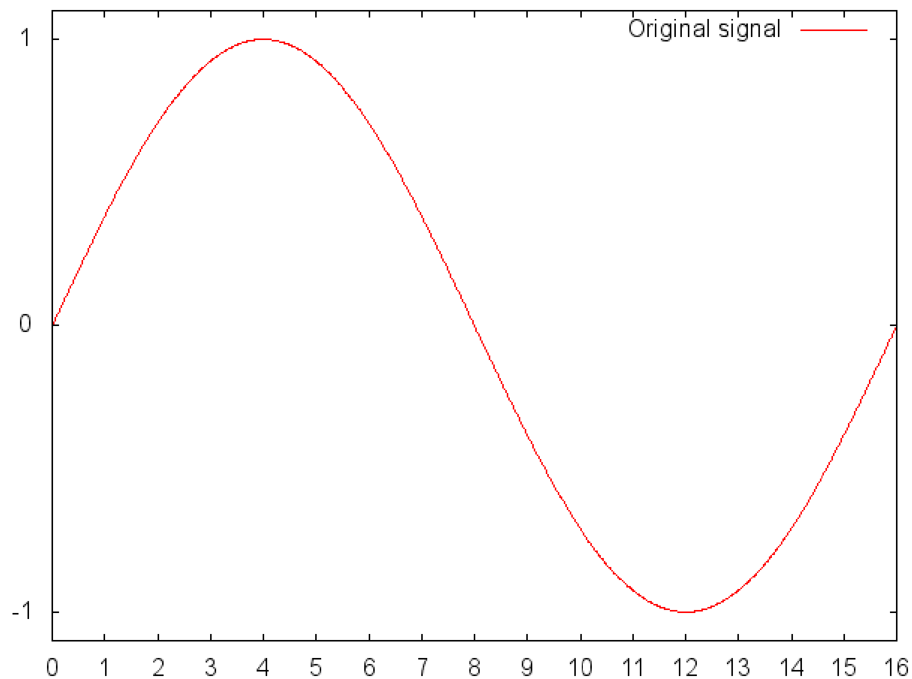




Quantization

- When correlation is low (almost always) even very coarse quantization is ok!



Sensitivity loss:

Quantization

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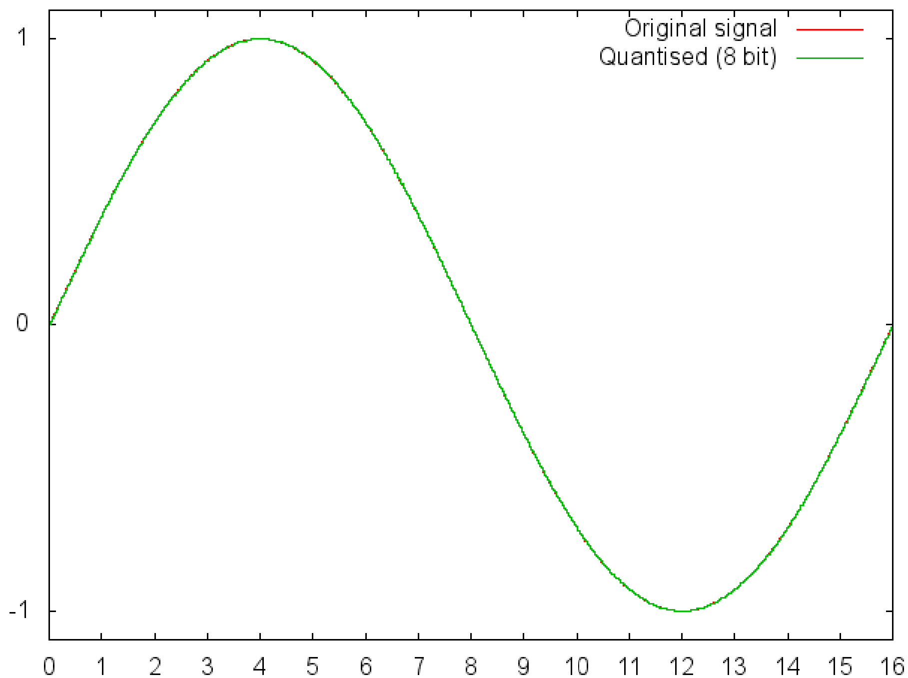

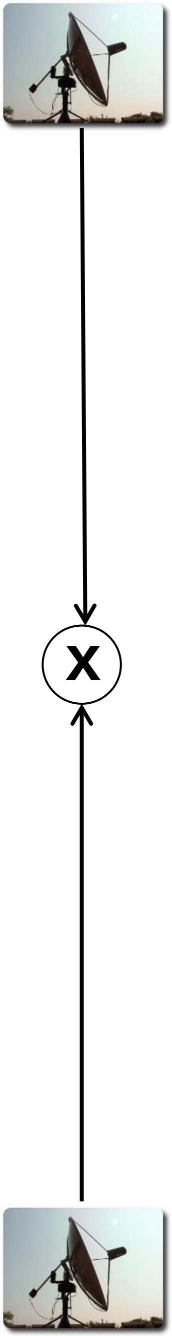

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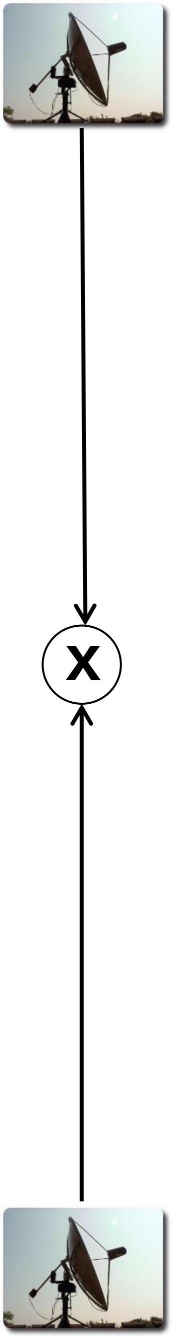

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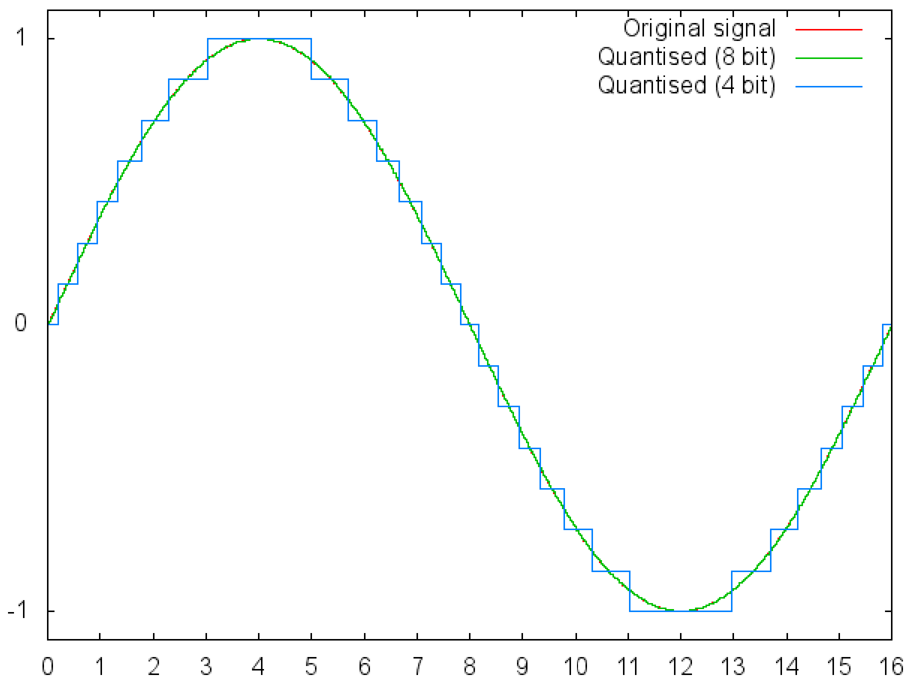

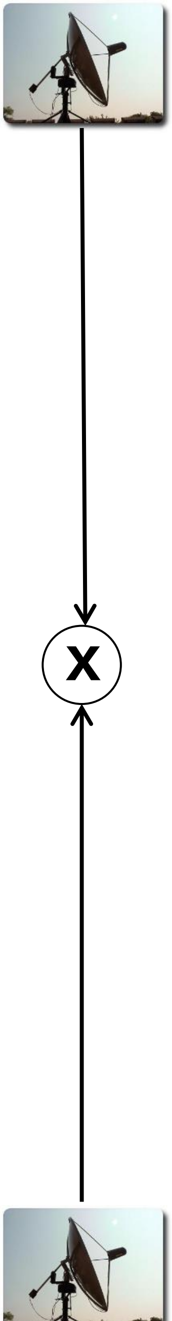

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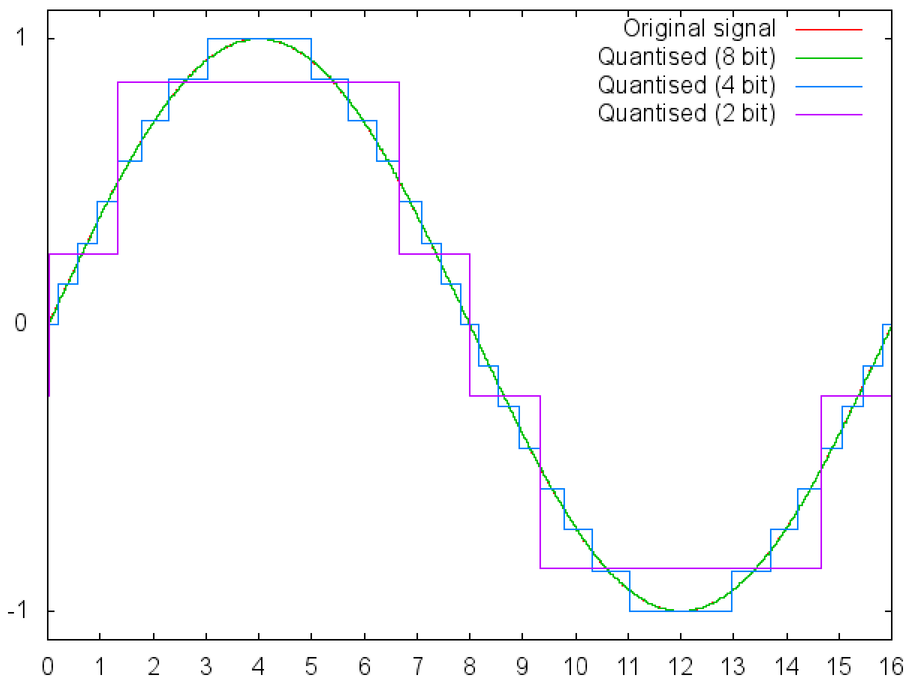
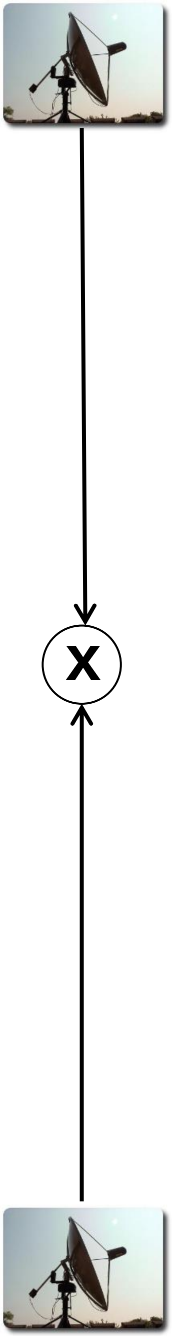


Quantization



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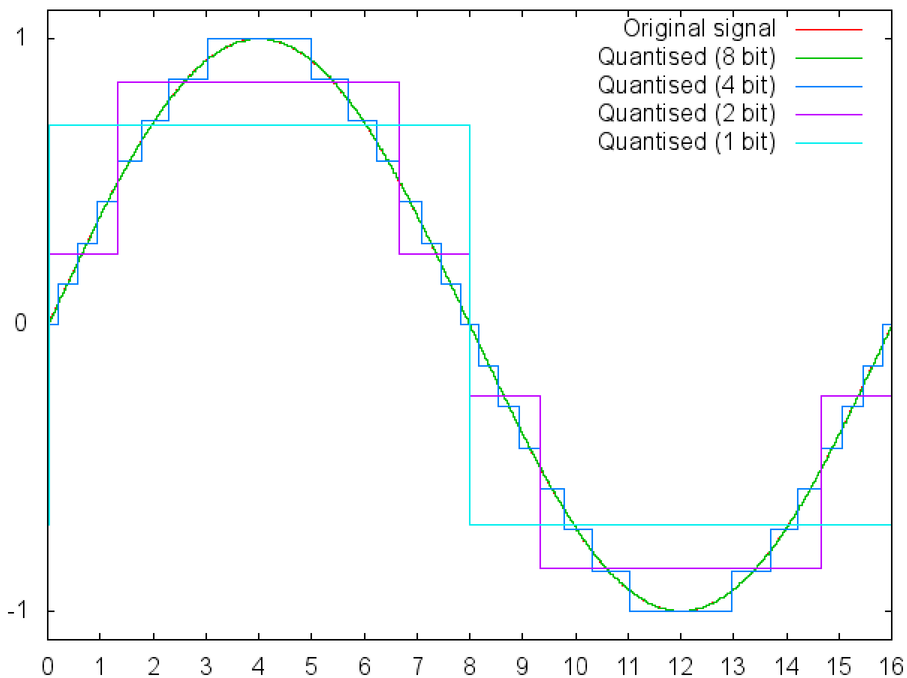
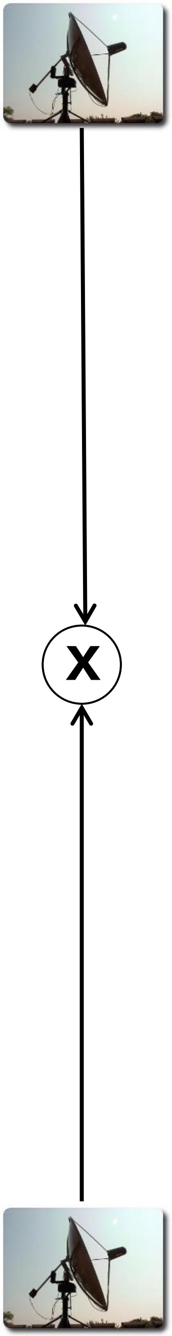


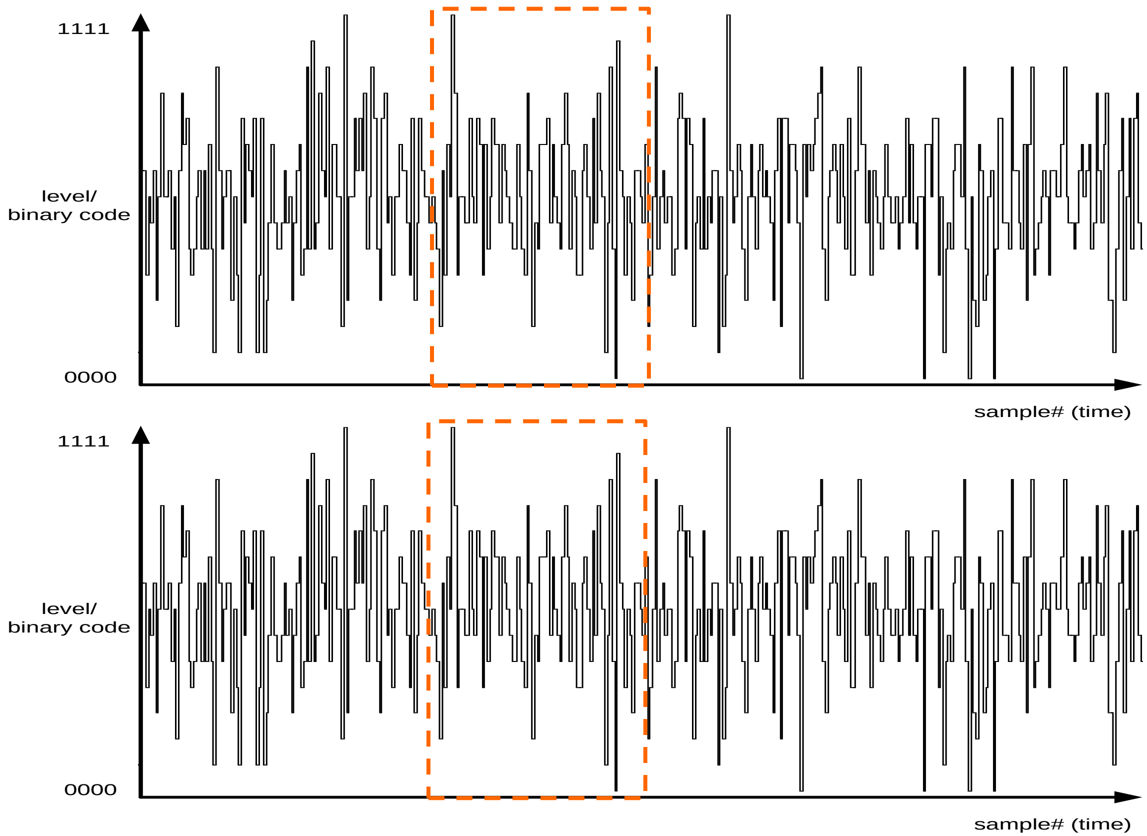
Sensitivity loss:
8 bit: 0.1%
4 bit: 1.3%
2 bit: 12%
1 bit: 36%

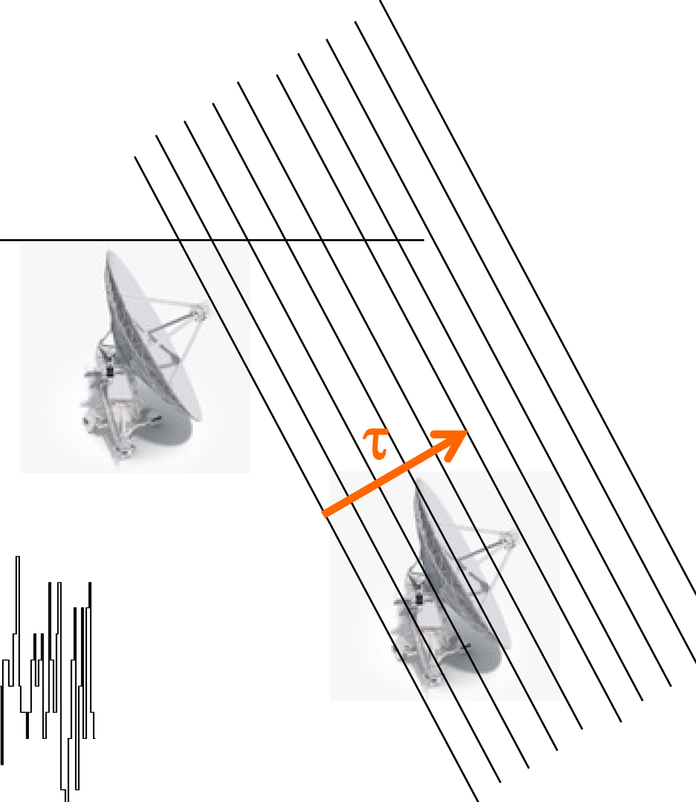
Correct visibility amplitudes for this sensitivity loss (done after correlation, exact correction depends on correlation level)

Delay compensation

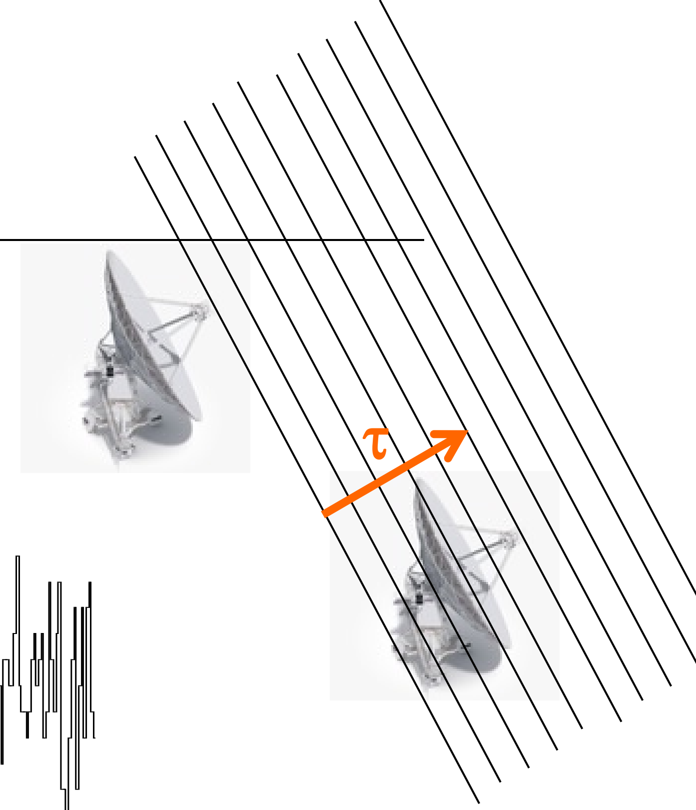
- Delay to the nearest sample is easy:







Delay compensation



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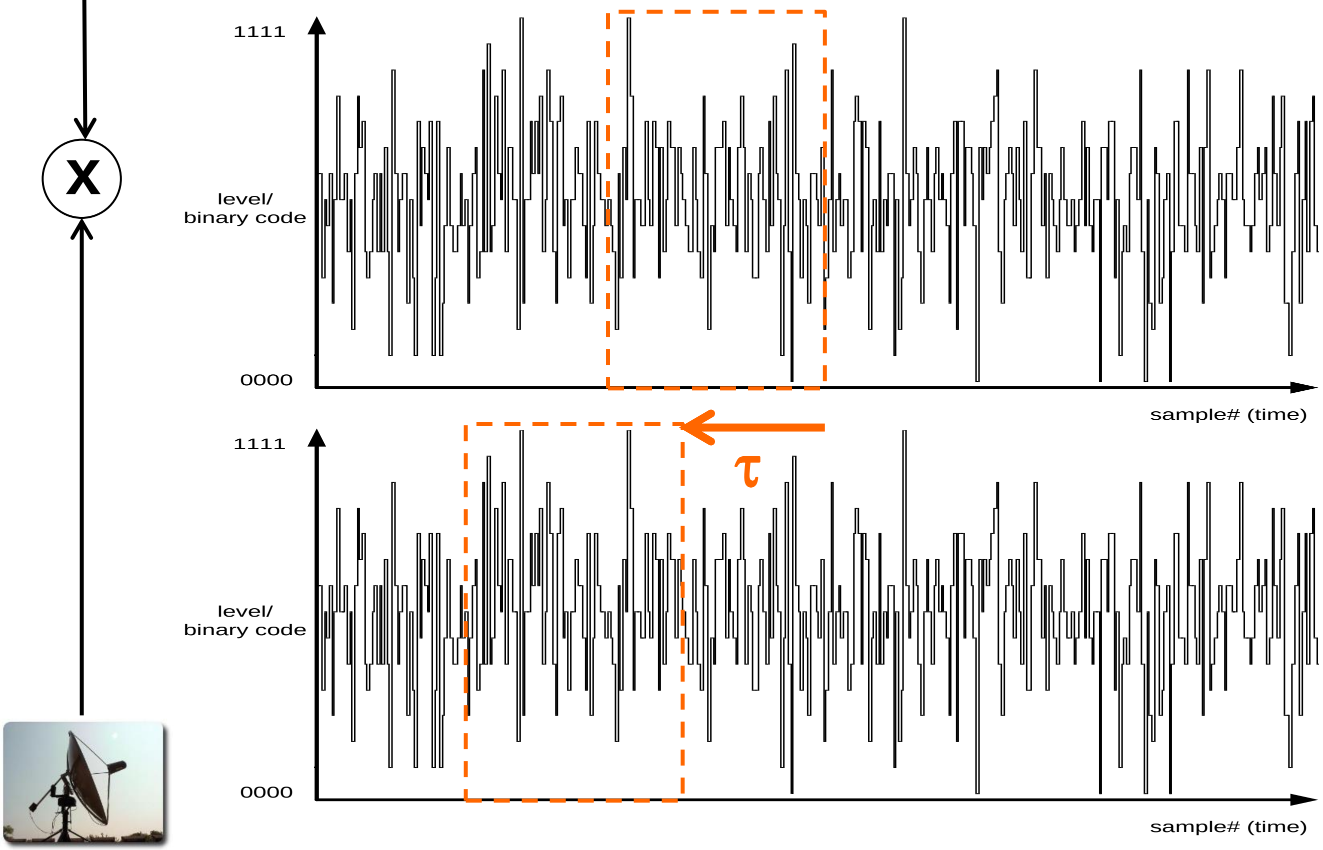
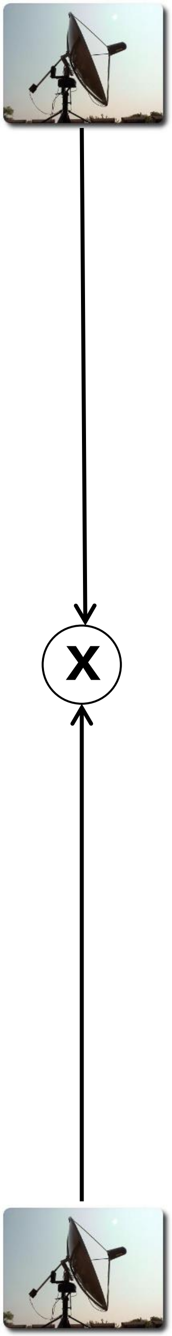
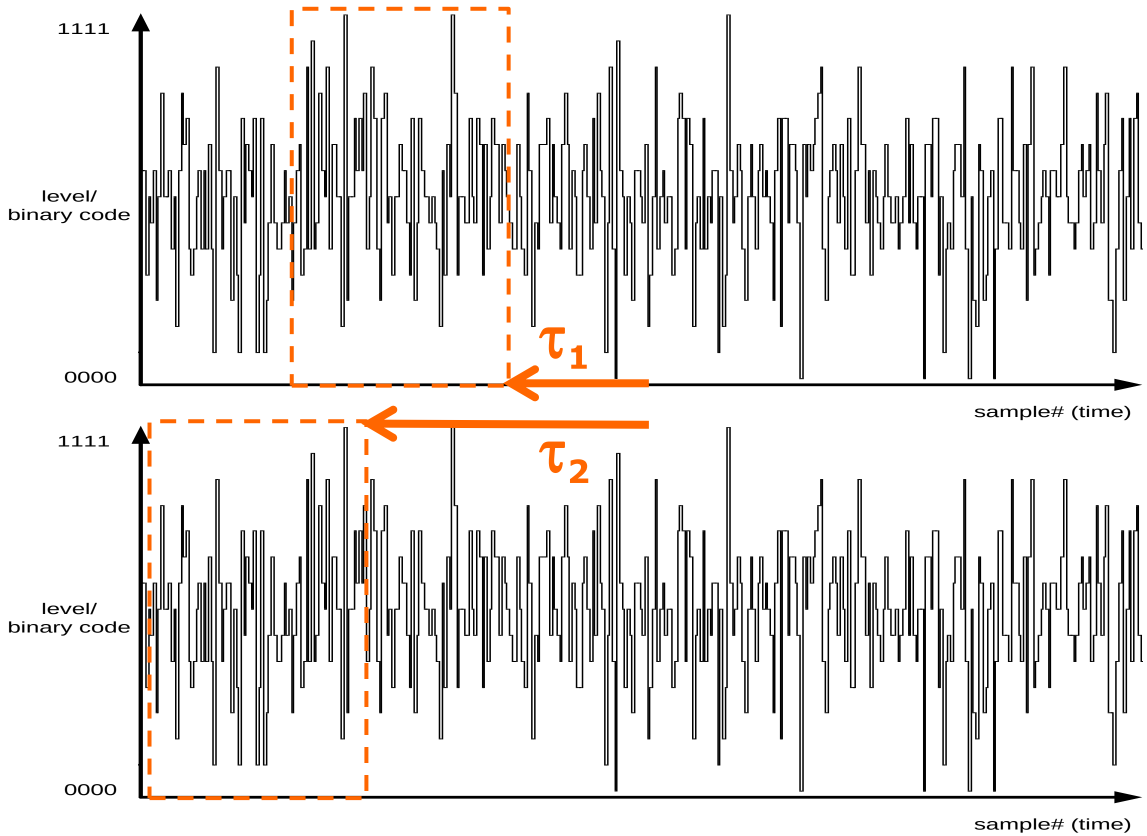
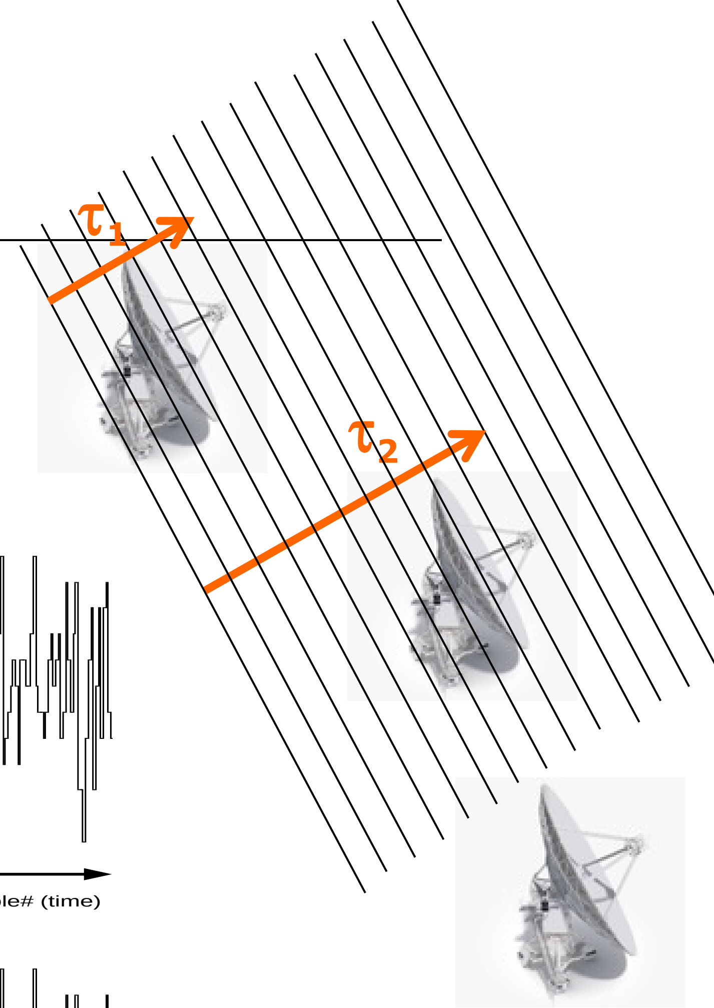


Delay compensation

- In practise, delay all to common reference

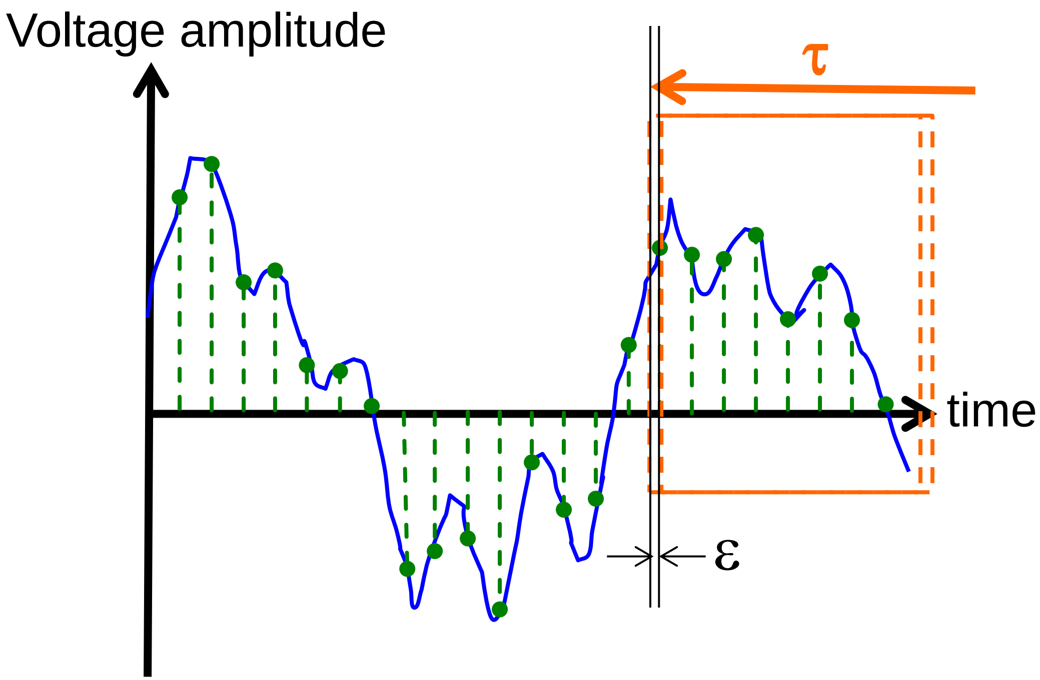






Fractional-sample correction

- Sampling prevents perfect alignment of datastreams; always a small error



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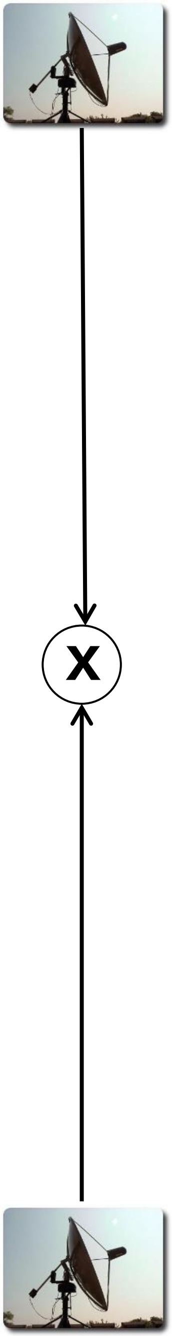

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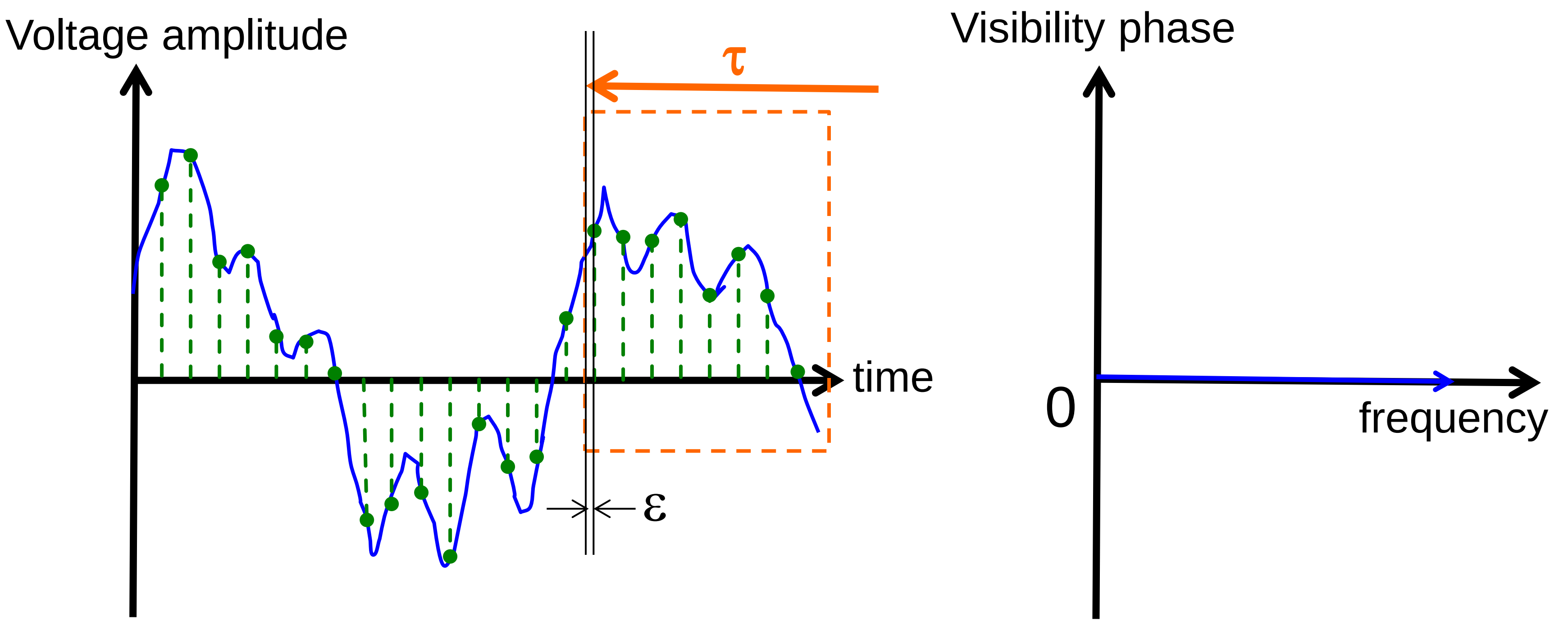

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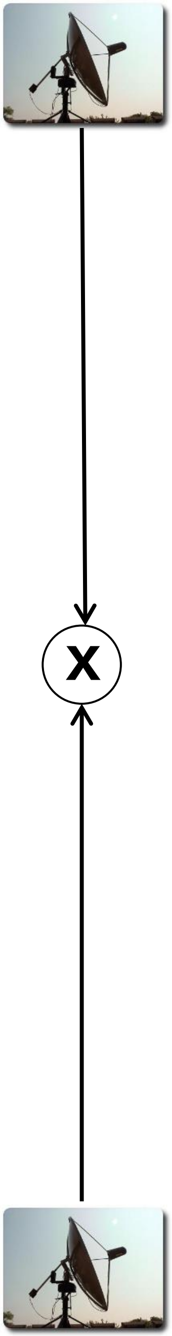

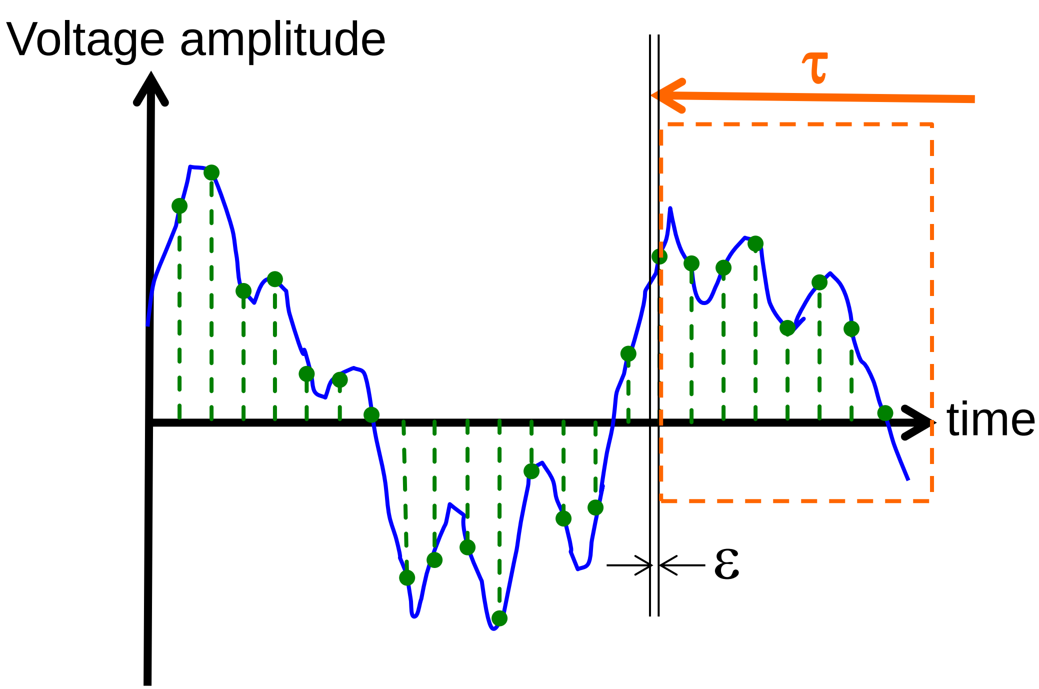

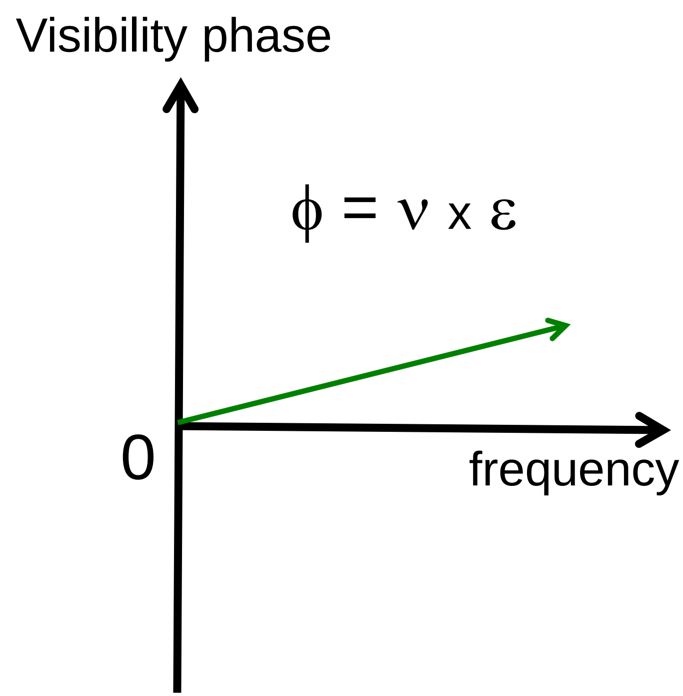

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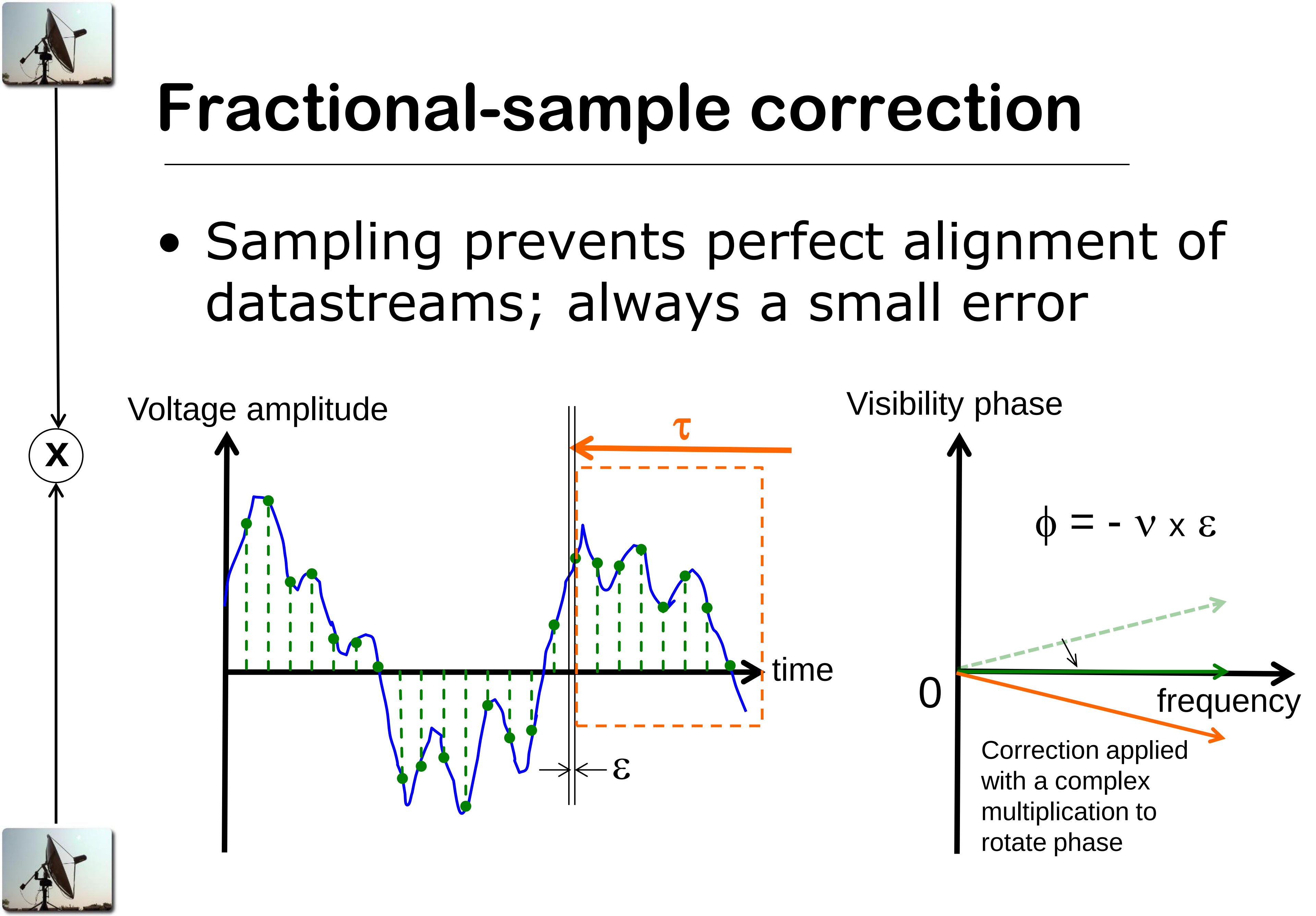
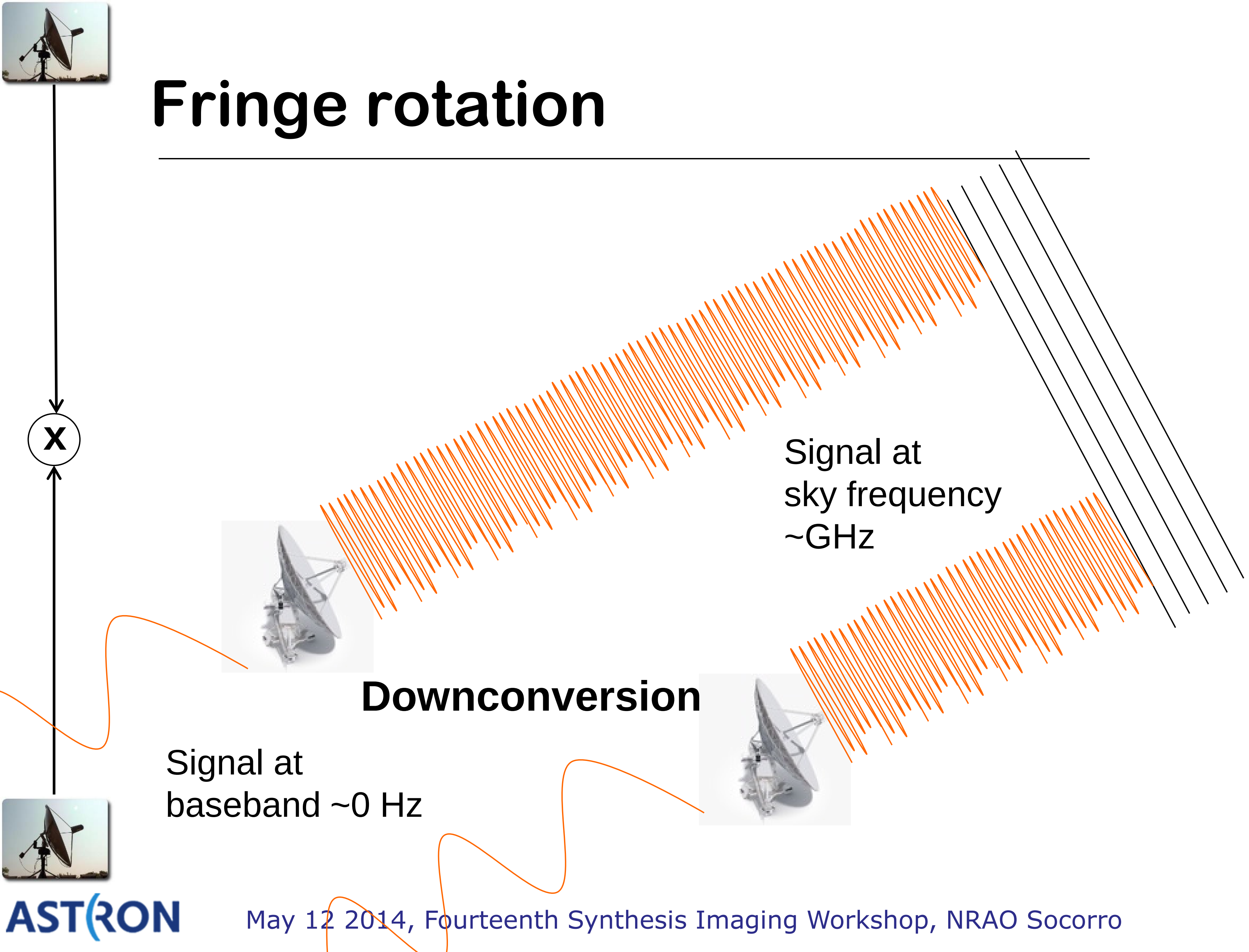
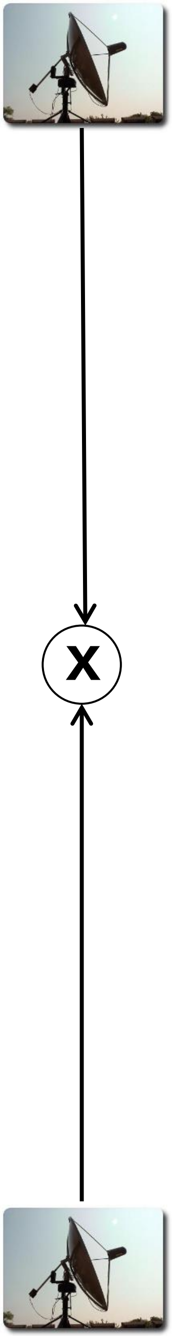


Fringe rotation

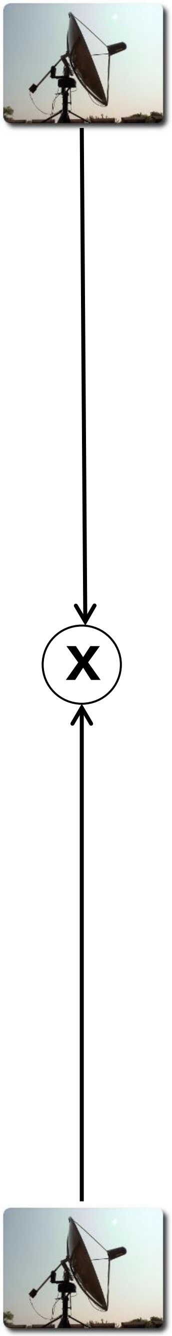




Fringe rotation

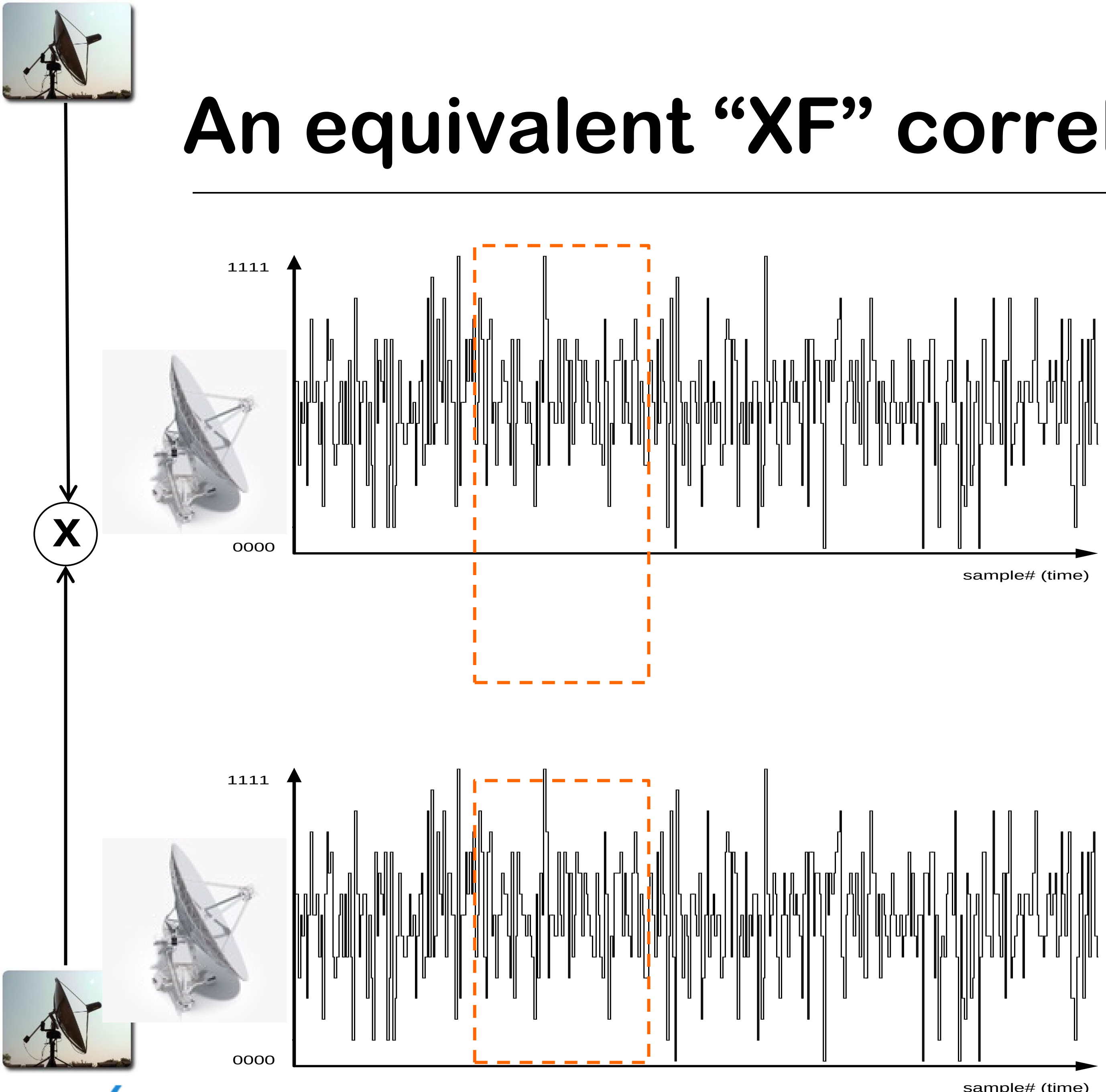
- Implementation: rotate phase using complex multiplier
- $\Delta\phi = 2\pi \nu_0 \tau_g$ ν_0 = sky frequency,
  τ_g = applied delay
- Most accurate: apply to voltages directly (time domain)
  - if τ_g is changing slowly (short baseline length), approximate as constant for short time, apply after FFT (frequency domain)

Alternate implementation



- We have shown how to build a practical FX correlator, which first Fourier transforms and then multiplies
- Convolution theorem: **Multiplication** in the frequency domain is equivalent to **convolution** in the time domain
- It is mathematically equivalent to convolve the two signals in the time domain and then Fourier transform

An equivalent “XF” correlator



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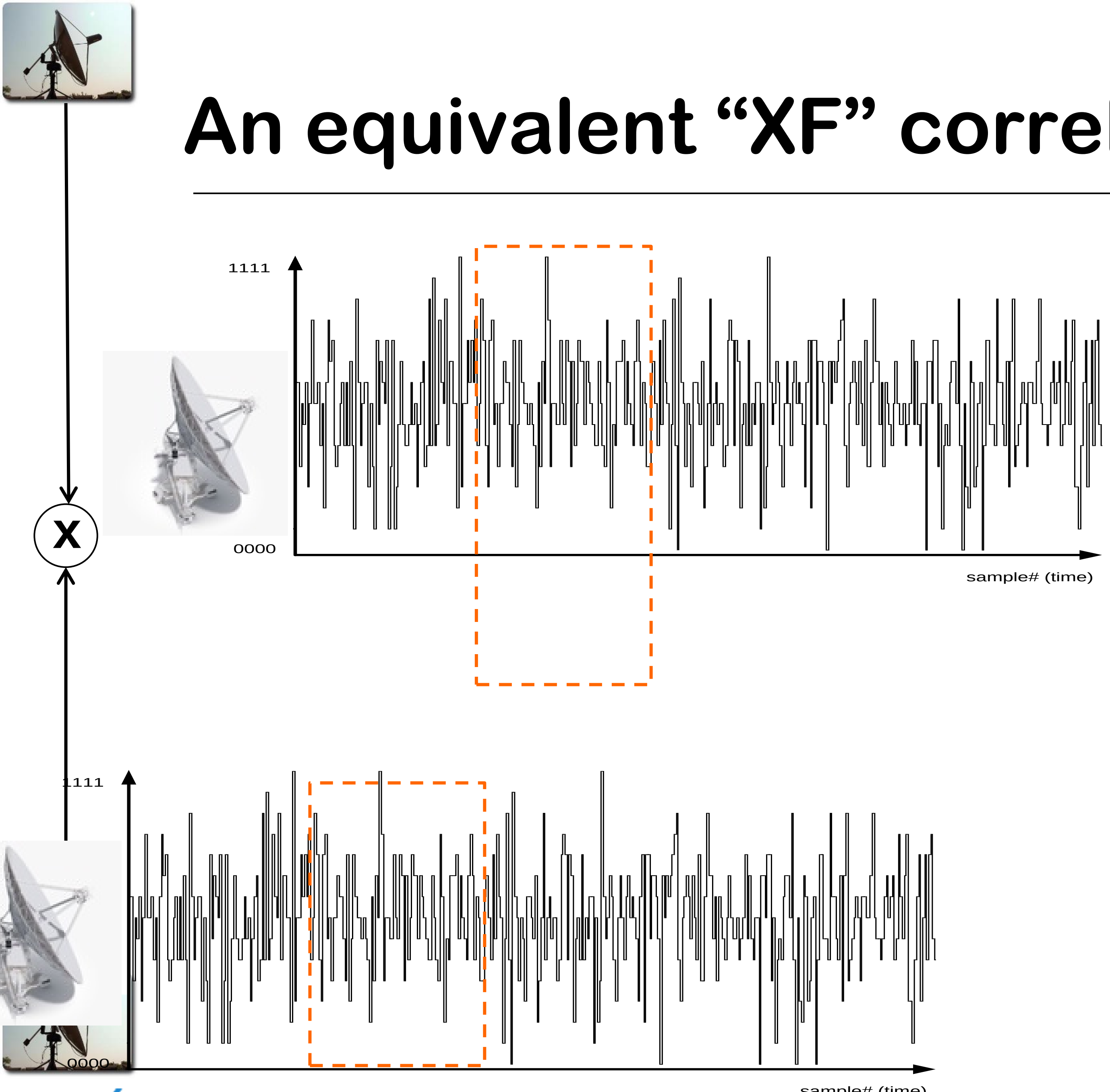

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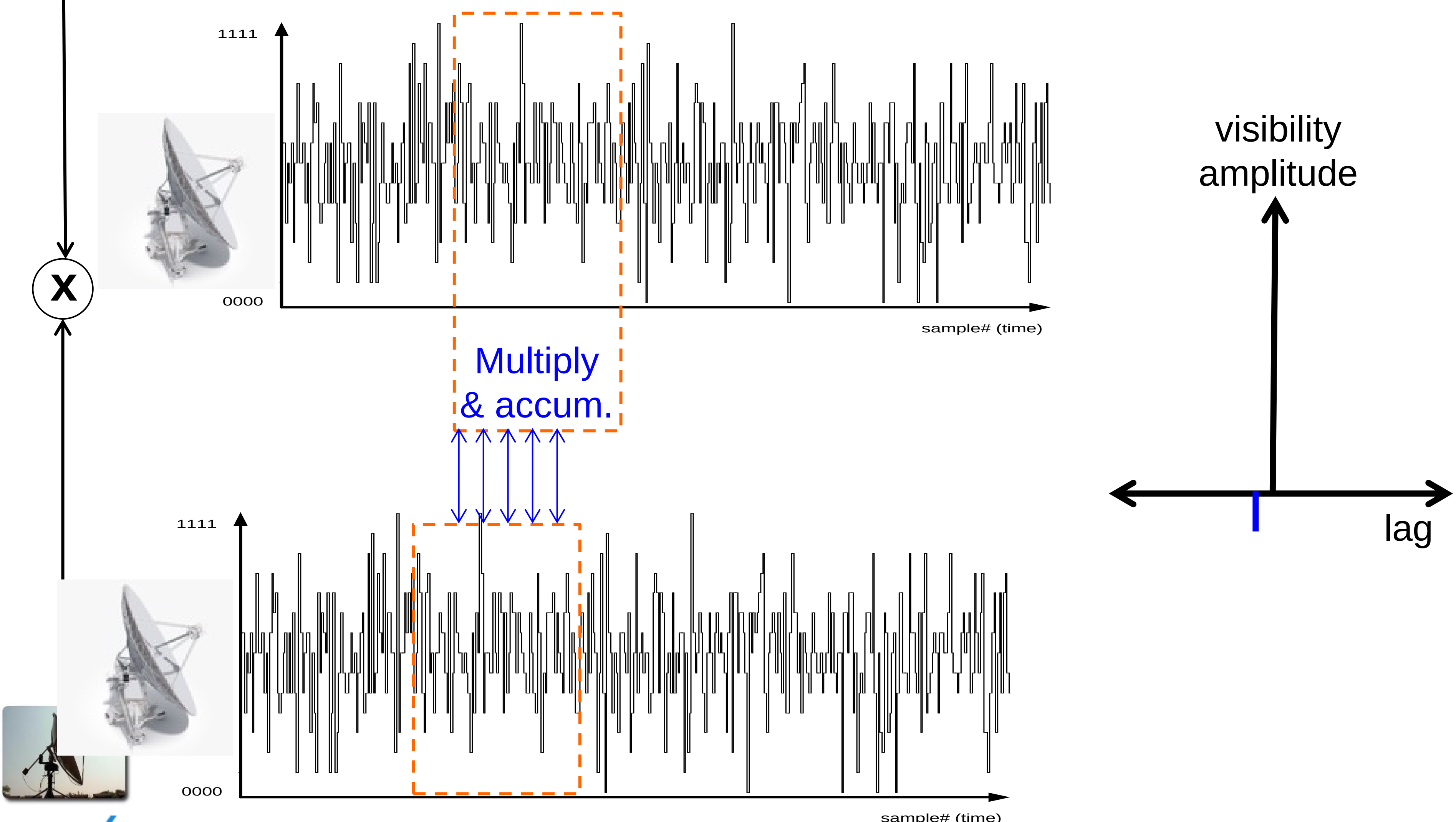

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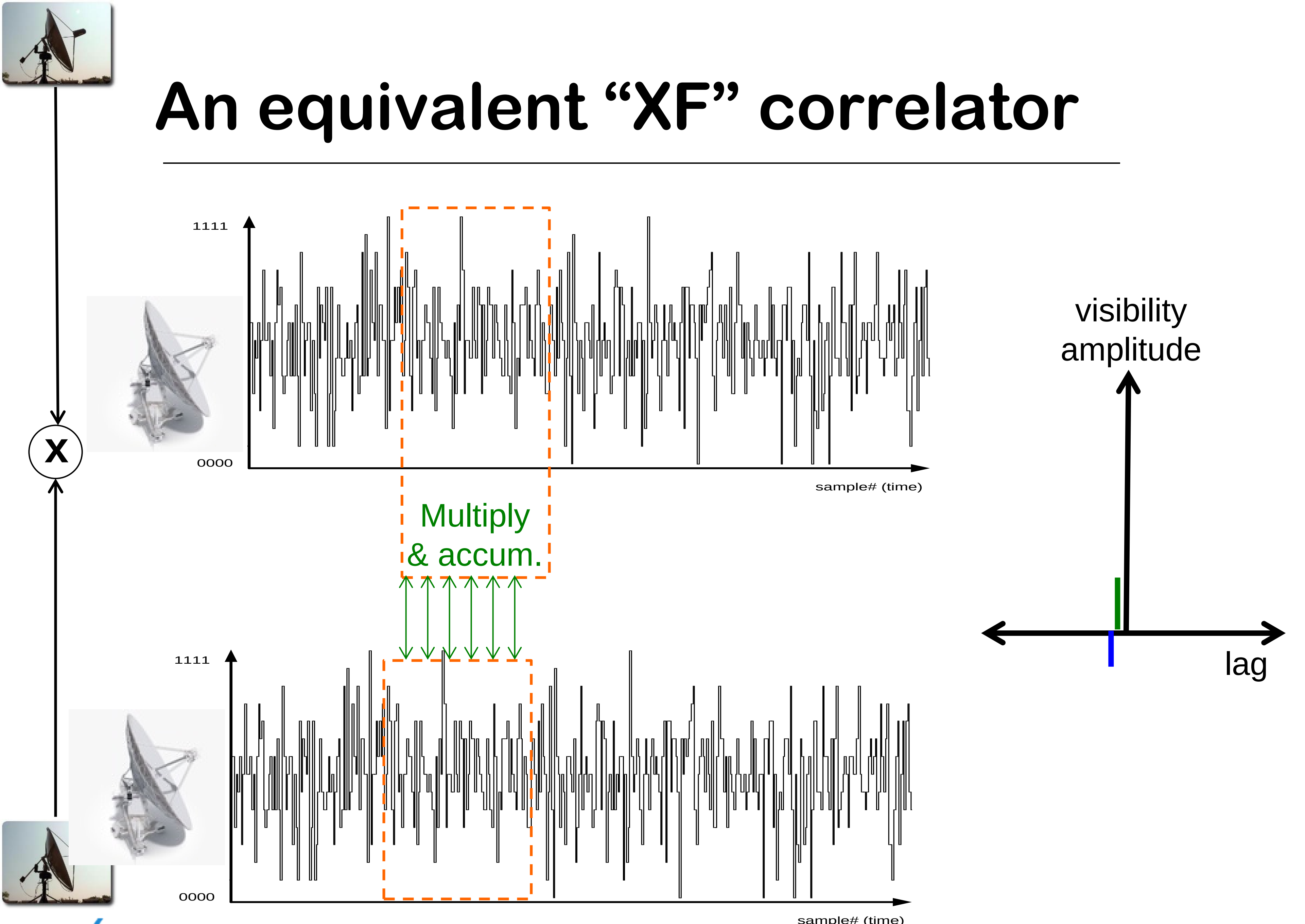
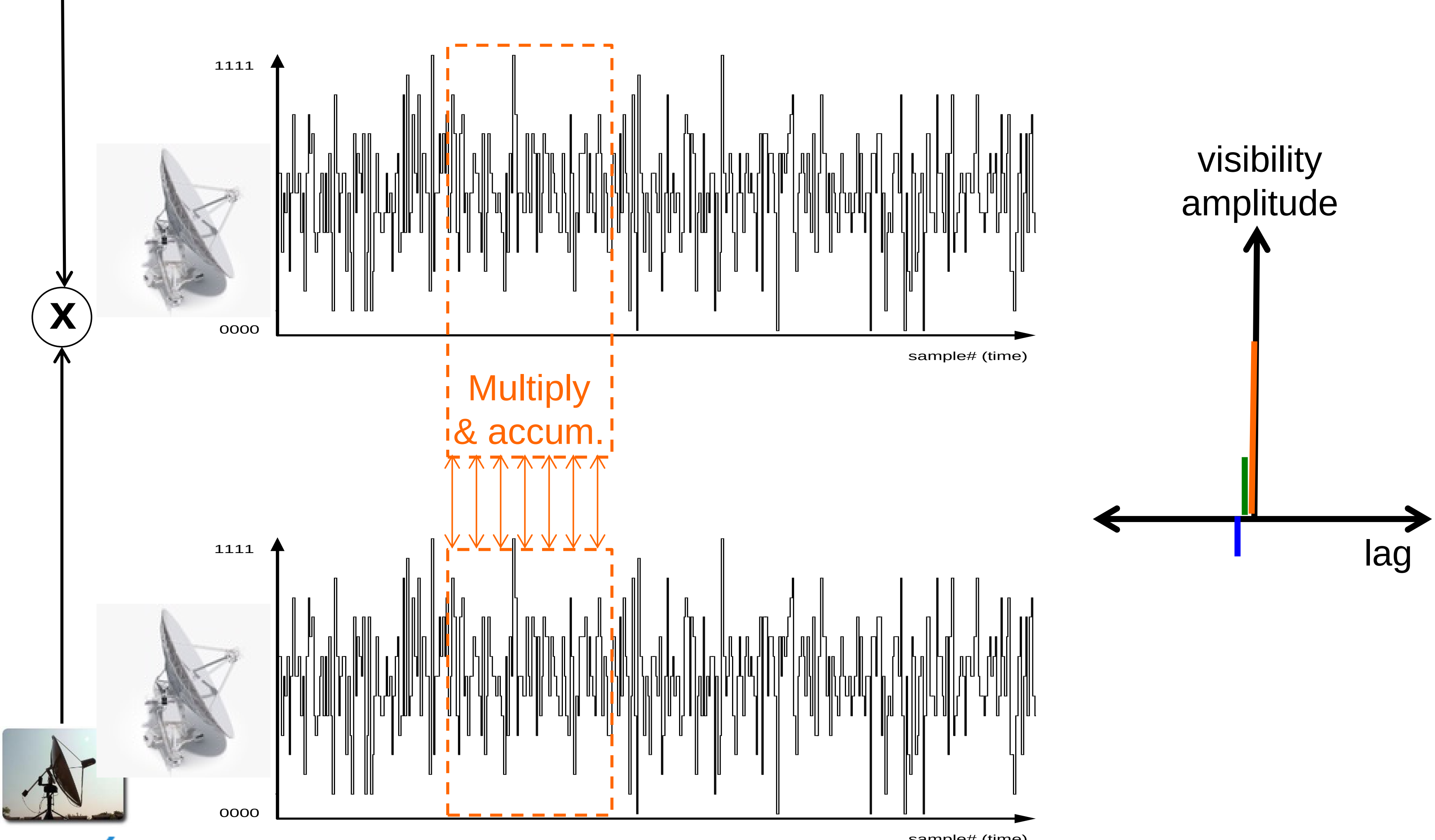
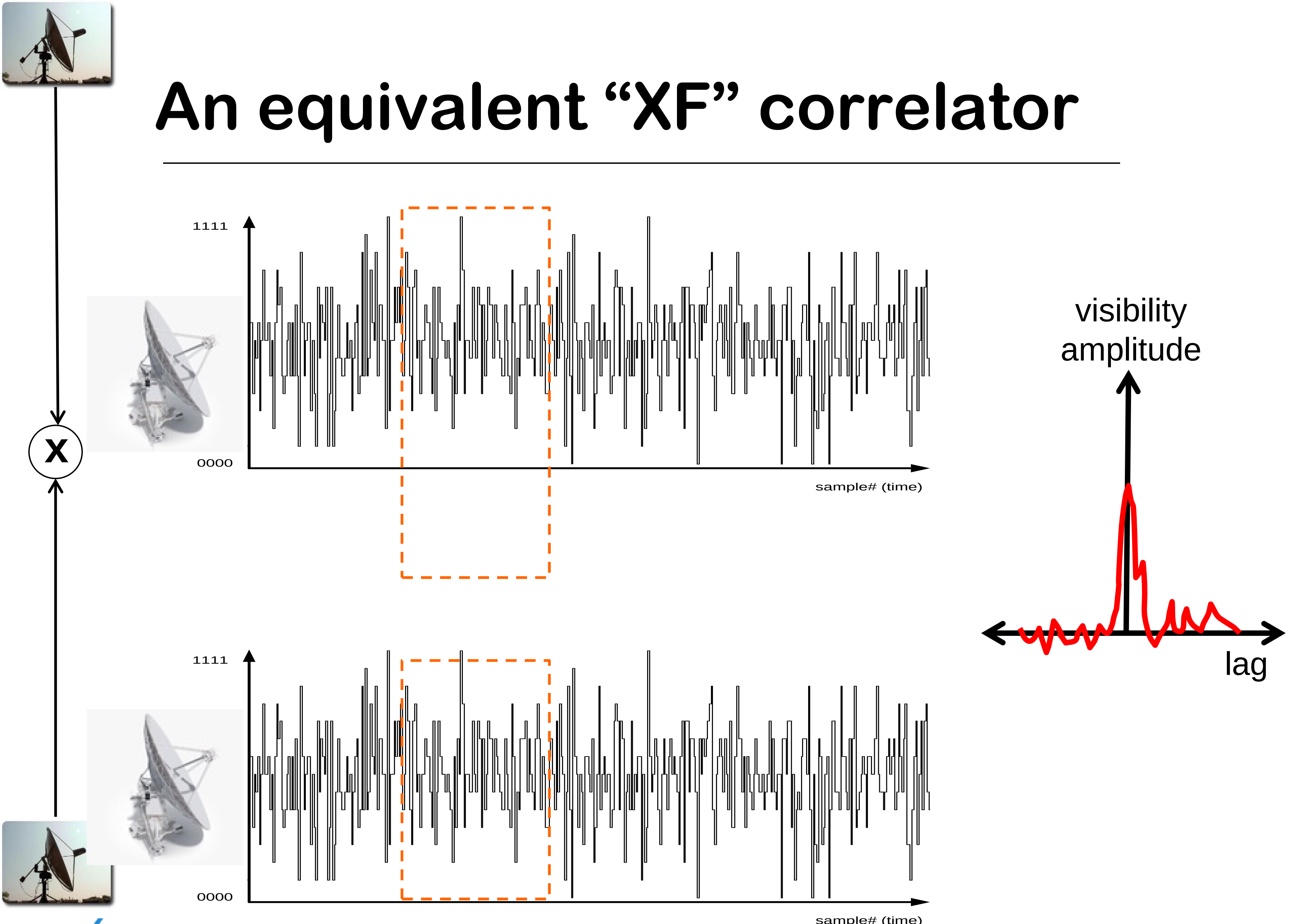


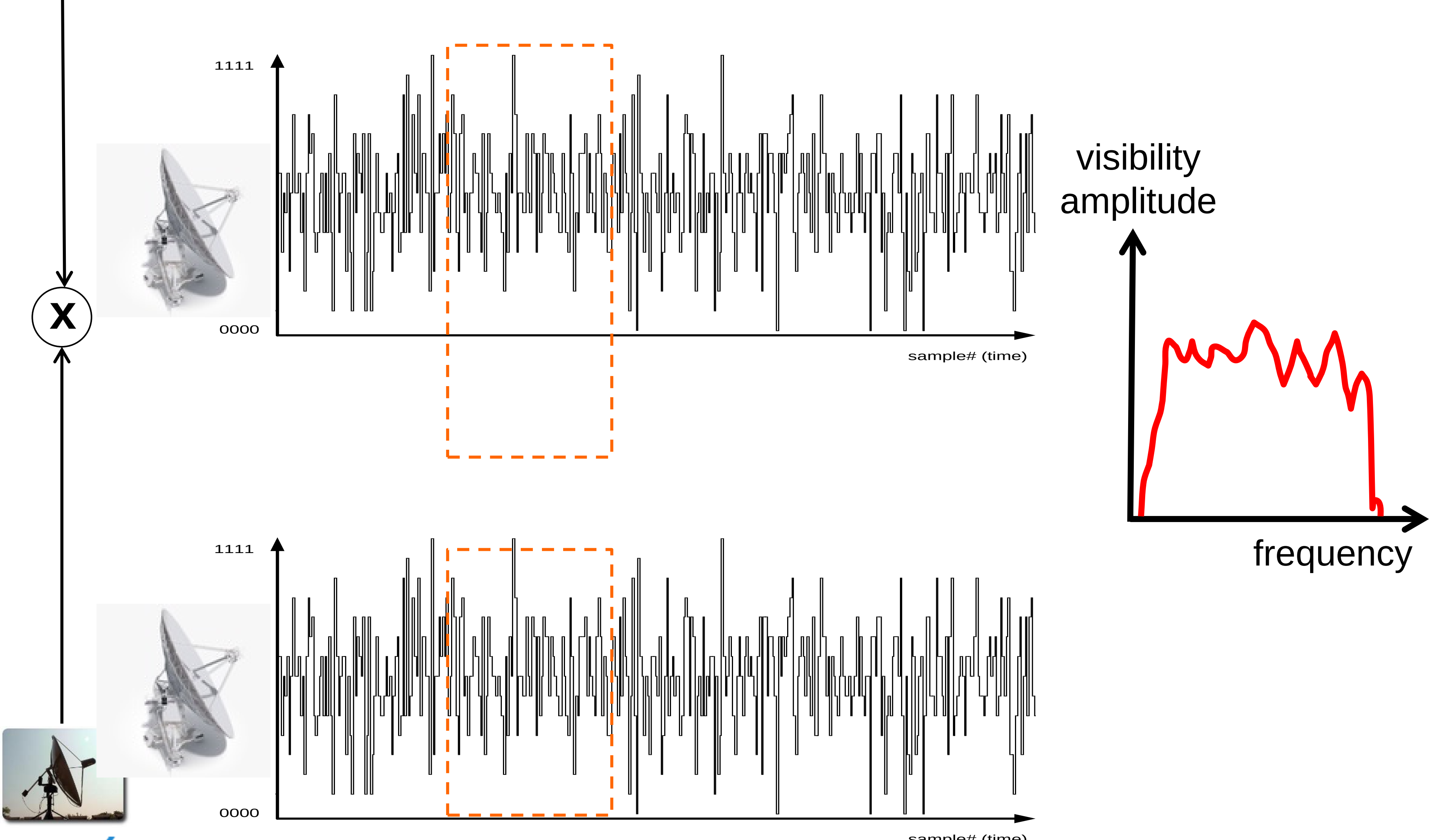
An equivalent “XF” correlator



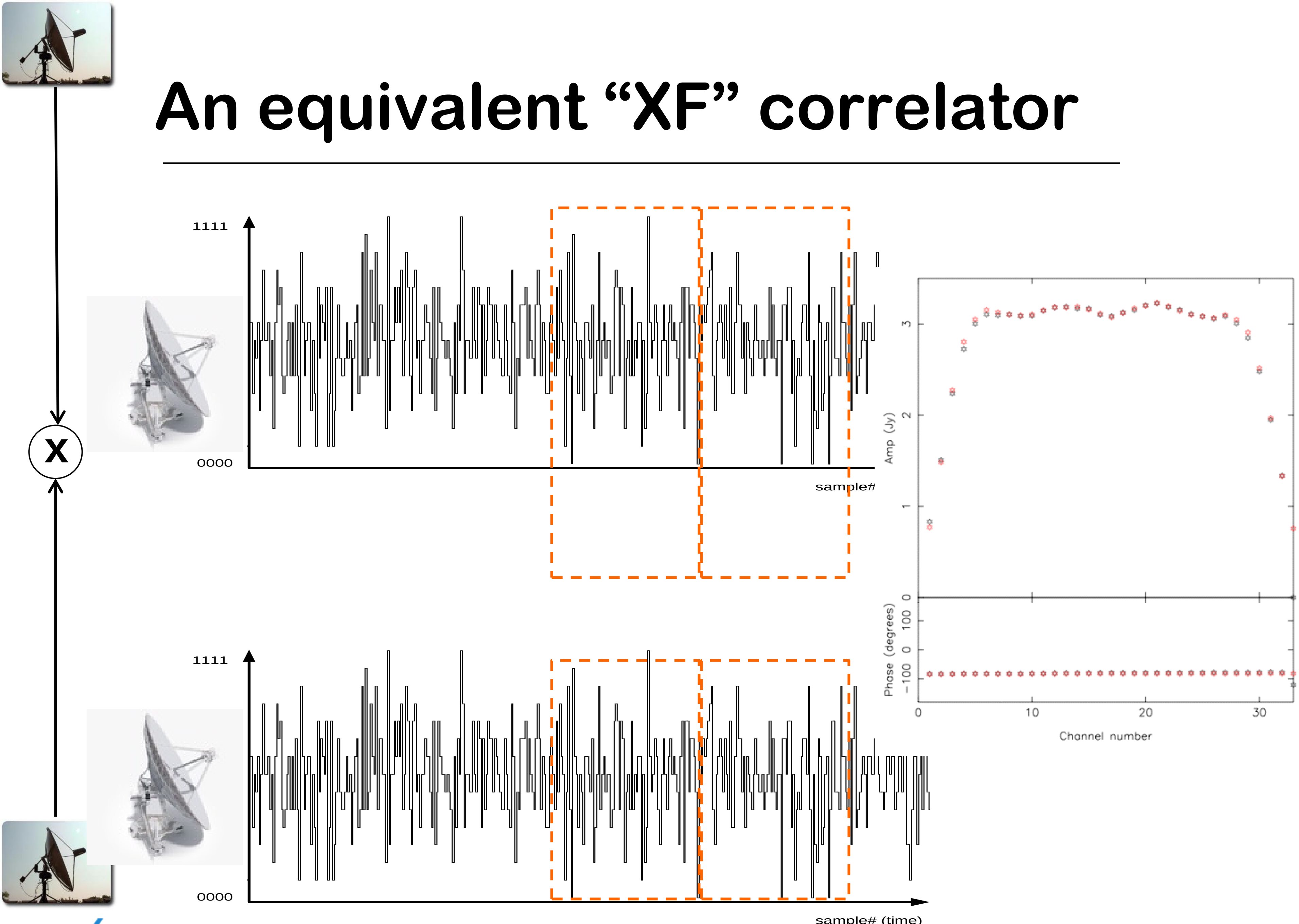
An equivalent "XF" correlator



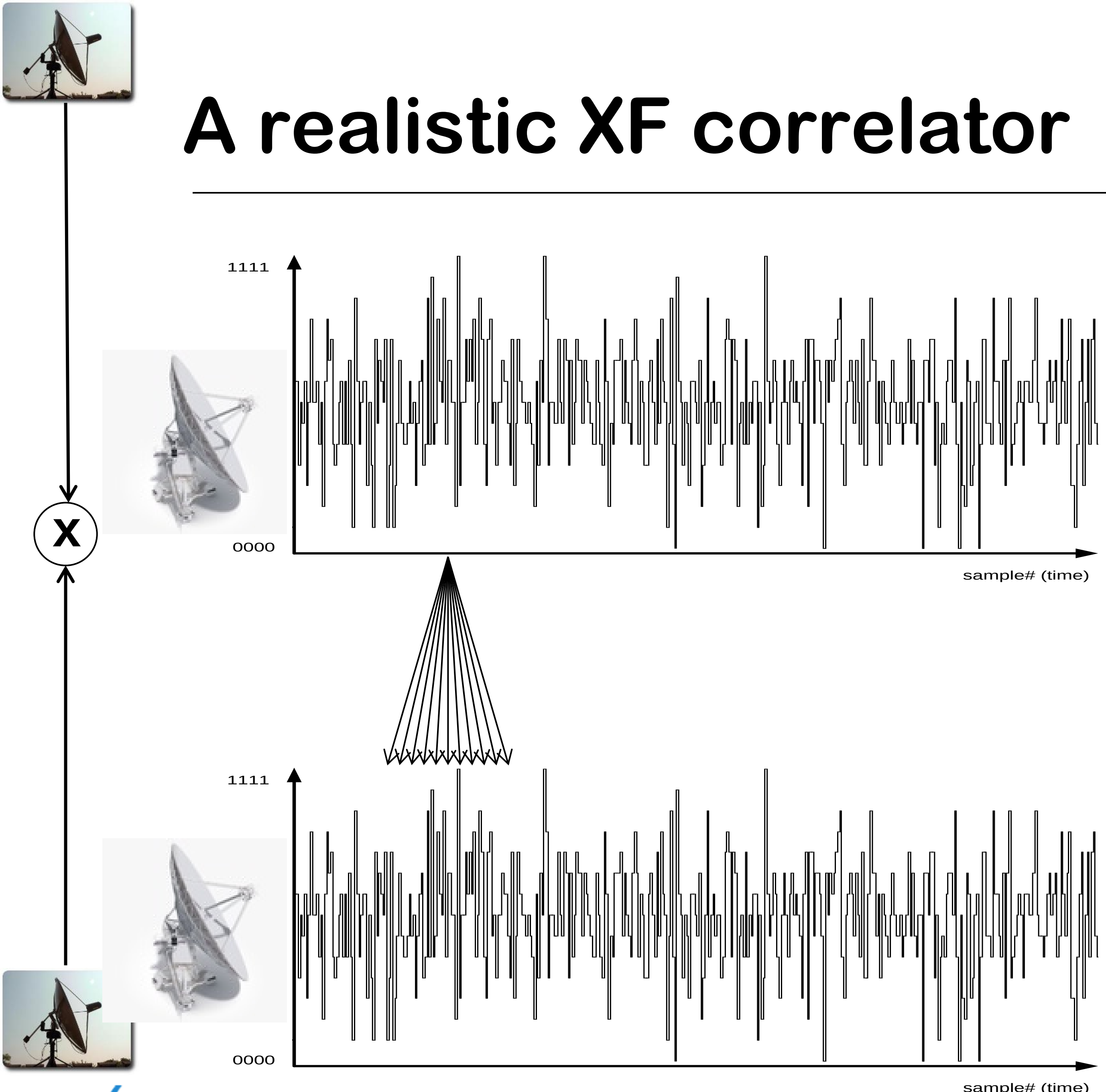
An equivalent “XF” correlator



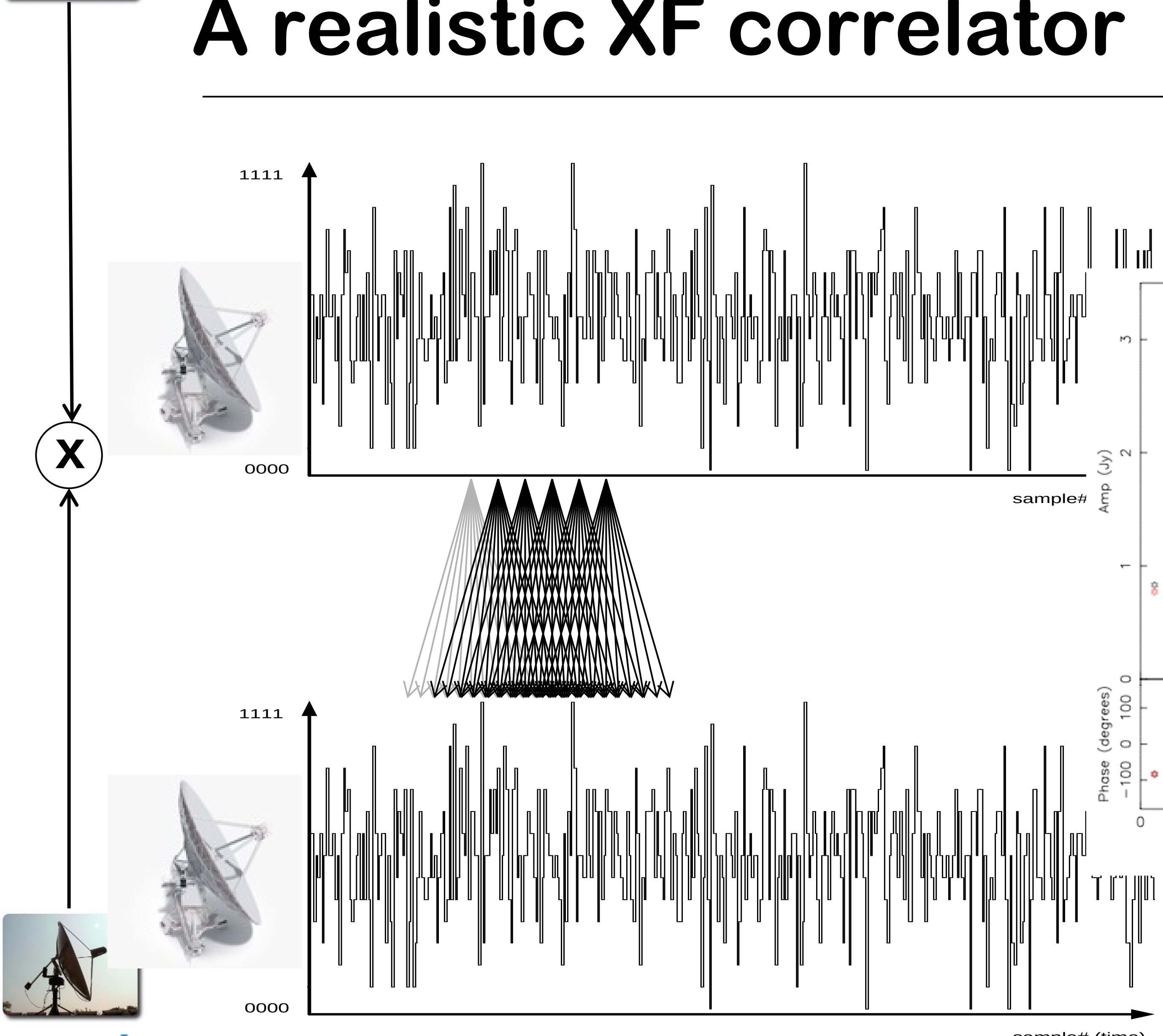
An equivalent "XF" correlator

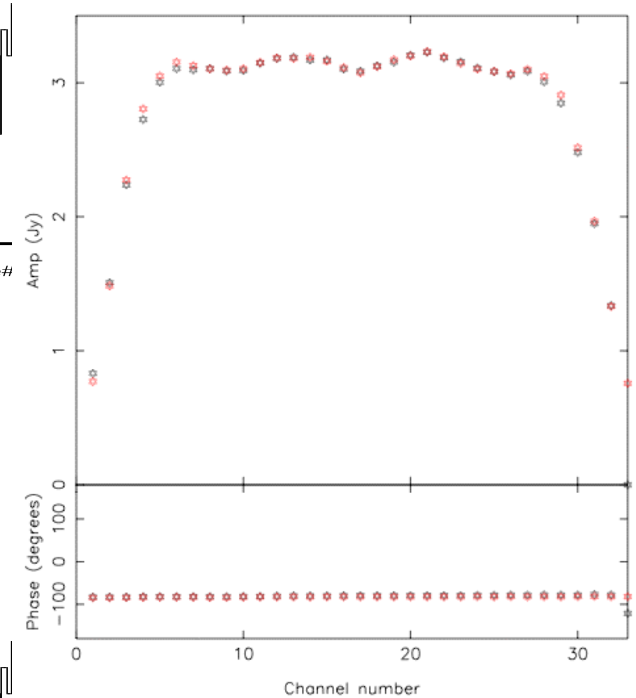


A realistic XF correlator



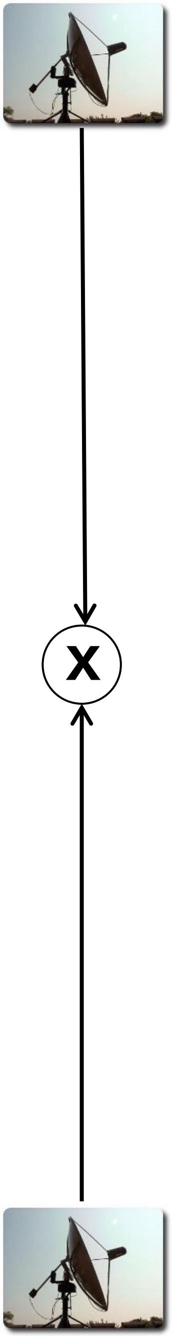
A realistic XF correlator

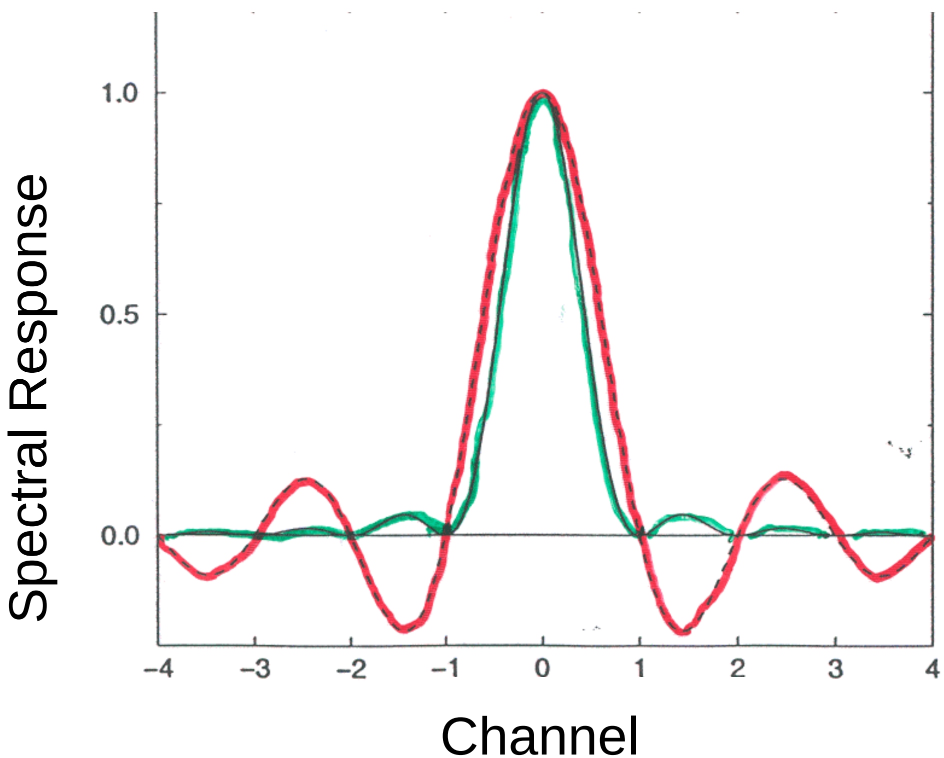


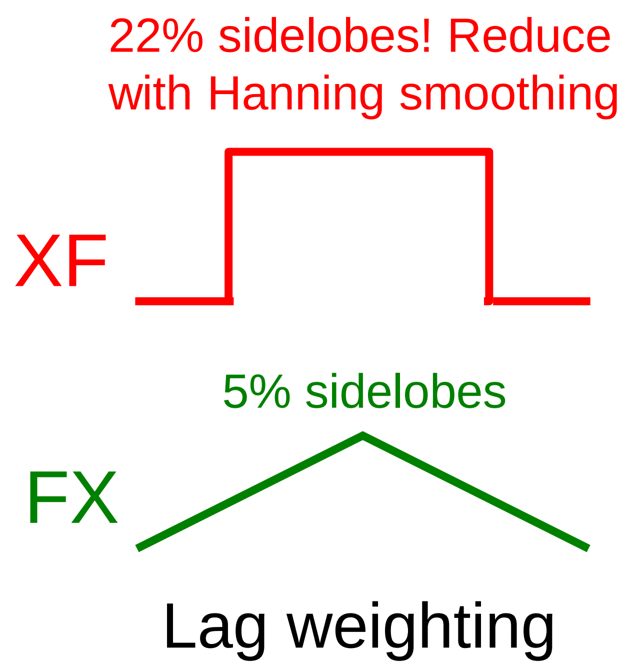


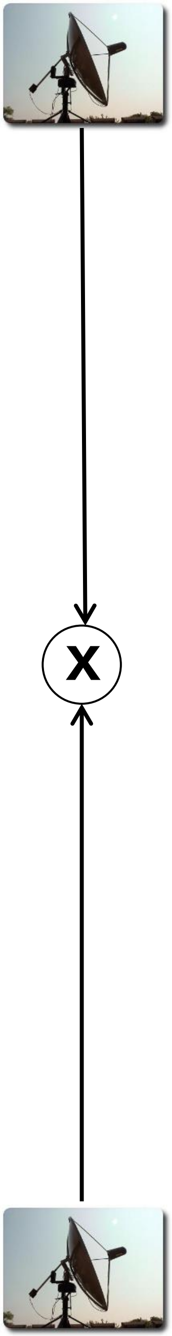
XF vs FX

- Different windowing in time domain gives different spectral response





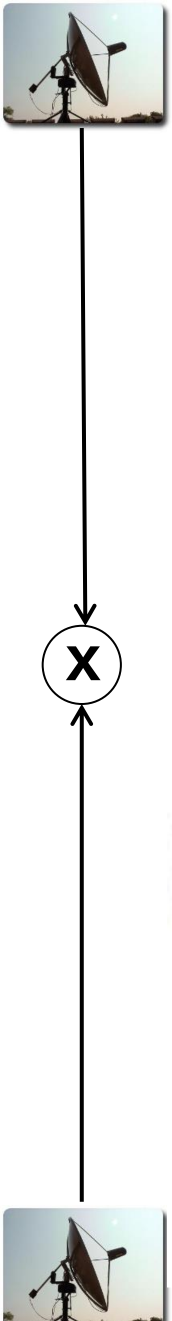




XF vs FX: which is better?

- Advantages and disadvantages to both
  - FX many fewer operations overall
  - XF can make use of very efficient low-precision integer multipliers up-front
  - FX: access to frequency domain at short timescale allows neat tricks and higher precision correction of delay effects
  - But issues with simple implementation of FX for very high spectral resolution

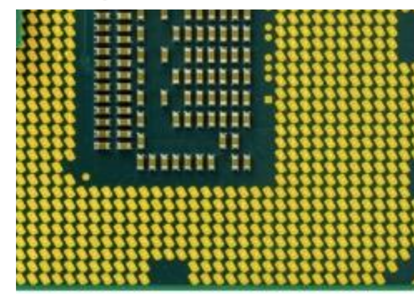
Correlator platforms

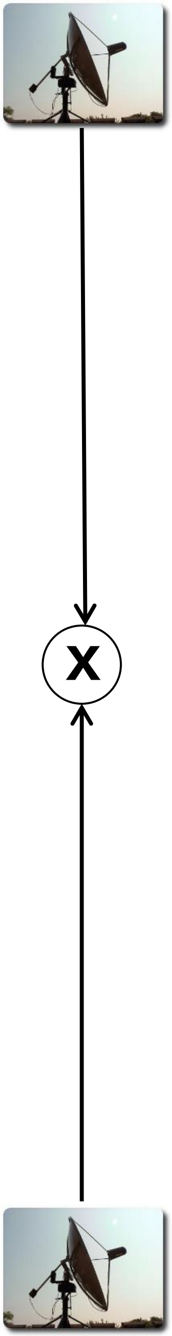


```
status = vectorFFT_CtoC_cf32(complexunpacked, fftd, pFFTSpecC, fftbuffer);
if(status != vecNoErr)
  csevere << startl << "Error doing the FFT!!!" << endl;

      ...

status = vectorAddProduct_cf32(vis1, vis2, &(scratchspace->threadcrosscorrs
```

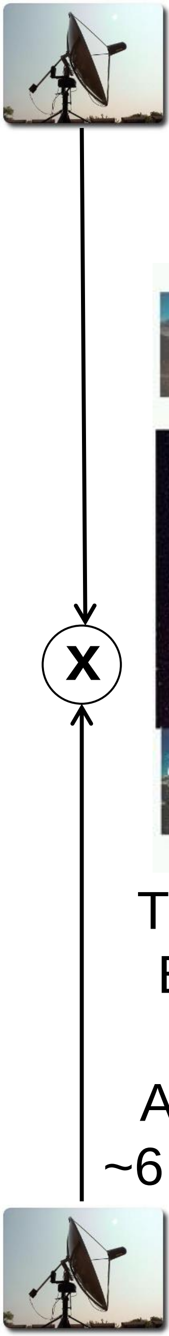




Correlators on CPUs

- Many positive points:
  - Can implement in “normal” code (e.g., C++); maintainable, many skilled coders
  - Development effort transferrable across generations of hardware
  - Incremental development is trivial
  - Natively good at floating point (good for FX), no cost to do high precision
- One major disadvantage:
  - CPUs not optimised for correlation; big system like JVLA would take **many** CPUs.

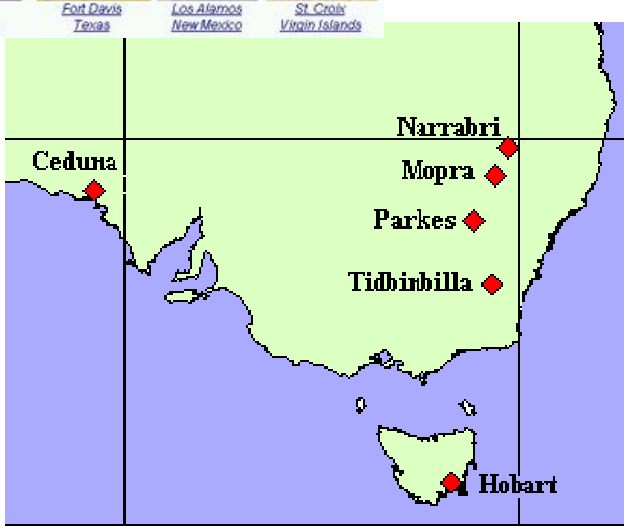
Correlators on CPUs



The Very Long Baseline Array, 10 stations

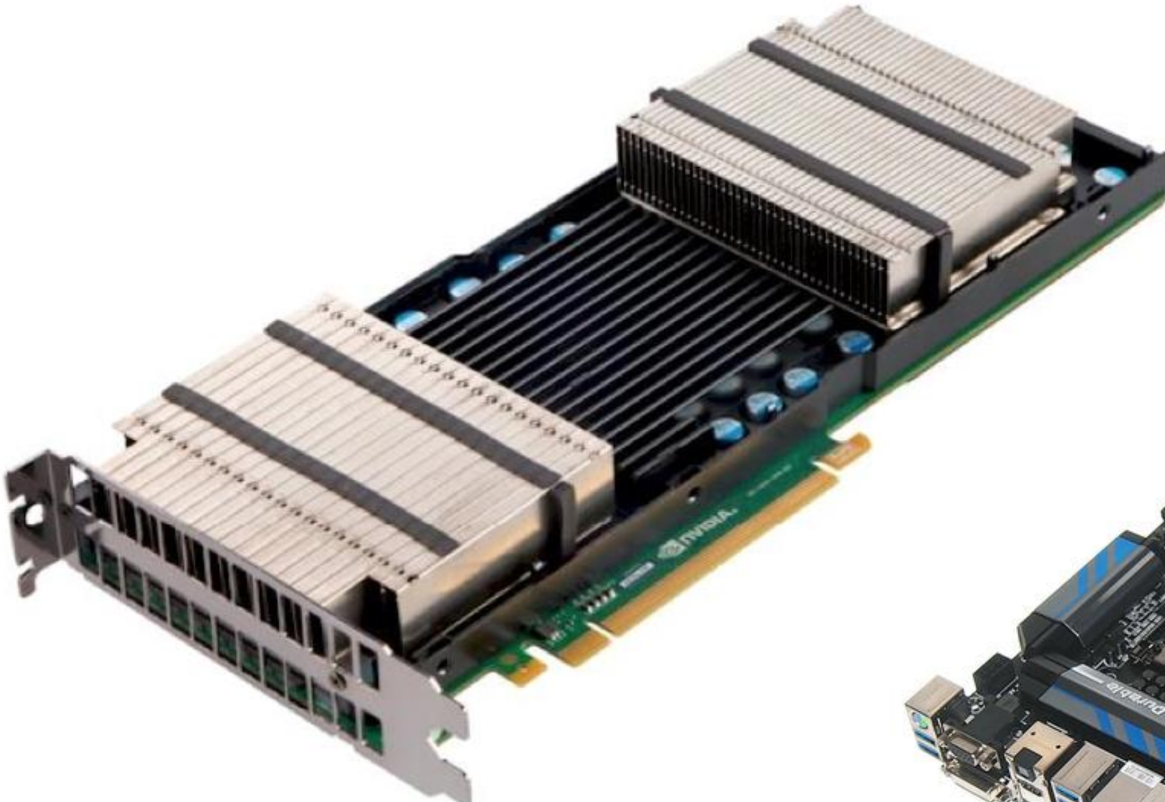
GMRT, India, 30 stations

The Long Baseline Array, Australia, ~6 stations



The European VLBI Network, ~20 stations

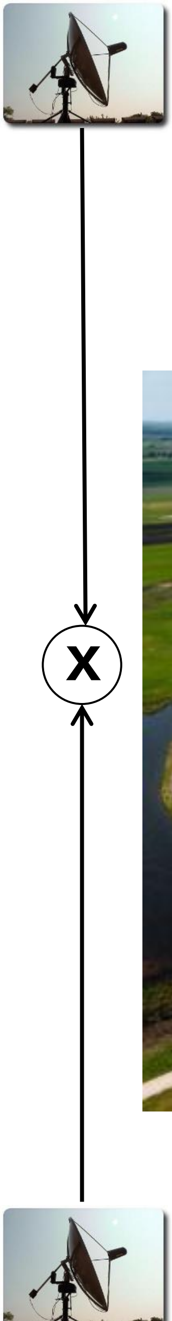
Correlators on GPUs



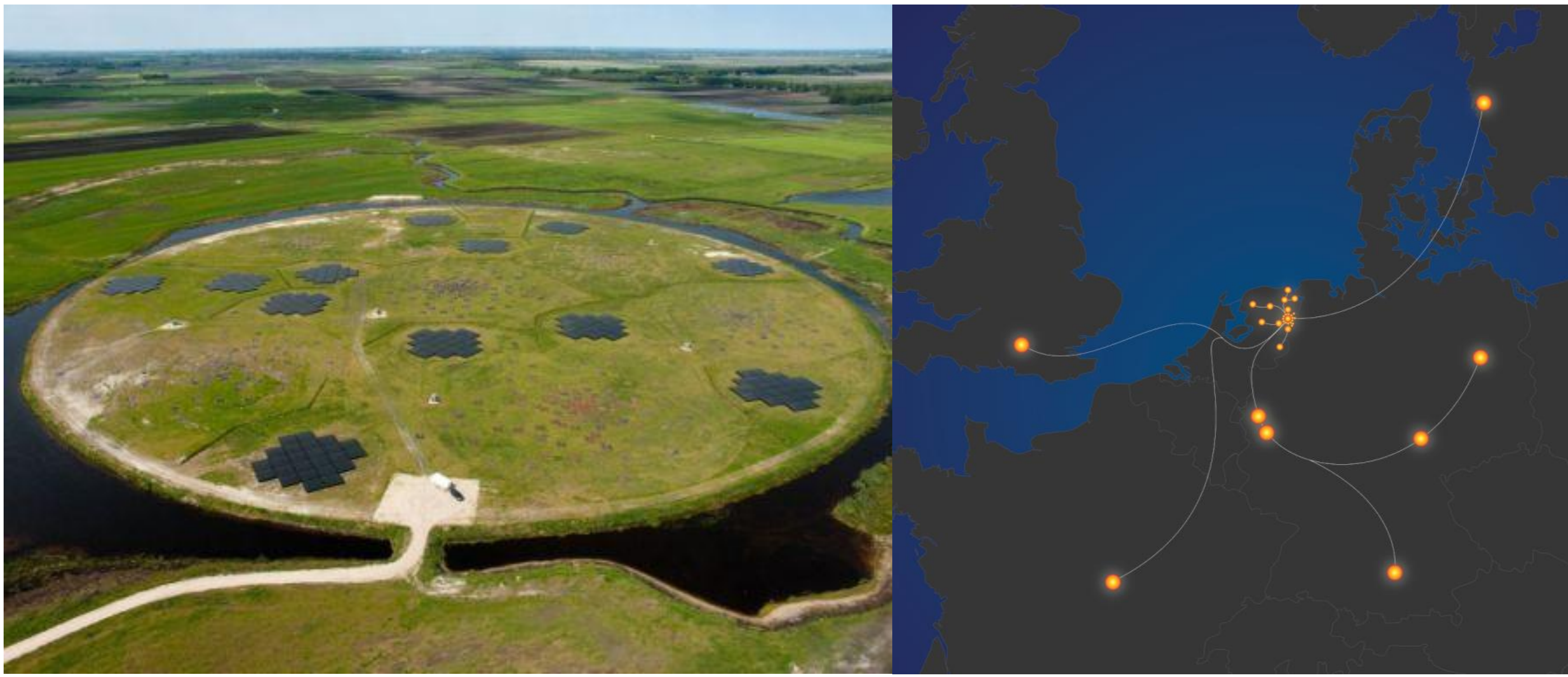
Like CPUs, GPUs are mounted
on a standard motherboard

Correlators on GPUs

- Advantages:
  - More powerful and more efficient than CPUs
  - Also good at floating point
- Disadvantages:
  - Writing code is more difficult (GPUs are more specialized, less flexible: need to carefully manage data transfers)
  - Fewer trained GPU programmers available
  - Transfer-ability of code across hardware generations not yet reliable (but close)

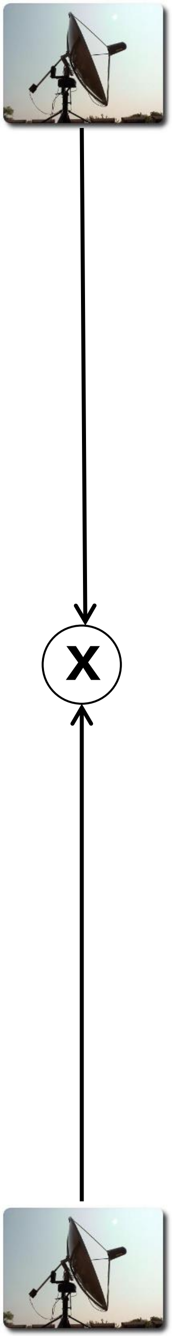


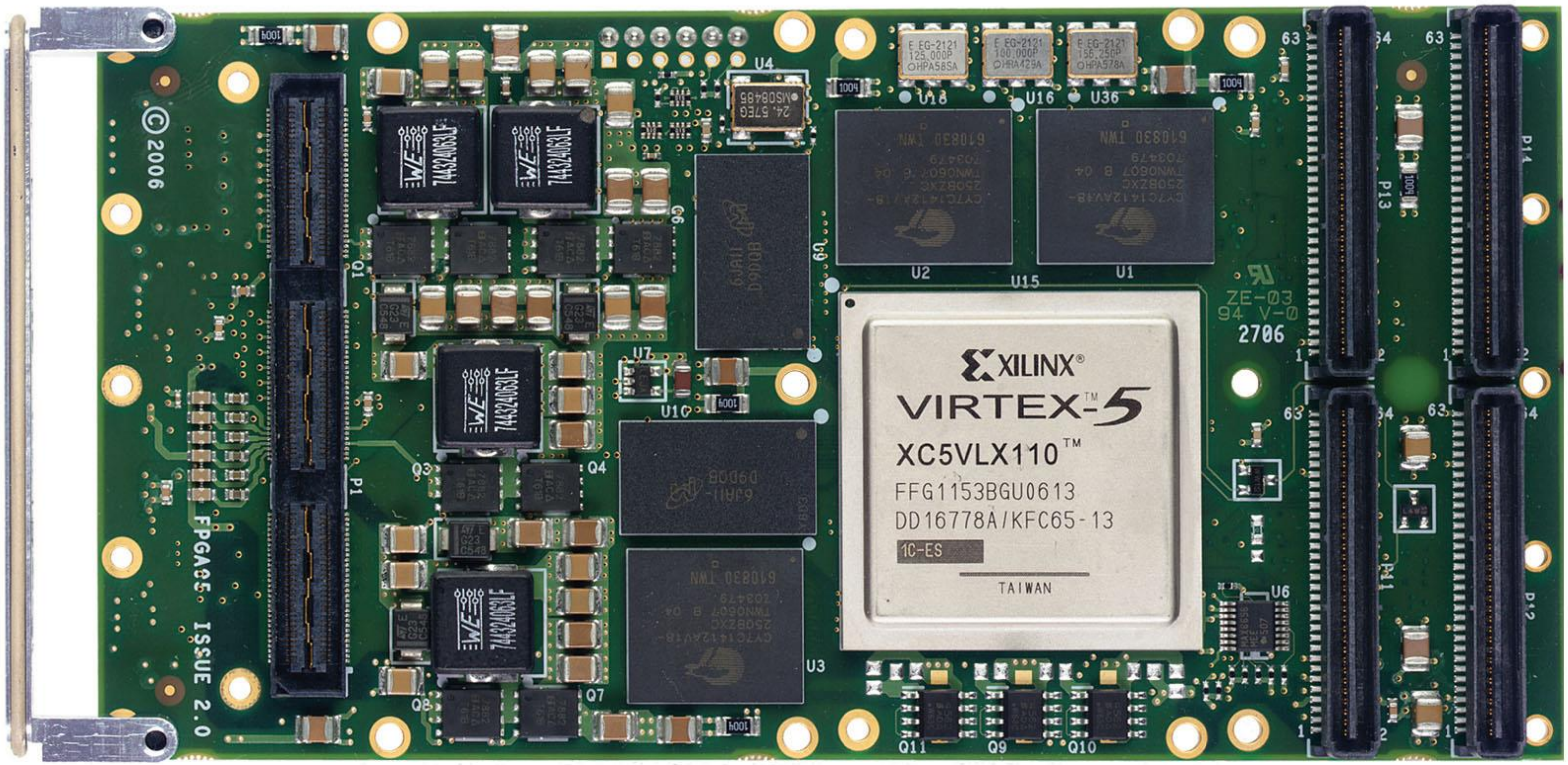
Correlators on GPUs

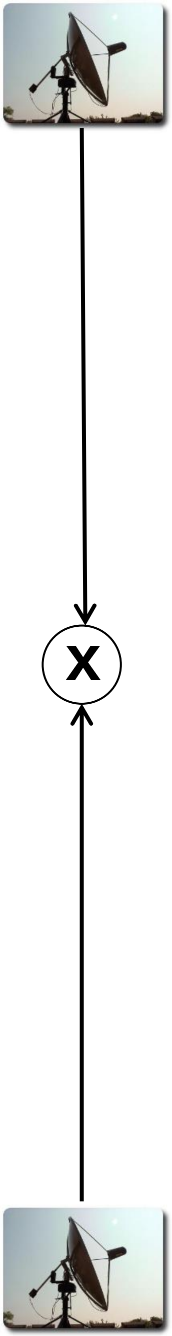


The Low Frequency Array (LOFAR), 70 stations

Correlators on FPGAs



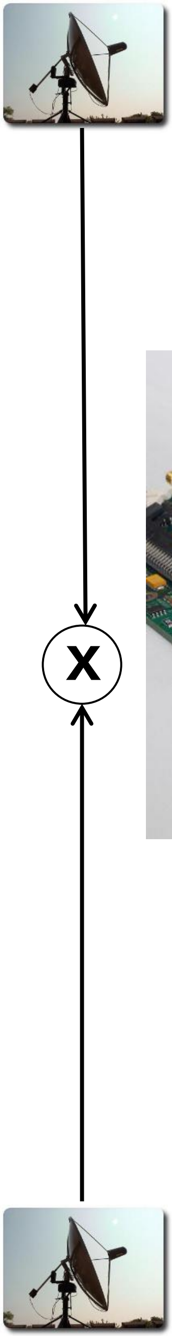


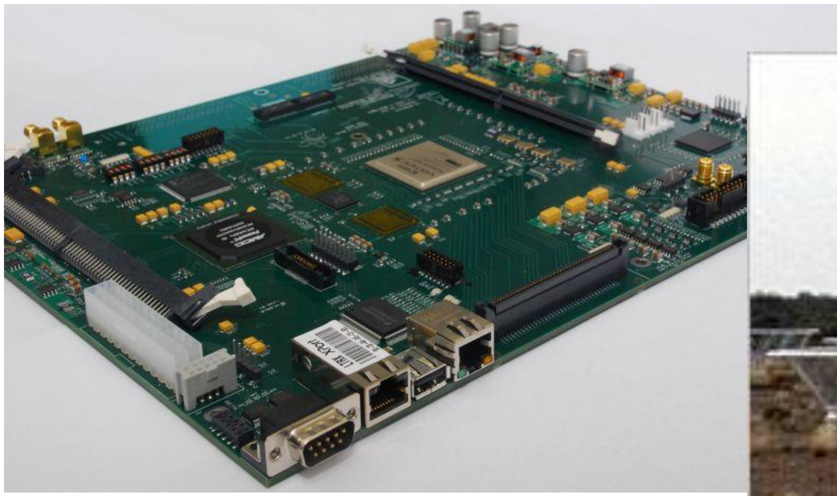


Correlators on FPGAs

- Advantages:
  - More efficient than CPUs or GPUs, particularly for integer multiplication
- Disadvantages:
  - Programming is harder again (especially debugging), yet fewer trained people
  - Transfer-ability across hardware generations even more limited
  - Synchronous (clocked) system, less robust to perturbations c.f. CPUs/GPUs

Correlators on FPGAs

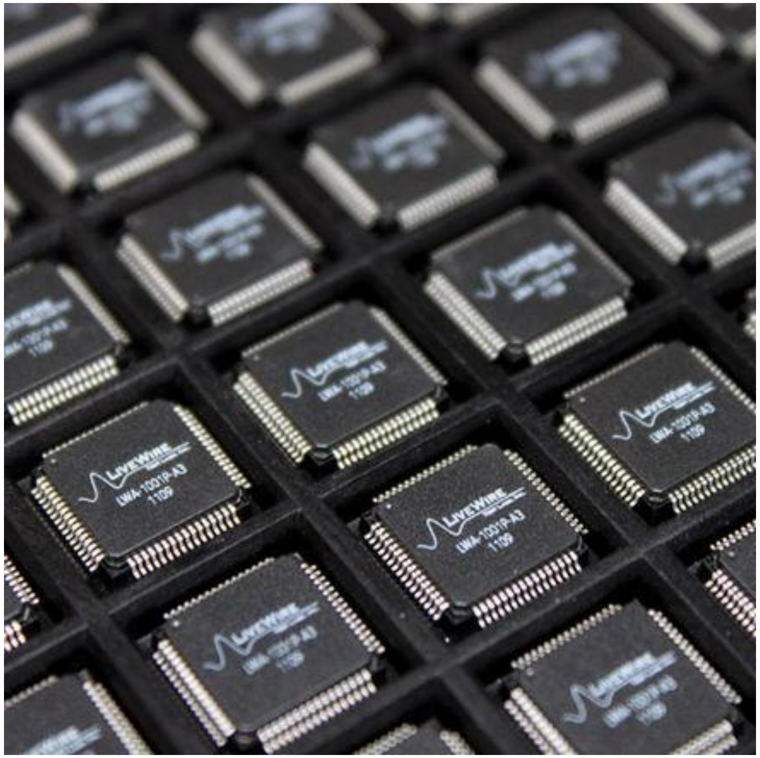


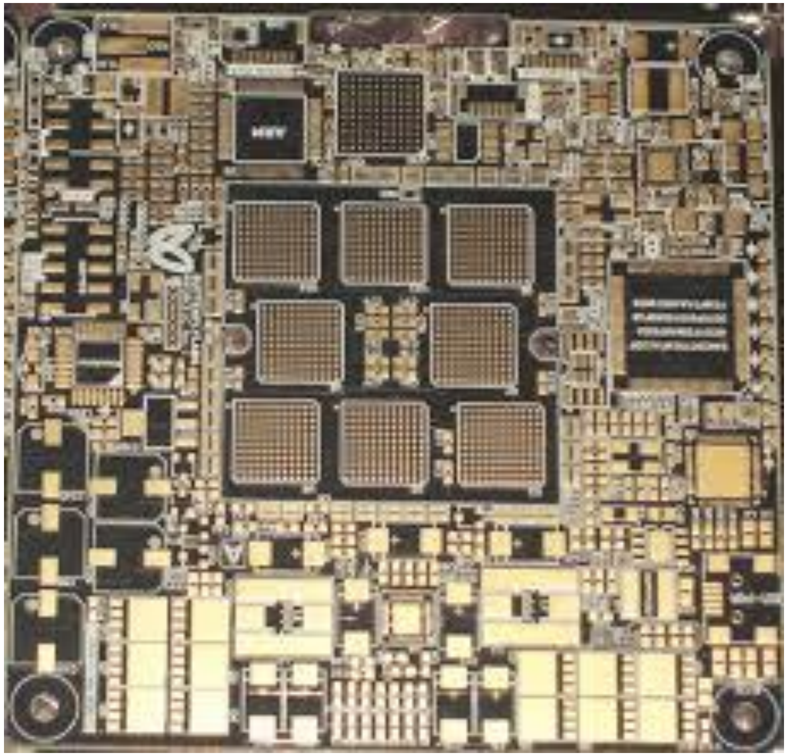


“Roach” reconfigurable FPGA board used for correlation

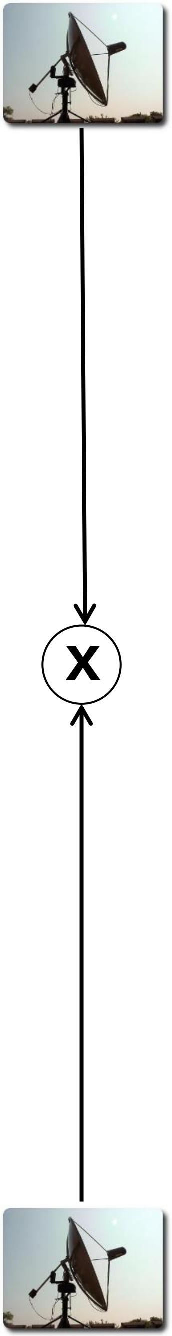
The Precision Array to Probe the Epoch of Reionization (PAPER), 128 stations

Correlators on ASICs



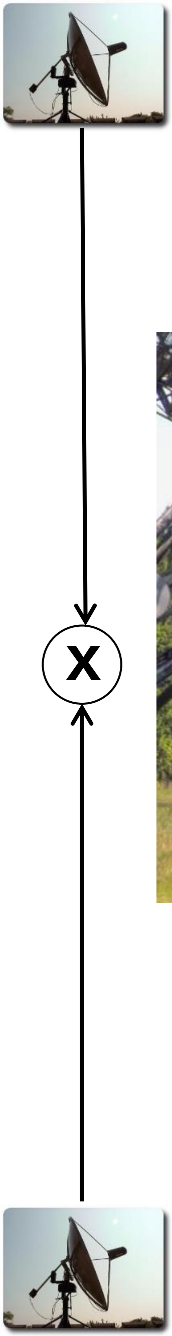


As with FPGAs, ASICs are mounted on boards



Correlators on ASICs

- Advantages:
  - Highest possible efficiency, low per-unit cost
- Disadvantages:
  - Highest development cost (time and manufacturing setup)
  - Specialized knowledge required
  - Can’t be changed / very difficult to upgrade during lifetime

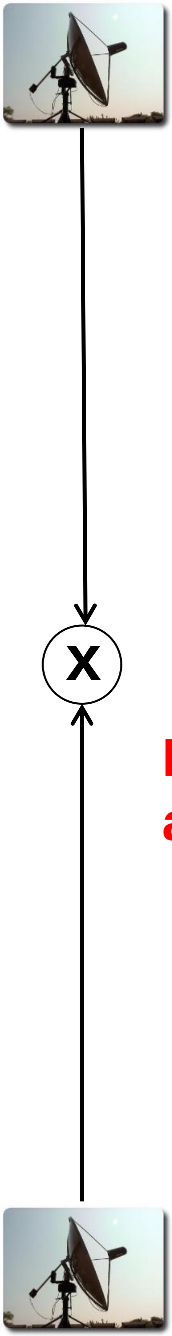


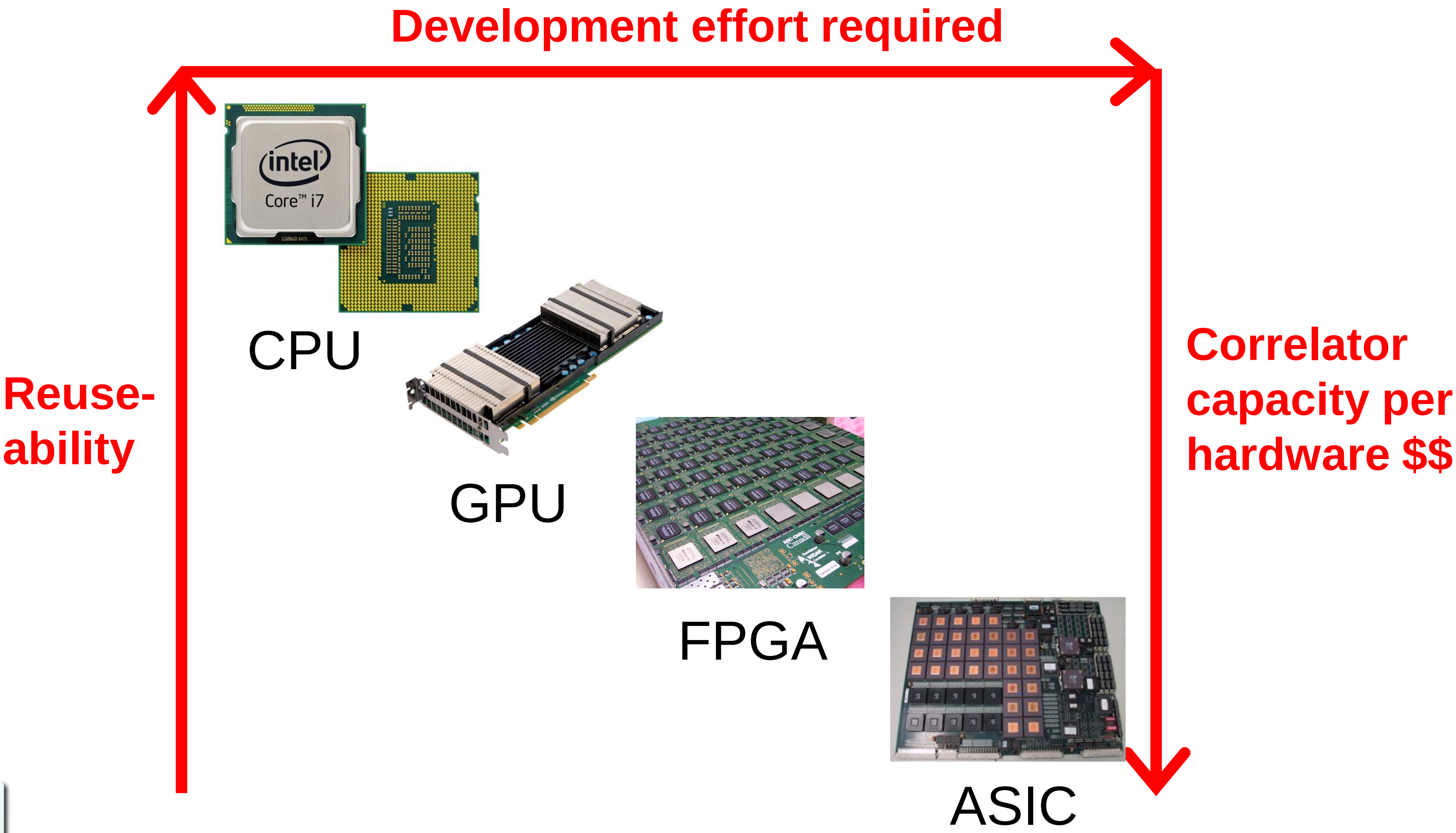
Correlators on ASICs

The Westerbork Synthesis Radio Telescope, Netherlands

The Very Large Array, New Mexico

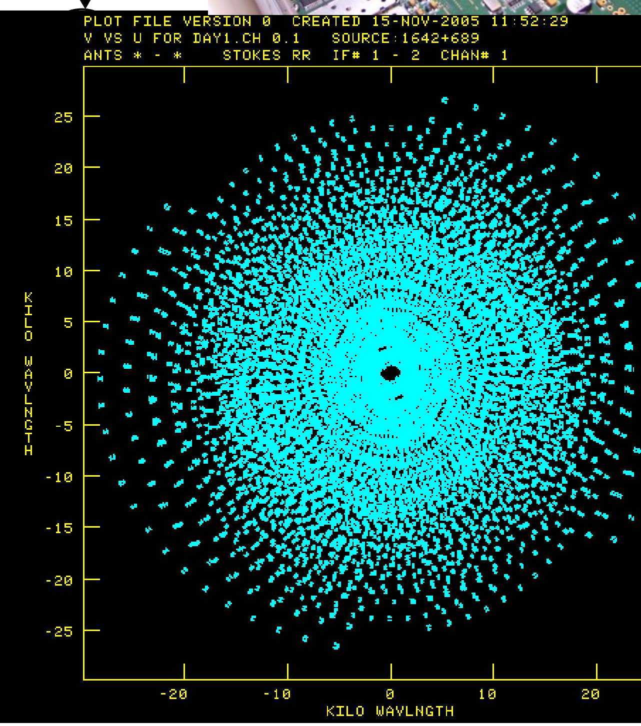
Correlator platform overview





Questions?

Canada



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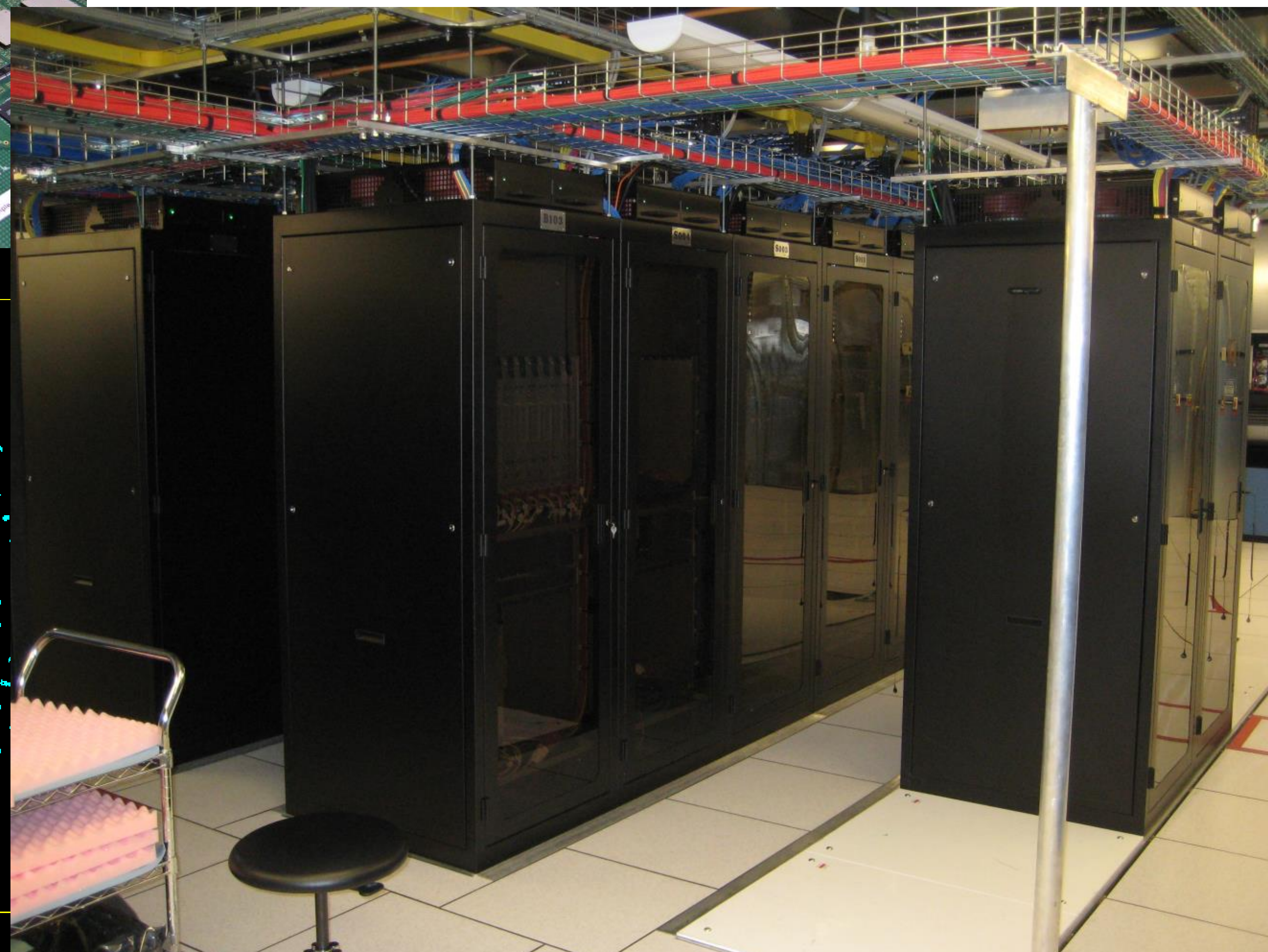

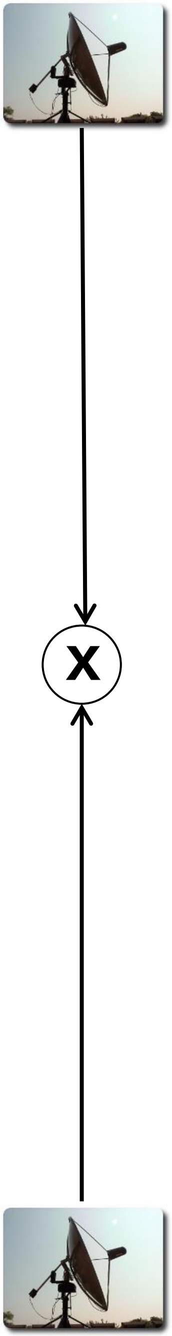

Trends in correlator design

- **Now:** Small scale systems completely dominated by CPU, medium-scale being taken over by “custom GPU”
- **Soon:** GPUs become more CPU-like; “prepackaged” GPU systems available
- **5+ years:** the mother of all correlators (Square Kilometre Array) must be built: will have to be highly optimized