# Imaging and Deconvolution

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Fourteenth Synthesis Imaging Workshop

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# References

- Thompson, A.R., Moran, J.M., Swensen, G.W. 2004 "Interferometry and Synthesis in Radio Astronomy", 2nd edition (Wiley-VCH)

- previous Synthesis Imaging Workshop proceedings
  - Perley, R.A., Schwab, F.R., Bridle, A.H. eds. 1989 ASP Conf. Series 6 "Synthesis Imaging in Radio Astronomy" (San Francisco: ASP)
    - Ch. 6 Imaging (Sramek & Schwab) and Ch. 8 Deconvolution (Cornwell)
  - www.aoc.nrao.edu/events/synthesis
    - lectures by Cornwell 2002 and Bhatnagar 2004, 2006
- IRAM Interferometry School proceedings
  - www.iram.fr/IRAMFR/IS/IS2008/archive.html
    - Ch. 13 Imaging Principles and Ch. 16 Imaging in Practice (Guilloteau)
    - lectures by Pety 2004-2012
- many other lectures and pedagogical presentations are available
  - ALMA primer, ATNF, CARMA, ASIAA, e-MERLIN, …

# Visibility and Sky Brightness

- *V(u,v), the complex visibility function, is the 2D Fourier transform of T(l,m), the sky brightness distribution (for incoherent source, small field of view, far field, etc.)*
  [for derivation from van Cittert-Zernike theorem, see TMS Ch. 14]

- mathematically

$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

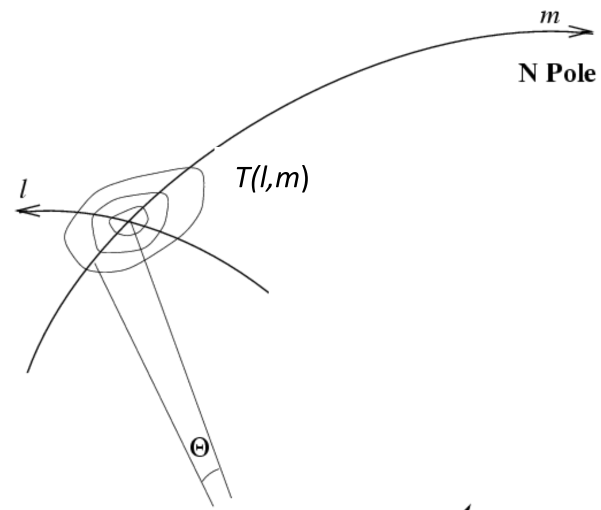
$$T(l,m) = \int\int V(u,v)e^{i2\pi(ul+vm)}dudv$$

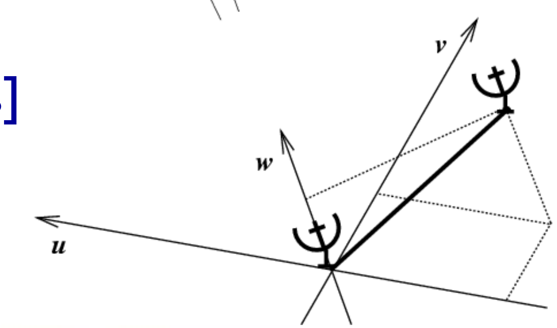
*u,v* are E-W, N-S spatial frequencies [wavelengths]

*l,m* are E-W, N-S angles in the tangent plane [radians]

(recall $e^{ix} = \cos x + i \sin x$)

$$V(u,v) \xrightarrow{\mathcal{F}} T(l,m)$$

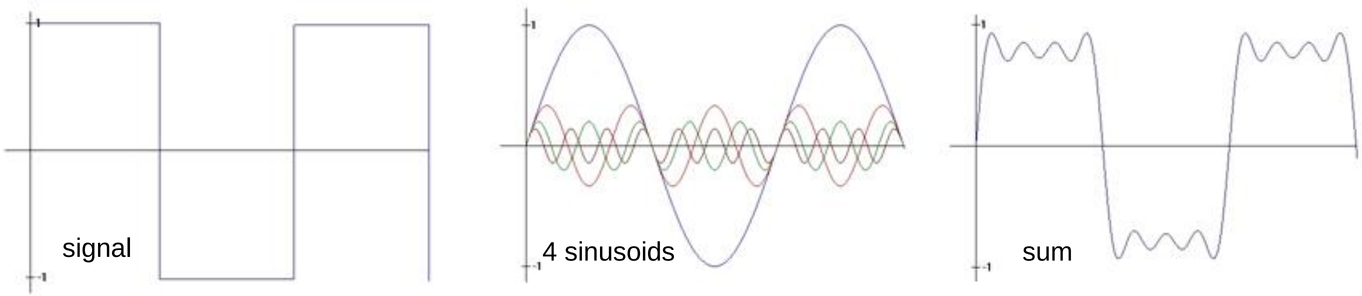


# The Fourier Transform

- Fourier theory states and any well behaved signal (including images) can be expressed as the sum of sinusoids

Jean Baptiste Joseph Fourier 1768-1830




$$x(t) = \frac{4}{\pi}\left(\sin(2\pi f t) + \frac{1}{3}\sin(6\pi f t) + \frac{1}{5}\sin(10\pi f t) + \cdots\right)$$

- the Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components
- the Fourier transform contains *all* of the information of the original signal

# The Fourier Domain

- acquire some comfort with the Fourier domain
- in older texts, functions and their Fourier transforms occupy *upper* and *lower* domains, as if "functions circulated at ground level and their transforms in the underworld" (Bracewell 1965)

- some properties of the Fourier transform $g(x) \xrightarrow{\mathcal{F}} G(s)$

  adding $g(x) + h(x) = G(s) + H(s)$

  scaling $g(\alpha x) = \alpha^{-1} G(s/\alpha)$

  shifting $g(x - x_0) = G(s) e^{i 2\pi x_0 s}$

  convolution/multiplication $g(x) = h(x) * k(x) \qquad G(s) = H(s)K(s)$

  Nyquist-Shannon sampling theorem

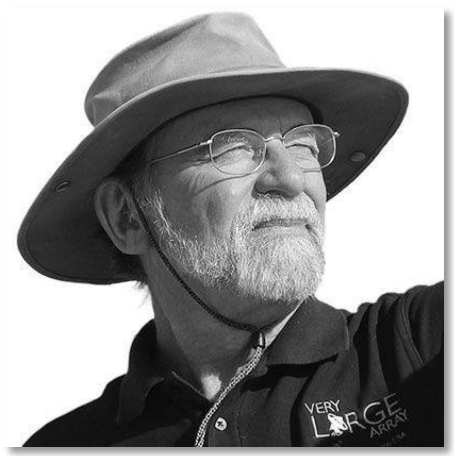
  $$g(x) \subset \Theta \text{ completely determined if } G(s) \text{ sampled at } \leq 1/\Theta$$

# Visibilities

- each *V(u,v)* contains information on *T(l,m) everywhere*, not just at a given *(l,m)* coordinate or within a particular subregion

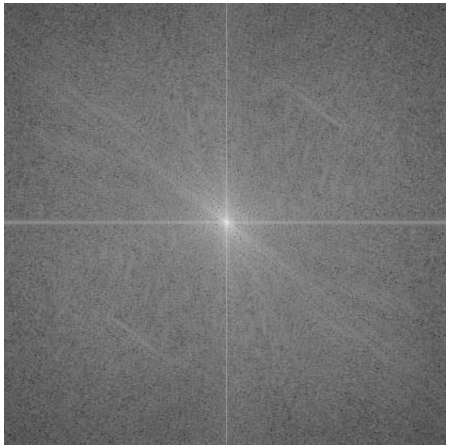
- each *V(u,v)* is a complex quantity
  - expressed as *(real, imaginary)* or *(amplitude, phase)*

*T(l,m)*

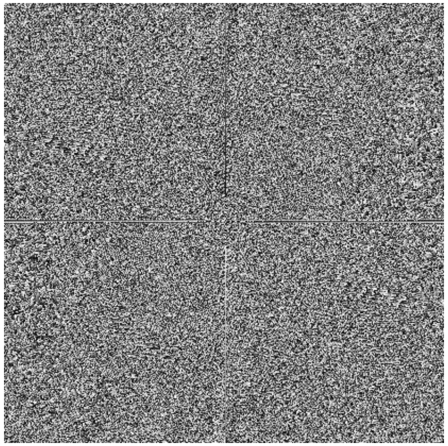



$\xrightarrow{\mathcal{F}}$

*V(u,v) amplitude*



*V(u,v) phase*



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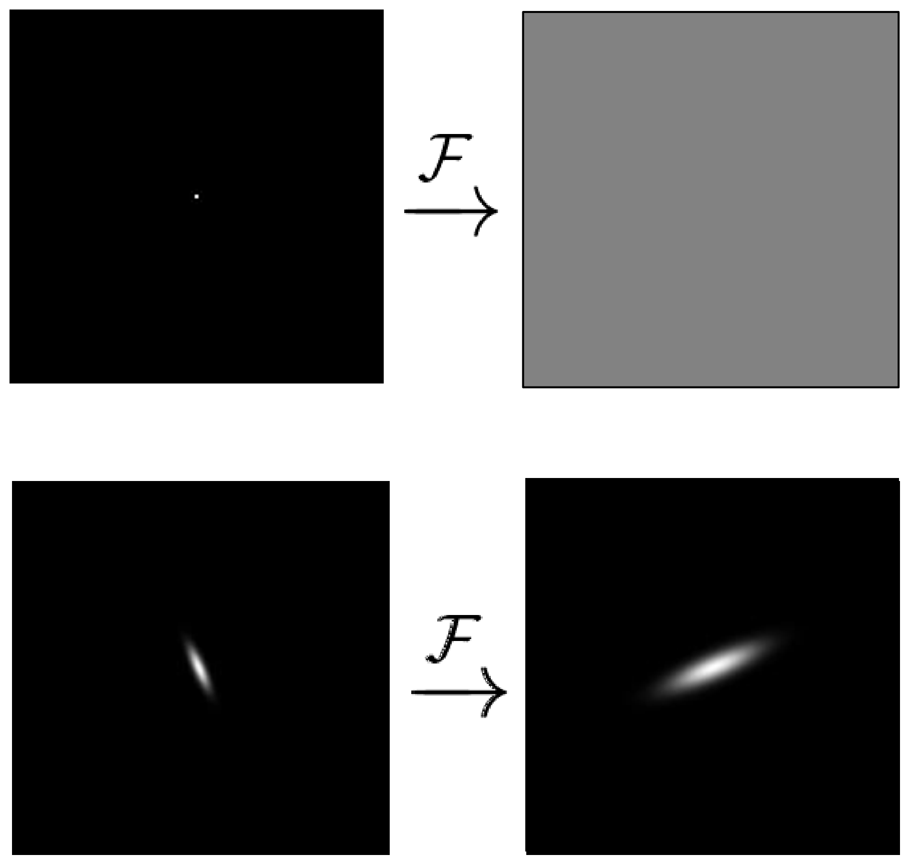
# Example 2D Fourier Transforms

*T(l,m)* *V(u,v) amplitude*

δ function → constant

elliptical Gaussian → elliptical Gaussian

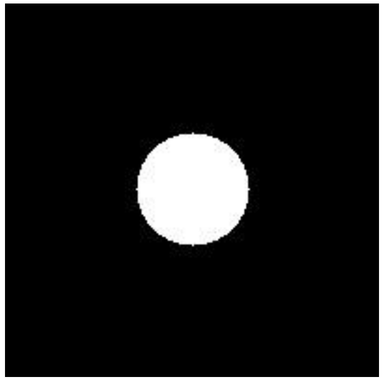



*narrow features transform into wide features (and vice-versa)*

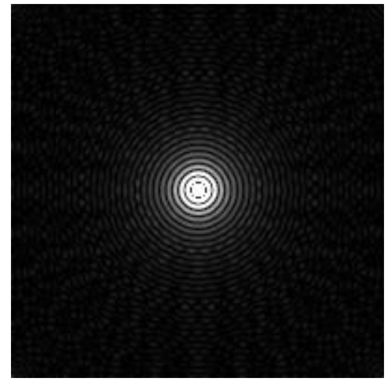
# Example 2D Fourier Transforms

*T(l,m)* *V(u,v) amplitude*

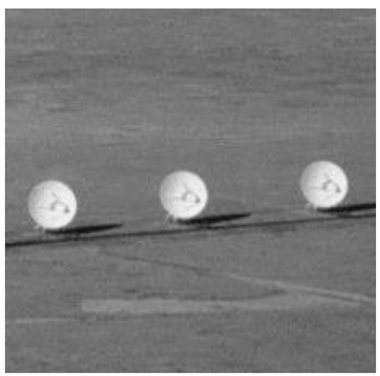
uniform disk



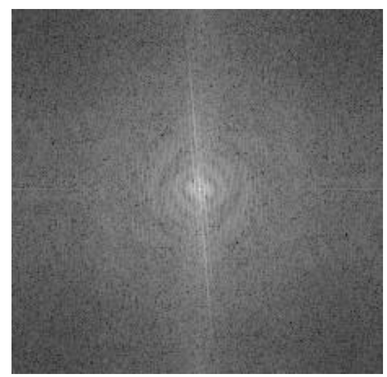
$\xrightarrow{\mathcal{F}}$



Bessel function



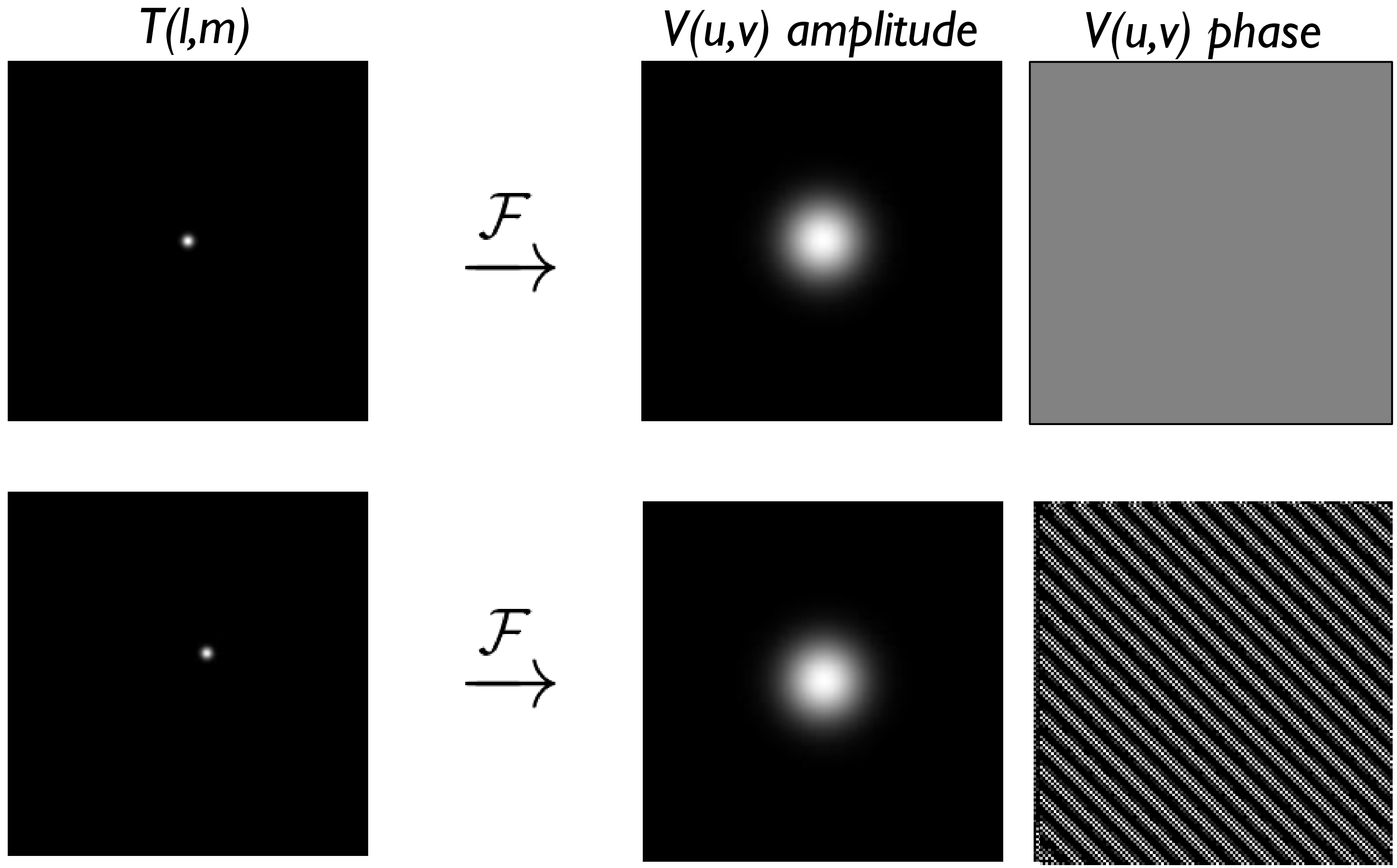
$\xrightarrow{\mathcal{F}}$



*sharp edges result in many high spatial frequencies*

# Amplitude and Phase

- amplitude tells “how much” of a certain spatial frequency
- phase tells “where” this spatial frequency component is located



# The Visibility Concept

$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

- visibility as a function of baseline coordinates *(u,v)* is the Fourier transform of the sky brightness distribution as a function of the sky coordinates *(l,m)*
- $V(u=0,v=0)$ is the integral of $T(l,m)dldm$ = total flux density
- since $T(l,m)$ is real, $V(-u,-v) = V^*(u,v)$
  - $V(u,v)$ is Hermitian
  - get two visibilities for one measurement

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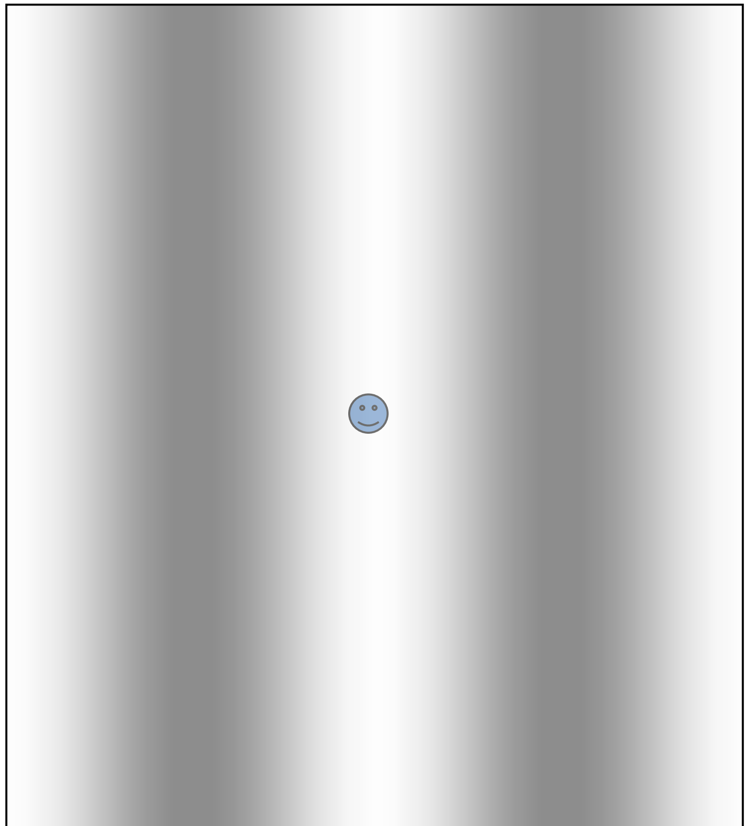
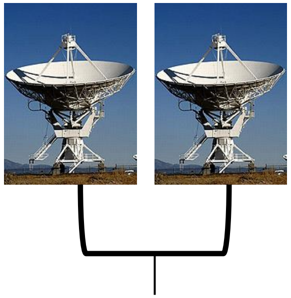
# The Visibility Concept

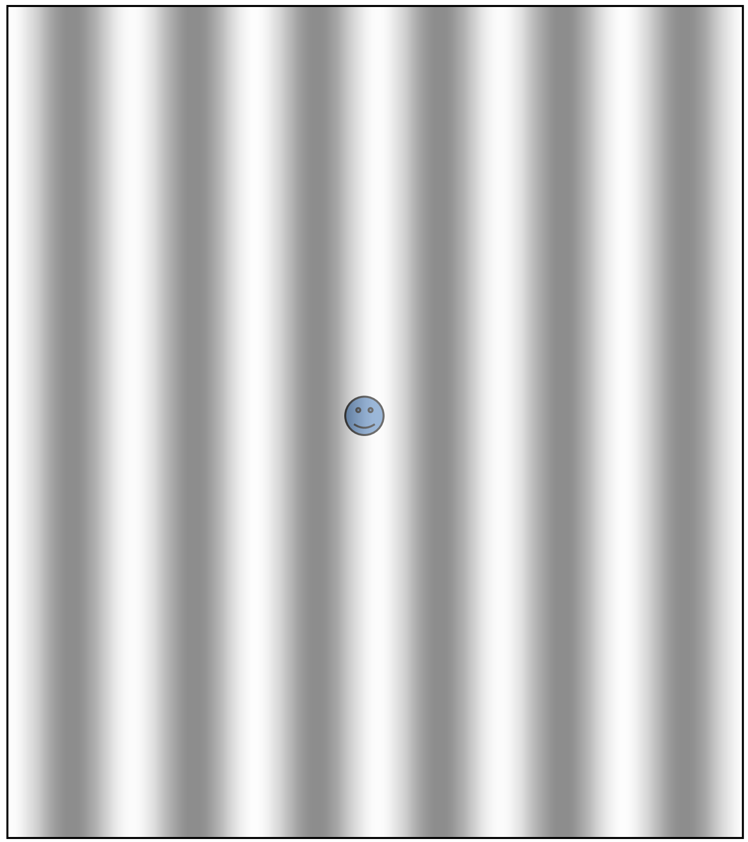
$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$

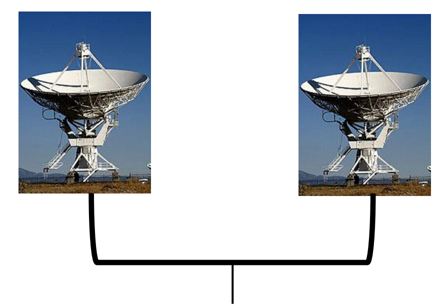




# The Visibility Concept

$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dl\,dm$$





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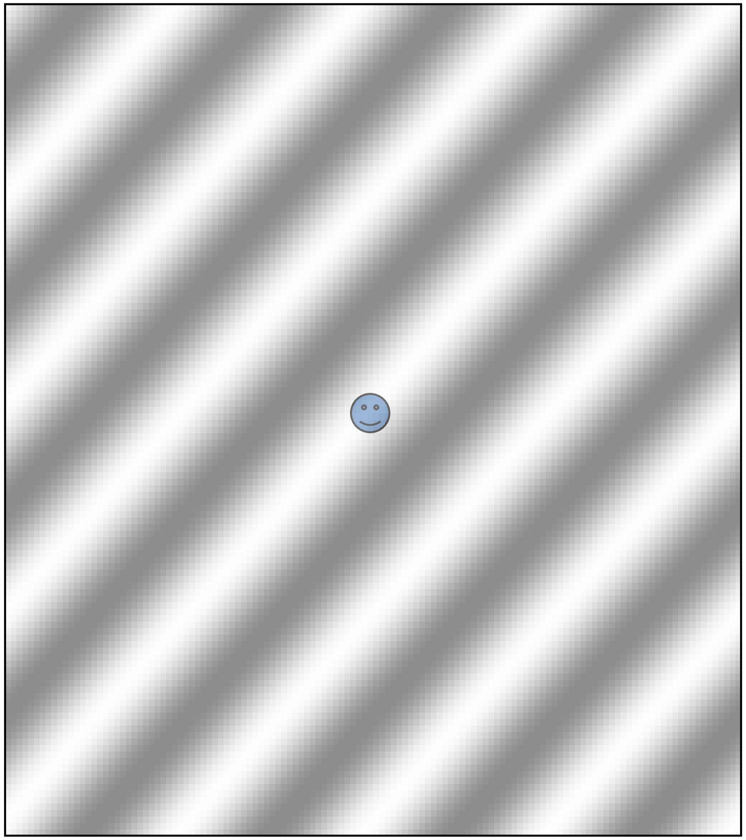
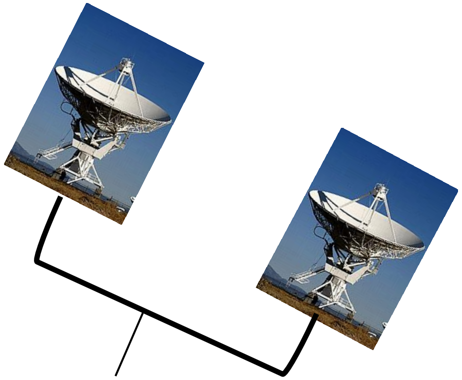
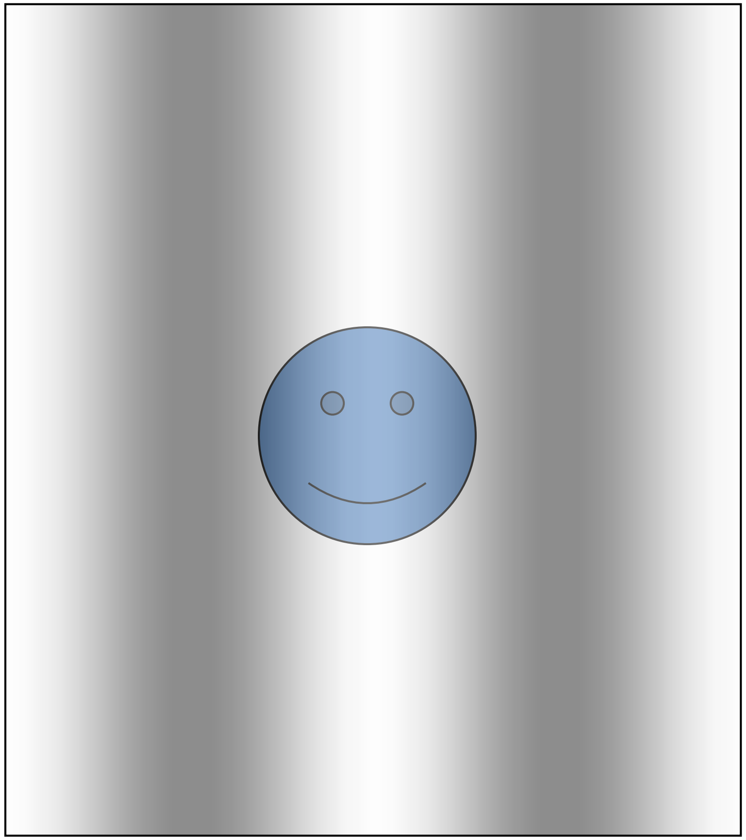
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

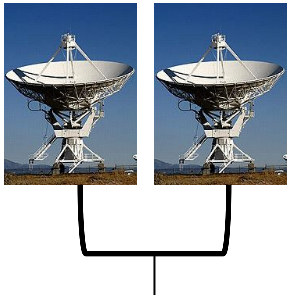




# The Visibility Concept

$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$





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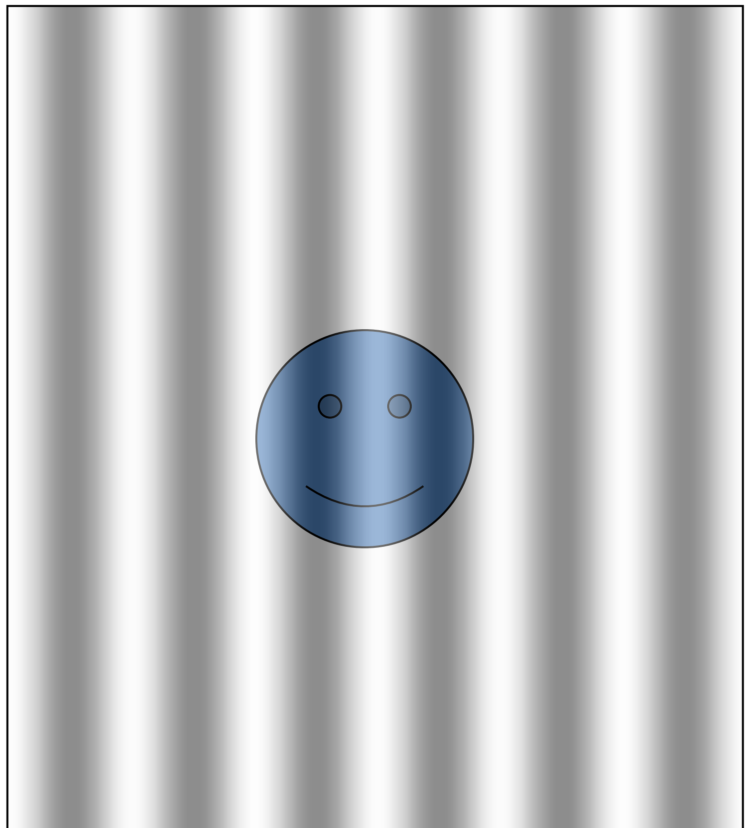
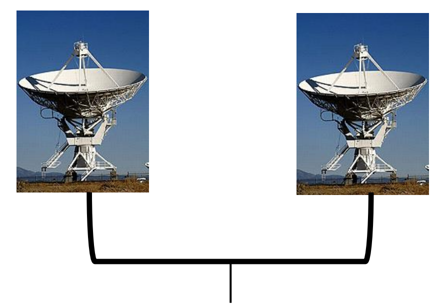
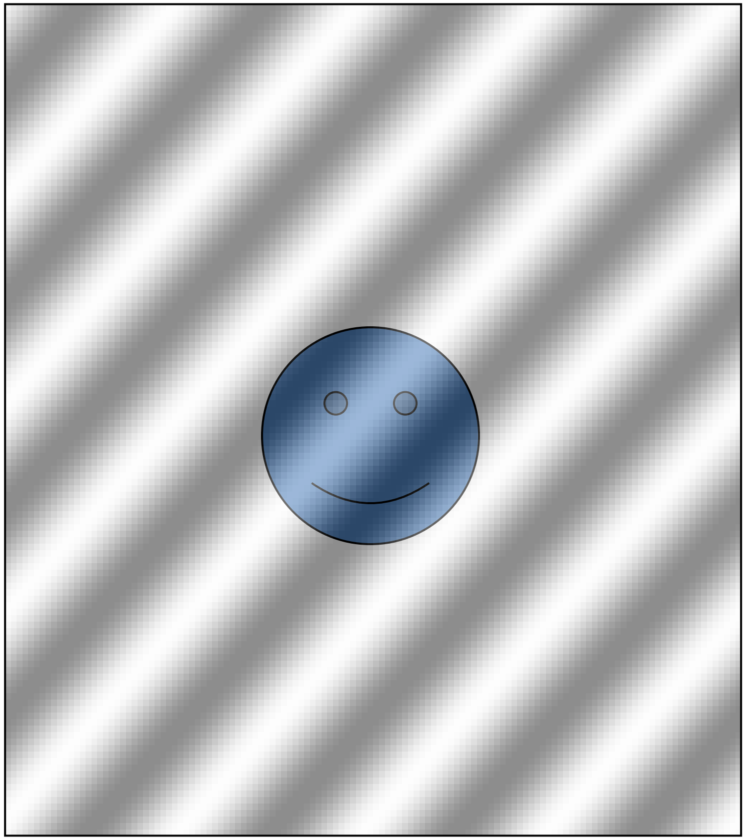
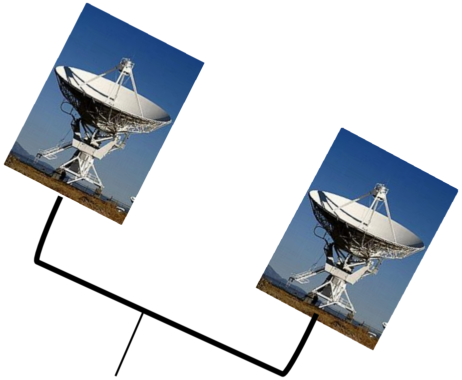




# The Visibility Concept

$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$



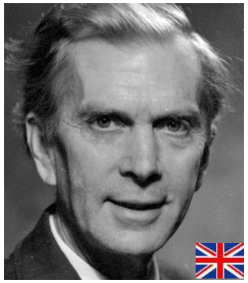


# Aperture Synthesis Basics

- idea: sample $V(u,v)$ at enough $(u,v)$ points using distributed small aperture antennas to synthesize a large aperture antenna of size $(u_{max}, v_{max})$

- one pair of antennas = one baseline
  = two $(u,v)$ samples at a time
- N antennas = N(N-1) samples at a time
- use Earth rotation to fill in $(u,v)$ plane over time
  (Sir Martin Ryle, 1974 Nobel Prize in Physics)
- reconfigure physical layout of N antennas for more samples
- observe at multiple wavelengths for $(u,v)$ plane coverage, for source spectra amenable to simple characterization ("multi-frequency synthesis")

- if source is variable, then be careful

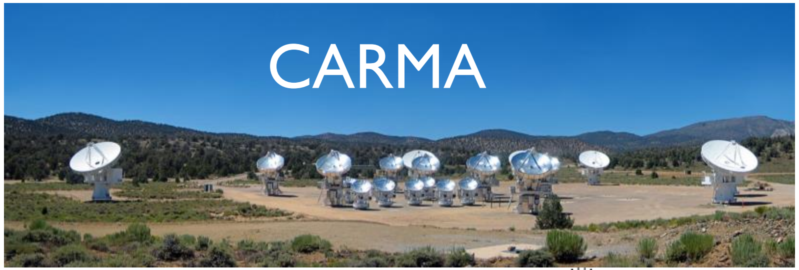


**Sir Martin Ryle**
**1918-1984**

# Examples of Aperture Synthesis Telescopes (for Millimeter Wavelengths)

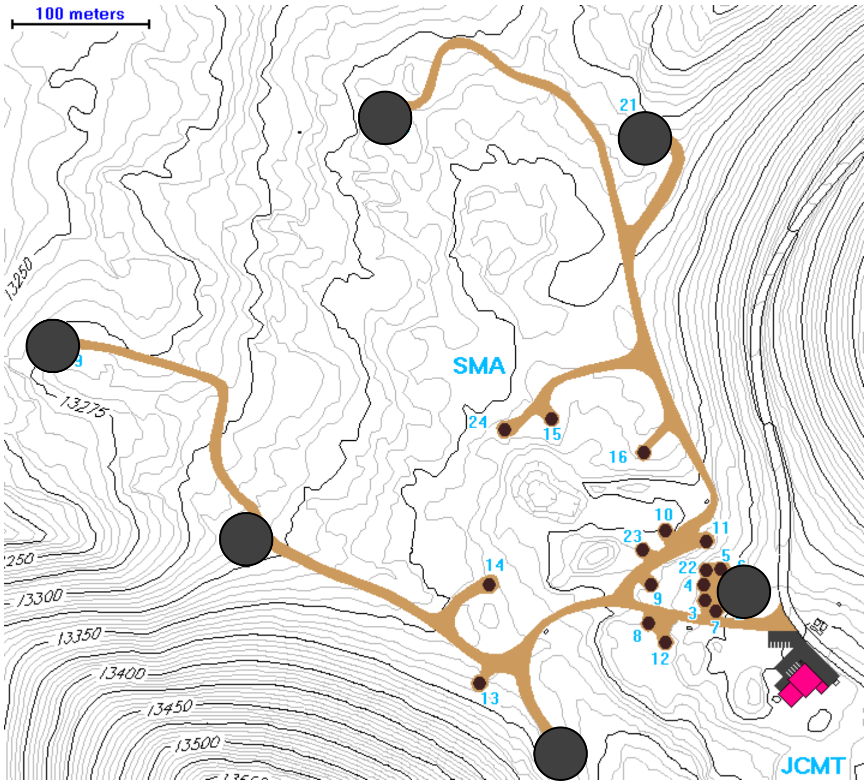


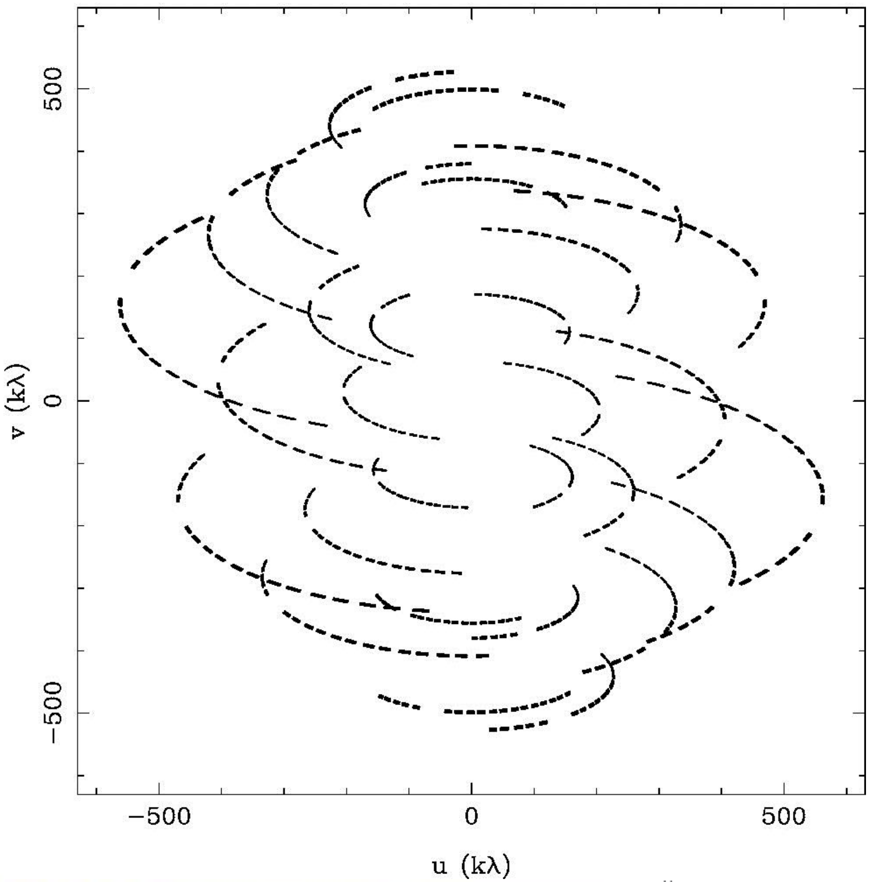


# An Example of (u,v) plane Sampling

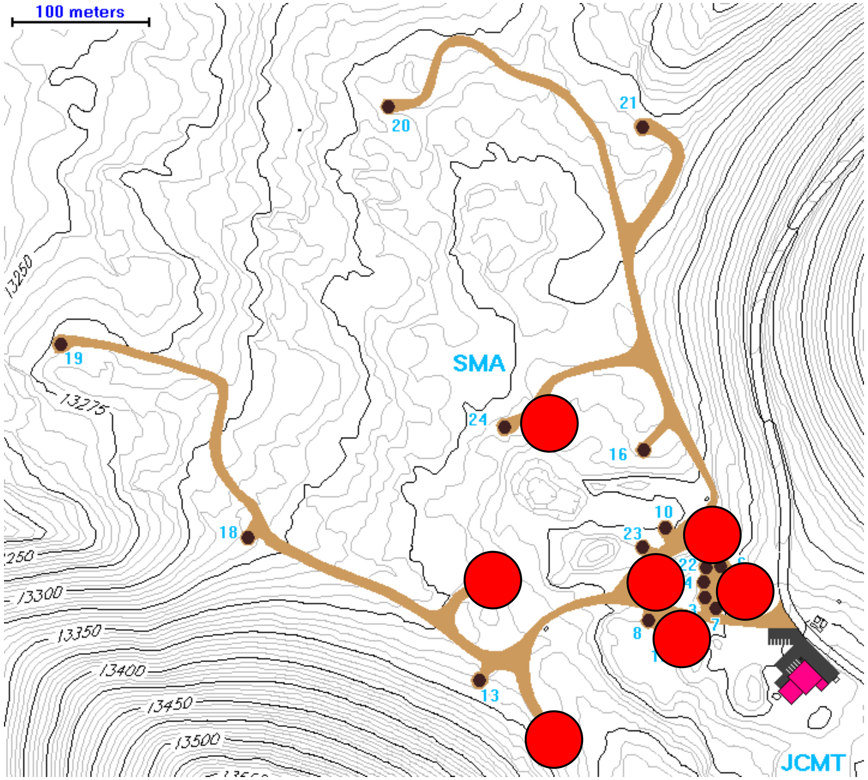
VEX configuration of 6 SMA antennas, $\nu$ = 345 GHz, dec = +22 deg

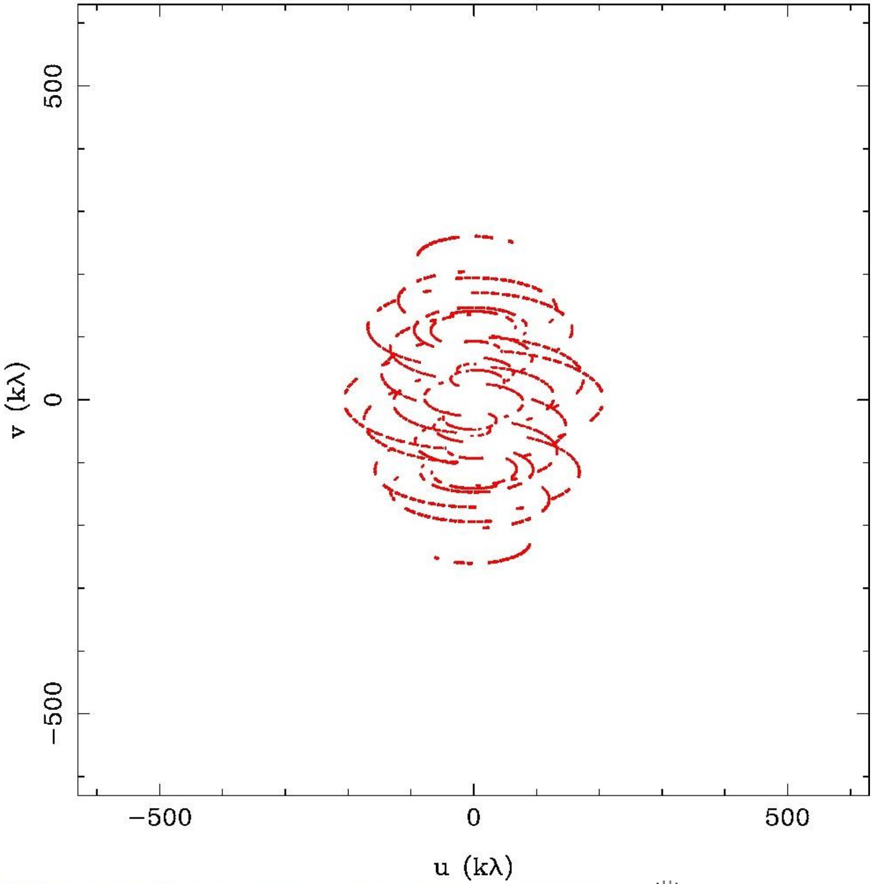


# An Example of (u,v) plane Sampling

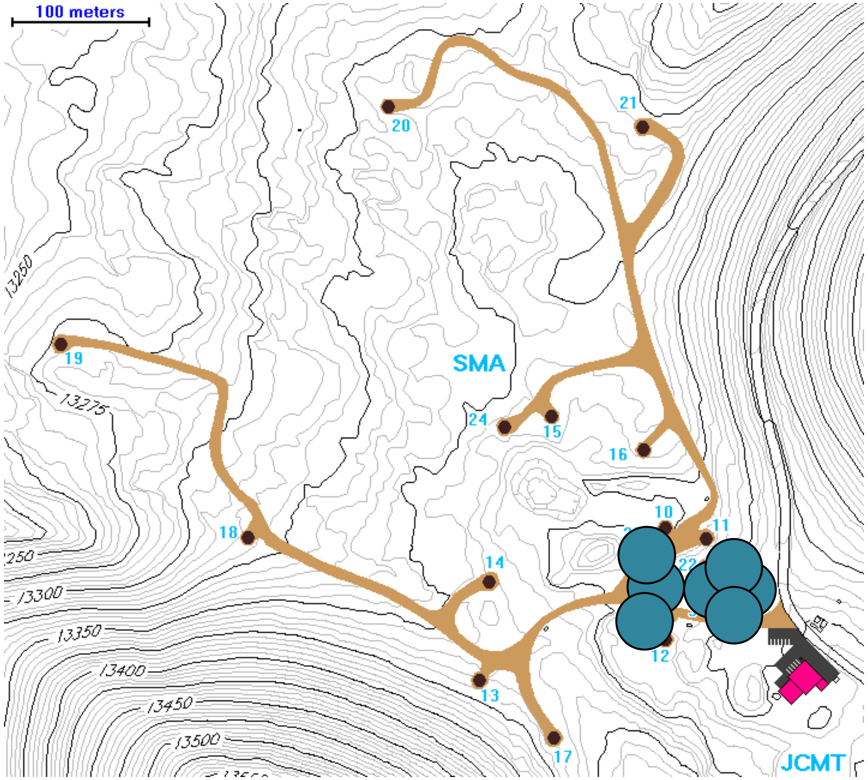
EXT configurations of 7 SMA antennas, ν = 345 GHz, dec = +22 deg

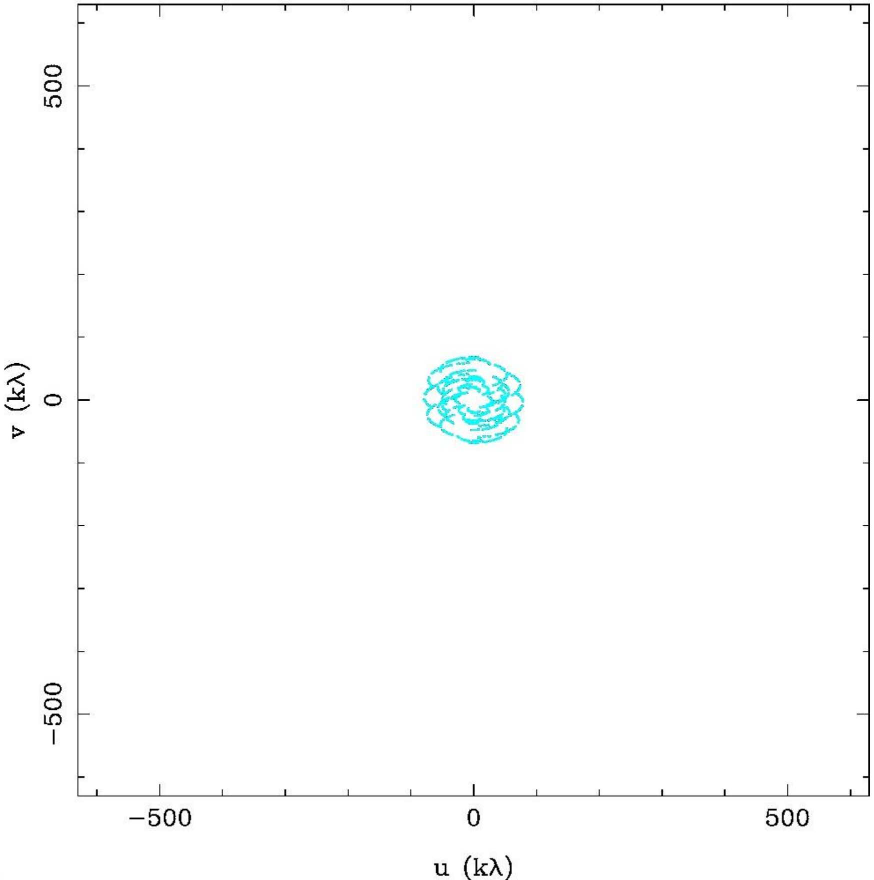


# An Example of (u,v) plane Sampling

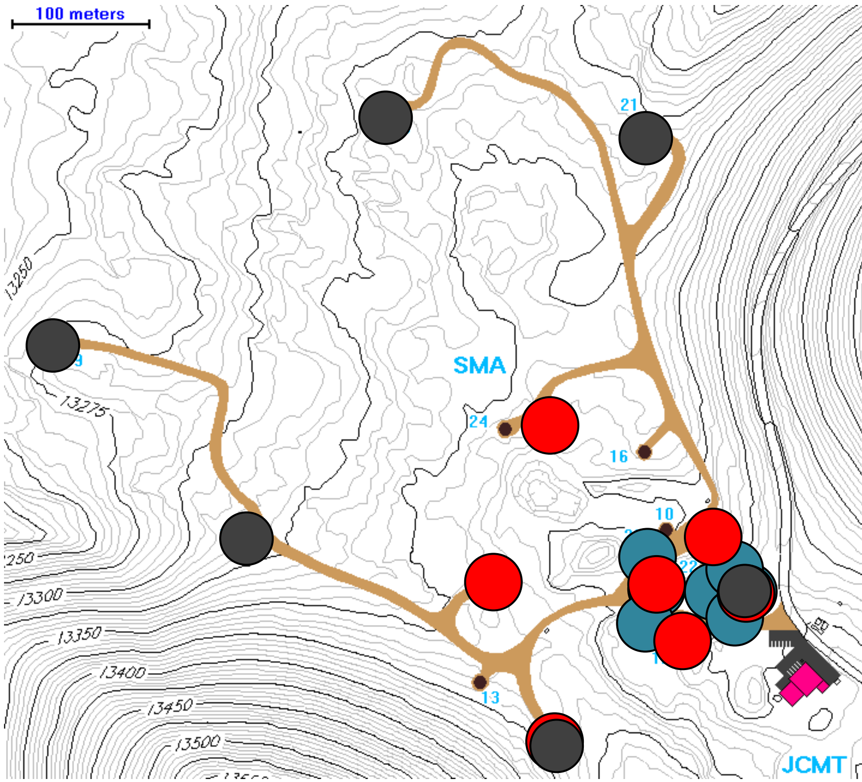
COM configurations of 7 SMA antennas, ν = 345 GHz, dec = +22 deg

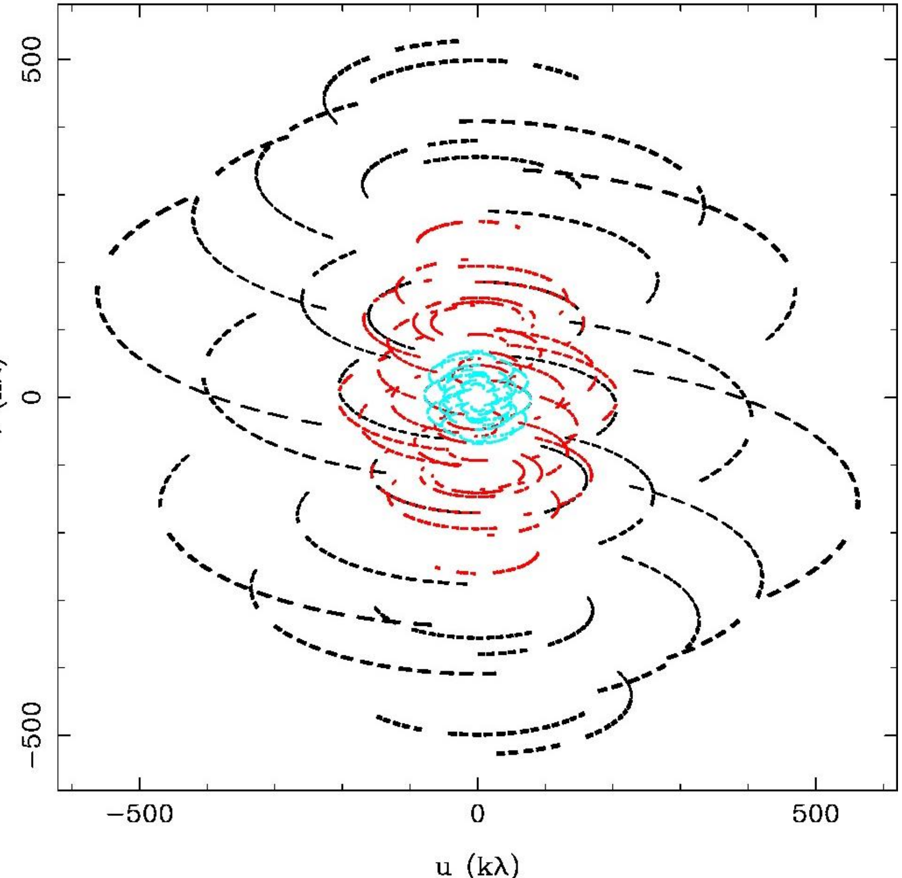


# An Example of (u,v) plane Sampling

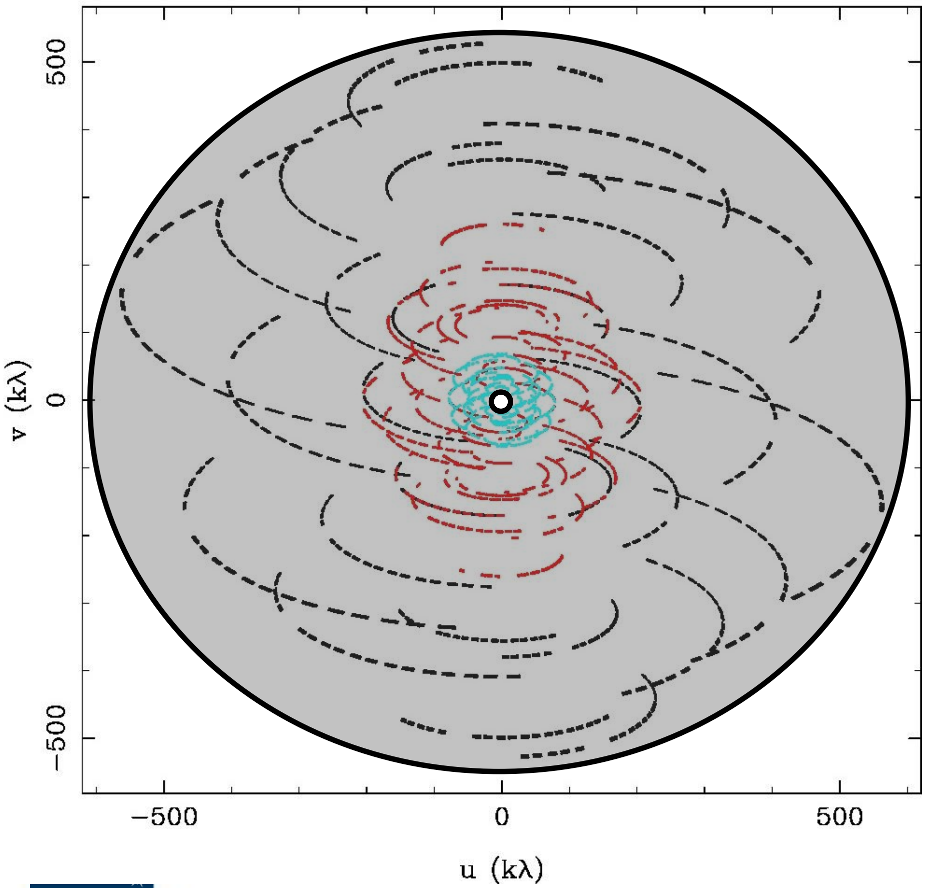
3 configurations of SMA antennas, ν = 345 GHz, dec = +22 deg



# Implications of (u,v) plane Sampling

samples of *V(u,v)* are limited by number of antennas and by Earth-sky geometry




- *outer boundary*
  - no information on smaller scales
  - resolution limit
- *inner hole*
  - no information on larger scales
  - extended structures invisible
- *irregular coverage between boundaries*
  - sampling theorem violated
  - information missing

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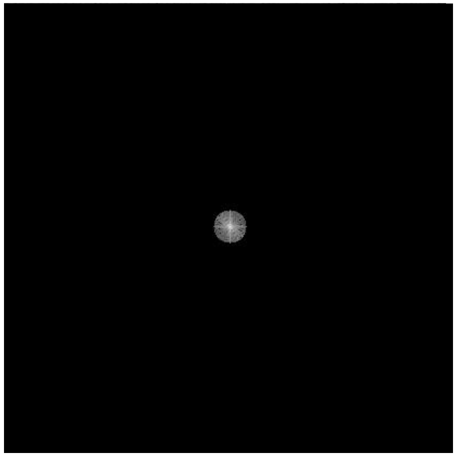
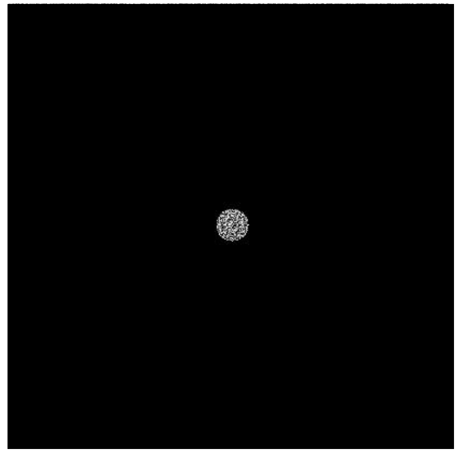
# Inner and Outer (u,v) Boundaries

*V(u,v) amplitude*



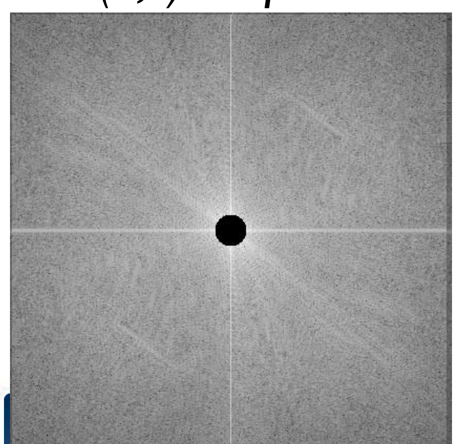
*V(u,v) phase*



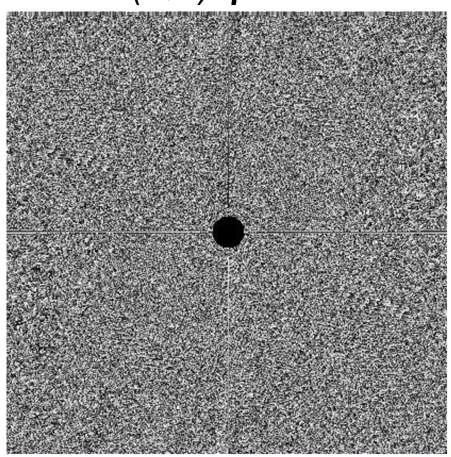
$\xrightarrow{\mathcal{F}}$

*T(l,m)*

*V(u,v) amplitude*

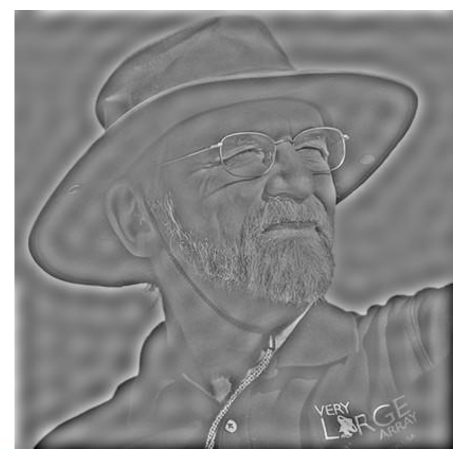


*V(u,v) phase*

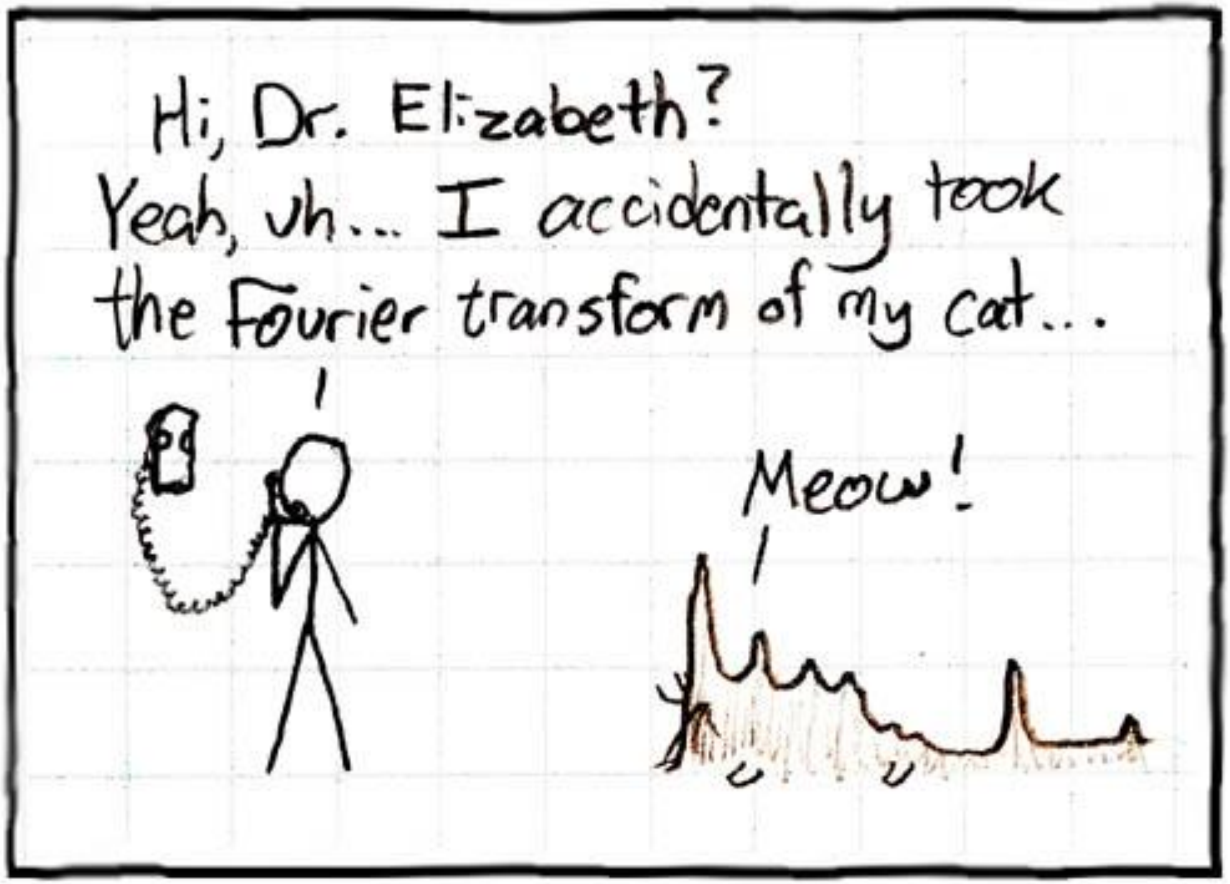


$\xrightarrow{\mathcal{F}}$

*T(l,m)*



# xkcd.com/26/



Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...
Meow!

# Formal Description of Imaging

$$V(u,v) \xrightarrow{\mathcal{F}} T(l,m)$$

- sample Fourier domain at discrete points $S(u,v) = \sum_{k=1}^{M} \delta(u-u_k, v-v_k)$
- Fourier transform sampled visibility function $V(u,v)S(u,v) \xrightarrow{\mathcal{F}} T^D(l,m)$
- apply the convolution theorem $T(l,m) * s(l,m) = T^D(l,m)$

where the Fourier transform of the sampling pattern $s(l,m) \xrightarrow{\mathcal{F}} S(u,v)$ is the "point spread function"

*the Fourier transform of the sampled visibilities yields the true sky brightness convolved with the point spread function*

radio jargon: the "dirty image" is the true image convolved with the "dirty beam"

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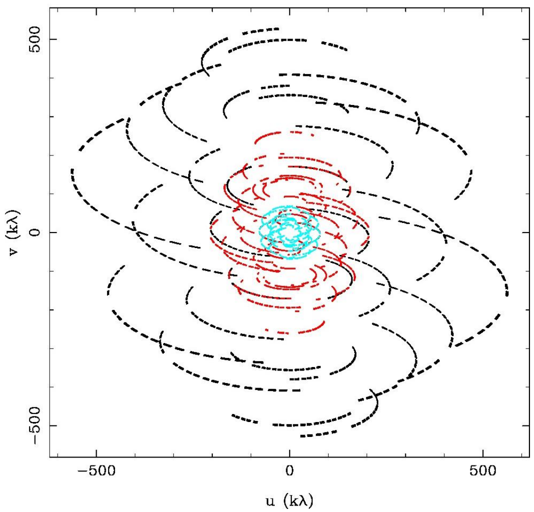
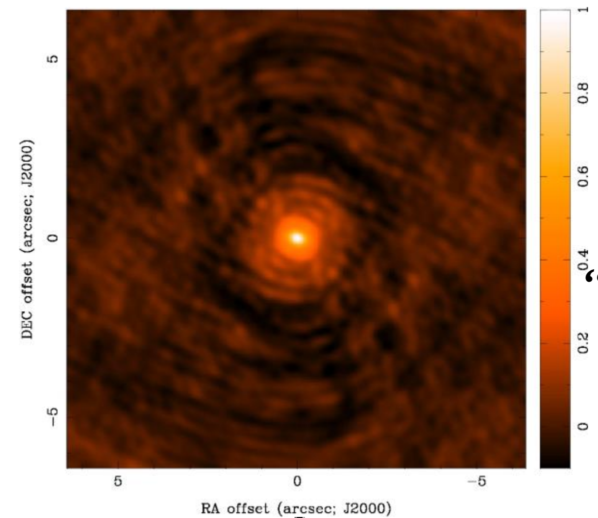
# Dirty Beam and Dirty Image

$S(u,v)$



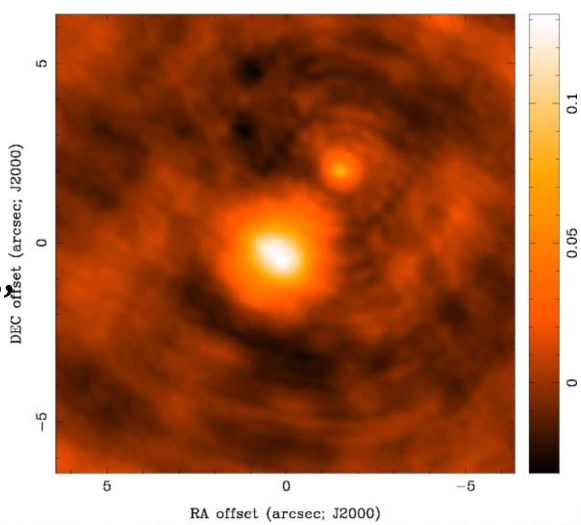

$\xrightarrow{\mathcal{F}}$




$s(l,m)$
*“dirty beam”*

$*$

$T^D(l,m)$
*“dirty image”*

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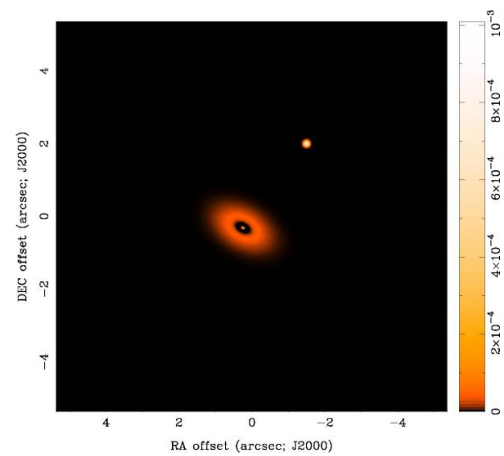
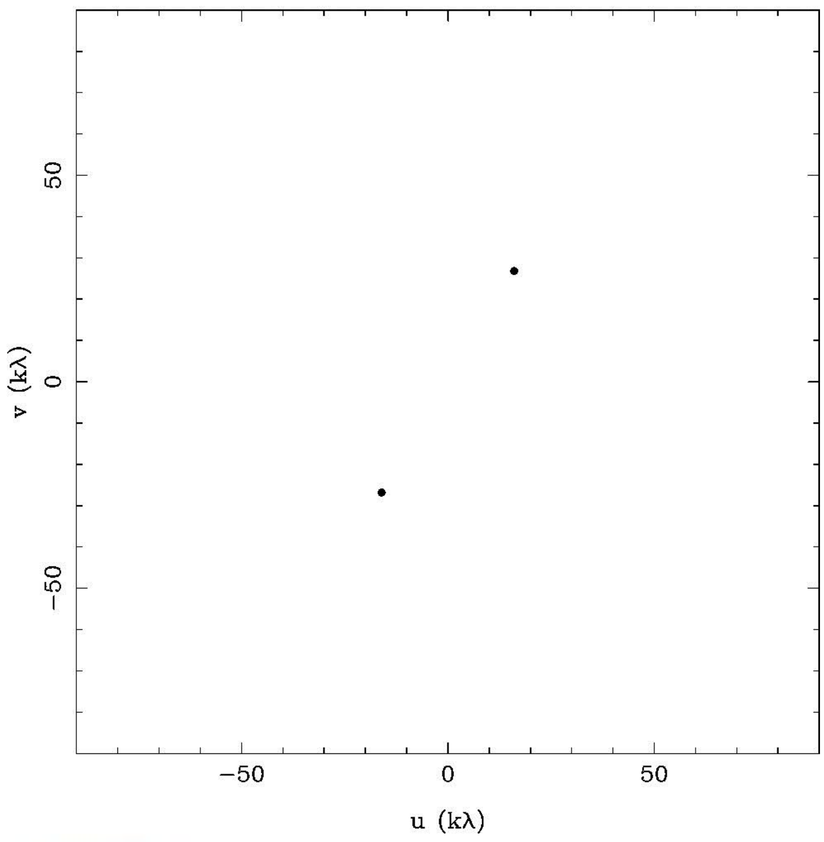
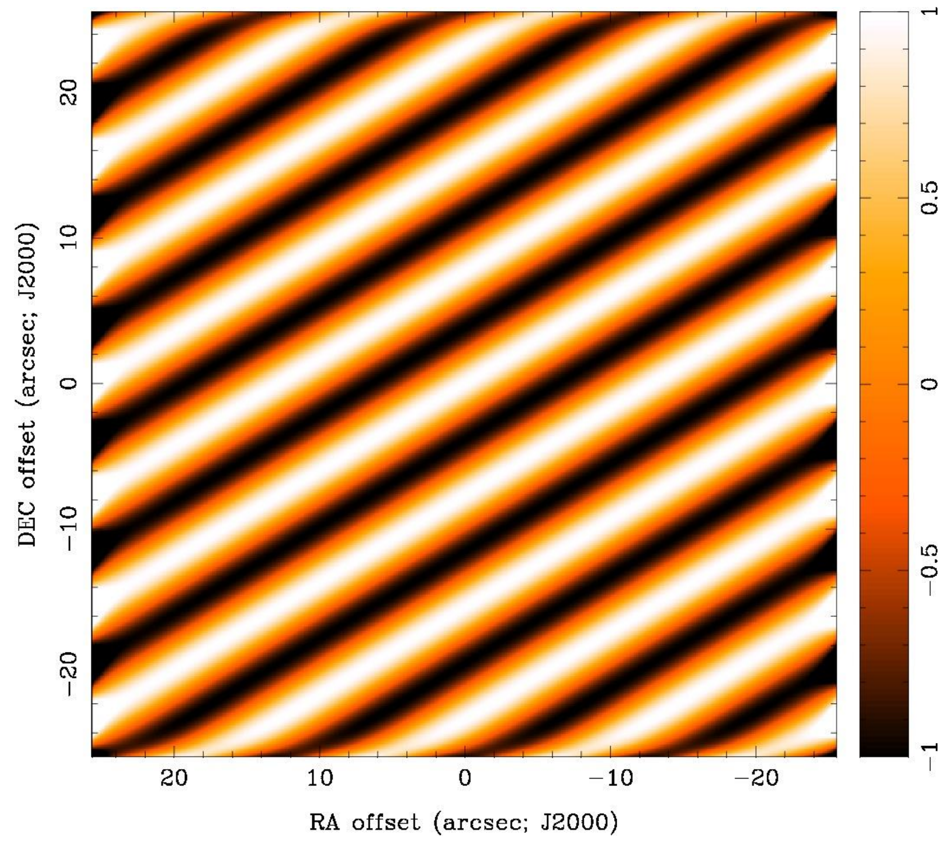



$T(l,m)$

# Dirty Beam Shape and N Antennas

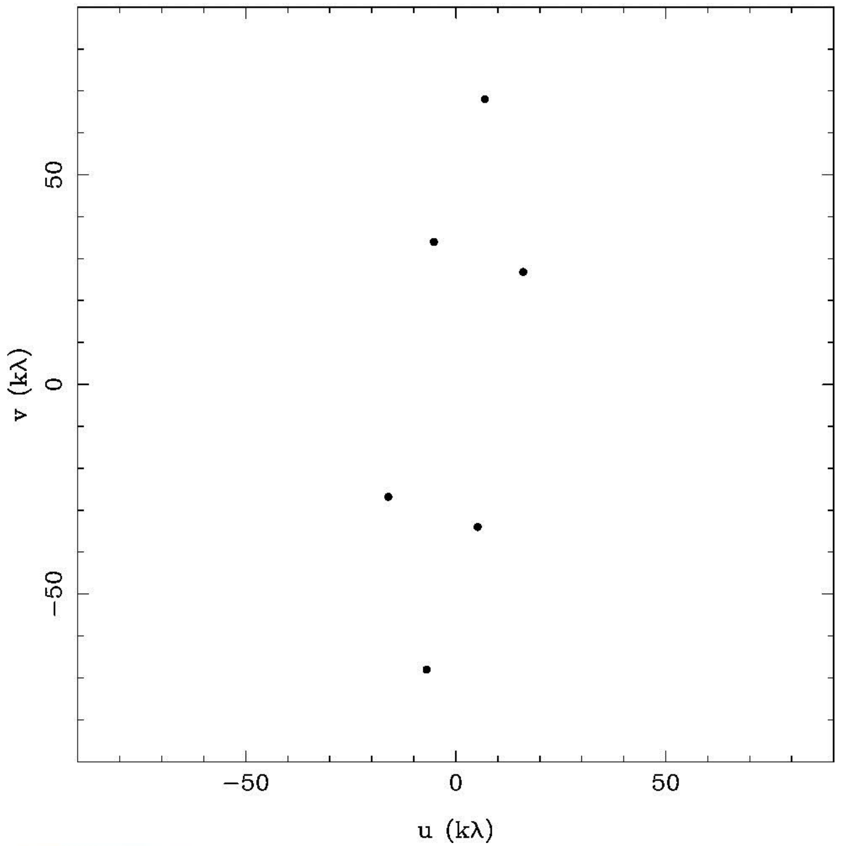
2 Antennas, 1 Sample

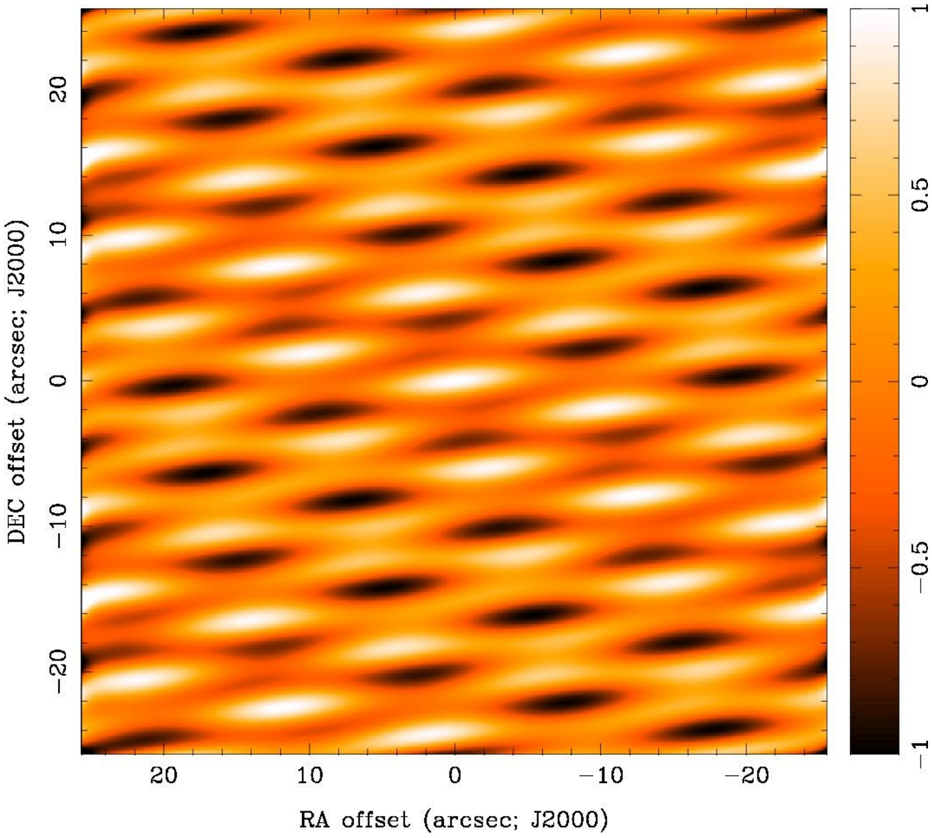


# Dirty Beam Shape and N Antennas

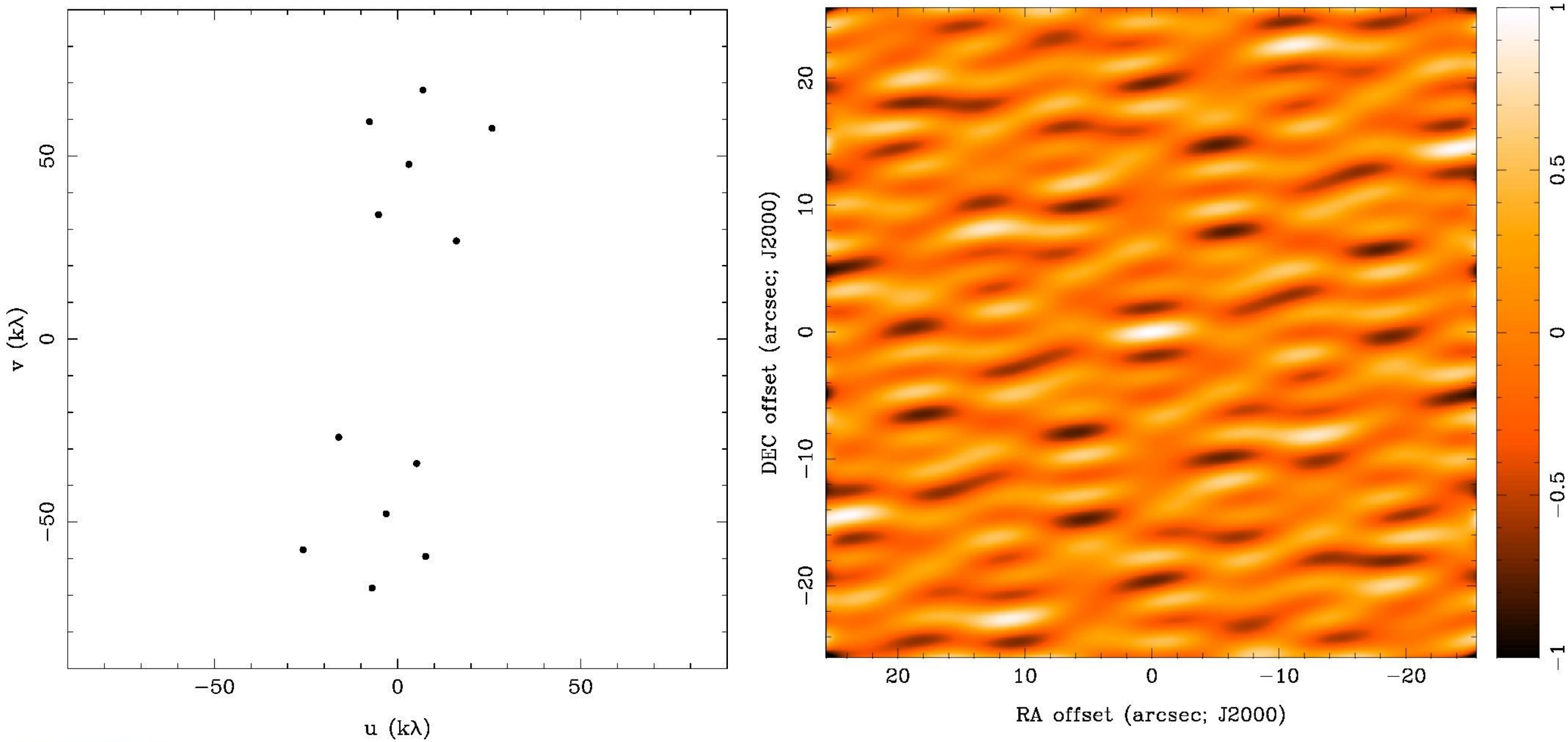
3 Antennas, 1 Sample



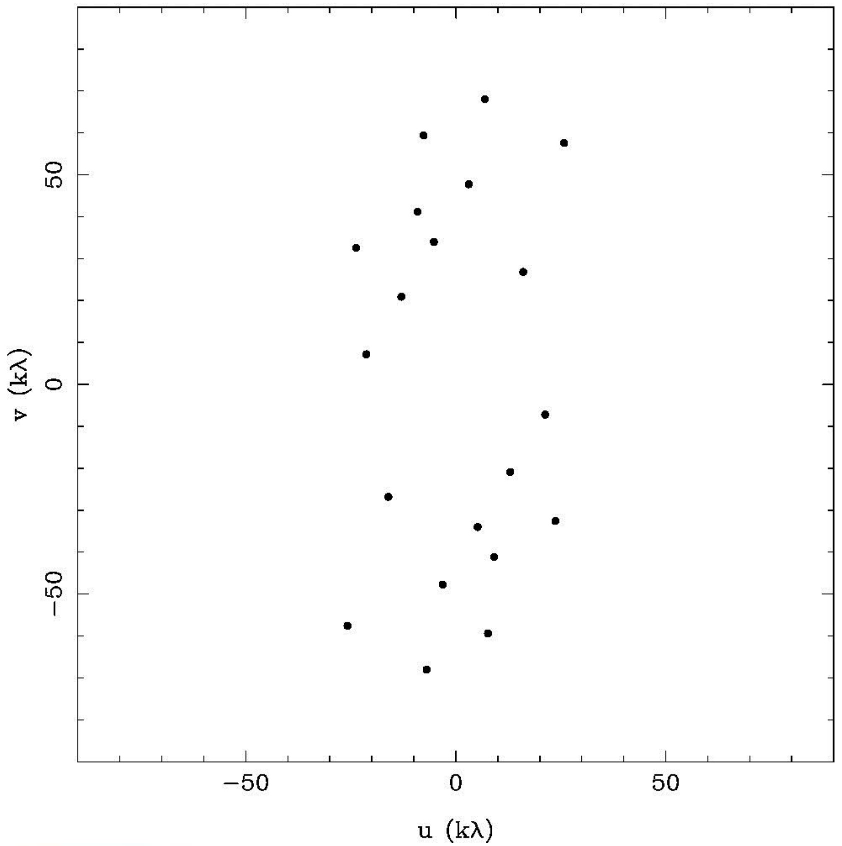
# Dirty Beam Shape and N Antennas

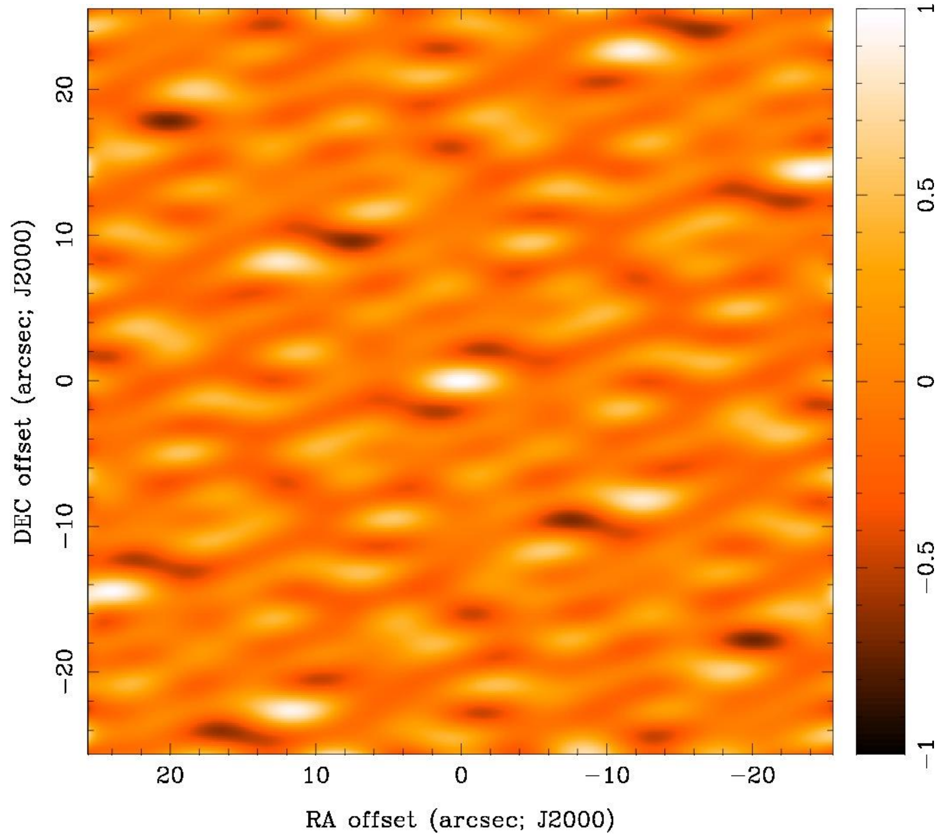
## 4 Antennas, 1 Sample



# Dirty Beam Shape and N Antennas

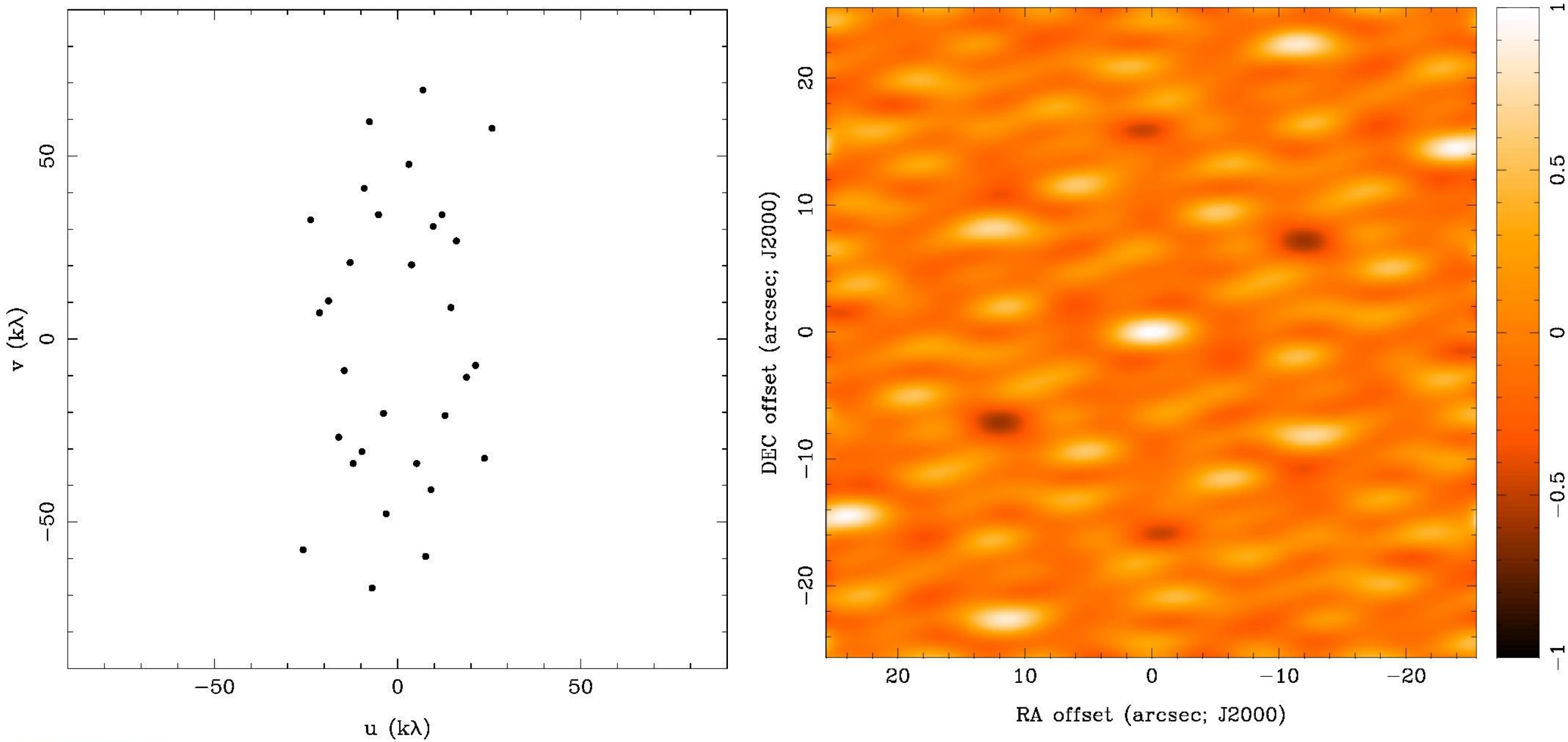
## 5 Antennas, 1 Sample



# Dirty Beam Shape and N Antennas

6 Antennas, 1 Sample

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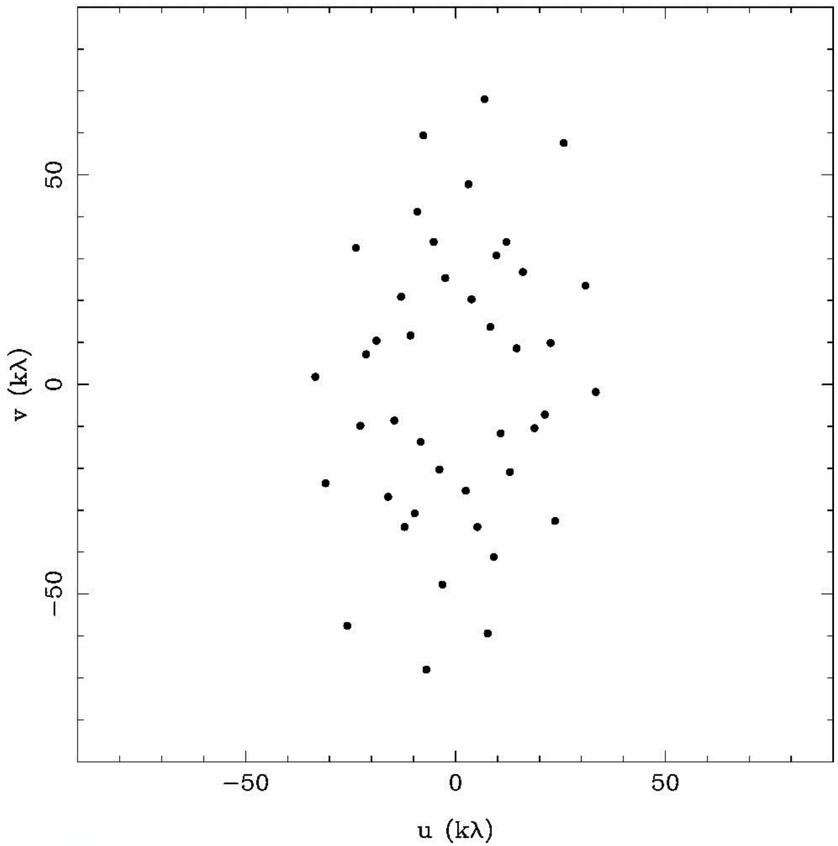
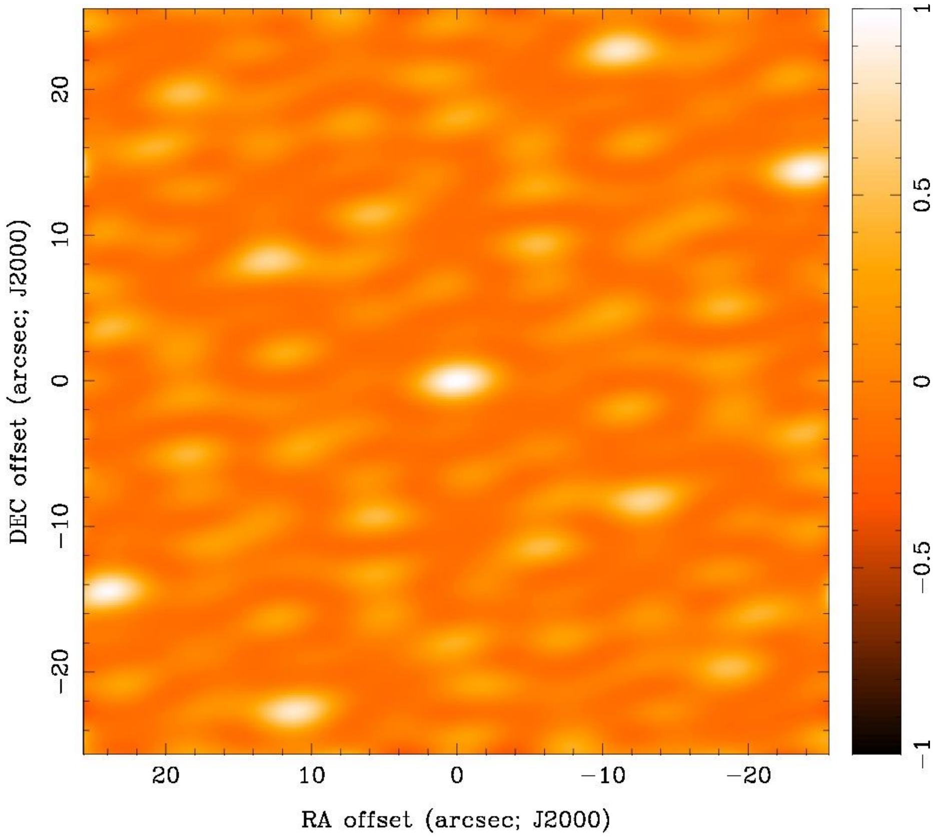
# Dirty Beam Shape and N Antennas

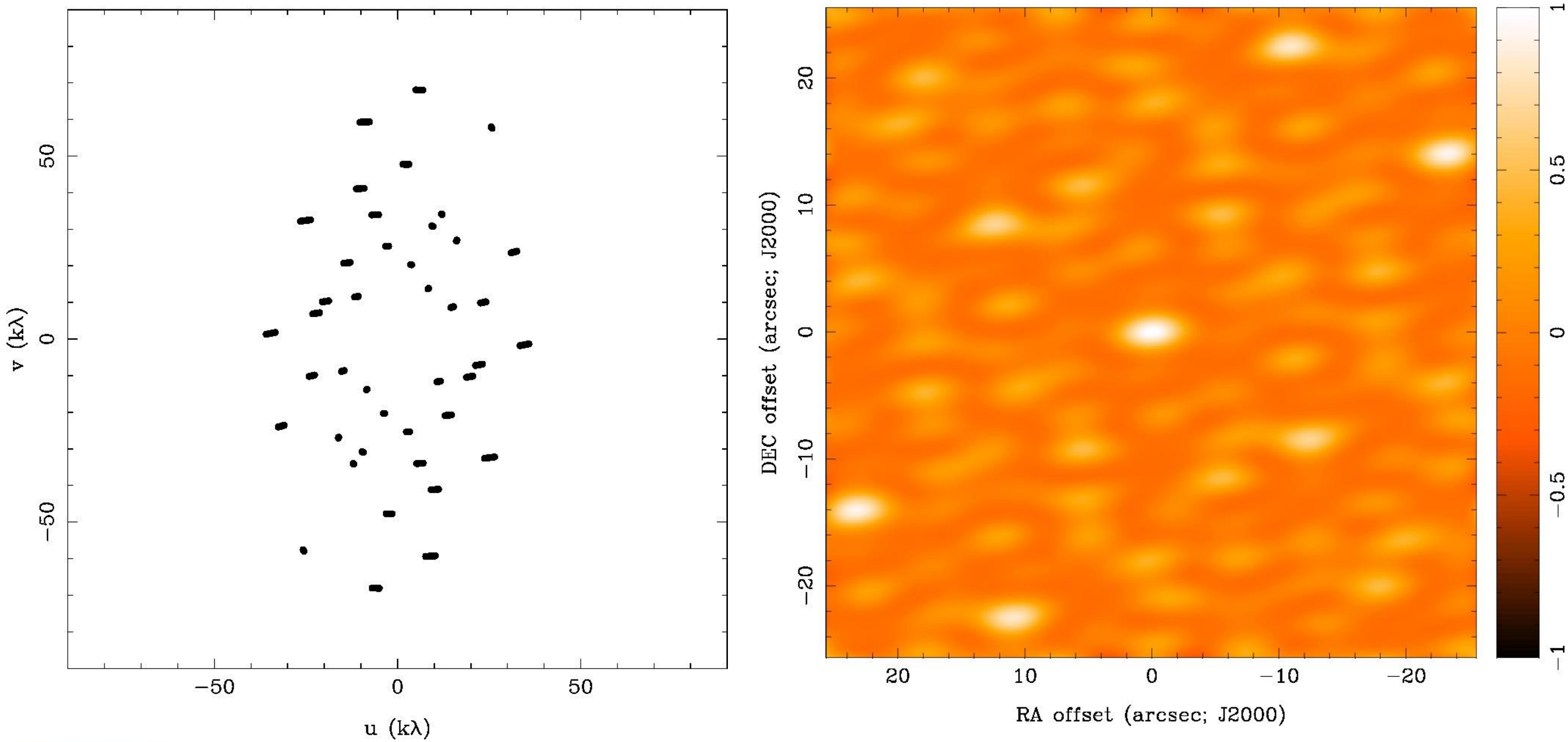
## 7 Antennas, 1 Sample



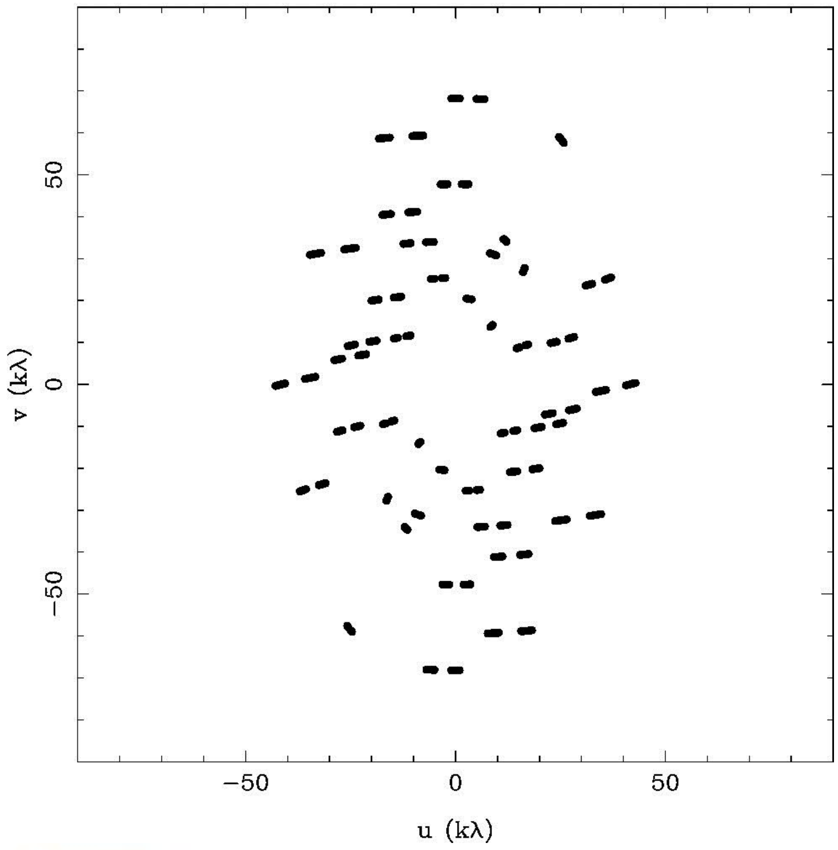
# Dirty Beam Shape and N Antennas

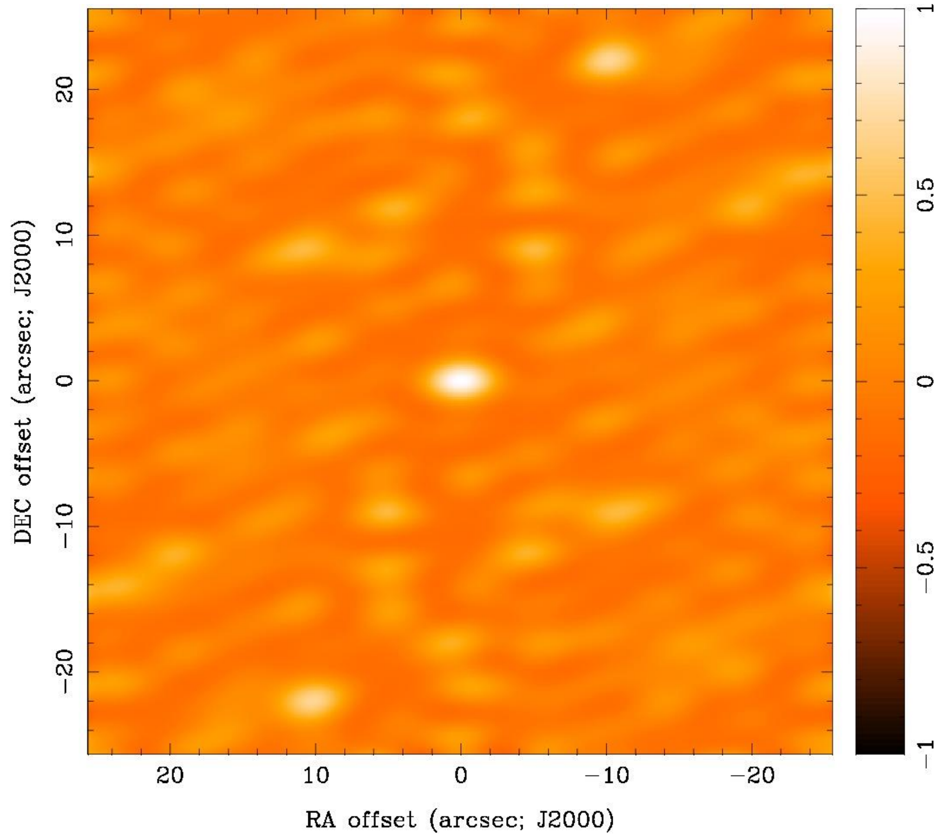
7 Antennas, 10 min



# Dirty Beam Shape and N Antennas

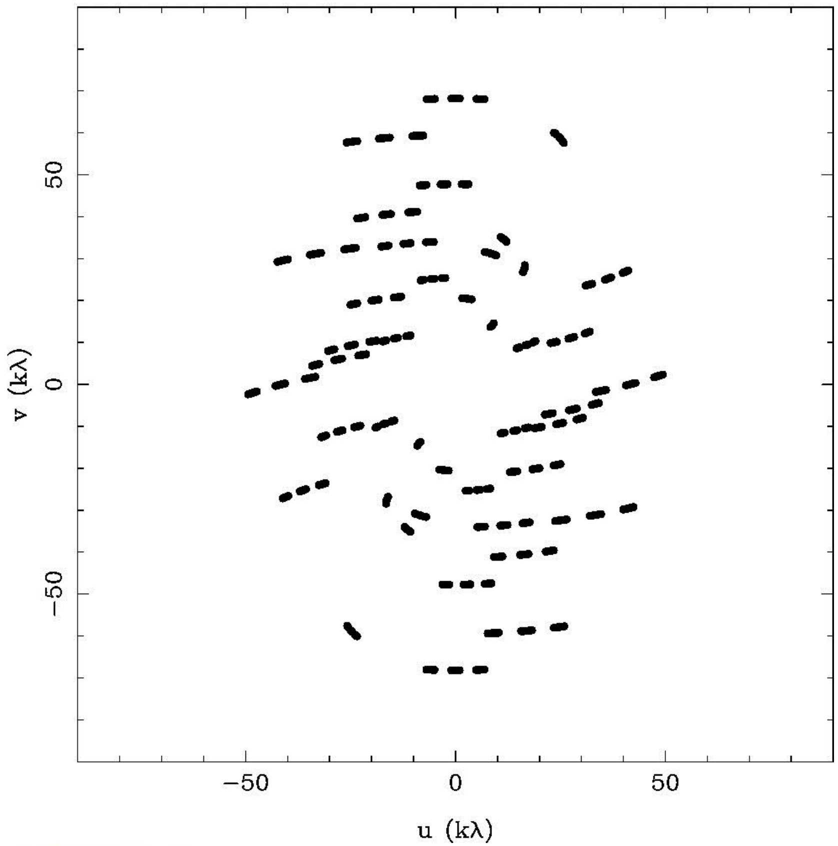
7 Antennas, 2 x 10 min

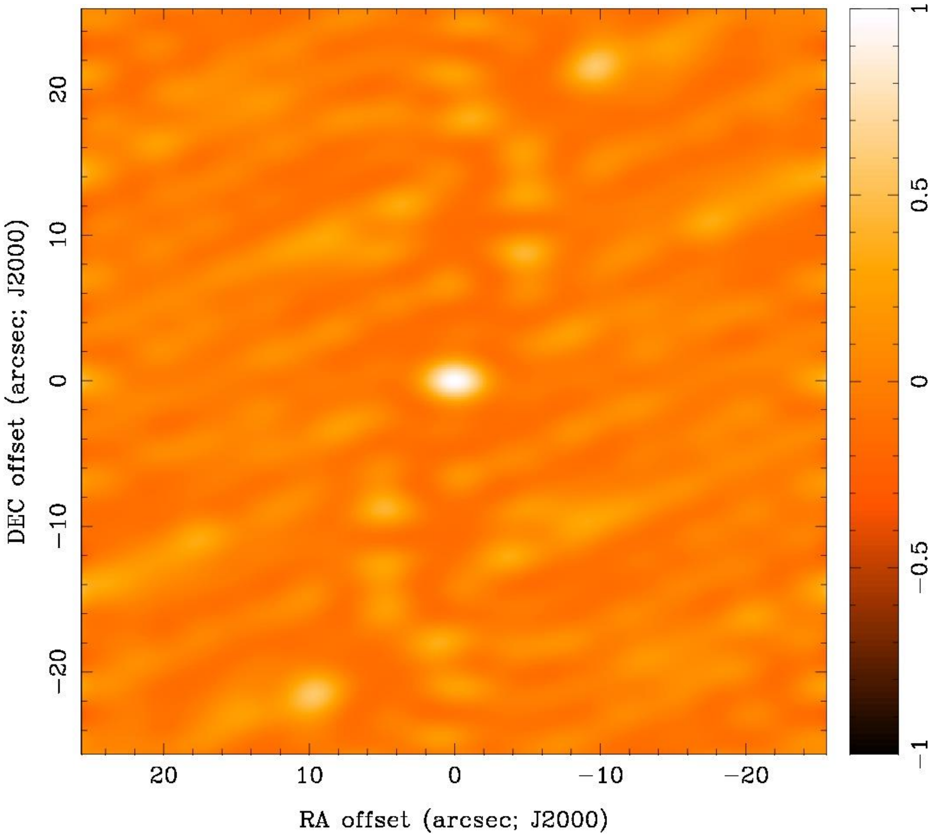


# Dirty Beam Shape and N Antennas

7 Antennas, 1 hour

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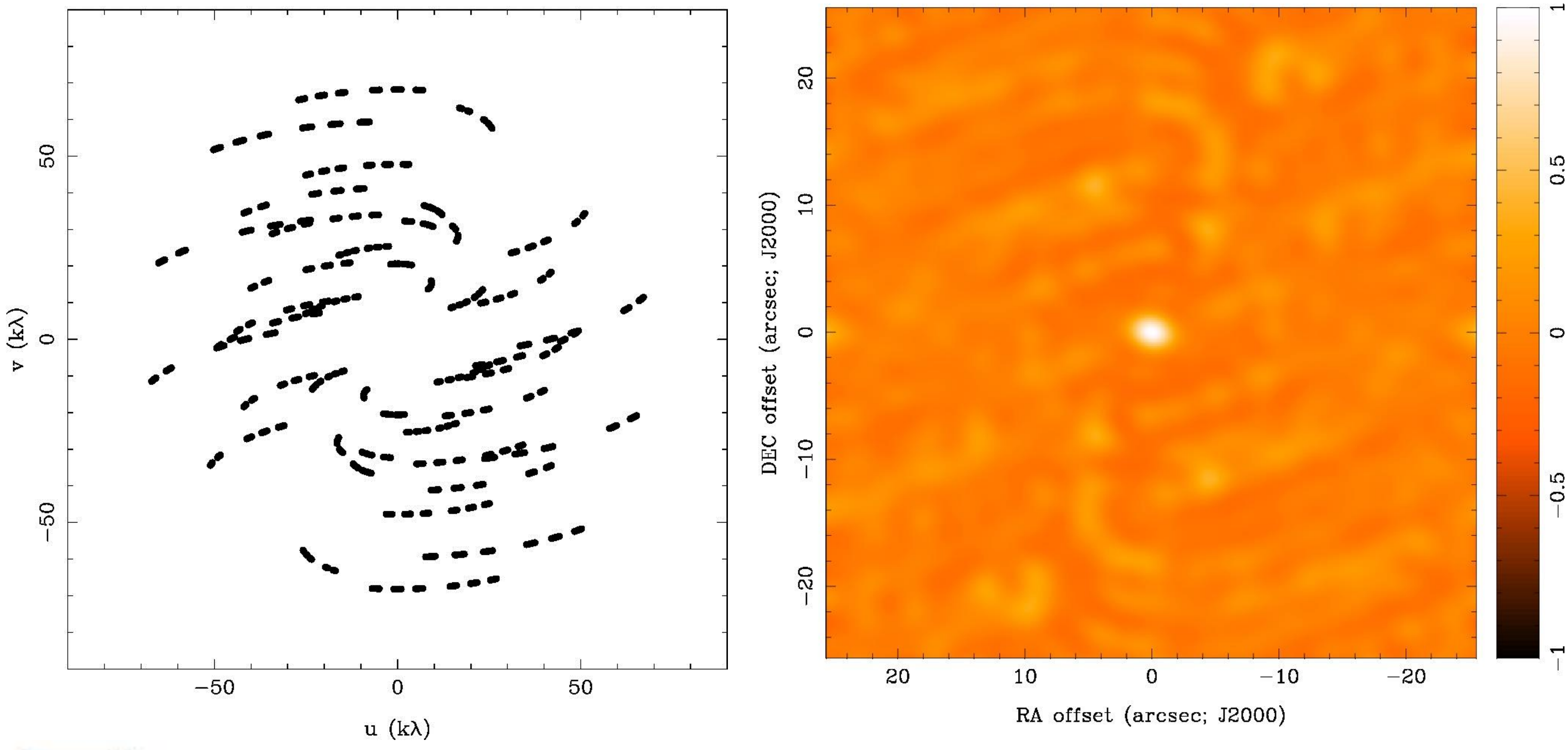
# Dirty Beam Shape and N Antennas

7 Antennas, 3 hours

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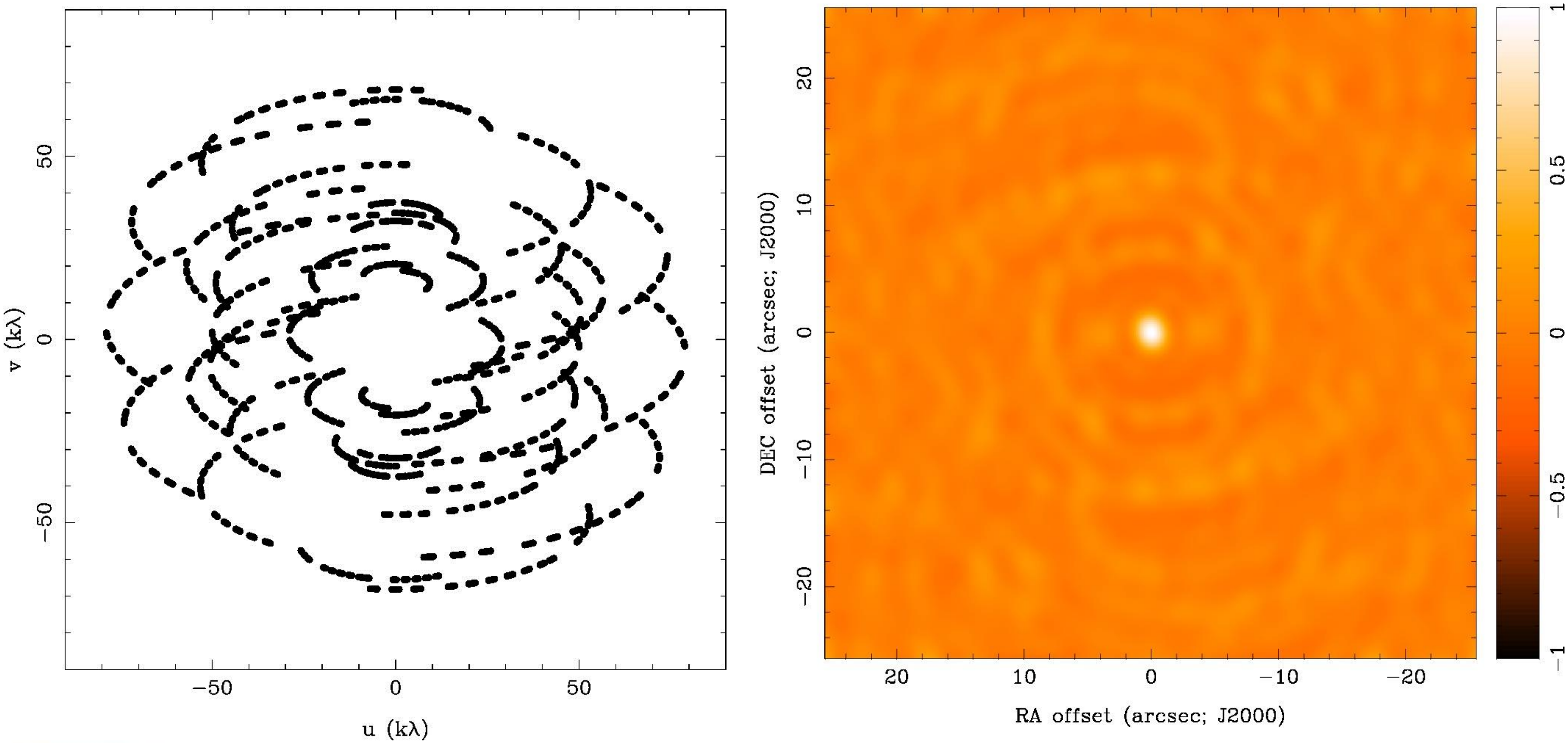
# Dirty Beam Shape and N Antennas

7 Antennas, 8 hours

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# Calibrated Visibilities: What's Next?

- analyze directly $V(u,v)$ samples by model fitting
  - good for simple structures, e.g. point sources, symmetric disks
  - sometimes for statistical descriptions of sky brightness
  - visibilities have very well defined noise properties

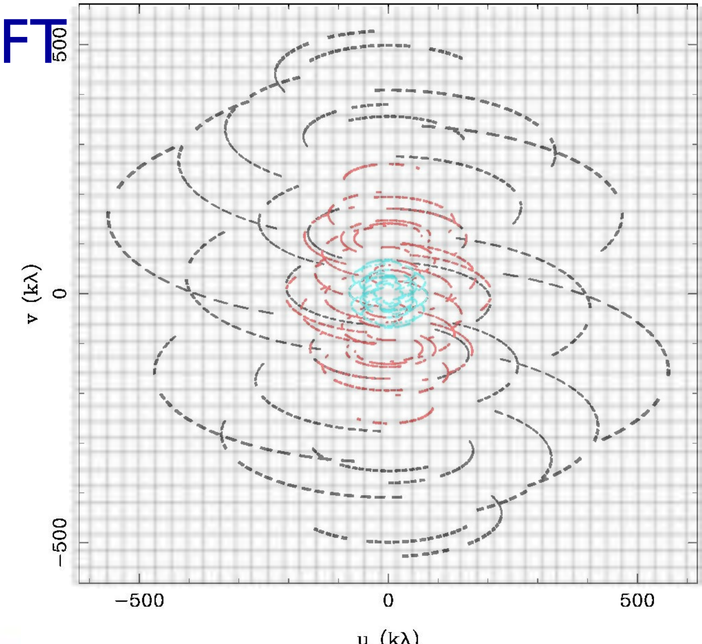
- recover an image from the observed incomplete and noisy samples of its Fourier transform for analysis
  - Fourier transform $V(u,v)$ to get $T^D(l,m)$
  - difficult to do science with the dirty image $T^D(l,m)$
  - deconvolve $s(l,m)$ from $T^D(l,m)$ to determine a model of $T(l,m)$
  - work with the model of $T(l,m)$

# Some Details of the Dirty Image

- "Fourier transform"
  - Fast Fourier Transform (FFT) algorithm is much faster than simple Fourier summation, $O(N\log N)$ for $2^N \times 2^N$ image
  - FFT requires data on a regularly spaced grid
  - aperture synthesis does not provide *V(u,v)* on a regularly spaced grid, so…

- "gridding" used to resample *V(u,v)* for FFT
  - customary to use a convolution method
  - special ("spheroidal") functions that minimize smoothing and aliasing

$$V^G(u,v) = V(u,v)S(u,v) * G(u,v)$$
$$\xrightarrow{F} T^D(l,m)g(l,m)$$

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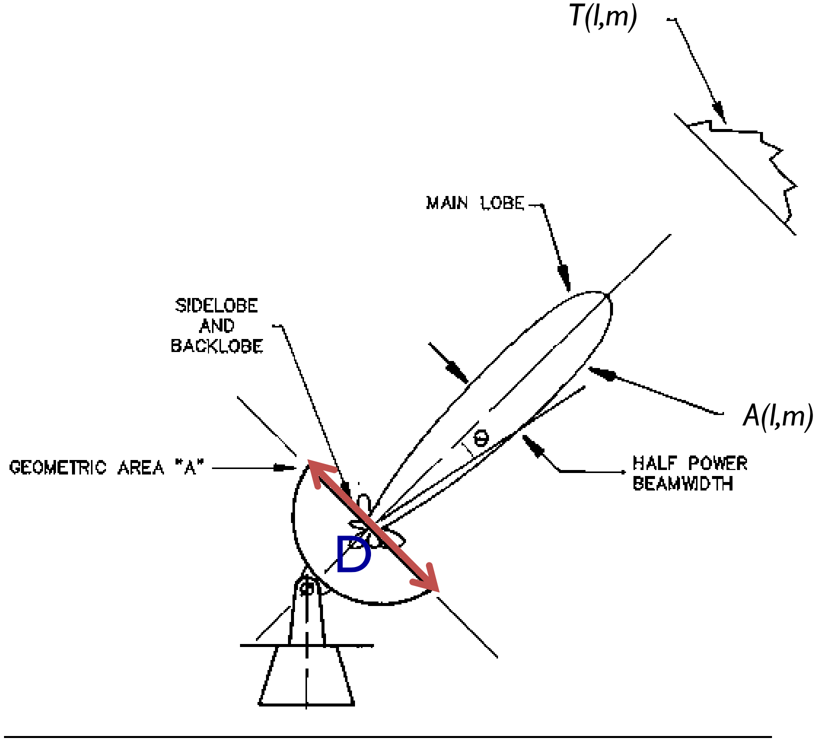
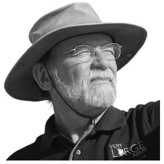
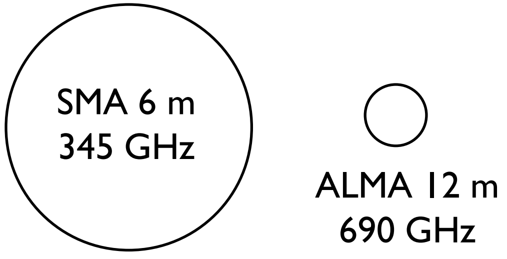
# Antenna Primary Beam Response

- antenna response *A(l,m)* is not uniform across the entire sky
  - main lobe = "primary beam" fwhm ~ λ/D
  - response beyond primary beam can be important ("sidelobes")




- antenna beam modifies the sky brightness distribution
  - $T(l,m) \rightarrow T(l,m)A(l,m)$
  - can correct with division by *A(l,m)* in the image plane
  - large source extents require multiple pointings of antennas = mosaicking





# Imaging Decisions: Pixel Size, Image Size

- pixel size
  - satisfy sampling theorem for longest baselines

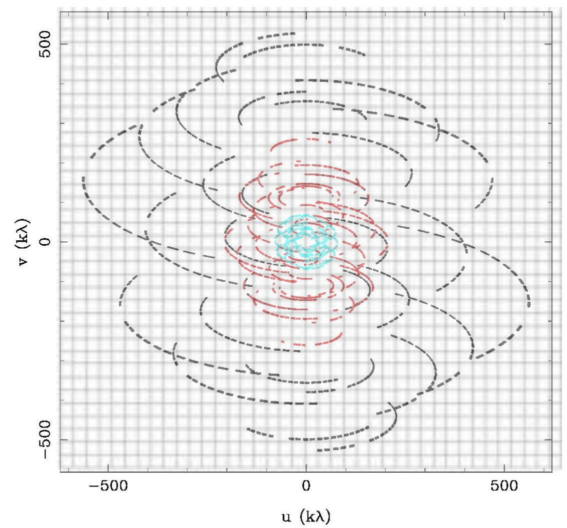
$$\Delta l < \frac{1}{2u_{max}} \qquad \Delta m < \frac{1}{2v_{max}}$$

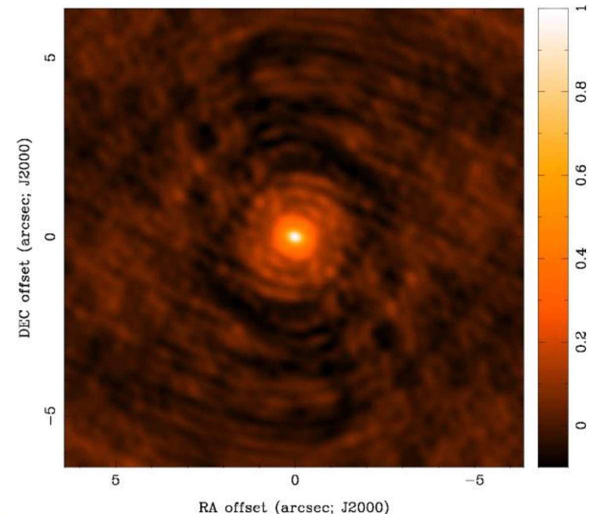
  - in practice, 3 to 5 pixels across main lobe of dirty beam to aid deconvolution
  - e.g. at 870 μm with baselines to 500 meters $\rightarrow$ pixel size < 0.1 arcsec
  - CASA "cell" size
- image size
  - natural choice is often the full extent of the primary beam *A(l,m)*
  - e.g. SMA at 870 μm, 6 meter antennas $\rightarrow$ image size 2 x 35 arcsec
  - if there are bright sources in the sidelobes of A(l,m), then the FFT will alias them into the image $\rightarrow$ make a larger image (or equivalent)
  - CASA "imsize"

# Imaging Decisions: Visibility Weighting

- introduce weighting function $W(u,v)$
  - modifies sampling function
  - $S(u,v) \rightarrow S(u,v)W(u,v)$
  - changes $s(l,m)$, the dirty beam shape

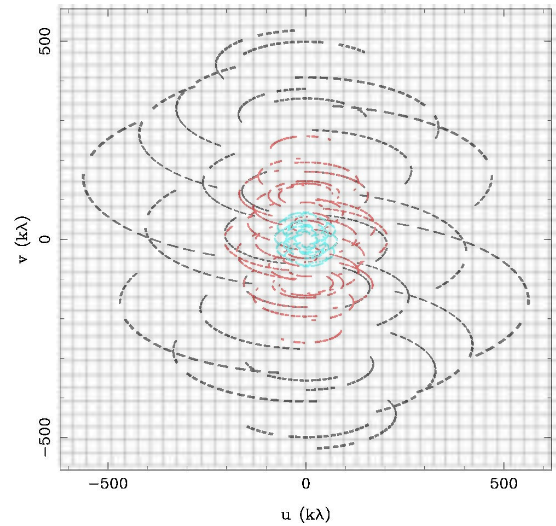
- natural weight
  - $W(u,v) = 1/\sigma^2$ in occupied $(u.v)$ cells, where $\sigma^2$ is the noise variance, and $W(u,v) = 0$ everywhere else
  - maximizes point source sensitivity
  - lowest rms in image
  - generally gives more weight to short baselines (low spatial frequencies), so angular resolution is degraded

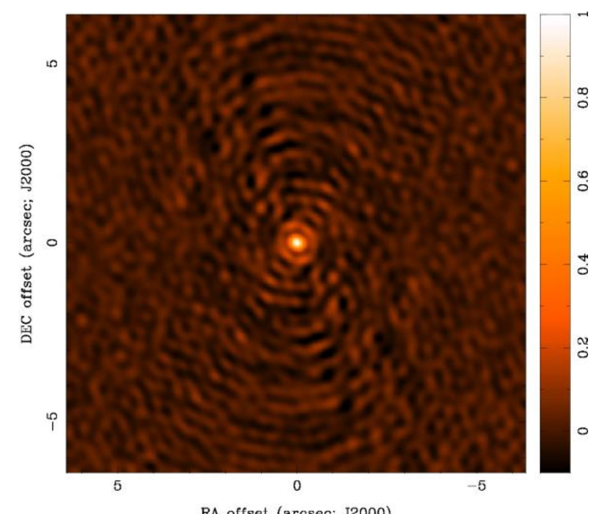


# Dirty Beam Shape and Weighting

- uniform weight
  - *W(u,v)* is inversely proportional to local density of *(u,v)* points
  - sum of weights in a *(u,v)* cell = const (and 0 for empty cells)
  - fills *(u,v)* plane more uniformly and dirty beam sidelobes are lower
  - gives more weight to long baselines (high spatial frequencies), so angular resolution is enhanced
  - downweights some data, so point source sensitivity is degraded
  - can be trouble with sparse sampling: cells with few data points have same weight as cells with many data points

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# Dirty Beam Shape and Weighting

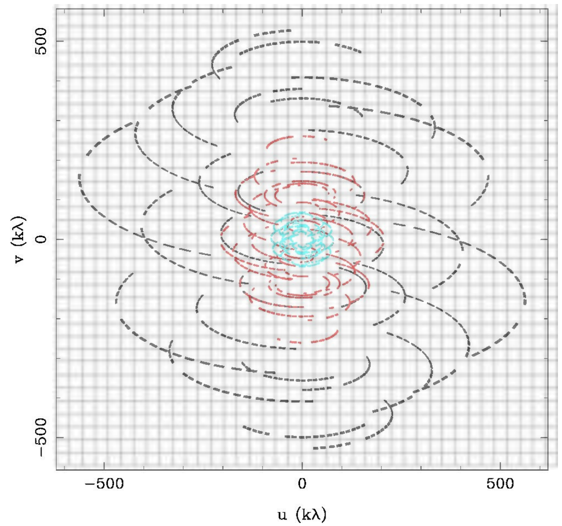
- robust (Briggs) weight
  - variant of uniform that avoids giving too much weight to *(u.v)* cells with low natural weight
  - software implementations differ
  - e.g. $$W(u,v) = \frac{1}{\sqrt{1 + S_N^2/S_{thresh}^2}}$$

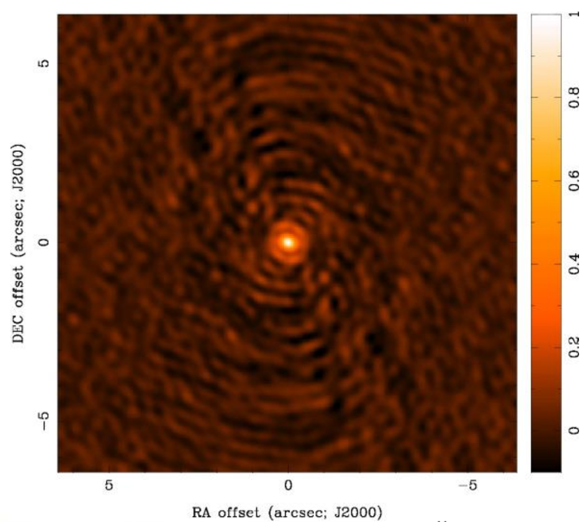
    $S_N$ is natural weight of cell

    $S_{thresh}$ is a threshold

    high threshold $\rightarrow$ natural weight

    low threshold $\rightarrow$ uniform weight
- *an adjustable parameter allows for continuous variation between maximum point source sensitivity and resolution*



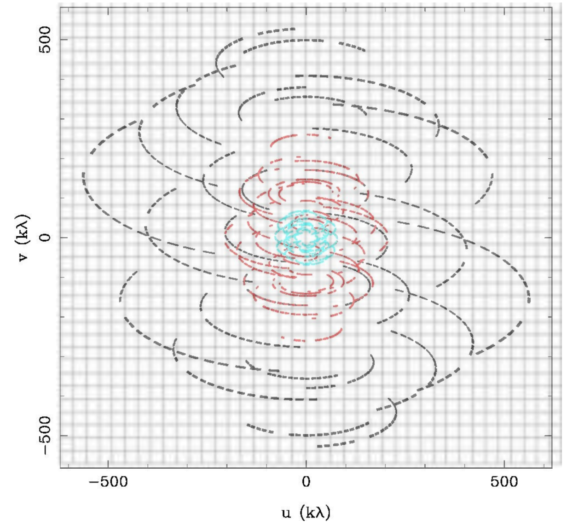



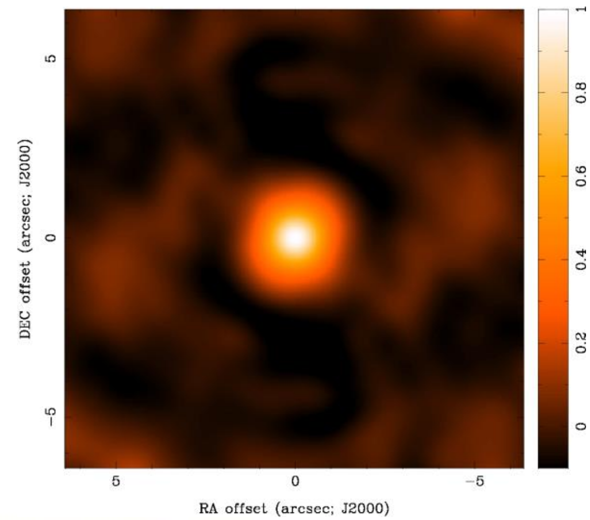
# Dirty Beam Shape and Weighting

- tapering
  - apodize *(u,v)* sampling by a Gaussian

$$W(u,v) = \exp\left(-\frac{(u^2+v^2)}{t^2}\right)$$

  *t* = adjustable tapering parameter
  - like smoothing in the image plane (convolution by a Gaussian)
  - gives more weight to short baselines, degrades angular resolution
  - downweights some data, so point source source sensitivity degraded
  - may improve sensitivity to extended structure sampled by short baselines
  - limits to usefulness

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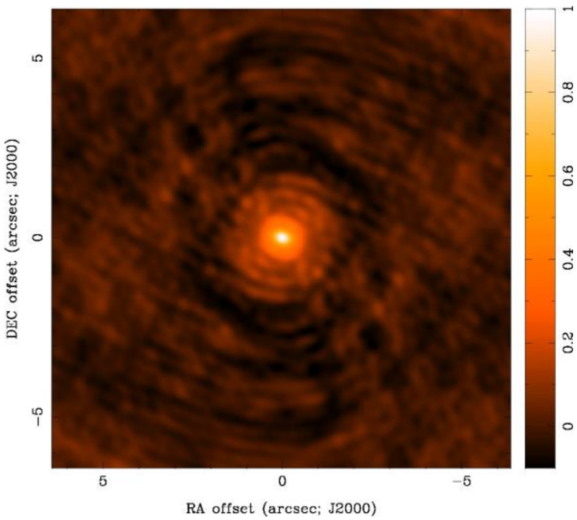
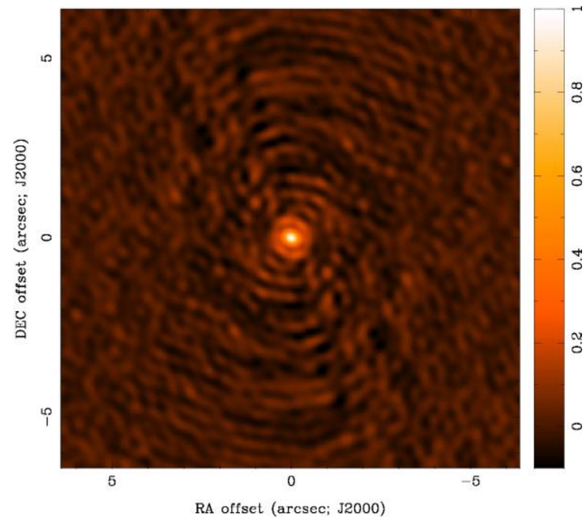
# Weighting and Tapering: Image Noise

natural
0.59x0.50

rms=1.0



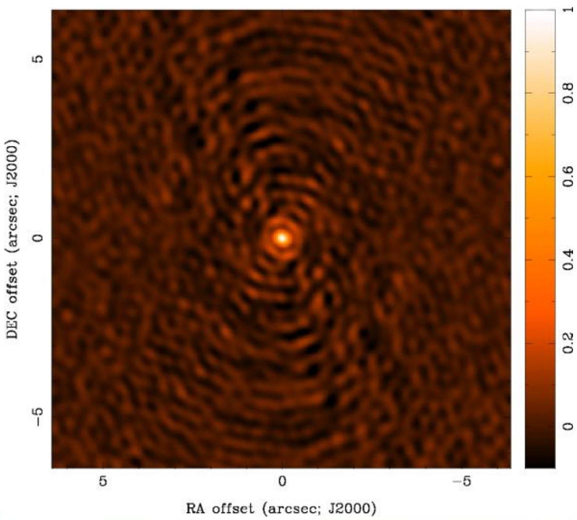



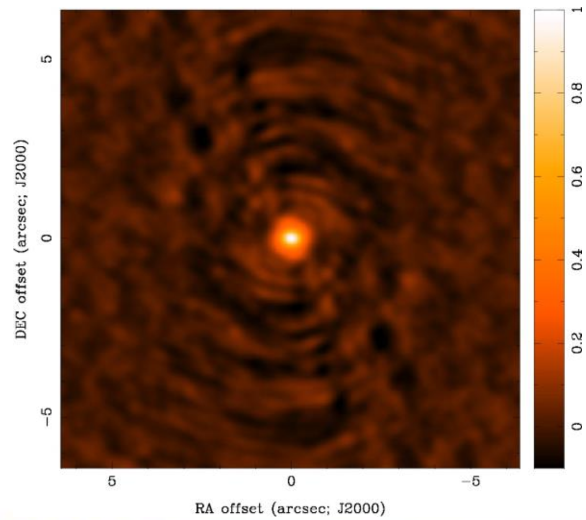

robust=0
0.40x0.34

rms=1.3

uniform
0.35x0.30

rms=2.1




robust=0
+ taper to
0.59x0.50

rms=1.2

# Weighting and Tapering: Summary

- imaging parameters provide a lot of freedom
- appropriate choices depend on science goals

| | Robust/Uniform | Natural | Taper |
|---|---|---|---|
| resolution | higher | medium | lower |
| sidelobes | lower | higher | depends |
| point source sensitivity | lower | maximum | lower |
| extended source sensitivity | lower | medium | higher |

# Beyond the Dirty Image: Deconvolution

- to keep you awake at night
  - $\exists$ an infinite number of *T(l,m)* compatible with sampled *V(u,v)*, with "invisible" distributions *R(l,m)* where *s(l,m)* * *R(l,m)* = 0
    - no data beyond $u_{max}$,$v_{max}$ $\rightarrow$ unresolved structure
    - no data within $u_{min}$,$v_{min}$ $\rightarrow$ limit on largest size scale
    - holes in between $\rightarrow$ synthesized beam sidelobes
  - noise $\rightarrow$ undetected/corrupted structure in *T(l,m)*
  - no unique prescription for extracting optimum estimate of *T(l,m)*

- deconvolution
  - uses non-linear techniques to interpolate/extrapolate samples of *V(u,v)* into unsampled regions of the *(u,v)* plane
  - aims to find a sensible model of *T(l,m)* compatible with data
  - requires *a priori* assumptions about *T(l,m)* to pick plausible "invisible" distributions to fill unmeasured parts of the Fourier plane

# Deconvolution Algorithms

- an active research area, e.g. compressive sensing methods

- clean: dominant deconvolution algorithm in radio astronomy
  - *a priori* assumption: *T(l,m)* is a collection of point sources
  - fit and subtract the synthesized beam iteratively
  - original version by Högbom (1974) purely image based
  - variants developed for higher computational efficiency, model visibility subtraction, to deal better with extended emission structure, etc.

- maximum entropy: a rarely used alternative
  - *a priori* assumption: *T(l,m)* is smooth and positive
  - define "smoothness" via a mathematical expression for entropy, e.g. Gull and Skilling (1983), find smoothest image consistent with data
  - vast literature about the deep meaning of entropy as information content

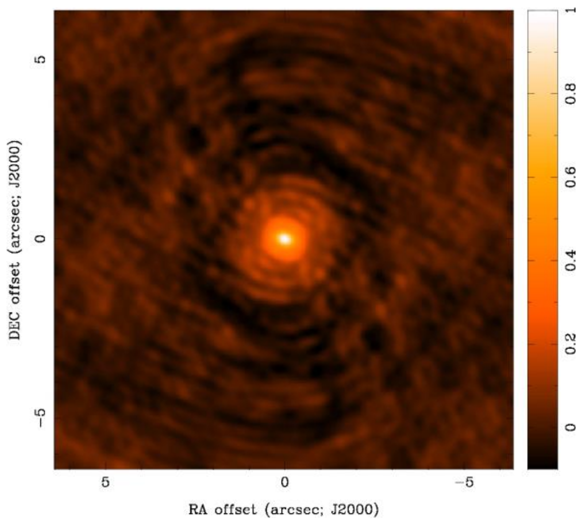
# Basic clean Algorithm

- initialize

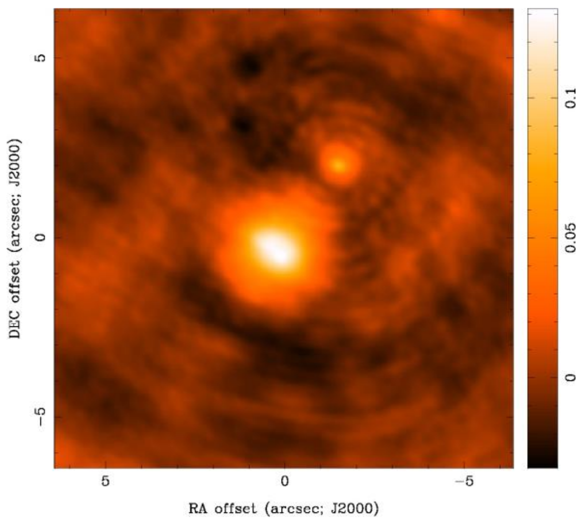
  a *residual map* to the dirty map

  a *Clean Component* list

1. identify the highest peak in the residual map as a point source
2. subtract a fraction of this peak from the *residual map* using a scaled dirty beam, *s(l,m)* x gain
3. add this point source location and amplitude to the *Clean Component* list
4. goto step 1 (an iteration) unless stopping criterion reached




*s(l,m)*




$T^D(l,m)$

# Basic clean Algorithm (continued)

- stopping criteria?
  - *residual map* maximum < threshold = multiple of rms (if noise limited)
  - *residual map* maximum < threshold = fraction of dirty map maximum (if dynamic range limited)
  - maximum number of *Clean Components* reached (no justification)
- loop gain?
  - good results for g=0.1 to 0.3
  - lower values can work better for smoother emission, g=0.05
- easy to include *a priori* information about where in dirty map to search for *Clean Components* (using "boxes" or "masks")
  - very useful but potentially dangerous

- Schwarz (1978) showed that the clean algorithm is equivalent to a least squares fit of sinusoids to visibilities in the case of no noise

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# Basic clean Algorithm (continued)

- last step: make "restored" image
  - make a model image with all point source *Clean Components*
  - convolve point sources with an elliptical Gaussian, fit to the main lobe of the dirty beam ("clean beam"); avoids super-resolution of model
  - add *residual map* of noise and source structure below the threshold
- resulting restored image is an estimate of the true sky brightness *T(l,m)*
- units of the restored image are (mostly) Jy per clean beam area

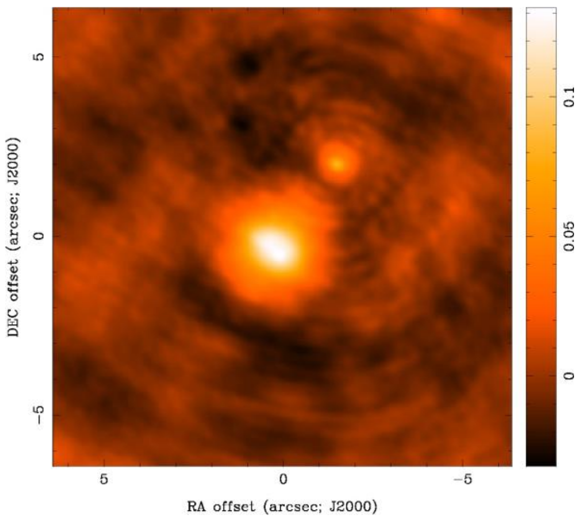
  = intensity (or brightness temperature)

- for most weighting schemes, there is information in the image from baselines that sample high spatial frequencies within the clean beam fwhm, so modest super-resolution may be OK

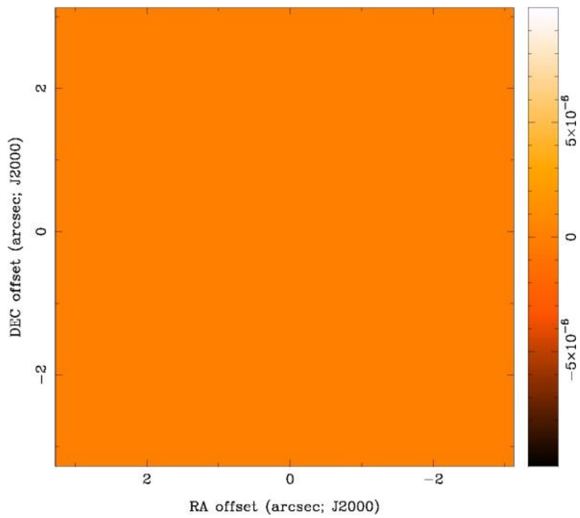
- the restored image does not actually fit the observed visibilities

# clean Example

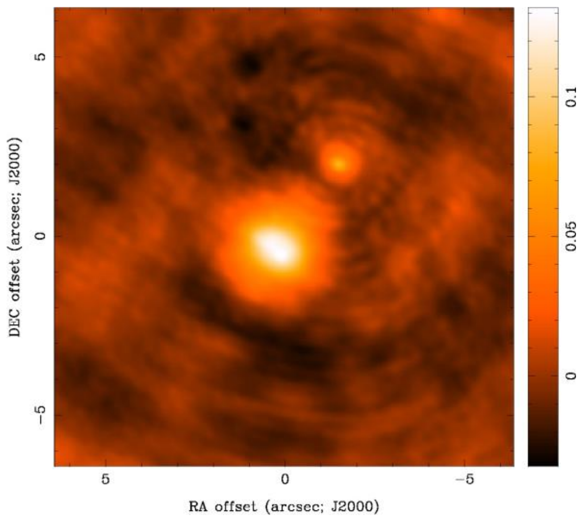
$T^D(l,m)$




*0 Clean Components*

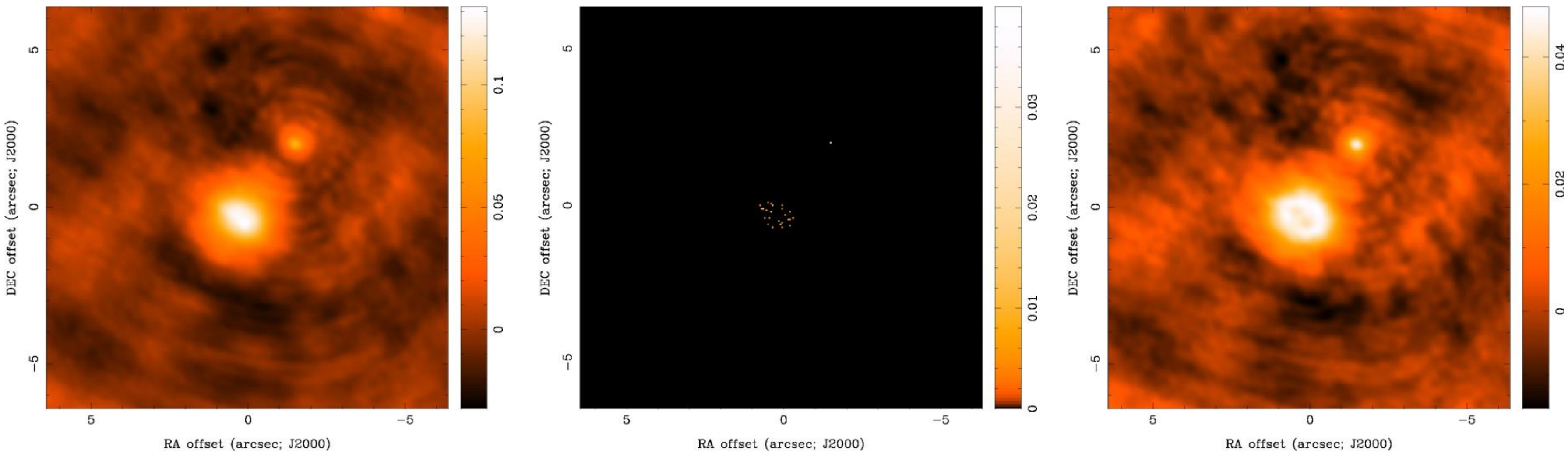



*residual map*



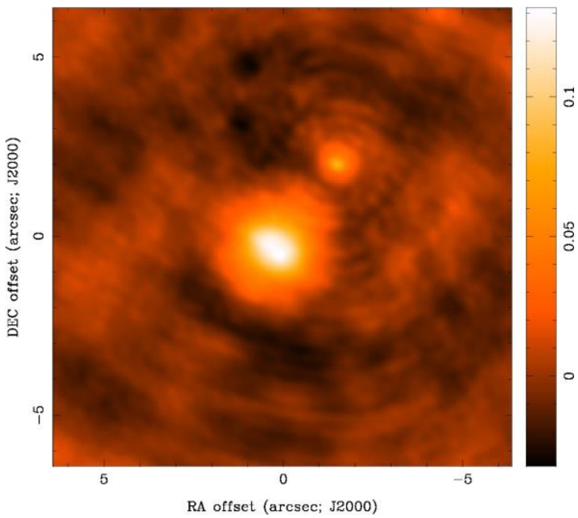
# clean Example

*$T^D(l,m)$* | *30 Clean Components* | *residual map*

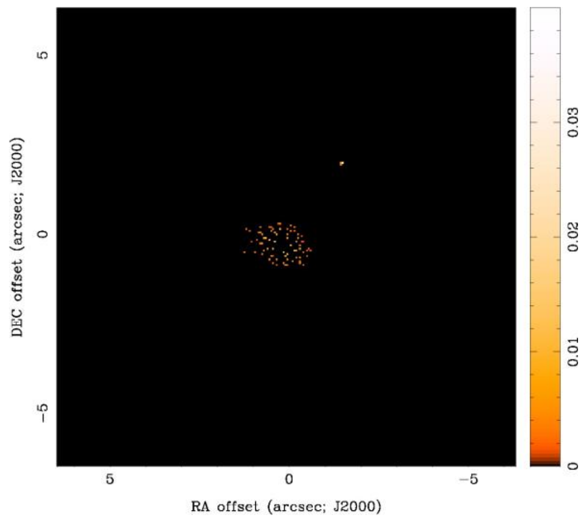


# clean Example

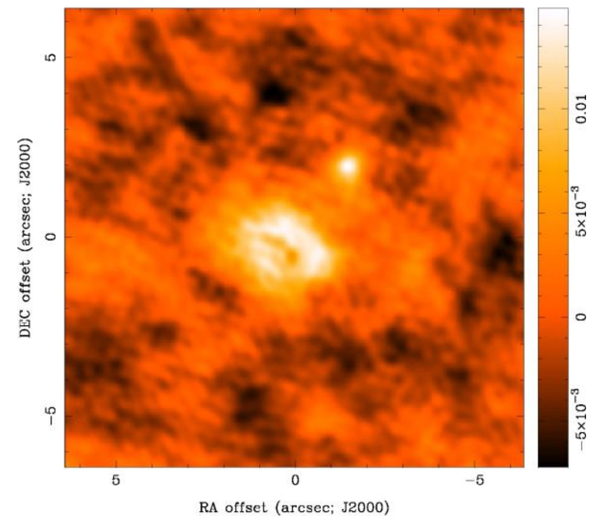
*$T^D(l,m)$*



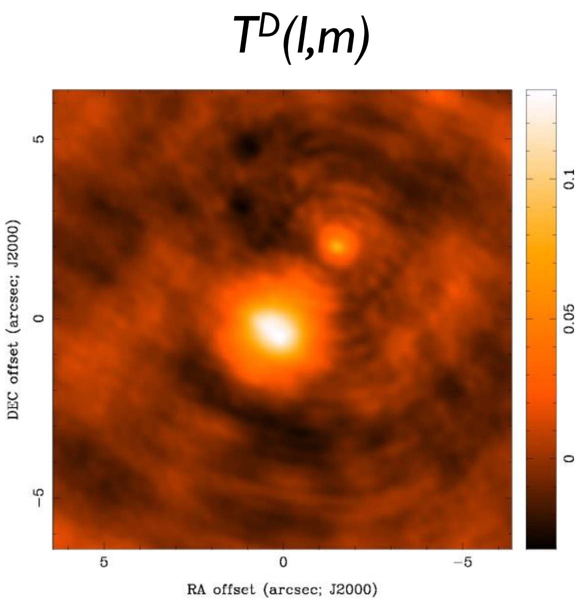

*100 Clean Components*

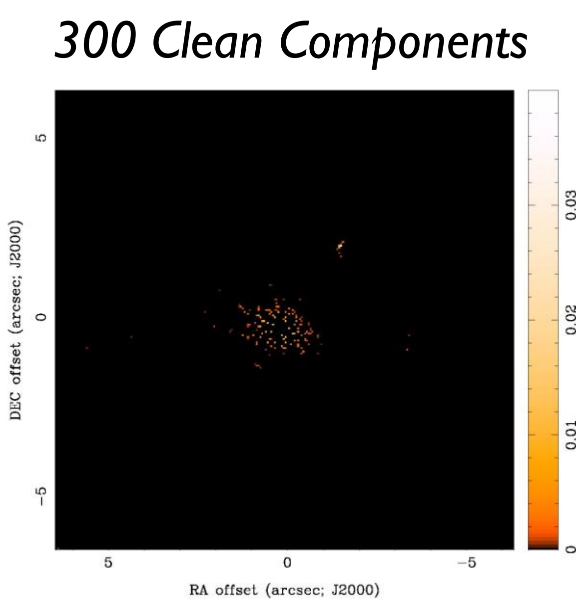


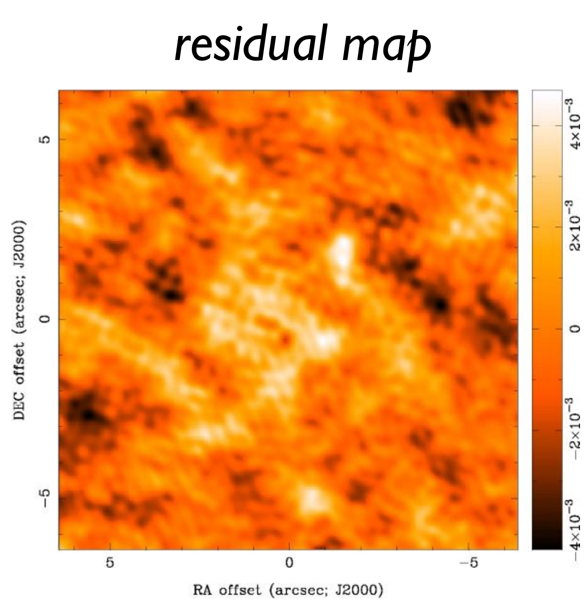

*residual map*



# clean Example

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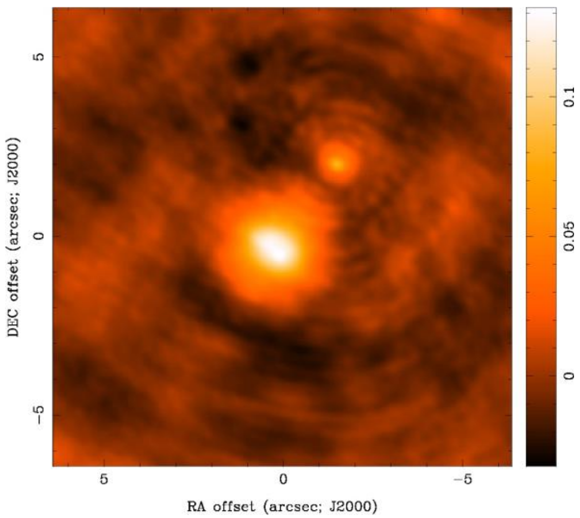
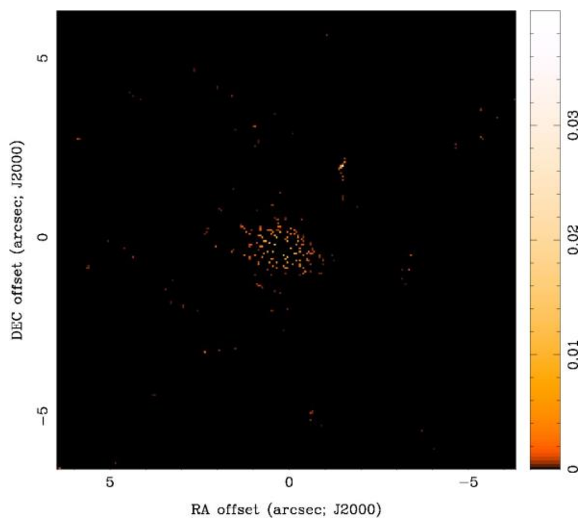
# clean Example

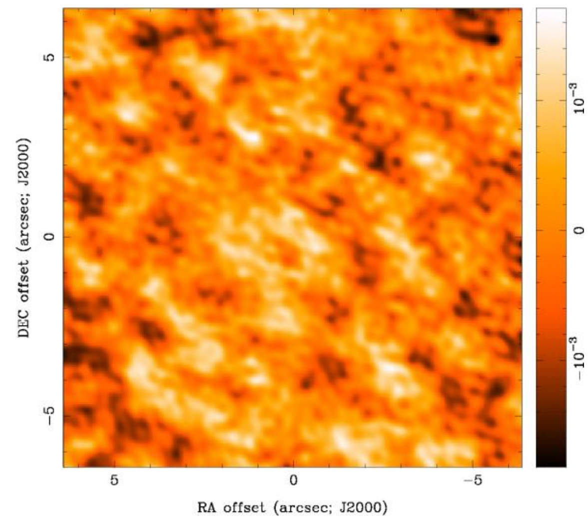
$T^D(l,m)$




*583 Clean Components*




*residual map*

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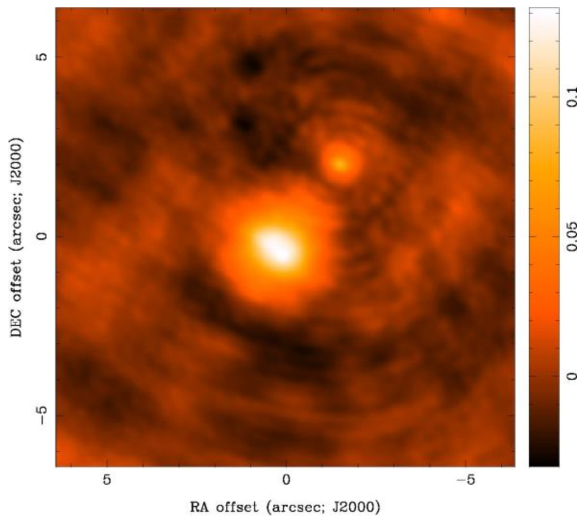
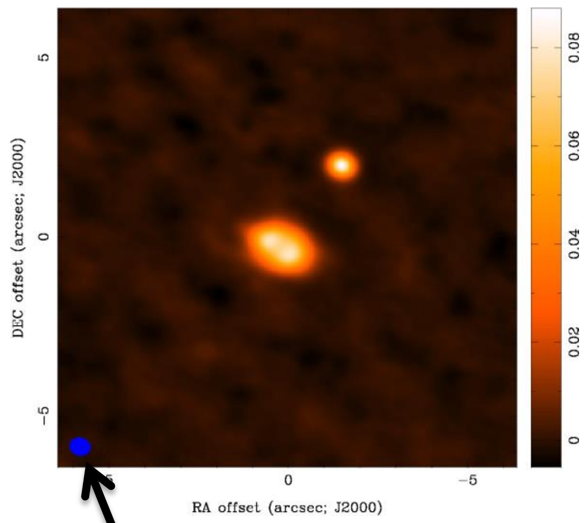
# clean Example

$T^D(l,m)$




*restored image*




ellipse = clean beam fwhm

*final image depends on*

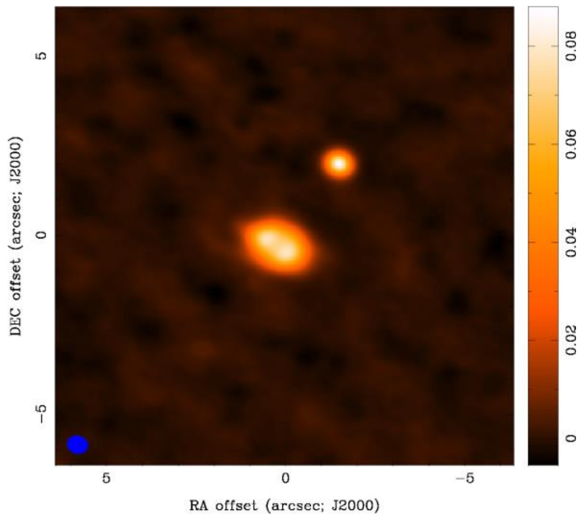
*imaging parameters (pixel size, visibility weighting scheme, gridding) and deconvolution (algorithm, iterations, masks, stopping criteria)*

# CASA clean filename extensions

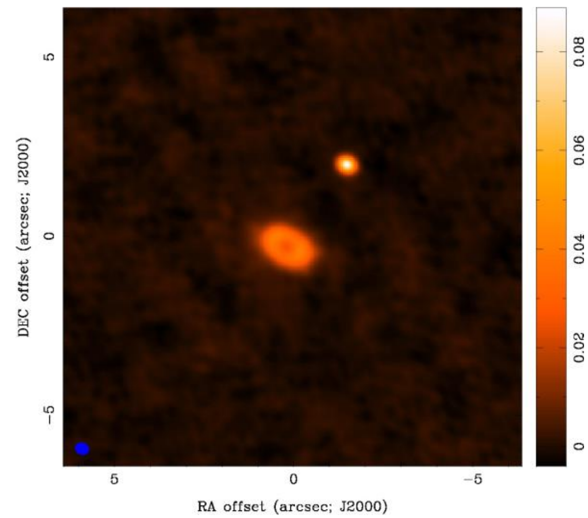
- <*imagename*>.image
  - final clean image (or dirty image if niter=0)
- <*imagename*>.psf
  - point spread function (= dirty beam)
- <*imagename*>.model
  - image of clean components
- <*imagename*>.residual
  - residual after subtracting clean components
    (use to decide whether or not to continue clean)
- <*imagename*>.flux
  - relative sensitivity on the sky
  - pbcor = True divides .image by .flux

# Results from Different Weighting Schemes

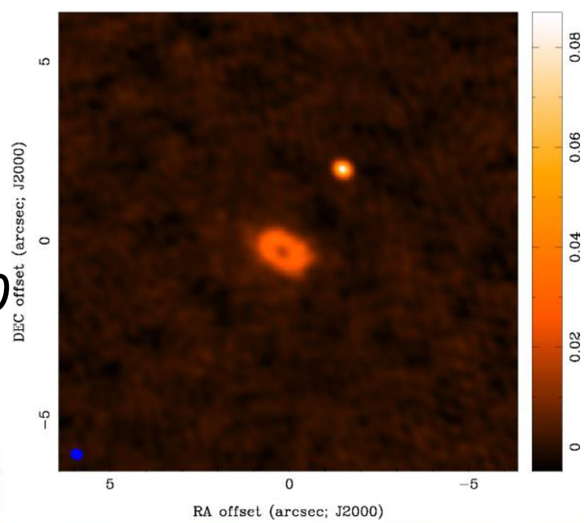
*natural*
*0.59x0.50*



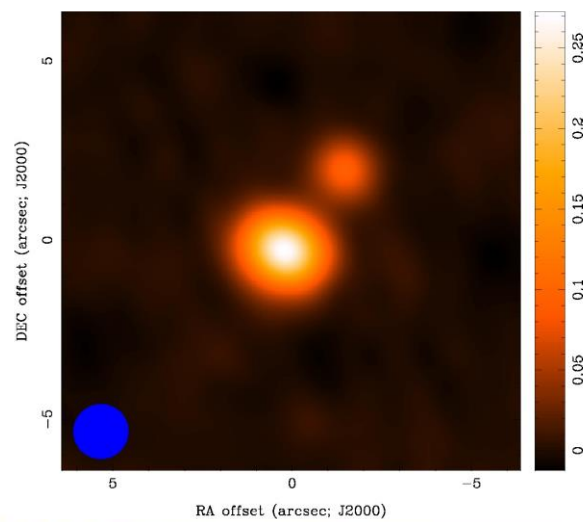

*robust=0*
*0.40x0.34*




*uniform*
*0.35x0.30*




*natural*
*+ taper to*
*1.5x1.5*

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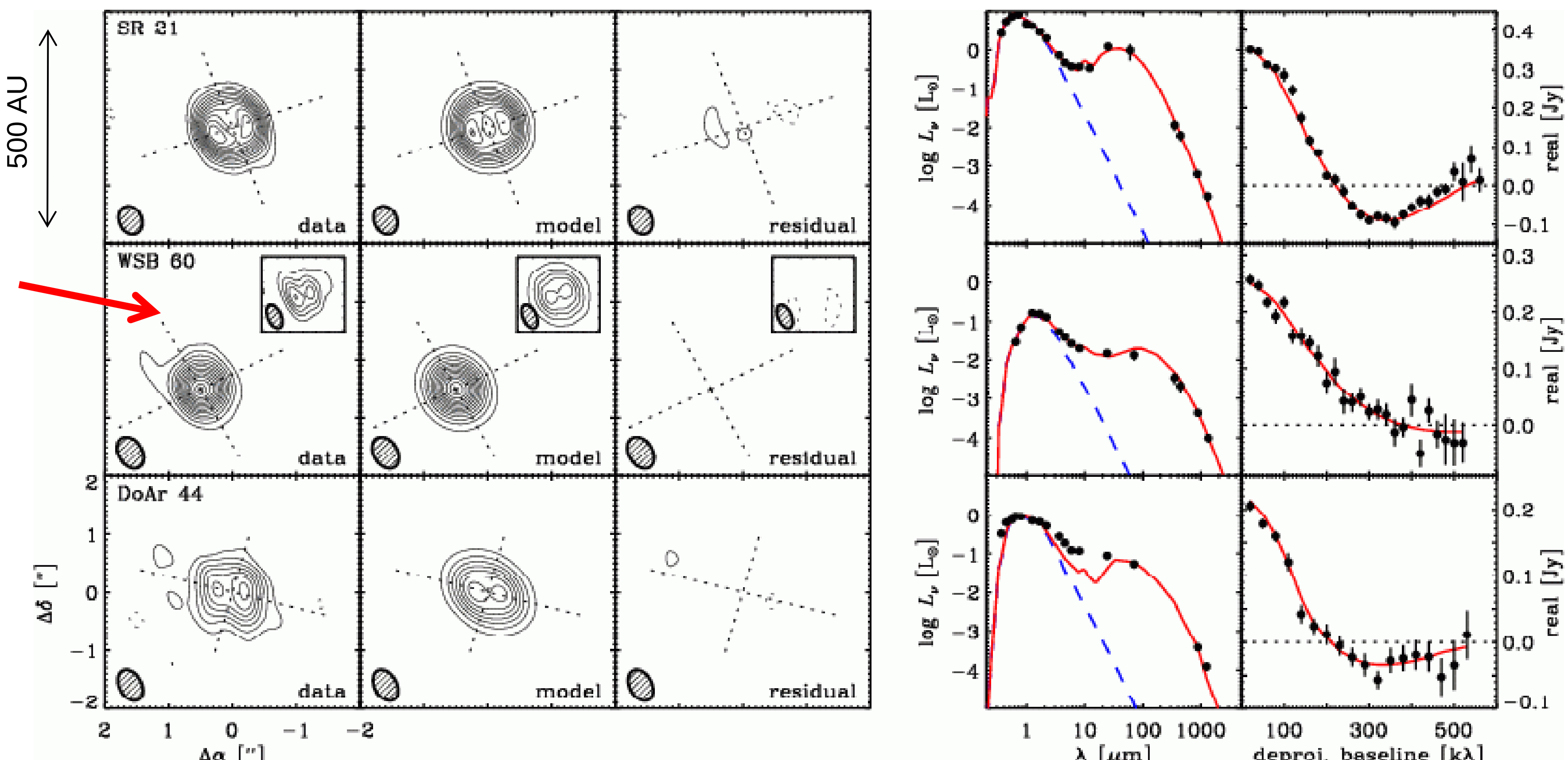
# Tune Resolution/Sensitivity to suit Science

- example: SMA 870 µm images of protoplanetary disk dust continuum emission with resolved inner cavities (Andrews et al. 2009, ApJ, 700, 1502)



# Scale Sensitive Deconvolution Algorithms

- basic clean (or Maximum Entropy) is scale-free and treats each pixel as an independent degree of freedom: no concept of source size

- adjacent pixels in an image are not independent
- an extended source covering 1000 pixels might be characterized by just a few parameters, not 1000 paramters (e.g. an elliptical Gaussian with 6 parameters: x, y, amp, major fwhm, minor fwhm, position angle)
- scale sensitive deconvolution algorithms try to employ fewer degrees of freedom to model plausible sky brightness distributions

- MS Clean (Multi-Scale Clean)
- Adaptive Scale Pixel (Asp) Clean

- yields promising results on extended emission

# "Invisible" Large Scale Structure

- missing short spacings can be problematic for large scale structure
- to estimate? simulate observations, or check simple expressions for a Gaussian or unform disk (appendix of Wilner & Welch 1994, ApJ, 427, 898)

*Homework Problem*

- Q: By what factor is the central brightness reduced as a function of source size due to missing short spacings for a Gaussian characterized by fwhm $\theta_{1/2}$ ?

- A: a Gaussian source central brightness is reduced 50% when

$$\theta_{1/2} = 18'' \left(\frac{\nu}{100\ GHz}\right)^{-1} \left(\frac{B_{min}}{15\ meters}\right)^{-1}$$

where $B_{min}$ is the shortest baseline [meters], $\nu$ is the frequency [GHz]

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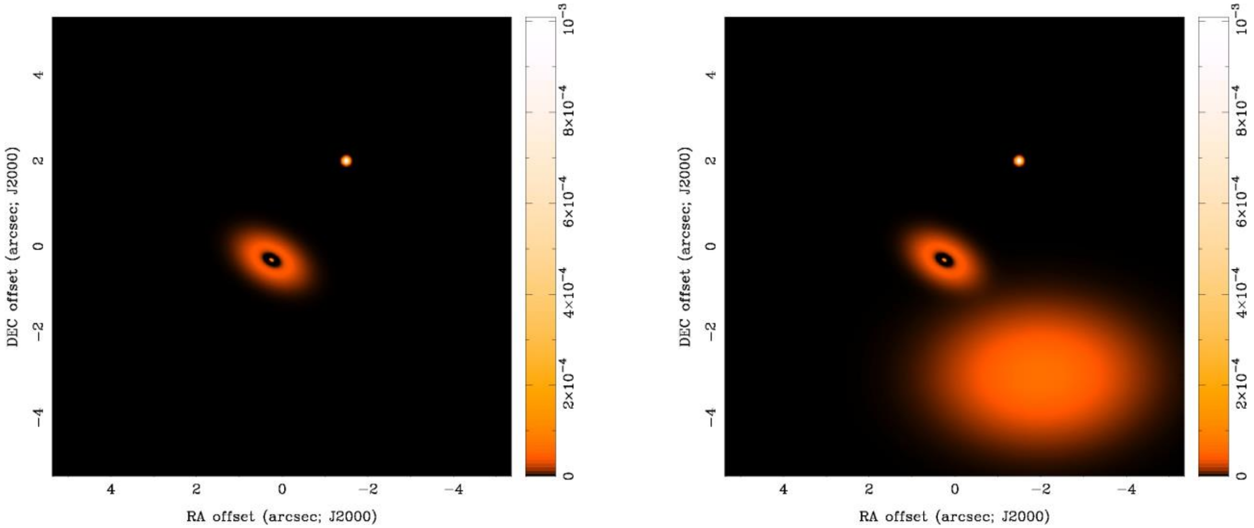
# Missing Short Spacings: Demonstration

- important structure may be missed in central hole of (u,v) coverage

- Do the visibilities observed in our example discriminate between these two models of the sky brightness distribution *T(l,m)*?



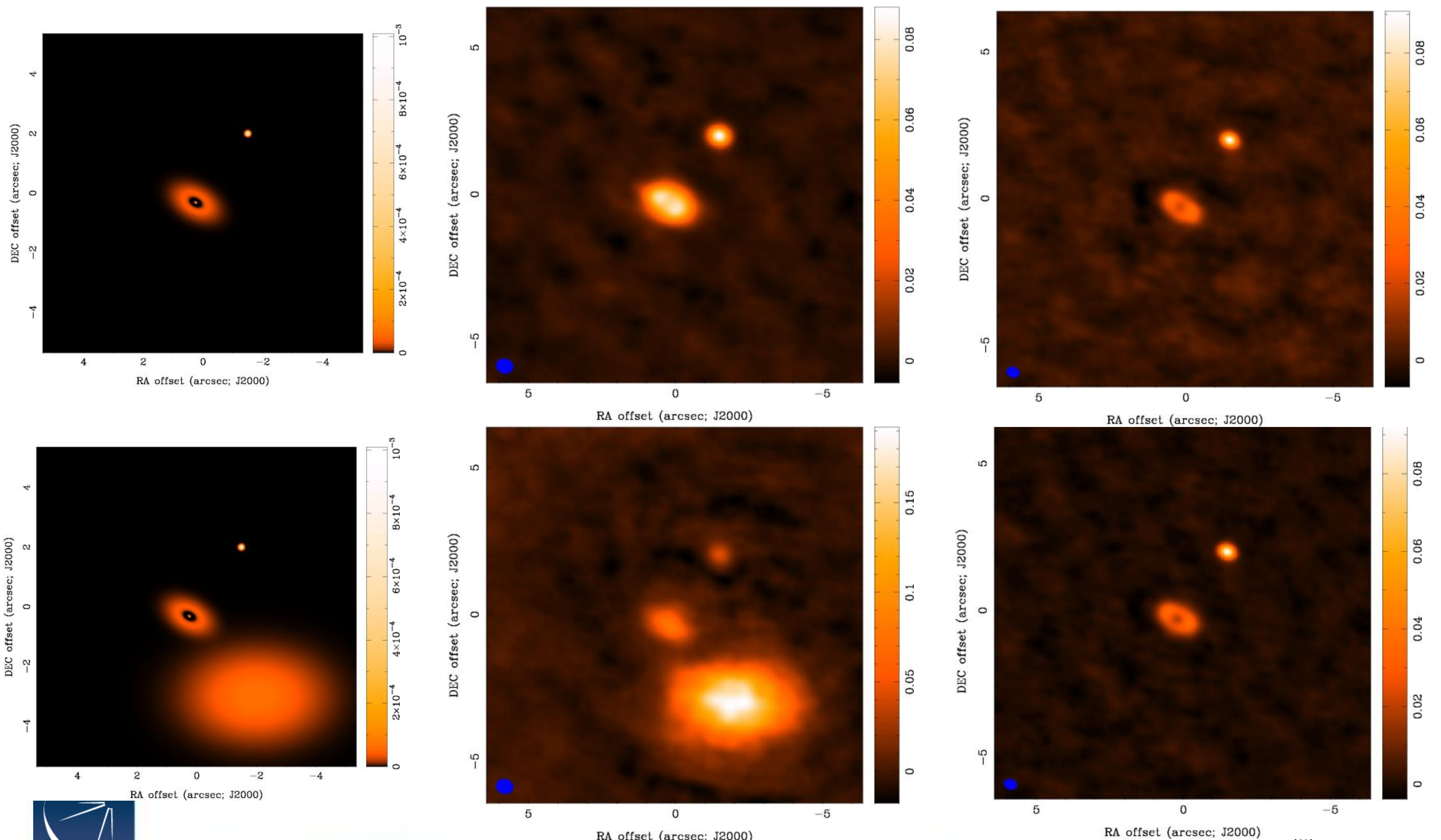

- Yes… but only on baselines shorter than about 75 kλ

# Missing Short Spacings: Demonstration

*T(l,m)*

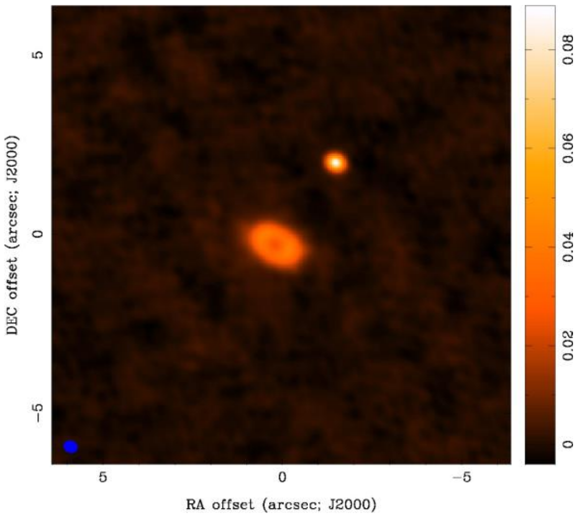
*natural weight*

*> 75 kλ natural weight*



# Measures of Image Quality

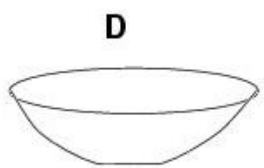
- *dynamic range*
  - ratio of peak brightness to rms noise in a region void of emission
  - easy way to calculate a lower limit to the error in brightness in a non-empty region
  - e.g. peak = 89 mJy/beam, rms = 0.9 mJy/beam
    $\rightarrow$ DR = 89/0.9 = 99

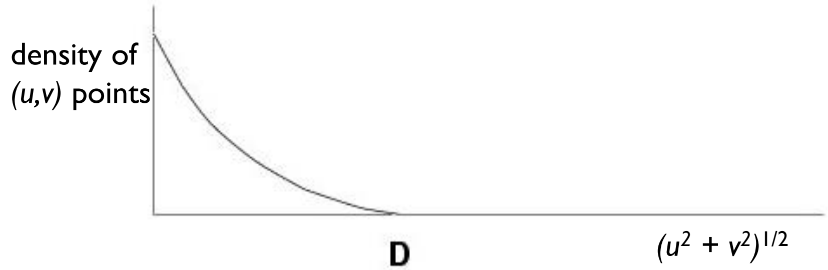



- *fidelity*
  - difference between any produced image and the correct image
  - fidelity image = input model / difference
    = model * beam / abs(model * beam – reconstruction)
    = inverse of the relative error
  - need knowledge of the correct image to calculate

# Techniques to Obtain Short Spacings

use a large single dish telescope




- *all* Fourier components from 0 to D sampled, where D is dish diameter (weighting depends on illumination)
- scan single dish across sky to make an image $T(l,m) * A(l,m)$
  where $A(l,m)$ is the single dish response pattern
- Fourier transform single dish image, $T(l,m) * A(l,m)$, to get $V(u,v)a(u,v)$
  and then divide by $a(u,v)$ to estimate $V(u,v)$ for baselines < D
- choose D large enough to overlap interferometer samples of $V(u,v)$
  and avoid using data where $a(u,v)$ becomes small, e.g. VLA & GBT

# Techniques to Obtain Short Spacings

use a separate array of smaller antennas

- small antennas can observe short baselines inaccessible to larger ones
- the larger antennas can be used as single dish telescopes to make images with Fourier components not accessible to the smaller antennas
- example: ALMA main array + ACA

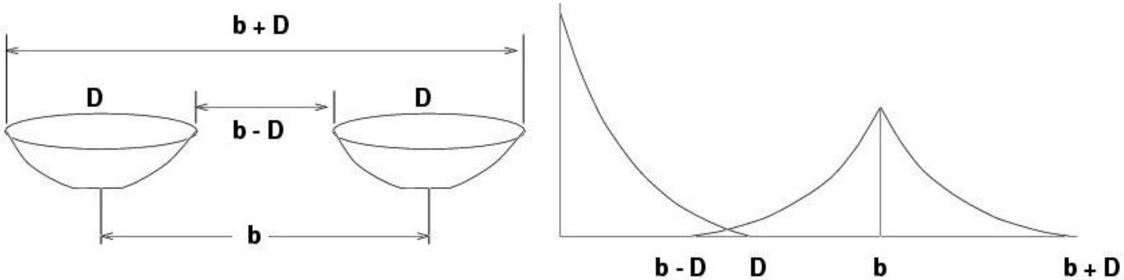
main array
50 x 12m: 12m to 14+ km

ACA
12 x 7m: covers 7-12m
4 x 12m single dishes: 0-7m

# Techniques to Obtain Short Spacings

mosaic with a homogeneous array

- recover a range of spatial frequencies around the nominal baseline b using knowledge of A(l,m), shortest spacings from single dishes (Ekers & Rots 1979)




- *V(u,v)* is a linear combination of baselines from b-D to b+D
- depends on pointing direction *($l_0$,$m_0$)* as well as on *(u,v)*

$$V(u,v;l_0,m_0) = \int\int T(l,m)A(l-l_0,m-m_0)e^{i2\pi(ul+vm)}dldm$$

- Fourier transform with respect to pointing direction *($l_0$,$m_0$)*

$$V(u-u_0,v-v_0) = \left(\int\int V(u,v;l_0,m_0)e^{i2\pi(u_0 l_0+v_0 m_0)}dl_0 dm_0\right)/a(u_0,v_0)$$

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# Self Calibration

- *a priori* calibration using external calibrators is not perfect
  - interpolated from different time, different sky direction from source
- basic idea of self calibration is to correct for antenna based phase and amplitude errors *together* with imaging to create a source model
- works because
  - at each time, measure N complex gains and N(N-1)/2 visibilities
  - source structure can be represented by a small number of parameters
  - a highly overconstrained problem if N large and source simple
- in practice, an iterative, non-linear relaxation process
  - assume source model → solve for time dependent gains → form new source model from corrected data using e.g. clean → solve for new gains
  - requires sufficient signal-to-noise at each solution interval
- loses absolute phase from calibrators and therefore position information
- dangerous with small N arrays, complex sources, marginal signal-to-noise

# Concluding Remarks

- interferometry samples Fourier components of sky brightness
- make an image by Fourier transforming sampled visibilities
- deconvolution attempts to correct for incomplete sampling

- remember
  - there are an infinite number of images compatible with the visibilities
  - missing (or corrrupted) visibilities affect the entire image
- astronomers must use judgement in the imaging and deconvolution process
- it's fun and worth the trouble → high angular resolution images!

*many, many issues not covered in this talk: see References and upcoming talks*

# END