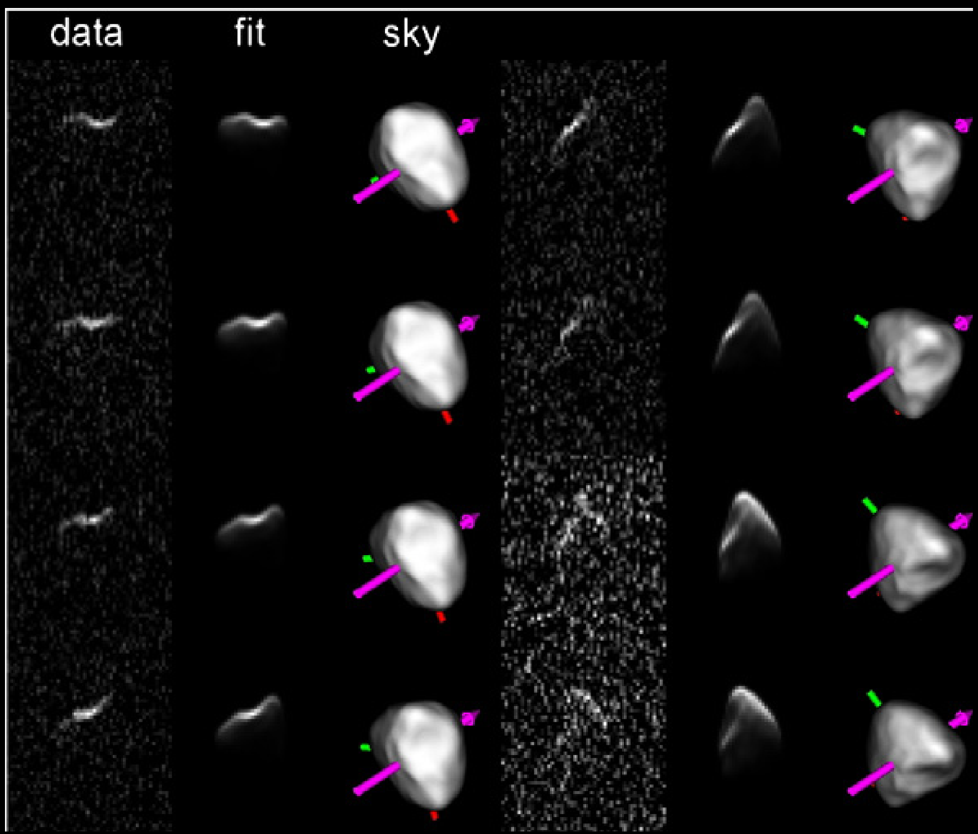
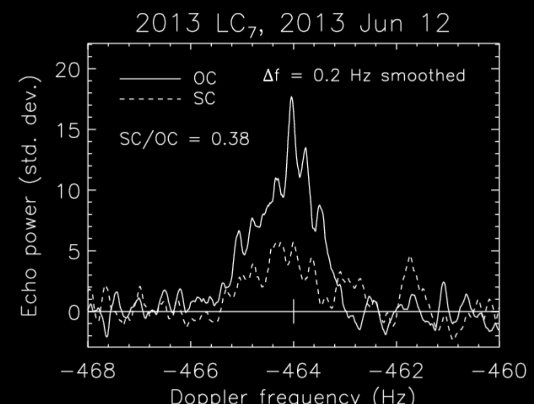
Uncertainties in Radar-Based Shape Modeling

Sean Marshall (Cornell)





Wednesday, June 10, 2015

Future of Planetary Radio Astronomy with Single-Dish Telescopes

Collaborators

- Donald Campbell, Tim Sears (Cornell U.)
- Ellen Howell, Michael Nolan, Patrick Taylor, Alessondra Springmann, Linda Ford, Jim Richardson, Edgard Rivera-Valentin (Arecibo Observatory)
- Christopher Magri (U. of Maine at Farmington)
- Yanga Fernandez, Jenna Crowell (University of Central Florida)
- Ronald Vervack (JHU APL)
- And others

How to Study Asteroids?

- Zap them with radar! Measure the echo

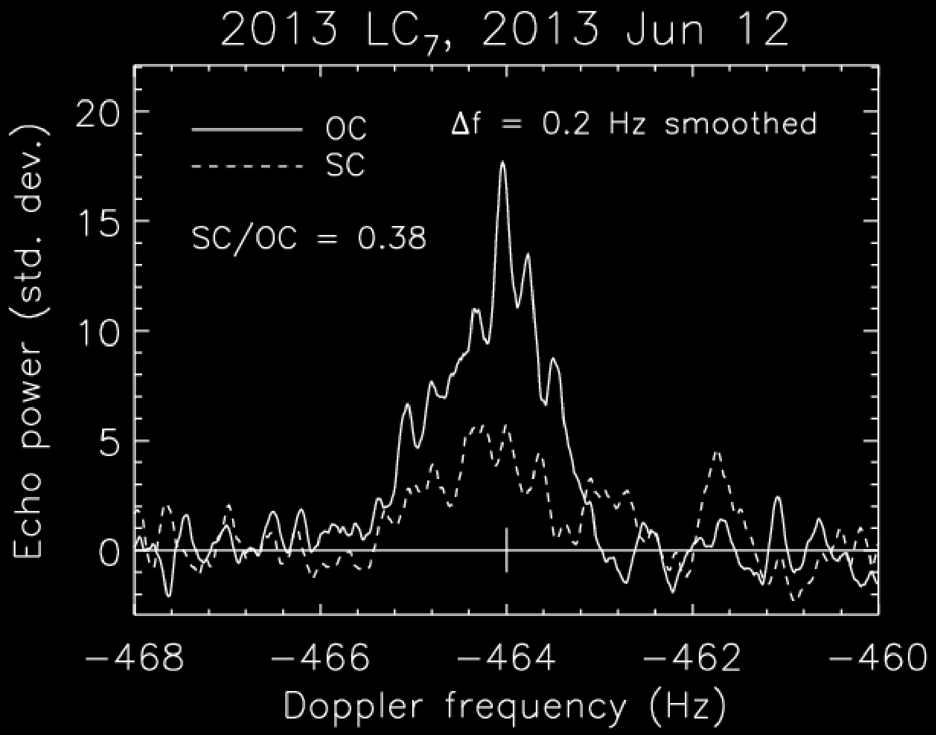
Arecibo (305 m diameter)
www.naic.edu/public/about/photos/hires/aoviews.html

GBT (100 m diameter)
science.nrao.edu/facilities/gbt

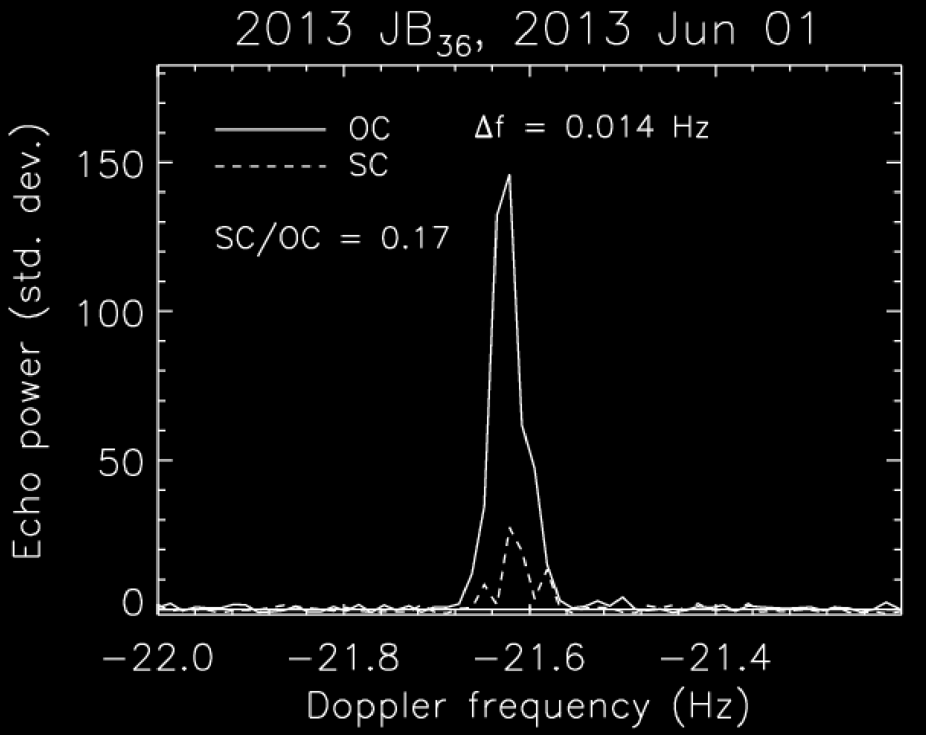
Goldstone (70 m diameter)
en.wikipedia.org/wiki/File:Goldstone_DSN_antenna.jpg

Measuring Frequency

- Transmit at a single frequency (CW)
- Receive radar echo at a range of frequencies



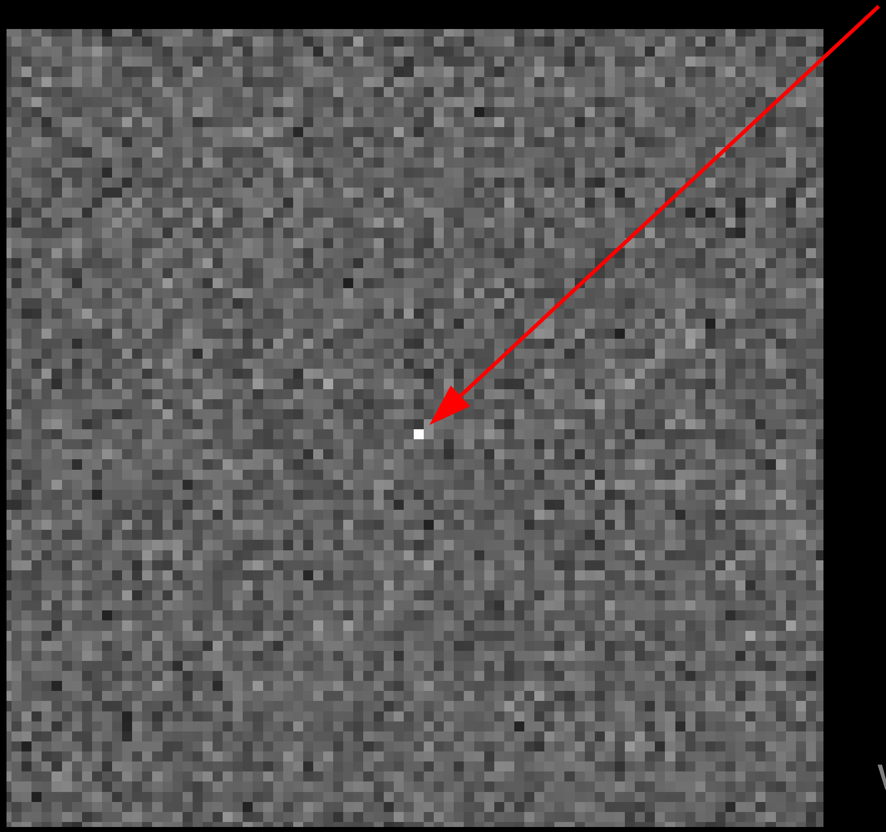
www.naic.edu/~radarusr/2013LC7/



www.naic.edu/~radarusr/2013JB36/

Measuring Delay (Time)

- Measure the length of time between radar transmission and echo reception
  - Measured to microsecond accuracy (or better)
  - Gives range (distance) to asteroid



www.naic.edu/~radarusr/2002GT/

Measure Both (Time and Frequency)

- Resolve radar echo into time (delay) and (Doppler-shifted) frequency

Doppler →

Delay →

www.naic.edu/~radarusr/2005NZ6
/apr28.gif

Doppler →

Delay →

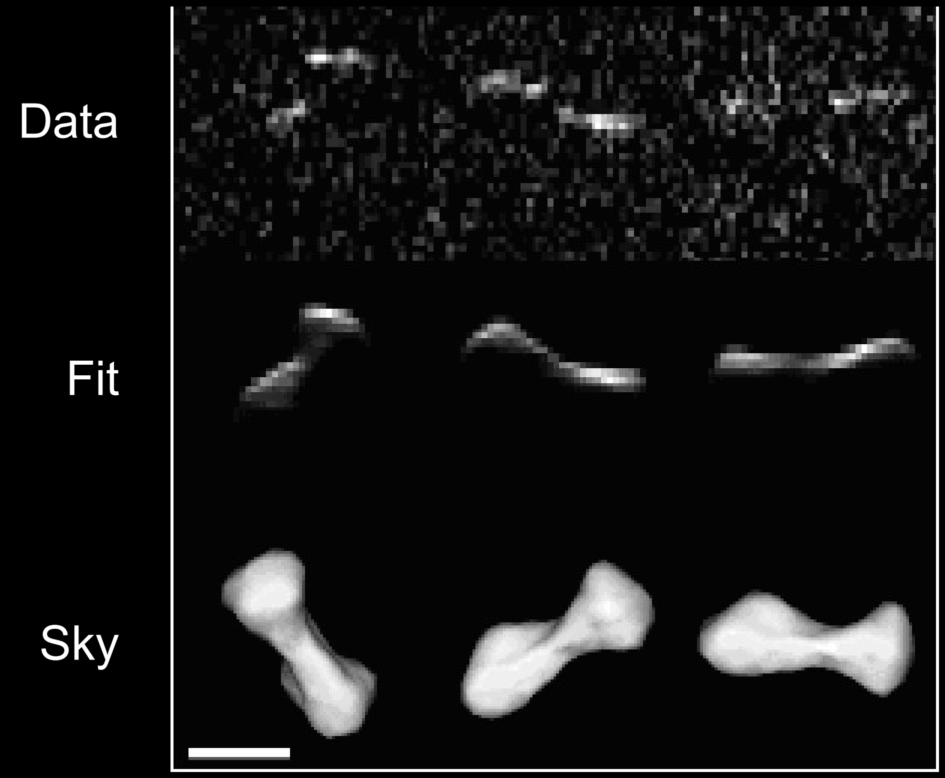
www.jpl.nasa.gov/video/
details.php?id=1359

Radar “Images”

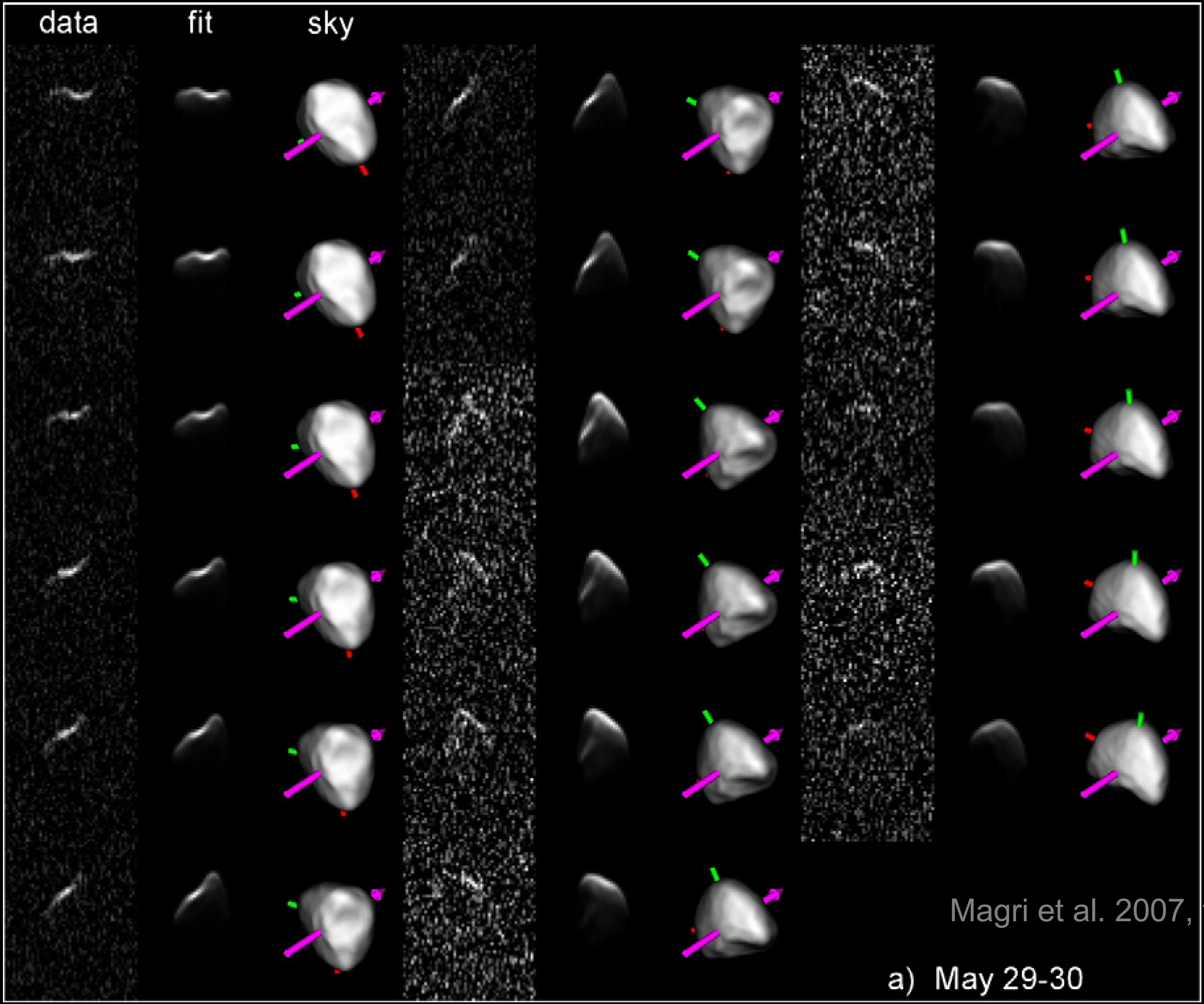
www.naic.edu/
~radarusr/
1998QE2/

Radar-Derived Shape Models

- Inverse problem: Find the shape that best fits the (noisy) observed Delay-Doppler “images”
- Put all observational data into the shape modeling software

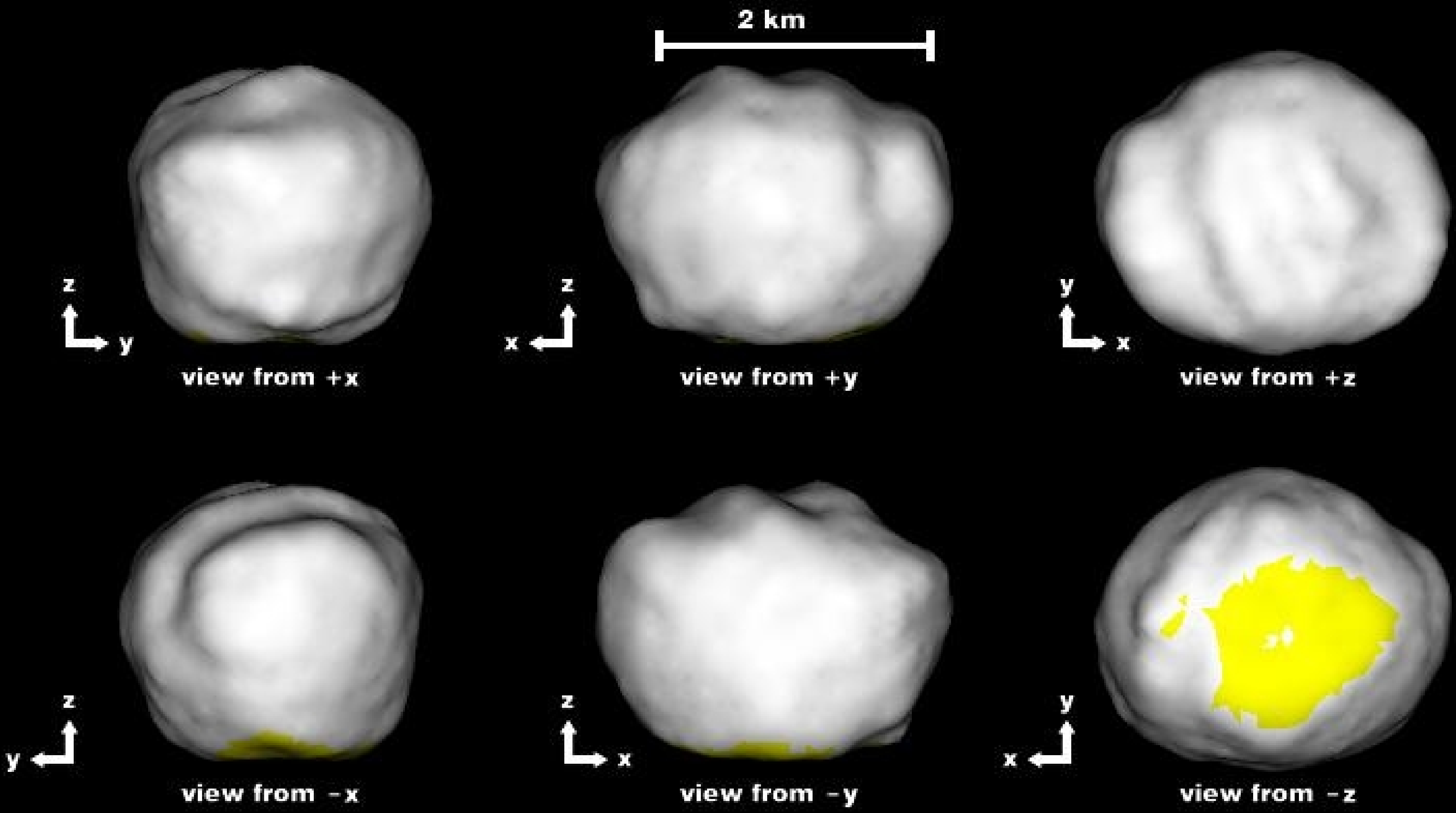


www.naic.edu/general/images/SCIENCE/Planetary/artts/kleo.jpg



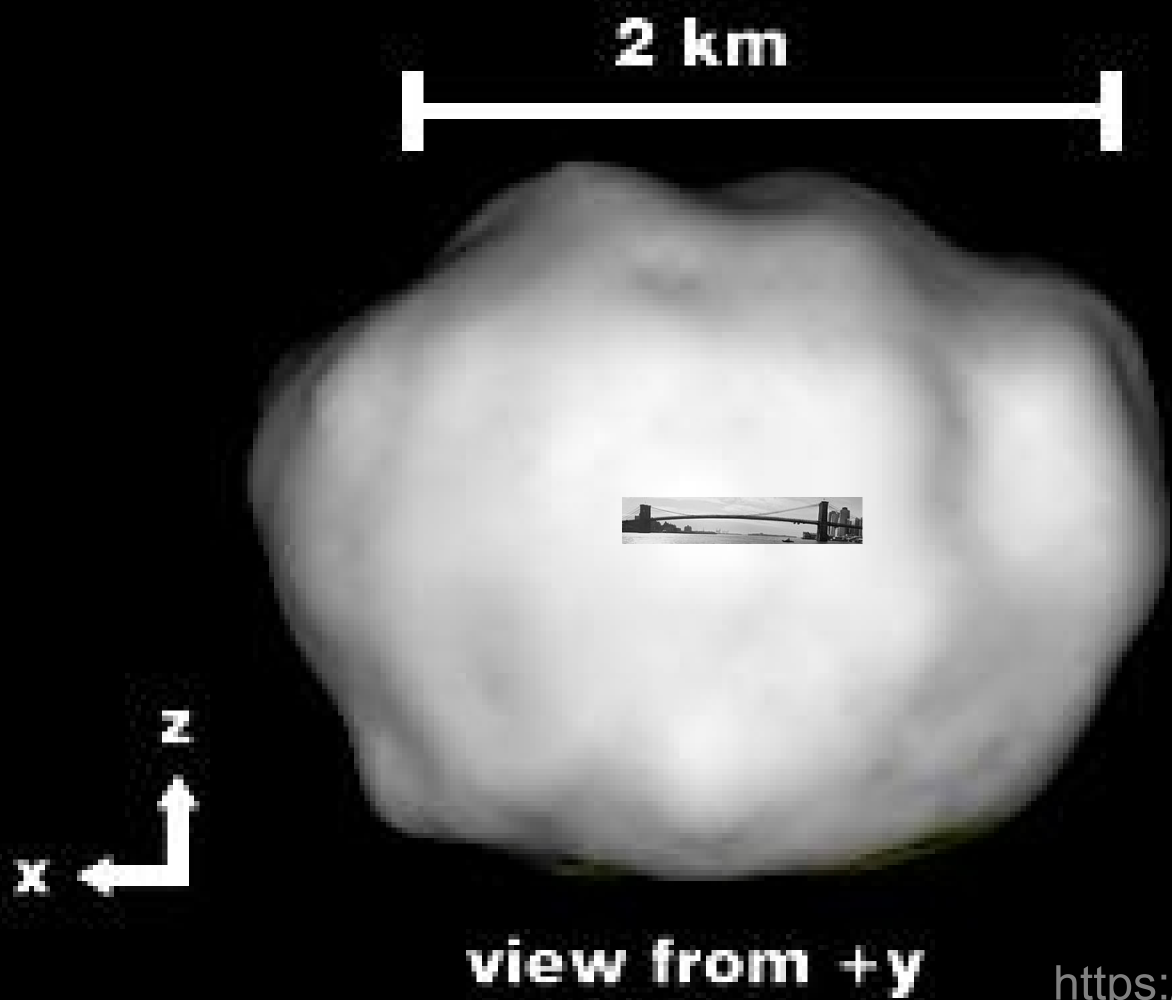
Magri et al. 2007, figure 1a

(162421) 2000 ET70



- Shape model from Naidu et al. 2013

(162421) 2000 ET70

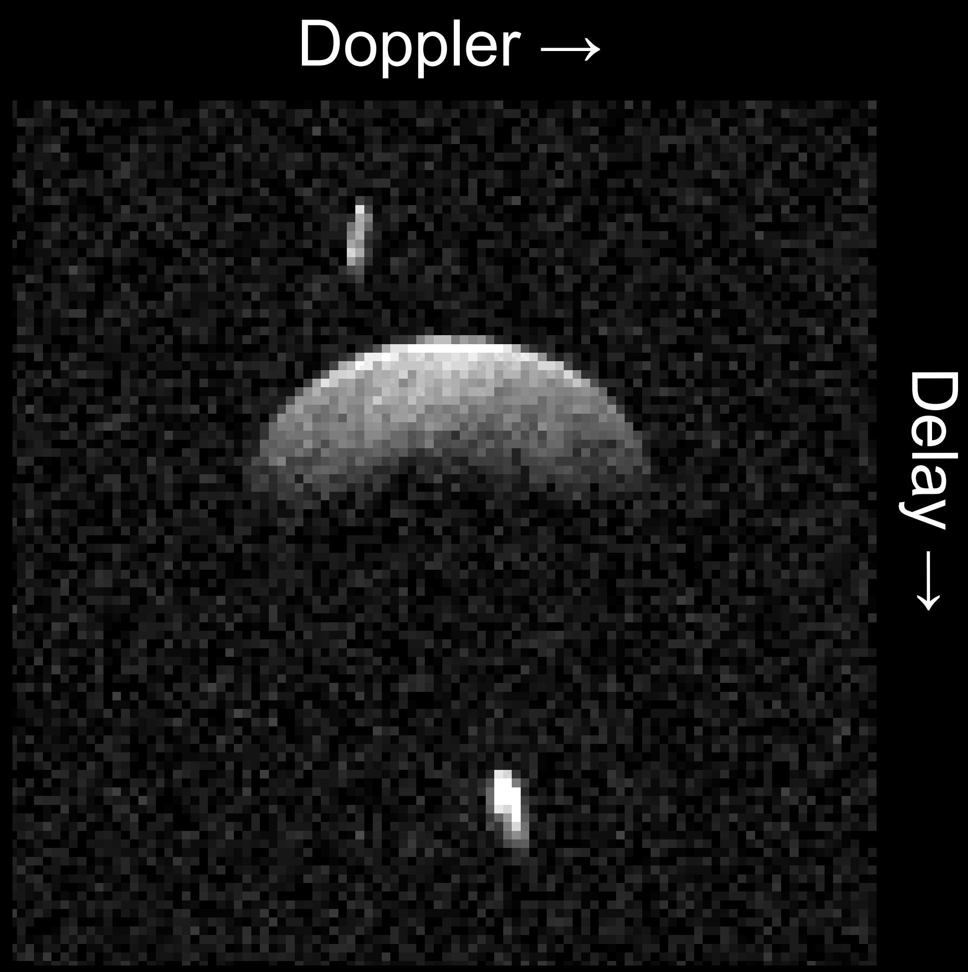


https://commons.wikimedia.org/wiki/File:Brooklyn_Bridge_bw.jpg

- With Brooklyn Bridge, for scale

Uncertainties in Shape Modeling

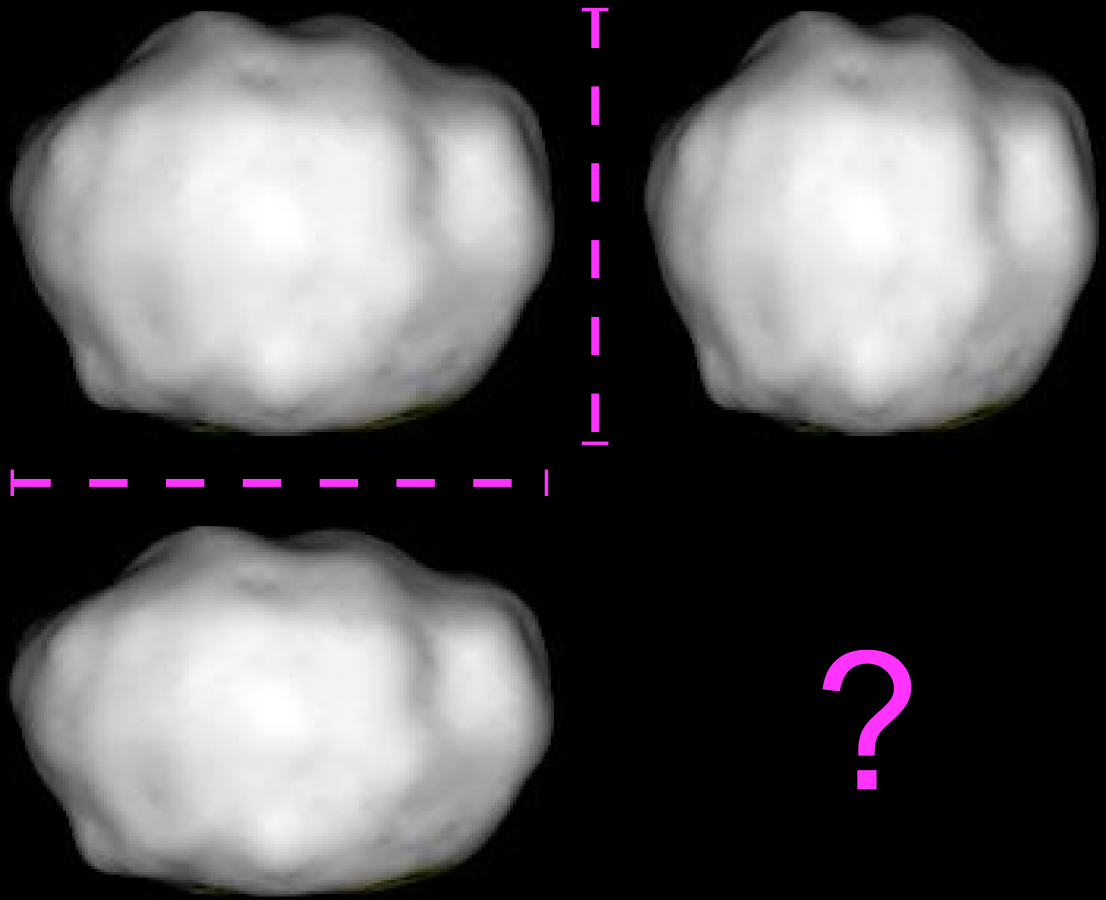
- Radar data have noise
- Noise is not just Gaussian!
  - Real data are more complicated
- Conversion from fits' chi-squared values to parameter error bars is not straightforward



www.news.cornell.edu/stories/Feb08/AreciboAsteroid.html

Uncertainties in Shape Modeling

- Difficult to determine uncertainties of shape model parameters
  - e.g. size
- Want a rigorous general method
- First step: characterize data uncertainties

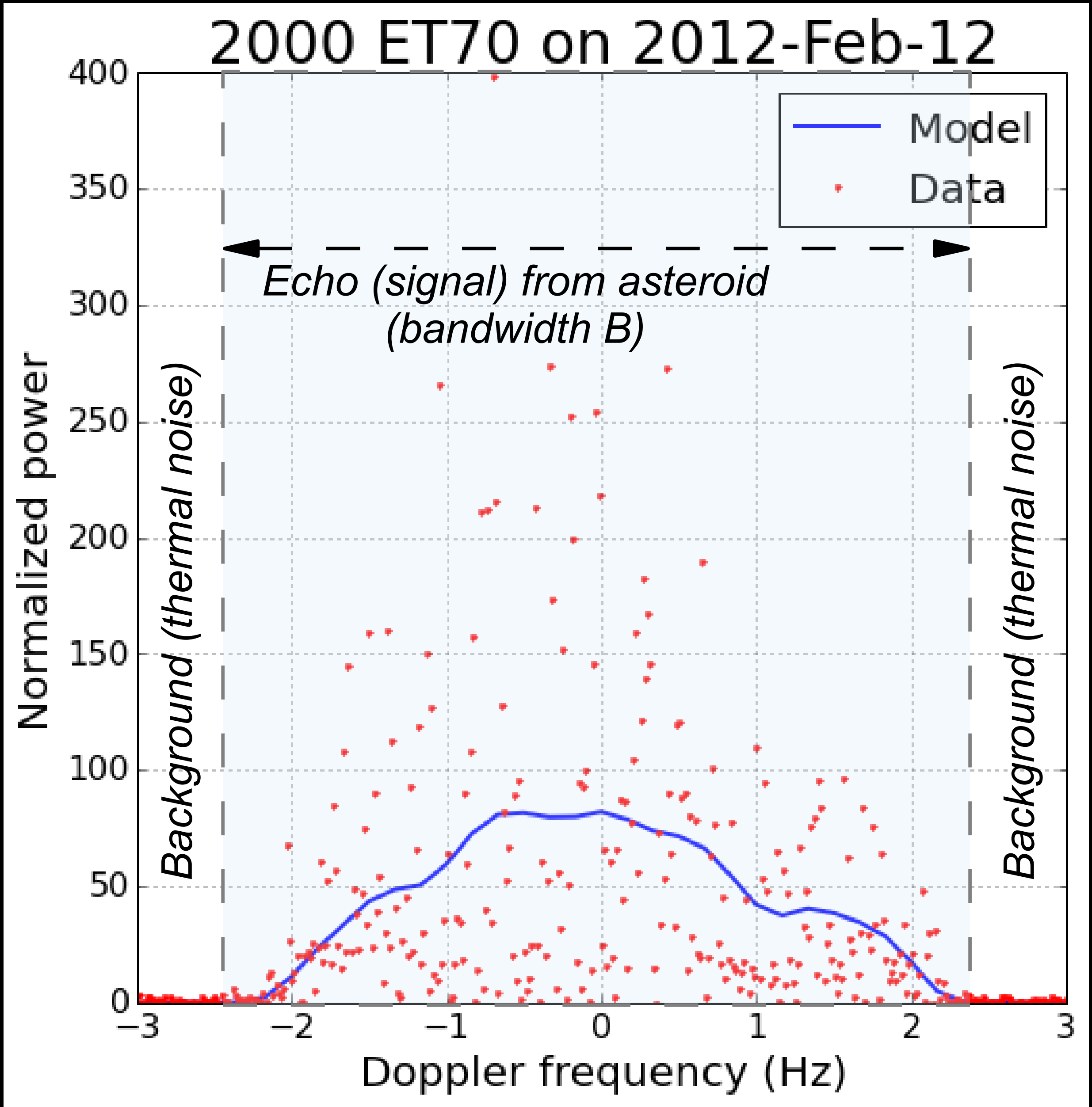


Frequency (CW) Spectrum

- Bandwidth of asteroid's radar echo constrains its size and rotation state

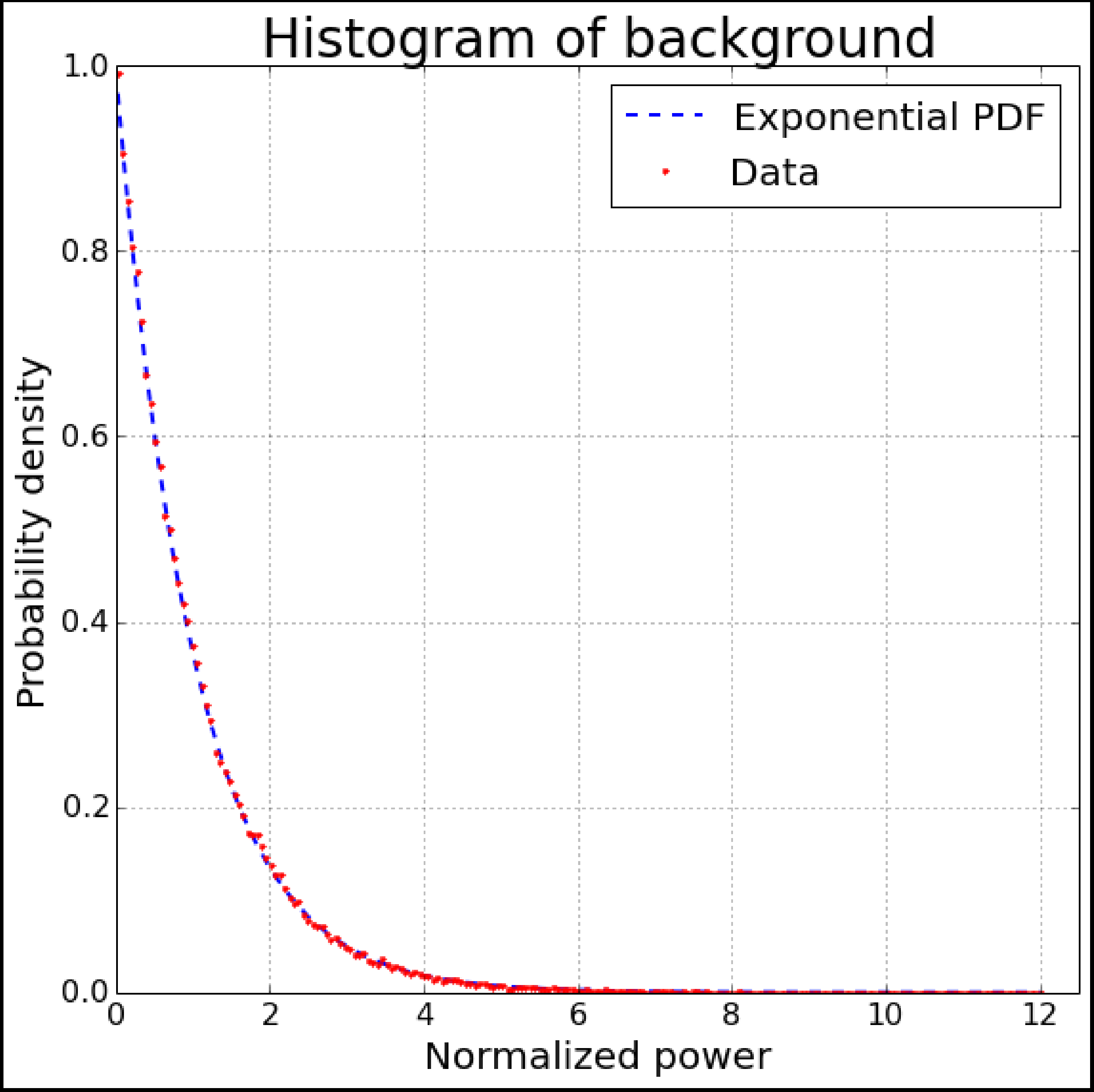
$$B = \frac{4\pi D \cos\varphi}{\lambda_0 P_{spin}}$$

- Frequency resolution of data is 0.0167 Hz
  - 60 points per Hz



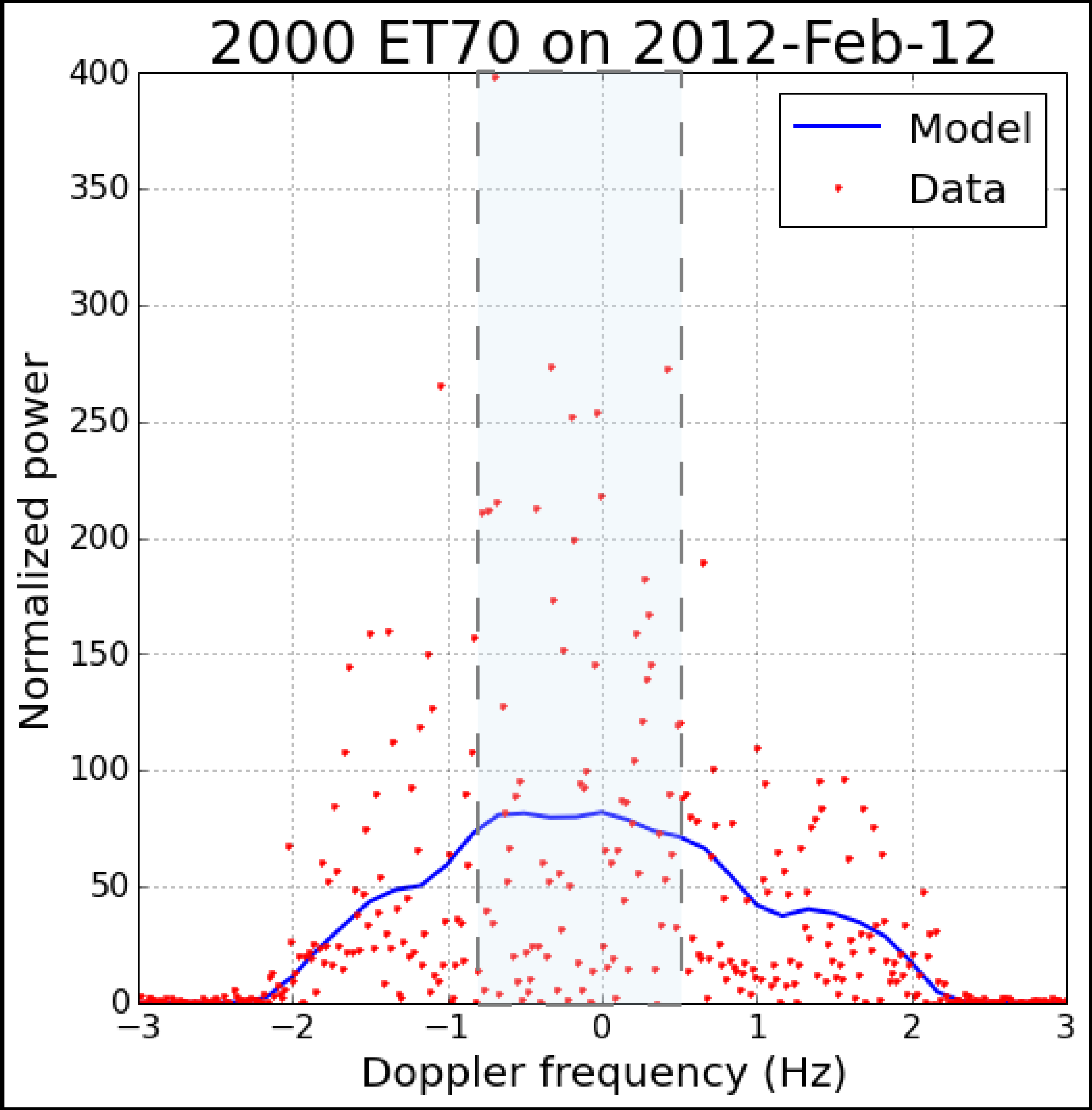
Statistics of Background Noise

- Values of background noise power follow exponential distribution
  - Mean equals standard deviation
  - So if you know one, you know the other



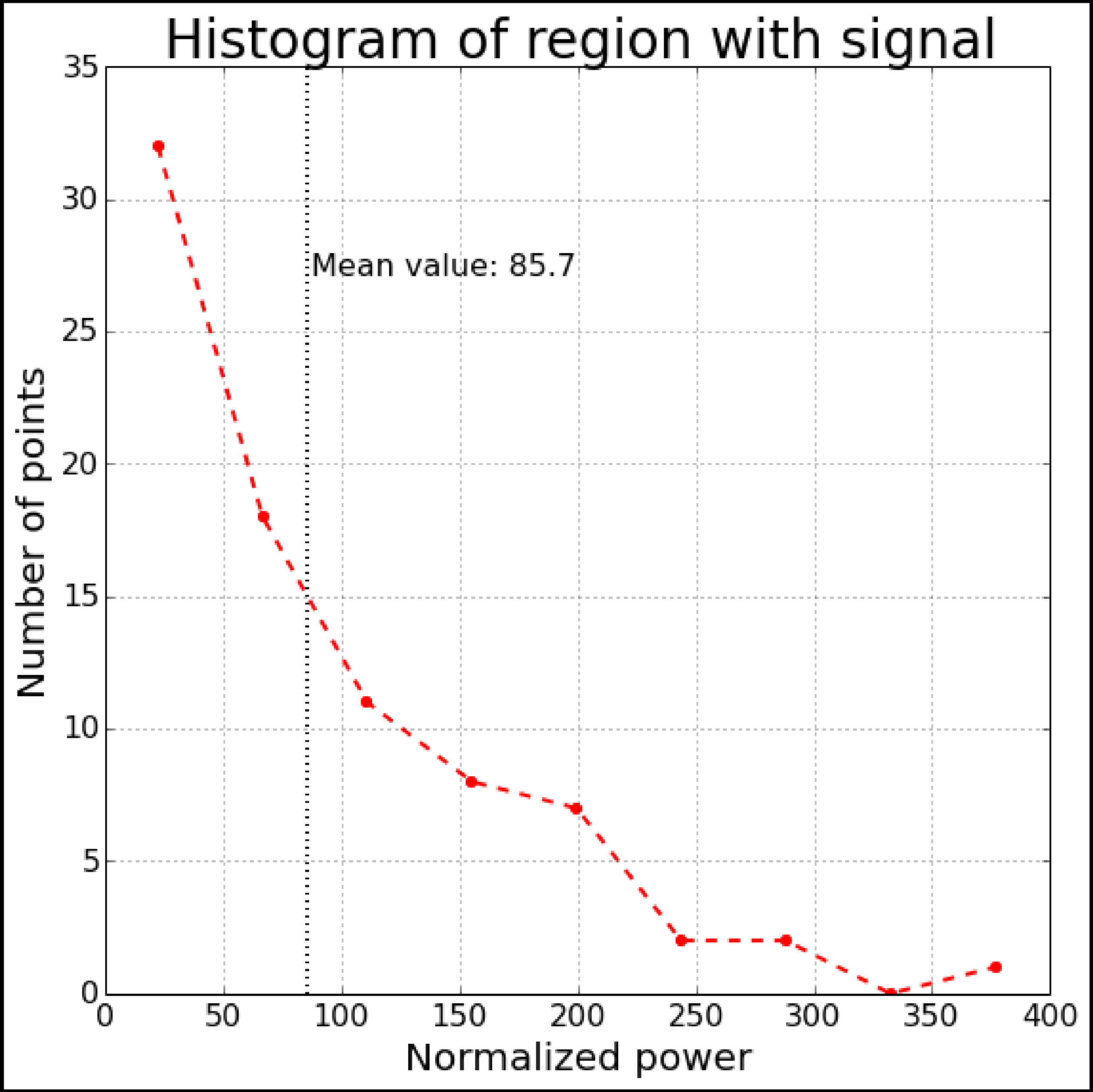
Statistics of Signal

- Look at a range of frequencies over which the model value (true mean?) is nearly constant
  - To about 10%
- Frequency resolution of data is 0.0167 Hz
  - 60 points per Hz



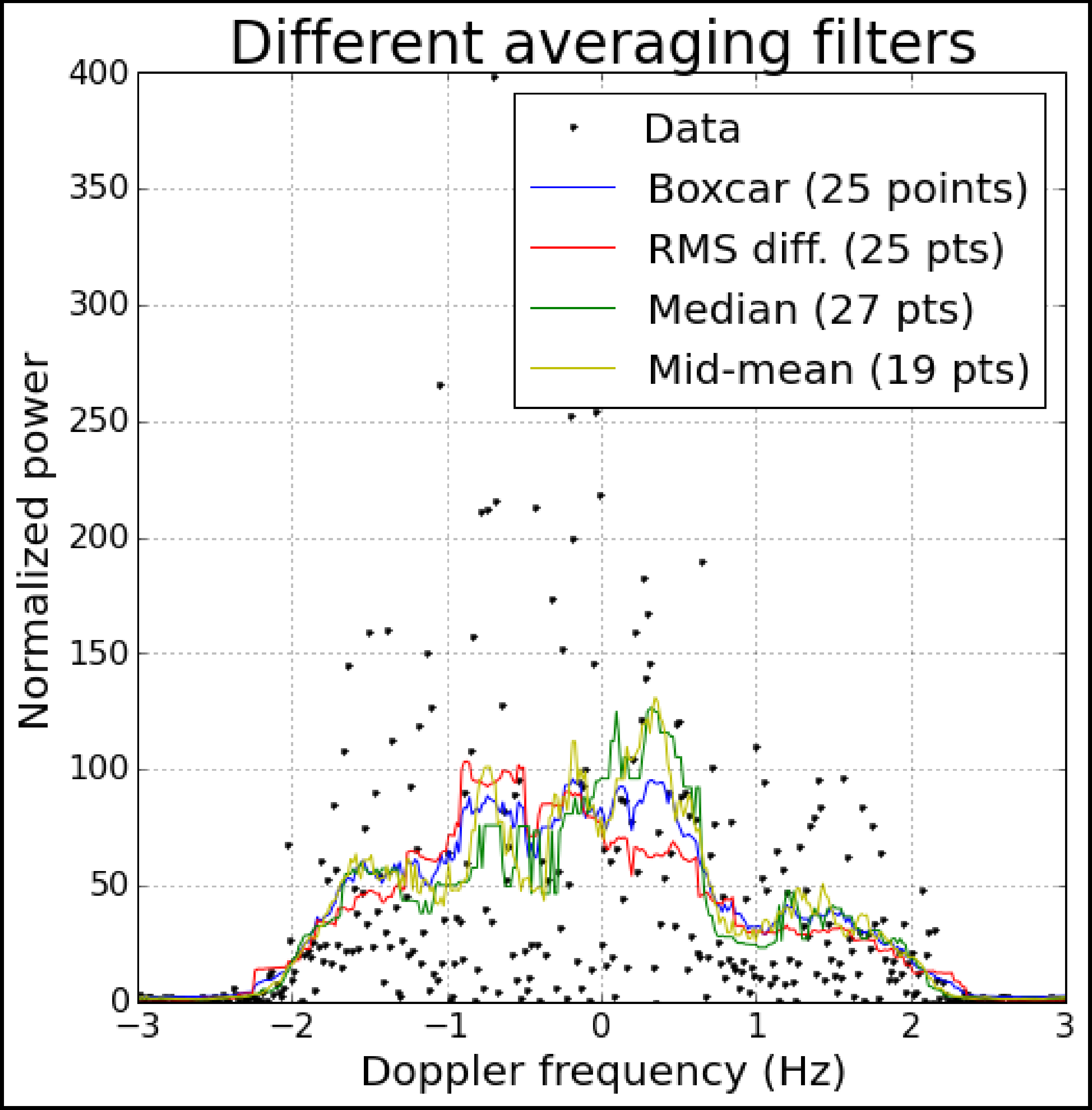
Statistics of Signal

- Signal values also follow exponential distribution
  - Mean equals std. dev.
- But, mean is unknown
- Want a method to estimate mean; then automate that



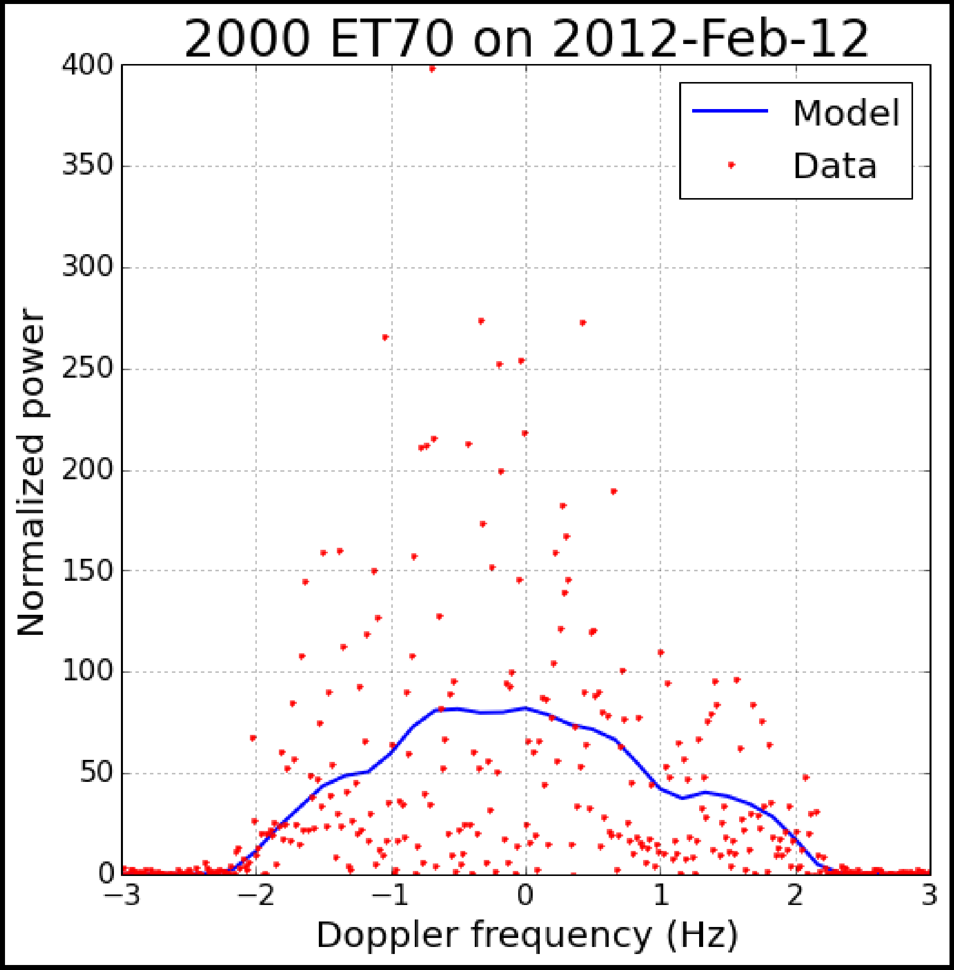
Filters to Estimate Local Mean

- Try different filters and widths to estimate std. dev. of each data point
  - 25 pts = 0.42 Hz
- Boxcar filter seems best
  - Gives sets of ratios closest to exponential distribution



Conclusions

- Background noise values in CW spectra follow exponential distribution, as expected
- Signal values also follow exp. distr.
  - But, unknown mean
- Filters to estimate local mean
- Images are more complicated
- First step in improved uncertainty analysis!



Acknowledgments

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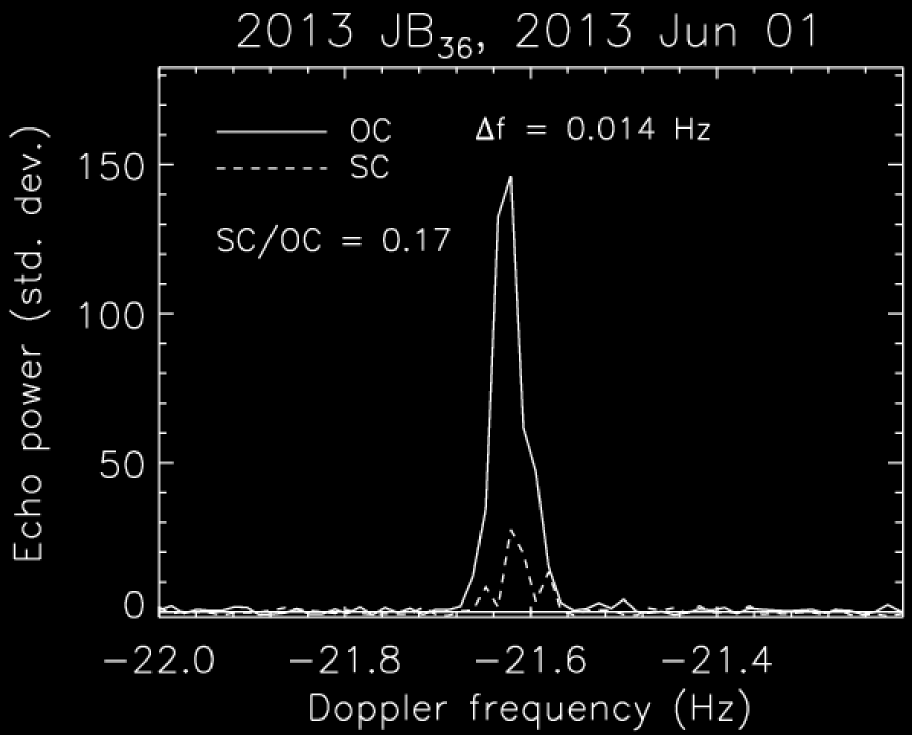
Backup Slides

Orbit Determination

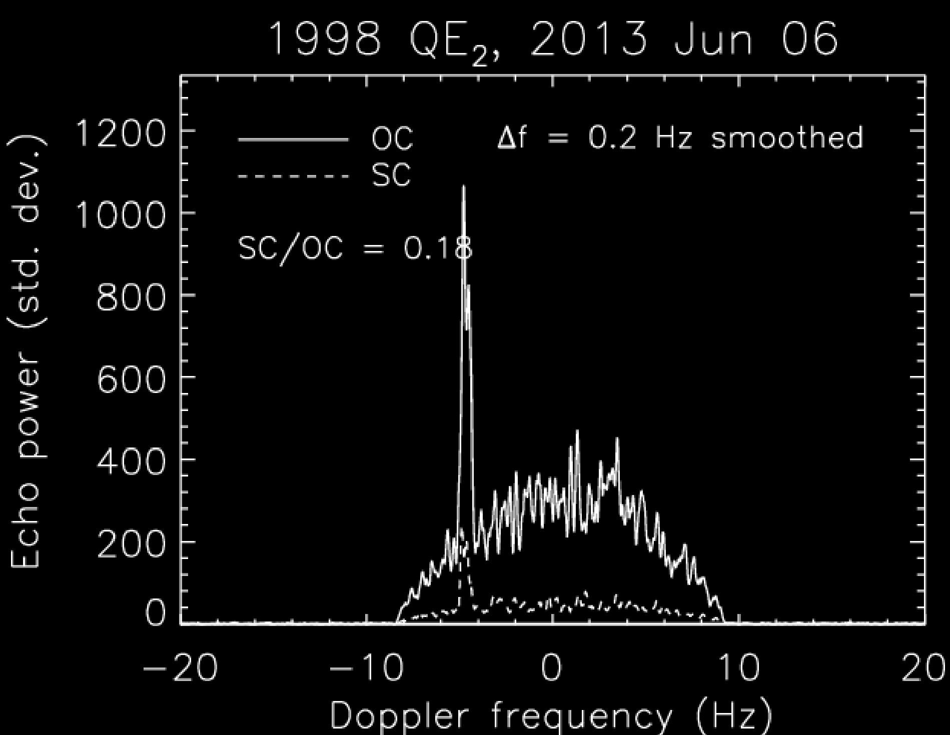
- Dramatic example: 2013 JB36
- Without Arecibo radar observations, it would have been lost after close approach.
- With radar data, its orbit can be accurately predicted over a thousand years into the future!



www.naic.edu/~radarusr/2013JB36/

Measure Both (Time and Frequency)

- For example: 1998 QE2
  - June 6 and 7, 2013



www.naic.edu/~radarusr/1998QE2/

Radar Equations: Signal

- Power received from radar echo (monostatic):

$$P_{rx} = \frac{G\,P_{tx}}{4\pi r^2}\,\frac{\sigma}{4\pi r^2}\,A_{eff} = \frac{G\,P_{tx}\,A_{eff}\,\sigma}{\left(4\pi r^2\right)^2}$$

- P_{tx} is transmitted power
- G is system gain (directionality) $\quad G = \frac{4\pi A_{eff}}{\lambda^2}$
- r is the distance to the target
- σ is the target's radar cross section $\quad \sigma = \hat{\sigma}\,A_{proj}$
- A_{eff} is the receiver's effective collecting area

Radar Equations: Noise

- Noise power:

$$N_{rms} = \frac{k\,T_{sys}\,B}{\sqrt{B\,\tau}} = k\,T_{sys}\sqrt{\frac{B}{\tau}}$$

- T_{sys} is the system temperature (convenient representation of the system's noise)
- τ is the integration time
- B is the target's (frequency) bandwidth

$$B = \nu_0 \frac{4\,v_{spin}\cos\varphi}{c} = \frac{4\,\nu_0}{c}\,\frac{\pi\,D\cos\varphi}{P_{spin}} = \frac{4\,\pi\,D\cos\varphi}{\lambda_0\,P_{spin}}$$

Radar Equations: Signal and Noise

$$P_{rx} = \frac{G\,P_{tx}}{4\pi r^2}\,\frac{\sigma}{4\pi r^2}\,A_{eff} = \frac{G\,P_{tx}\,A_{eff}\,\sigma}{\left(4\pi r^2\right)^2}$$

en.wikipedia.org/wiki/File:Goldstone_DSN_antenna.jpg

$$N_{rms} = \frac{k\,T_{sys}\,B}{\sqrt{B\tau}} = k\,T_{sys}\sqrt{\frac{B}{\tau}}$$

- Signal-to-noise ratio:

$$SNR = \frac{P_{rx}}{N_{rms}} = \frac{G\,P_{tx}\,A_{eff}\,\sigma\,\sqrt{\tau}}{\left(4\pi r^2\right)^2 k\,T_{sys}\sqrt{B}}$$

www.naic.edu/public/about/photos/hires/aoviews.html