The Advantages of Massive, Redundant Arrays

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Why are redundant arrays are traditionally avoided in radio astronomy?

The weaknesses of redundant arrays...

- Measure many fewer unique visibilities than total visibilities.
- Measure the same visibilities repeatedly, focusing sensitivity on limited modes.

...are really the strengths of redundant arrays.

Traditional correlation scales as $O(N^2)$

- With 40,000 antennas, 10 second integrations at 20 kHz resolution from 50 - 250 MHz, the correlator output is:
  - 8×10^{12} visibilities (60 TB) per integration
  - 6×10^{18} visibilities (50 EB) per season

Because redundant arrays measure many fewer unique visibilities than total visibilities, correlation need not scale as $O(N^2)$.

FFT Correlation

Measured voltages:

$$d_n = \int e^{-i(\mathbf{k}\cdot\mathbf{x}_n + \omega t)} \frac{B_n(\mathbf{k}) s(\mathbf{k}) dk_x dk_y}{k\sqrt{k^2 - k_x^2 - k_y^2}}$$

Different directions are uncorrelated

$$\begin{aligned}\langle s(\mathbf{k}) s(\mathbf{k}')\rangle &= S(\mathbf{k})\delta(\mathbf{k} - \mathbf{k}') \\ &= S(\mathbf{k})\delta(k_x - k_x')\delta(k_y - k_y')k\sqrt{k^2 - k_x^2 - k_y^2}\end{aligned}$$

Visibilities:

$$\langle d_n d_m^* \rangle = \int e^{i\mathbf{k}\cdot(\mathbf{x}_n - \mathbf{x}_m)} \frac{B_n(\mathbf{k}) I(\mathbf{k}) B_n^*(\mathbf{k}) dk_x dk_y}{k\sqrt{k^2 - k_x^2 - k_y^2}}$$

Visibilities are just measured voltages, Fourier transformed, squared, and Fourier transformed back.

Tegmark & Zaldarriaga

FFT Correlation

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If all positions are on a grid, then this Fourier transform can be done with an FFT in O(N log N) rather than $O(N^2)$!

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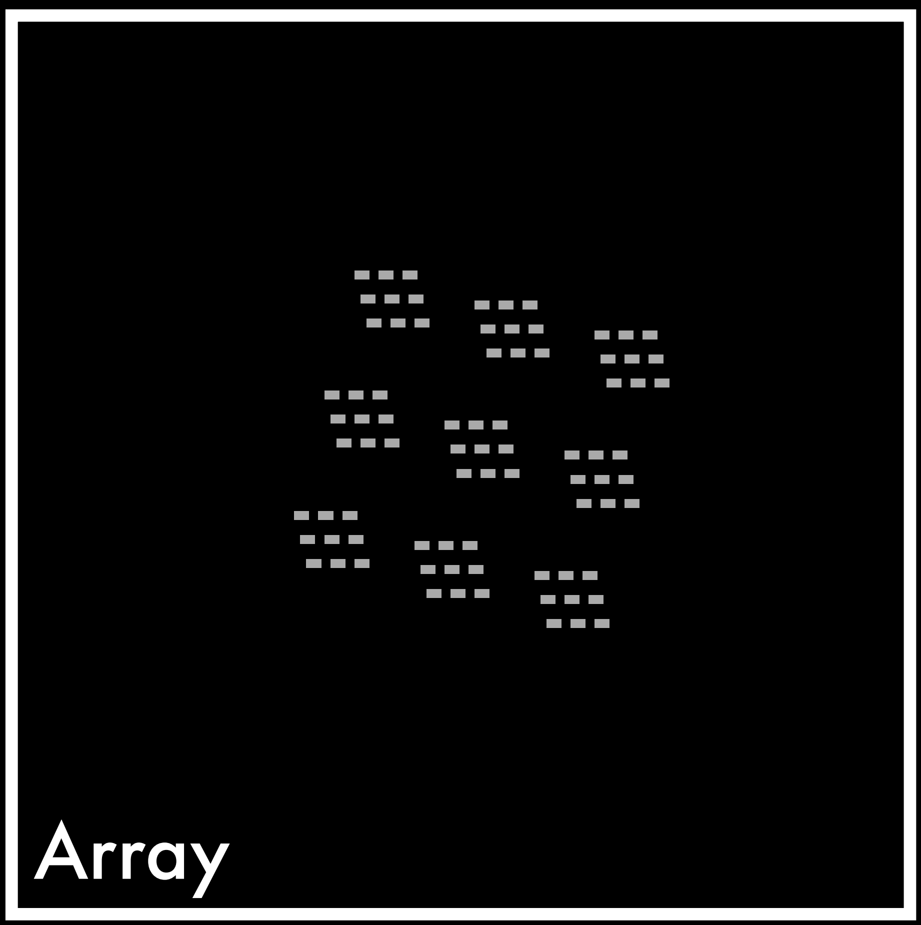
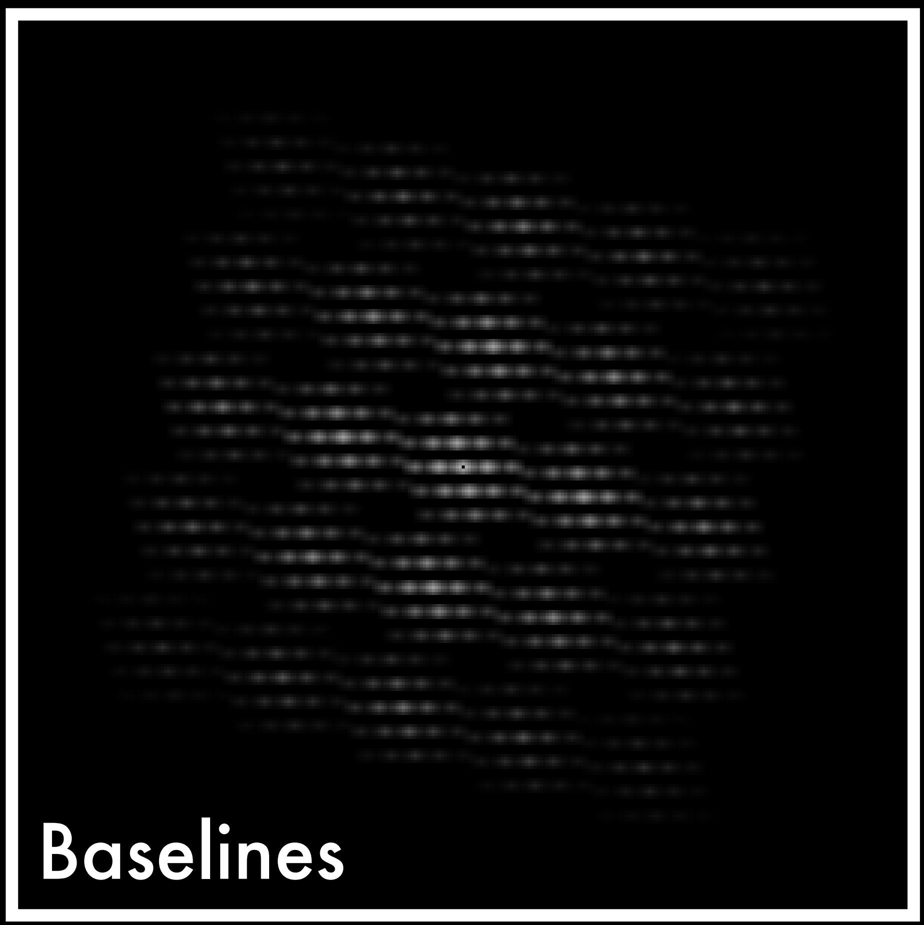
Visibilities are just measured voltages, Fourier transformed, squared, and Fourier transformed back.

Hierarchically regular arrays are also FFT-correlatable.





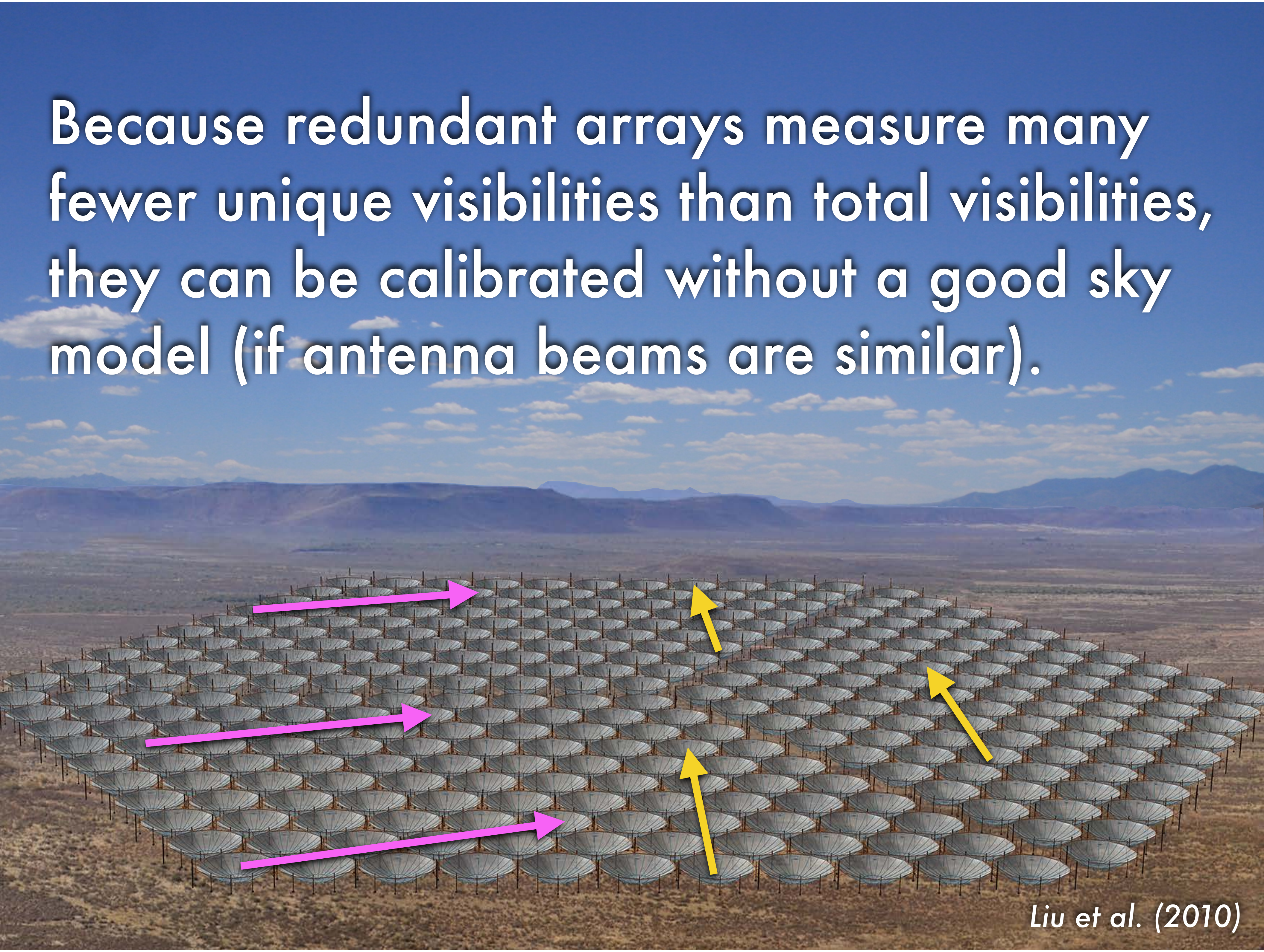
Tegmark & Zaldarriaga

FFT Correlation Requires Real-Time Calibration

Each antenna multiplies it's measurement by some complex gain:

$$V_{nm}^{\mathrm{measured}} = g_n g_m^* V_{nm}^{\mathrm{true}}$$

FFT correlation assumes antennas are the same (same beam, same gain) because *FFT correlation calculates average visibilities.*

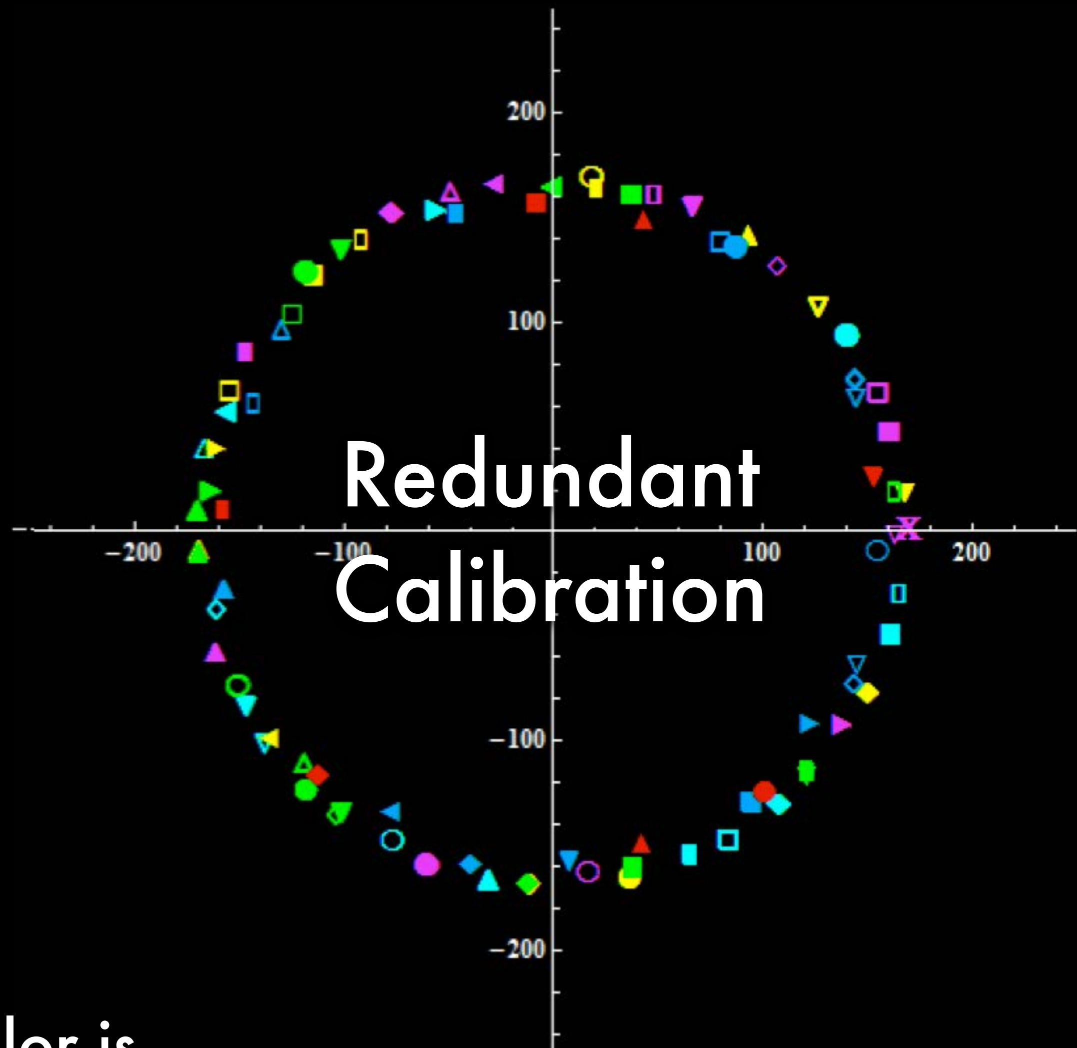

Because redundant arrays measure many fewer unique visibilities than total visibilities, they can be calibrated without a good sky model (if antenna beams are similar).
Liu et al. (2010)

Wait! Doesn't redundant calibration require $O(N^2)$ visibilities?

- Nominally, yes. But…
  - We don't need to calibrate every frequency every integration.
  - We don't need all $O(N^2)$ visibilities for redundant calibration, only $N_{vis} >> N_{ant}$.

MITEoR: a prototype highly-scalable interferometer for 21 cm cosmology.

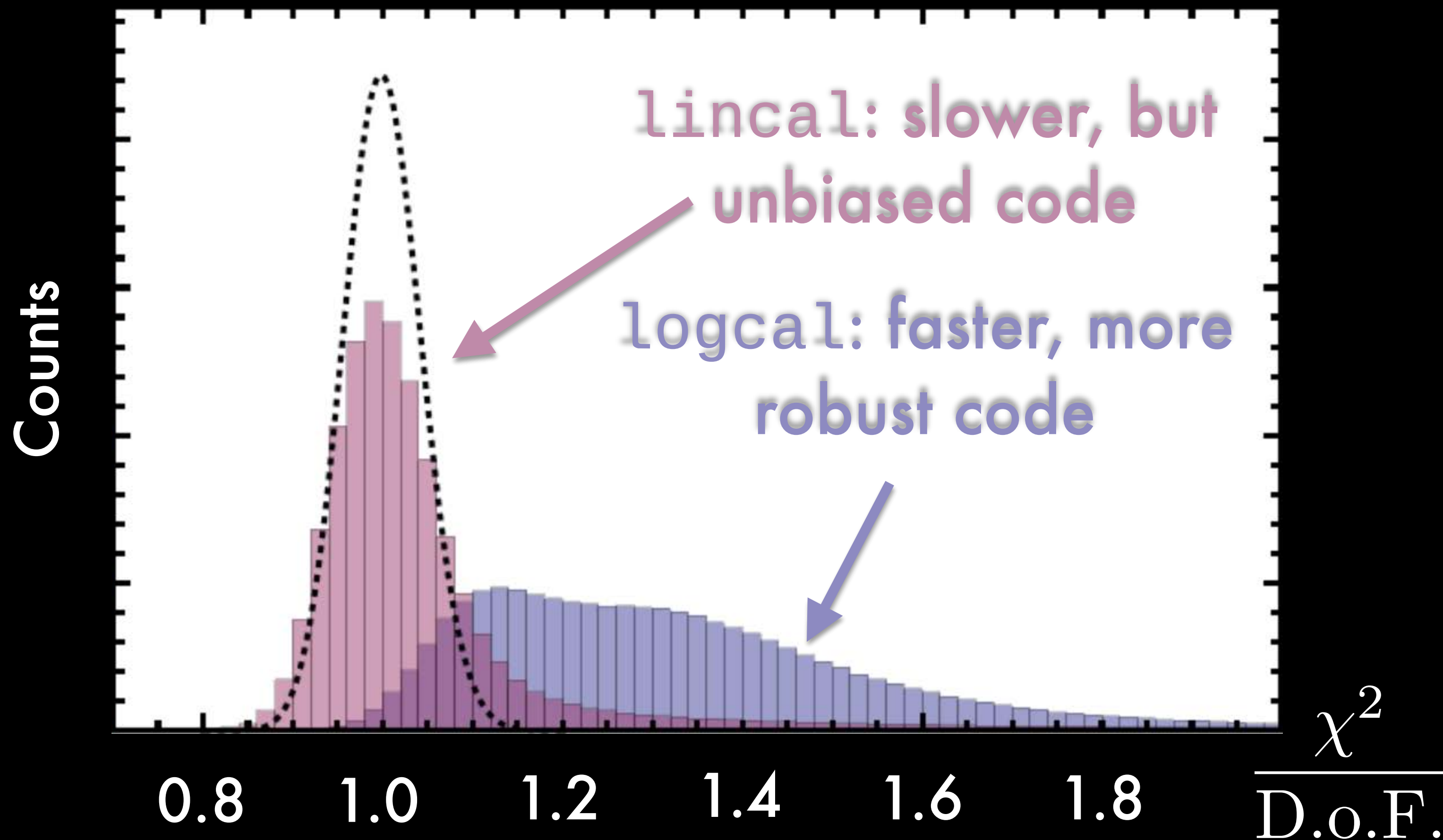
Redundant baselines allow us to quickly and precisely calibrate the amplitudes and phases of every antenna.



Each shape/color is
a unique baseline.

Zheng et al. (2014)

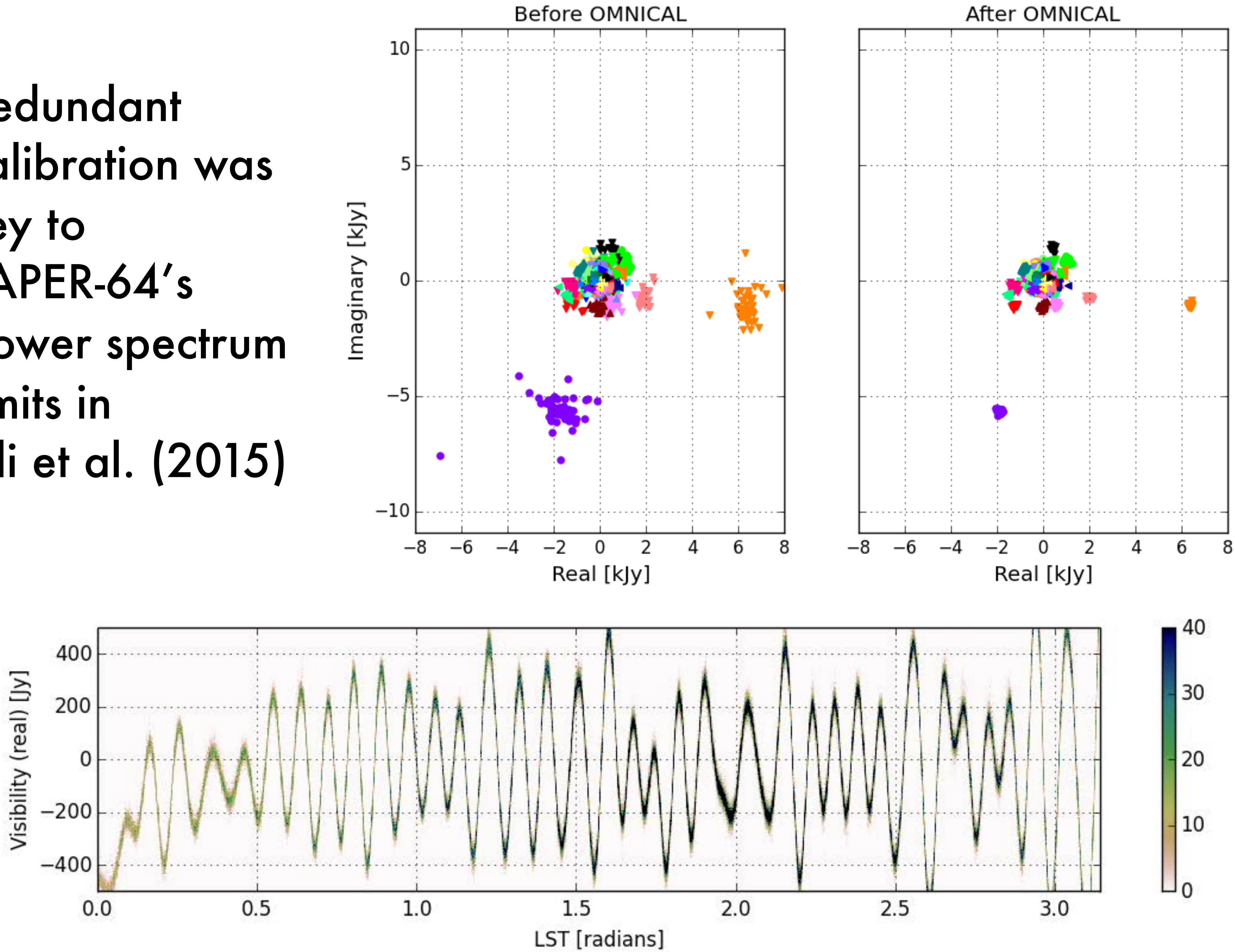
Redundant baselines allow for a quantitative test of calibration and the real-time identification of problems.



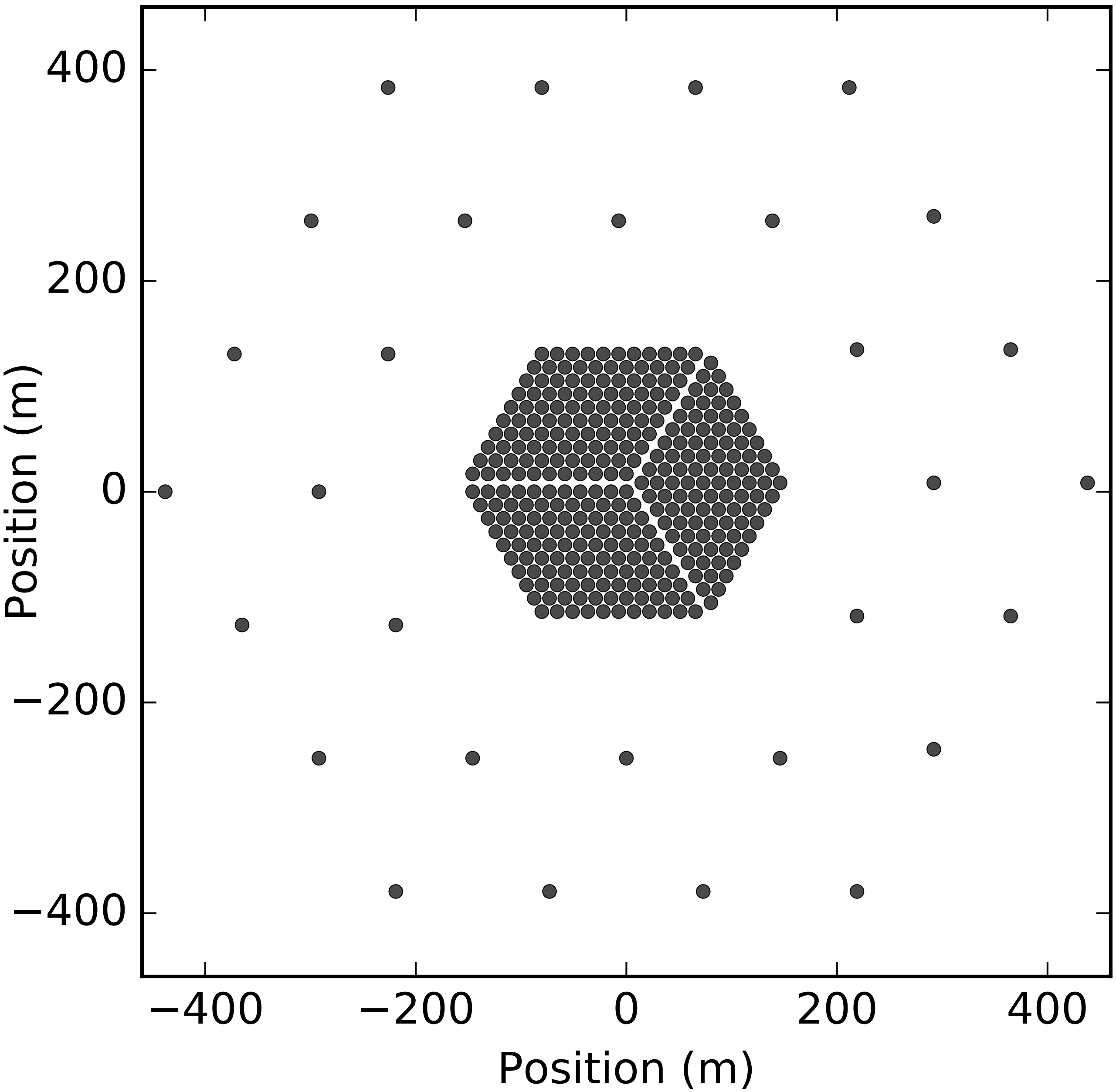
$$\chi^2 = \sum_{\text{all baselines}} \left[(g_i g_j^*) V_{i,j}^{\text{fit}} - V_{i,j}^{\text{measured}} \right] / \sigma^2$$

Zheng et al. (2014)

Redundant calibration was key to PAPER-64's power spectrum limits in Ali et al. (2015)

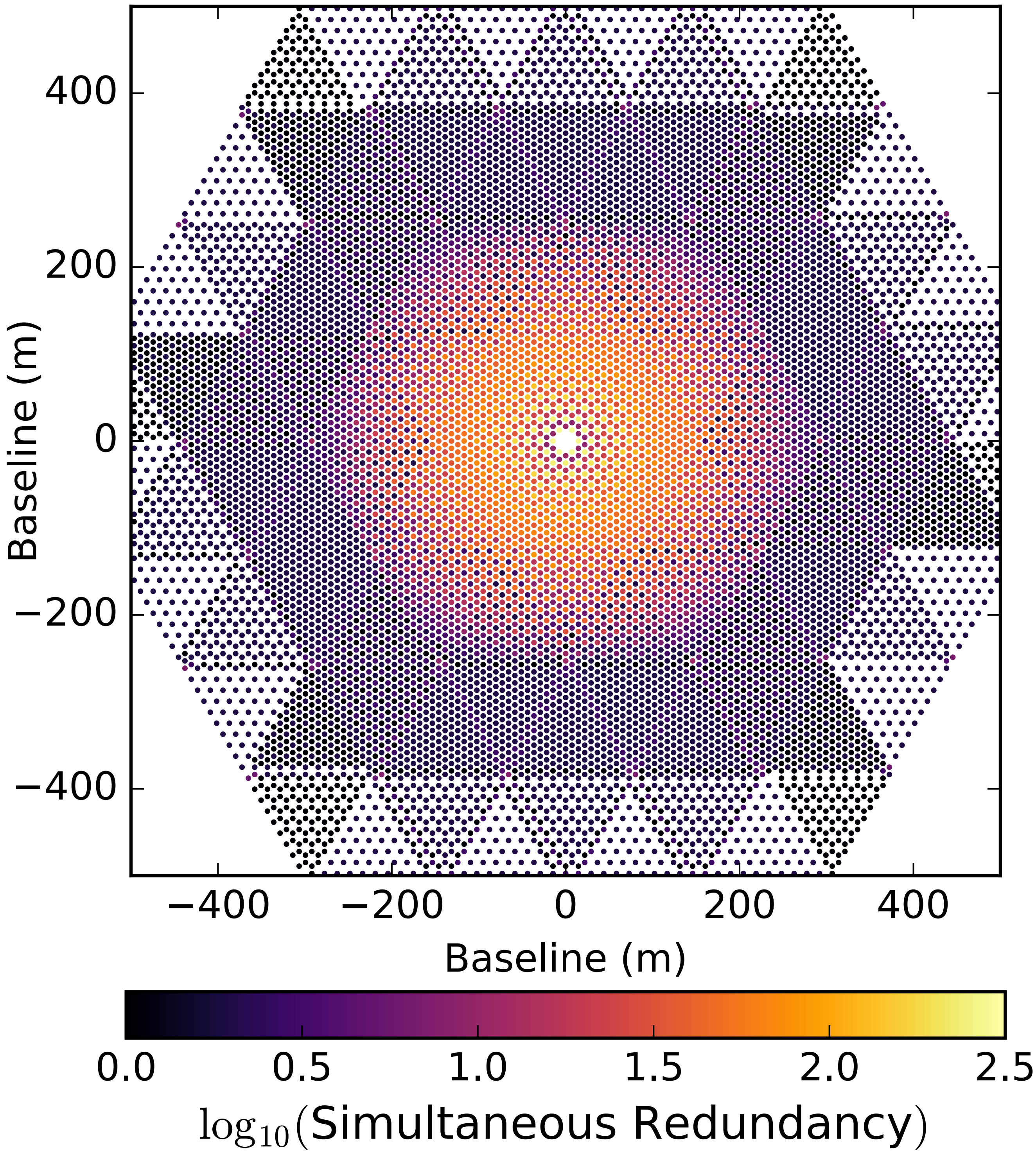


The need for redundant baselines and high sensitivity to short baselines drove HERA’s design.

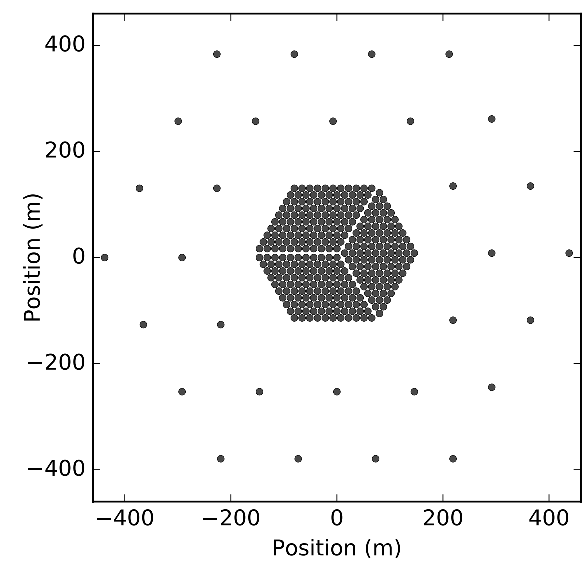


HERA's split configuration enables

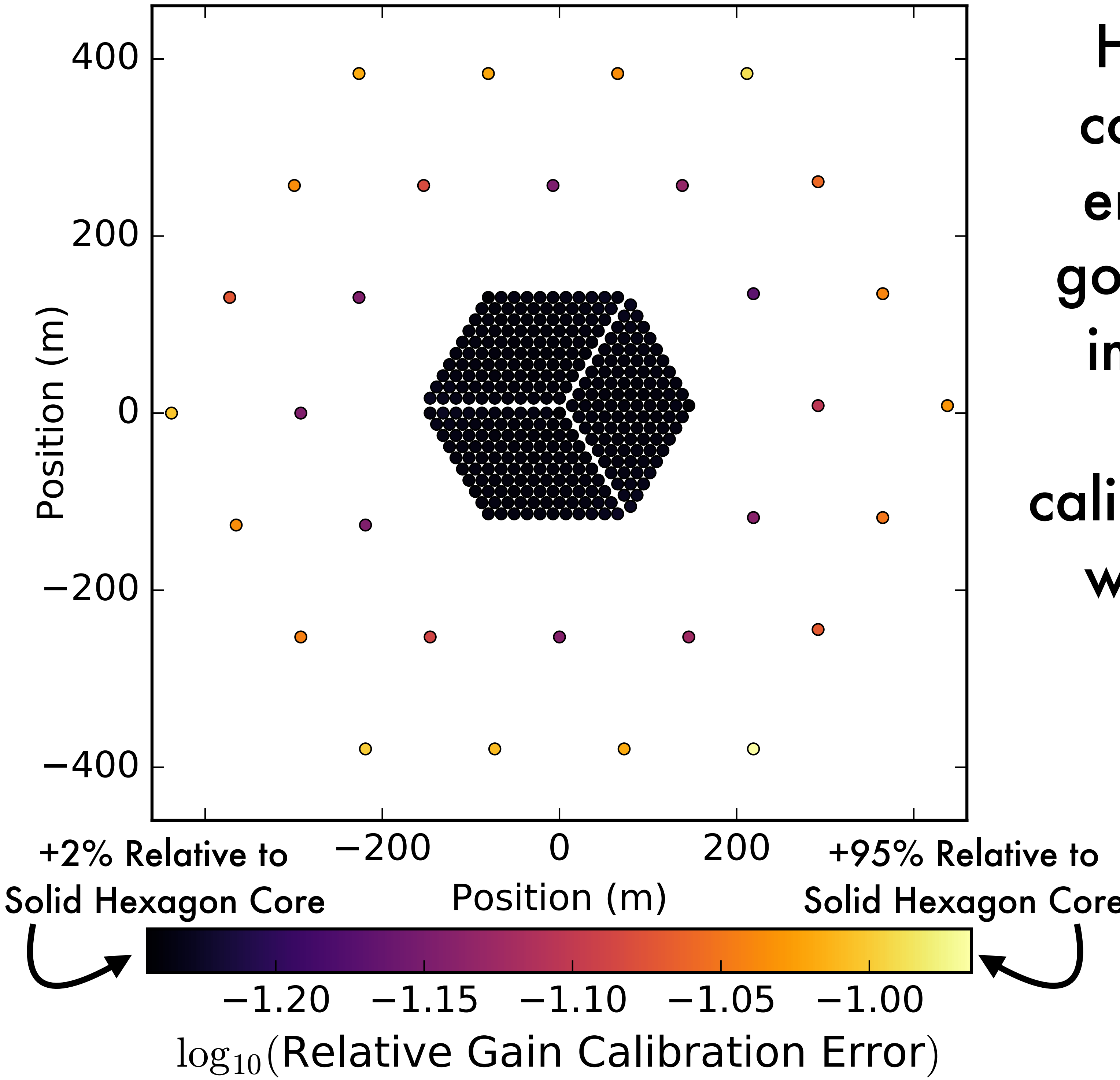
Dillon & Parsons (2016)



HERA's split configuration enables both good widefield imaging



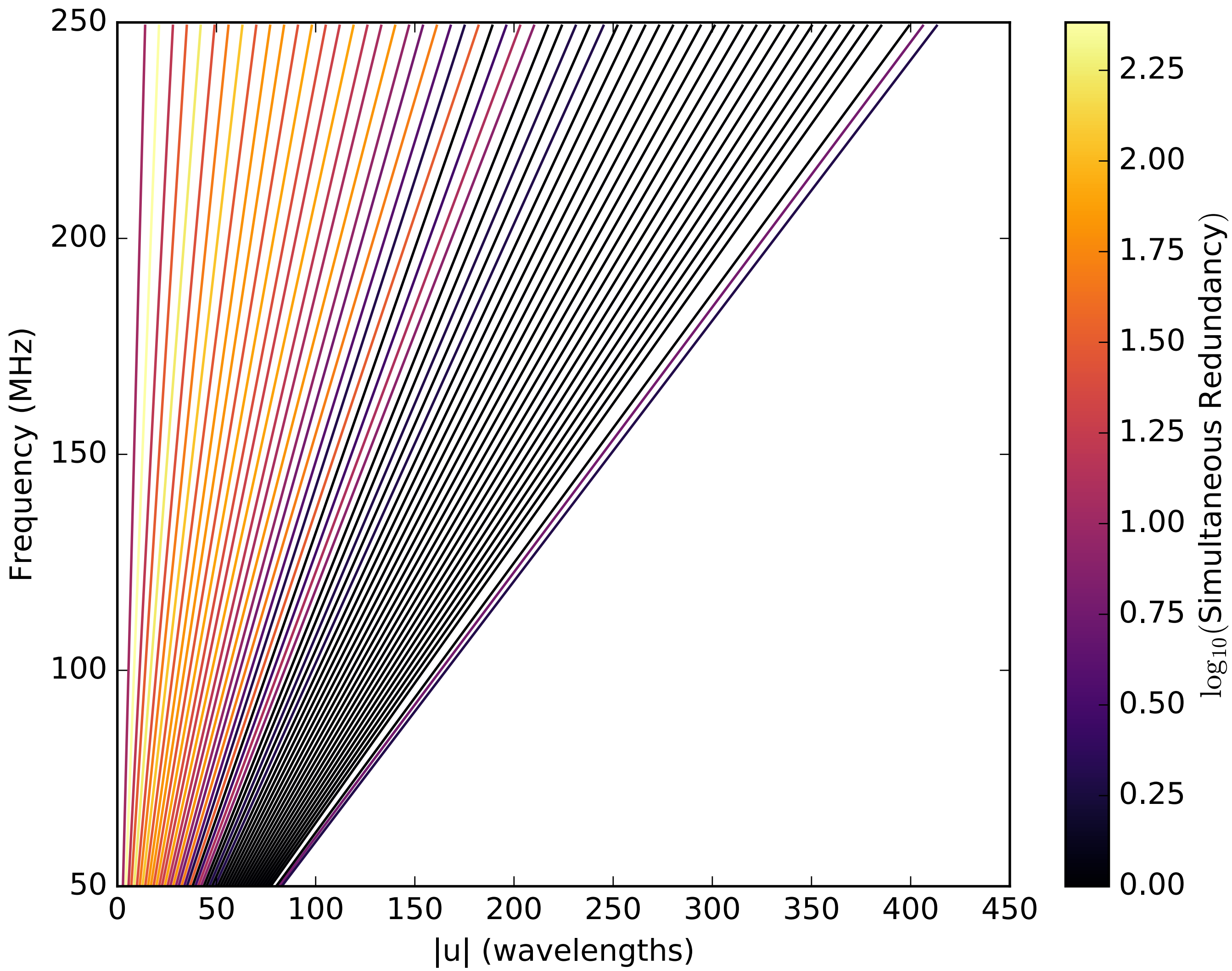
Dillon & Parsons (2016)

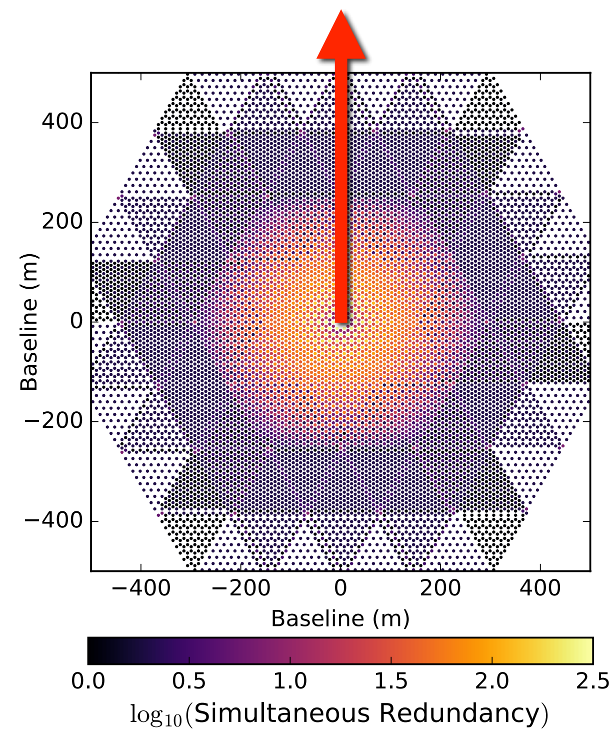


HERA's split configuration enables both good widefield imaging and redundant calibration of the whole array.

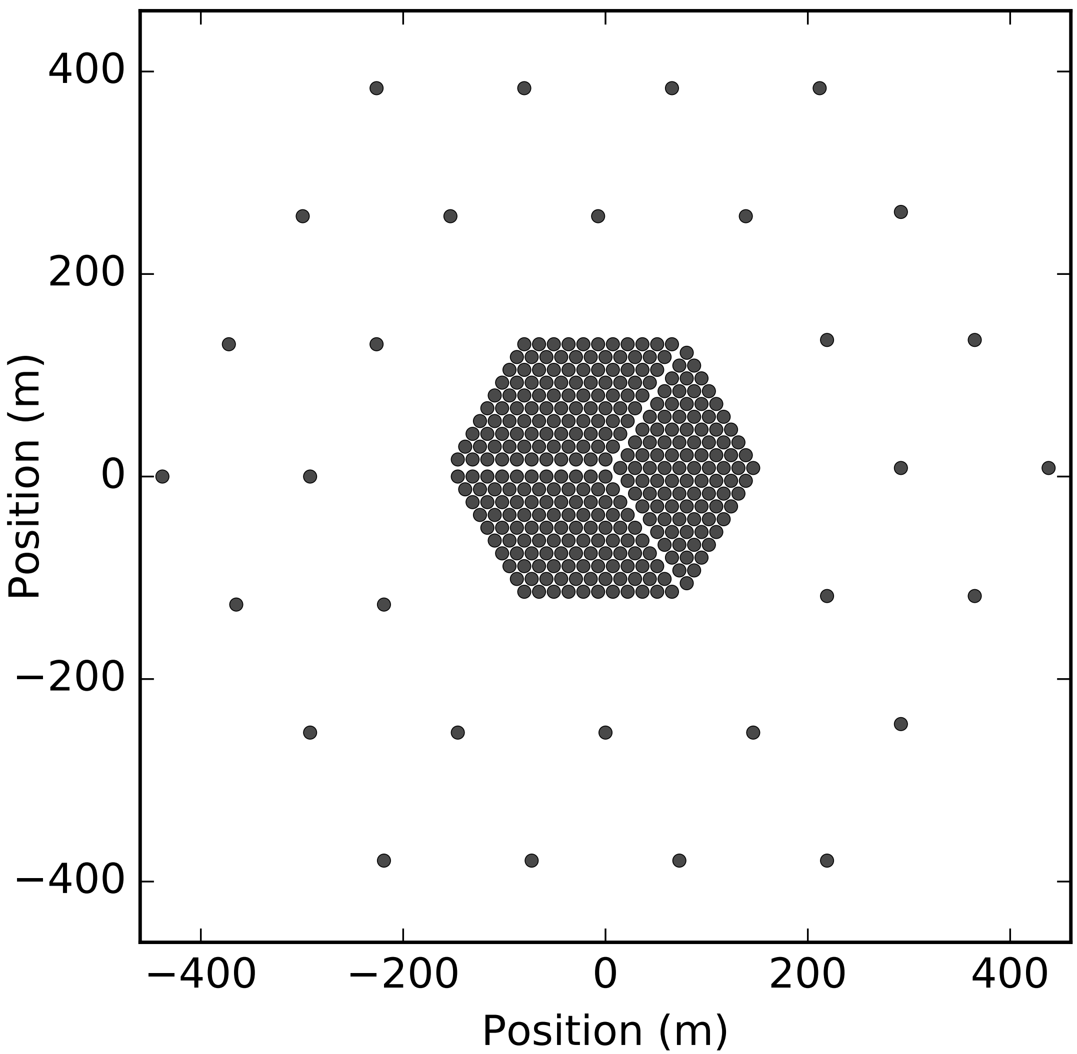
Dillon & Parsons (2016)

The split core also increases the frequency sampling at fixed (u,v).



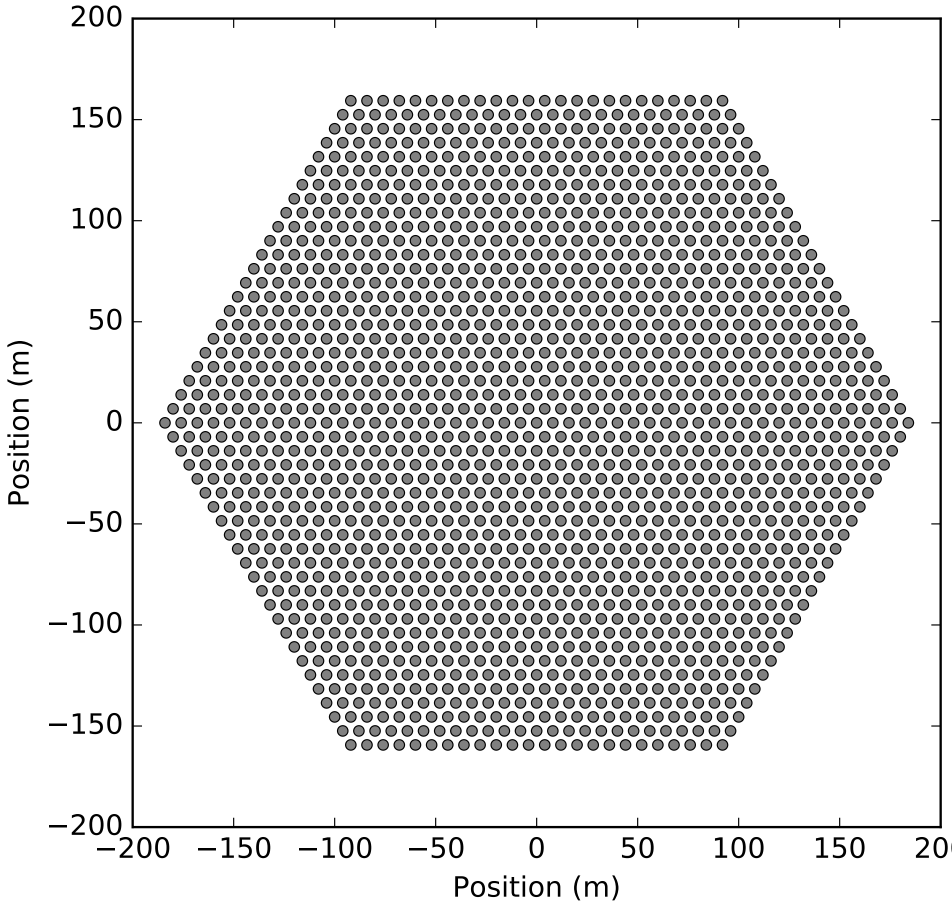


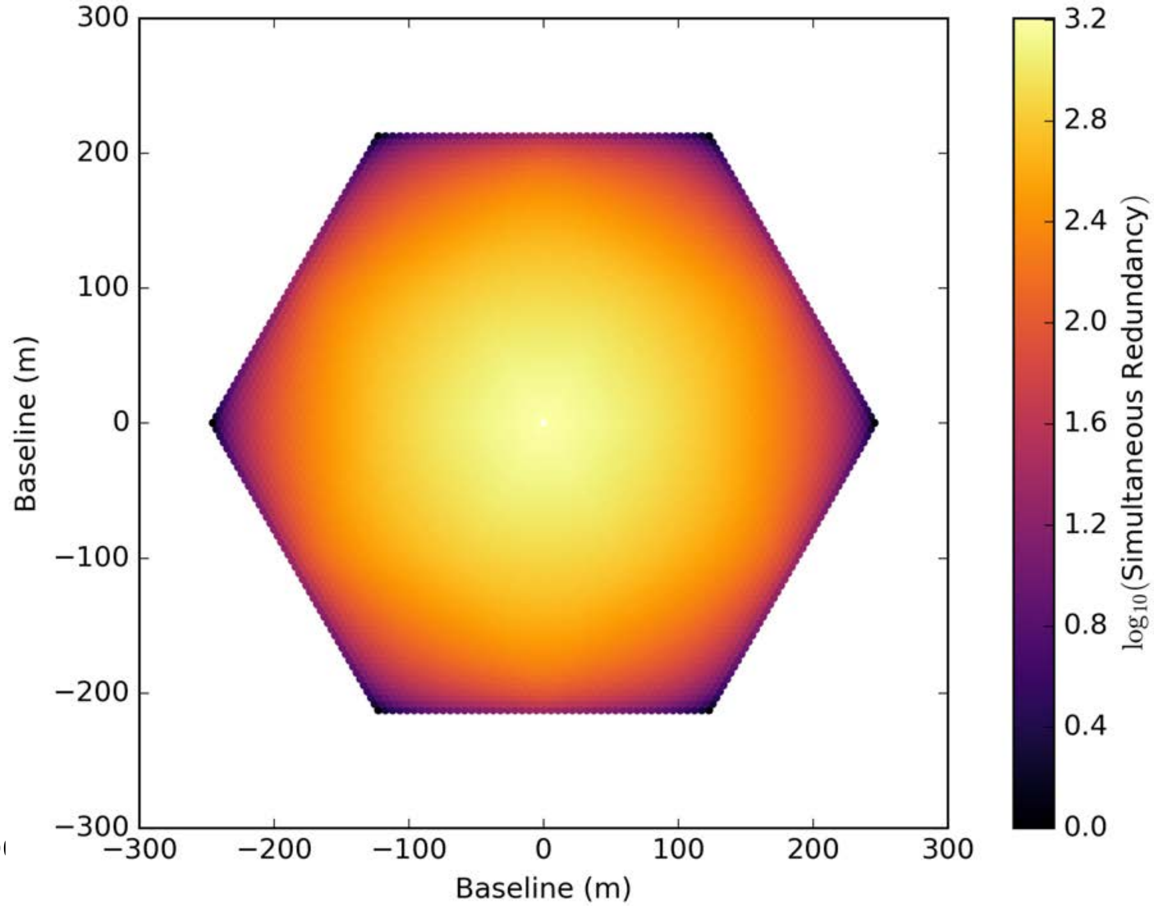
Looking forward, we could just scale up the HERA design.



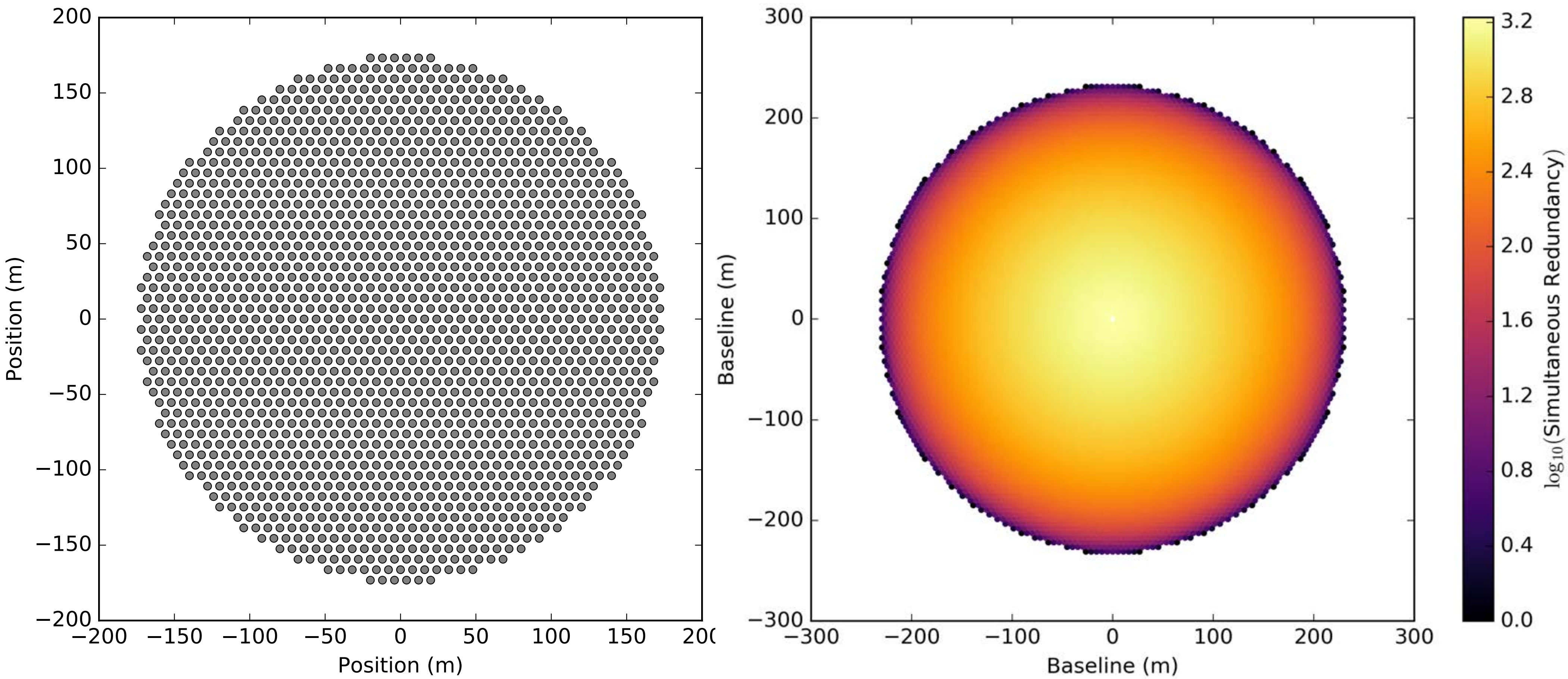
- Scale up core.
- Scale up outriggers
- Smaller individual elements
- More splits to the core (9 ways rather than 3)

Or maybe we should build a giant core (the original FFTT)

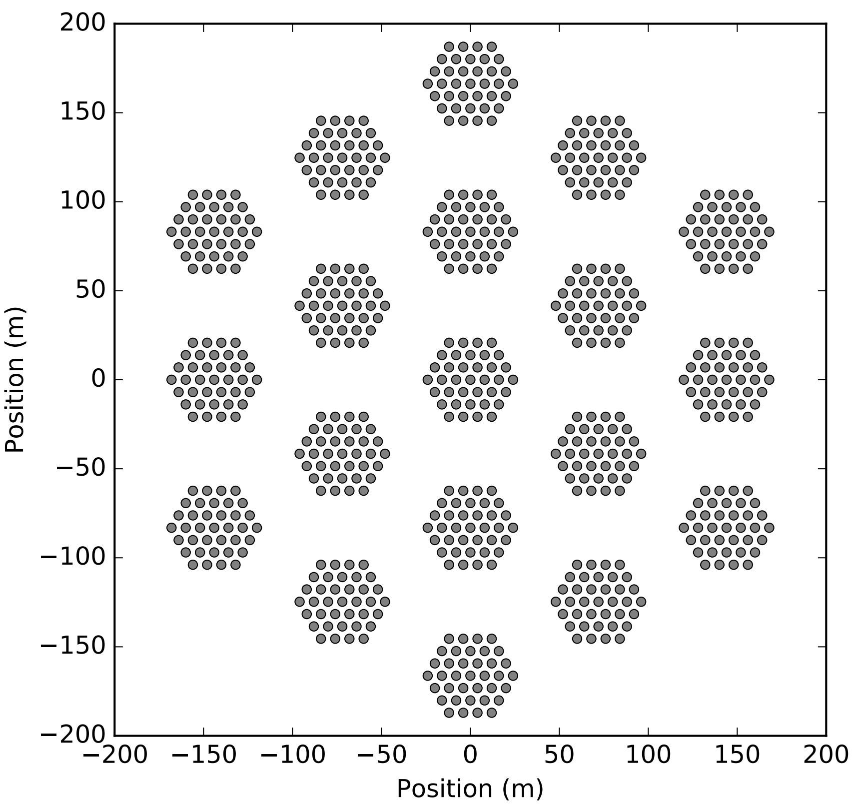


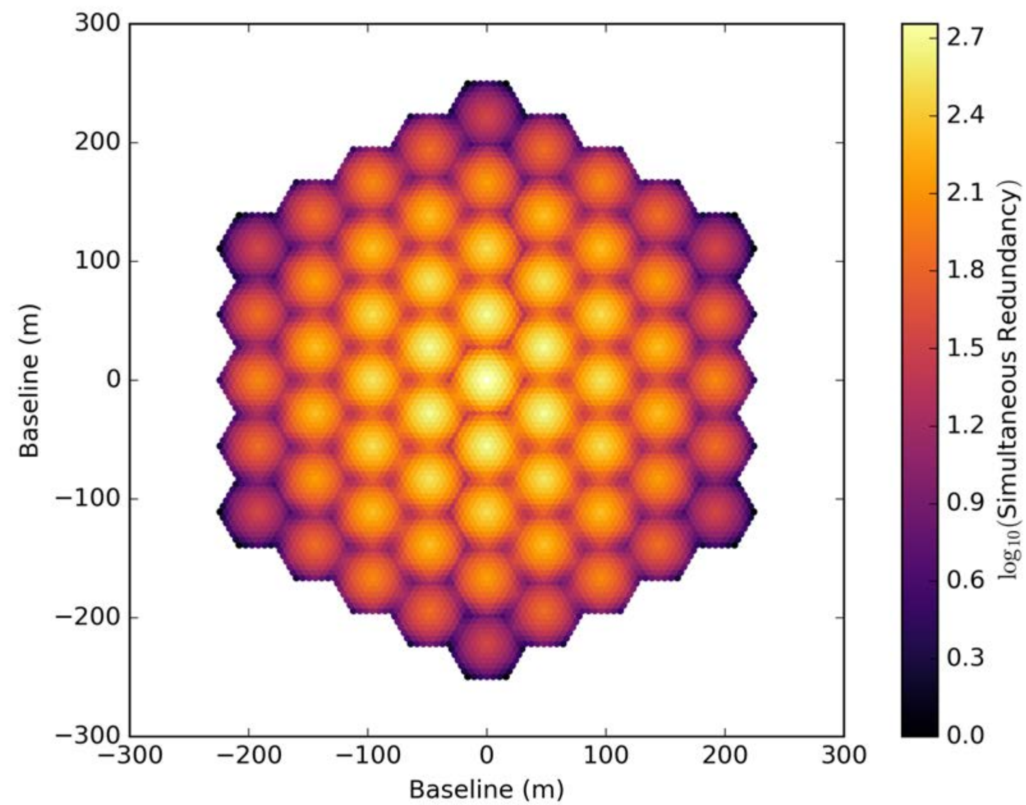


Though the perimeter doesn't have to be a hexagon.

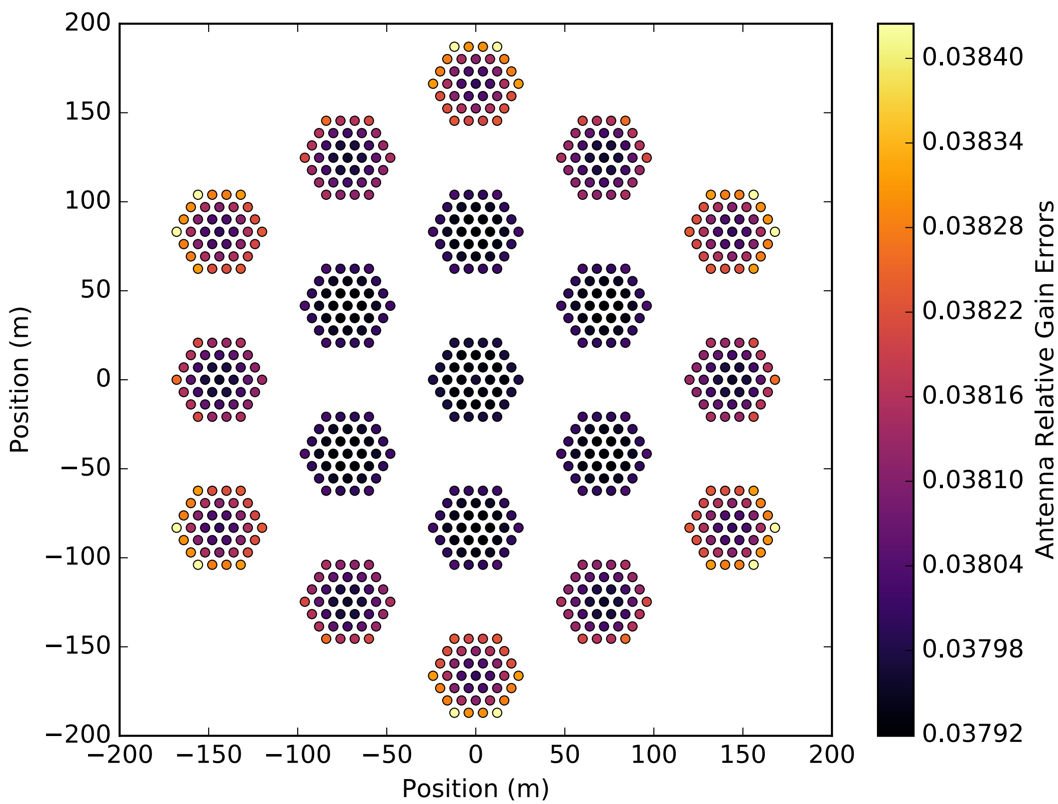


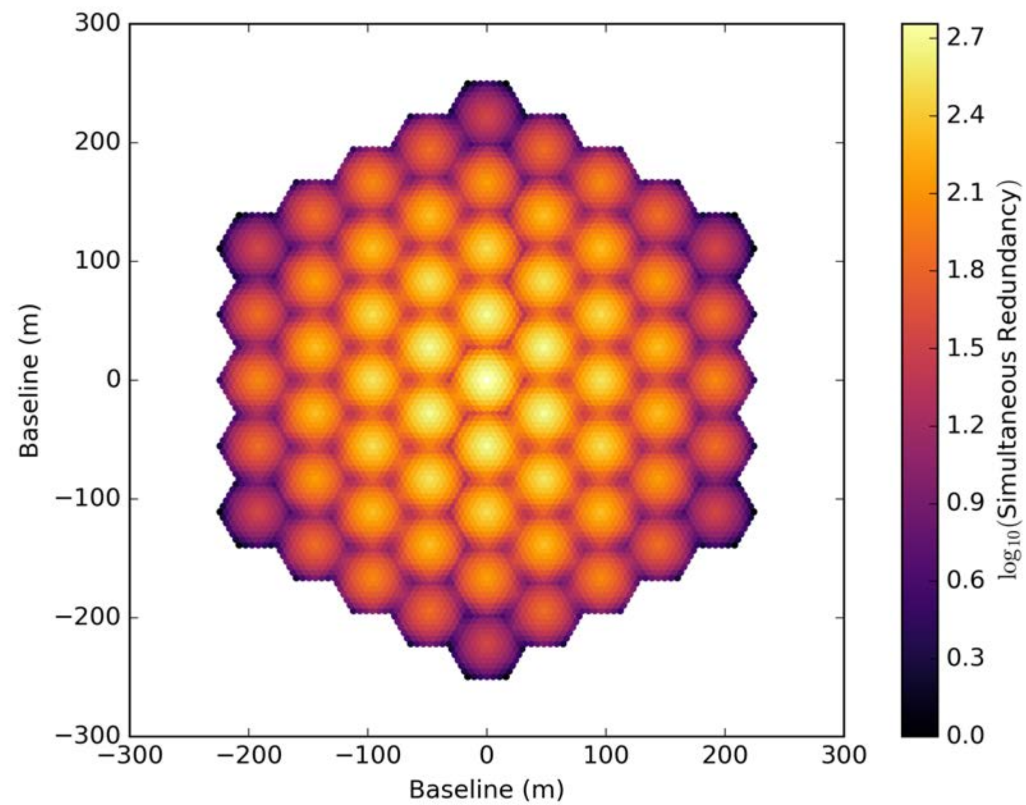
We can also explore hierarchically redundant arrays.



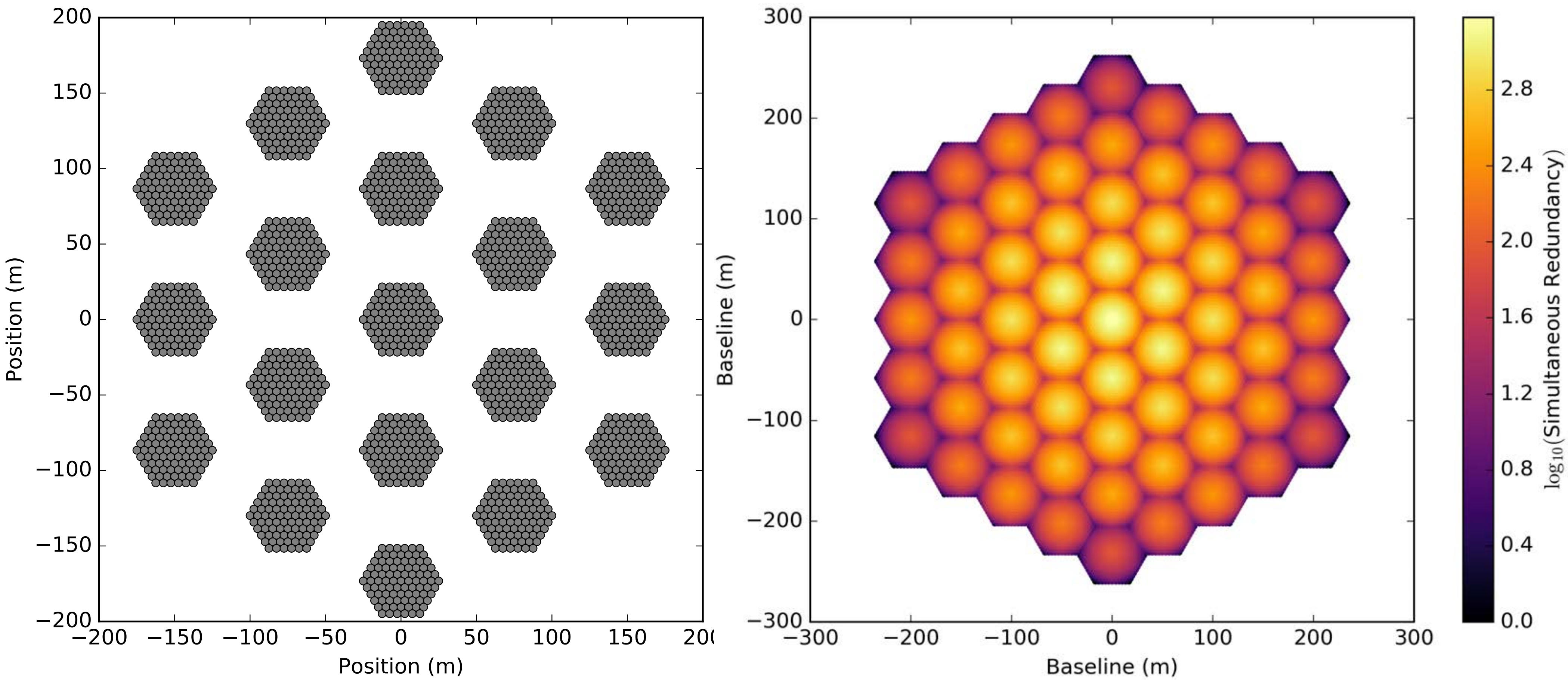


Baseline overlap makes hierarchical arrays redundantly calibratable.





We can do a small number of large hexagonal arrays.



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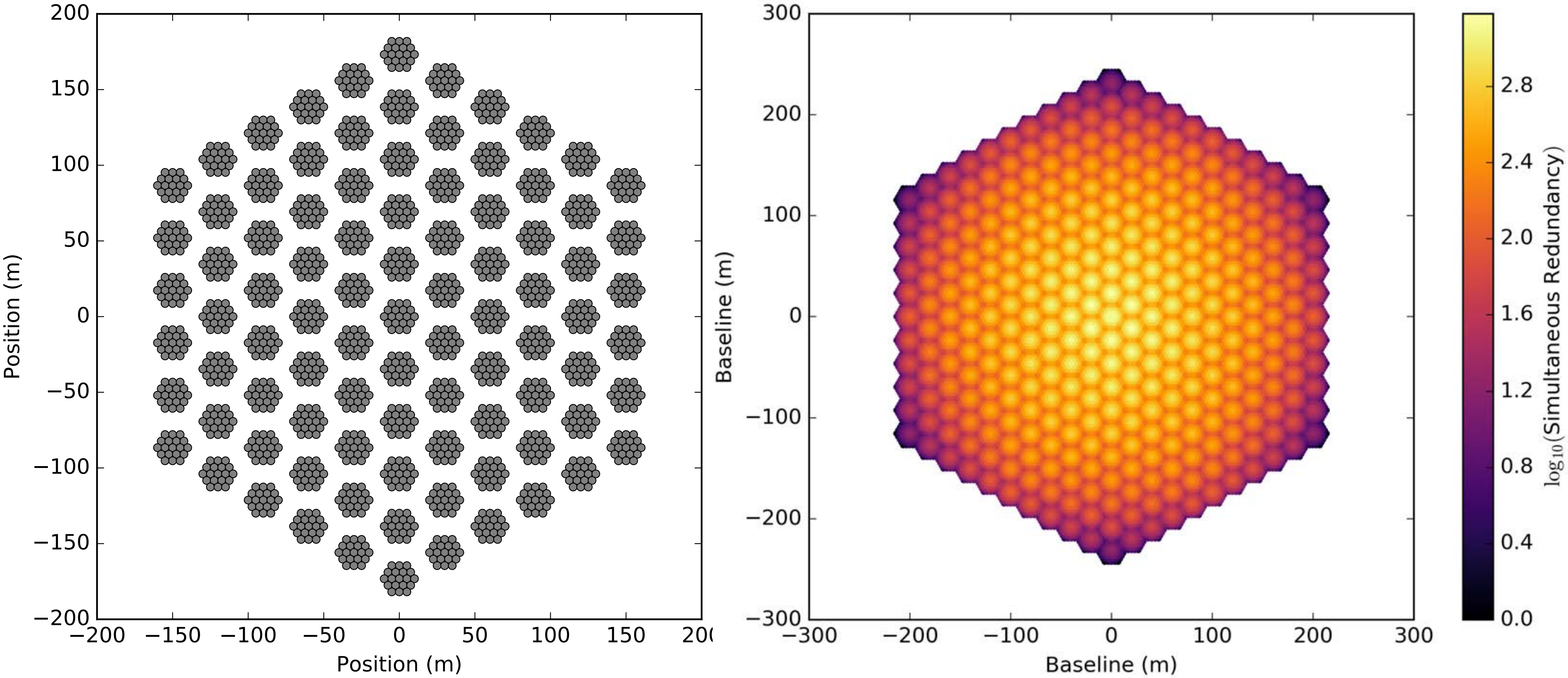
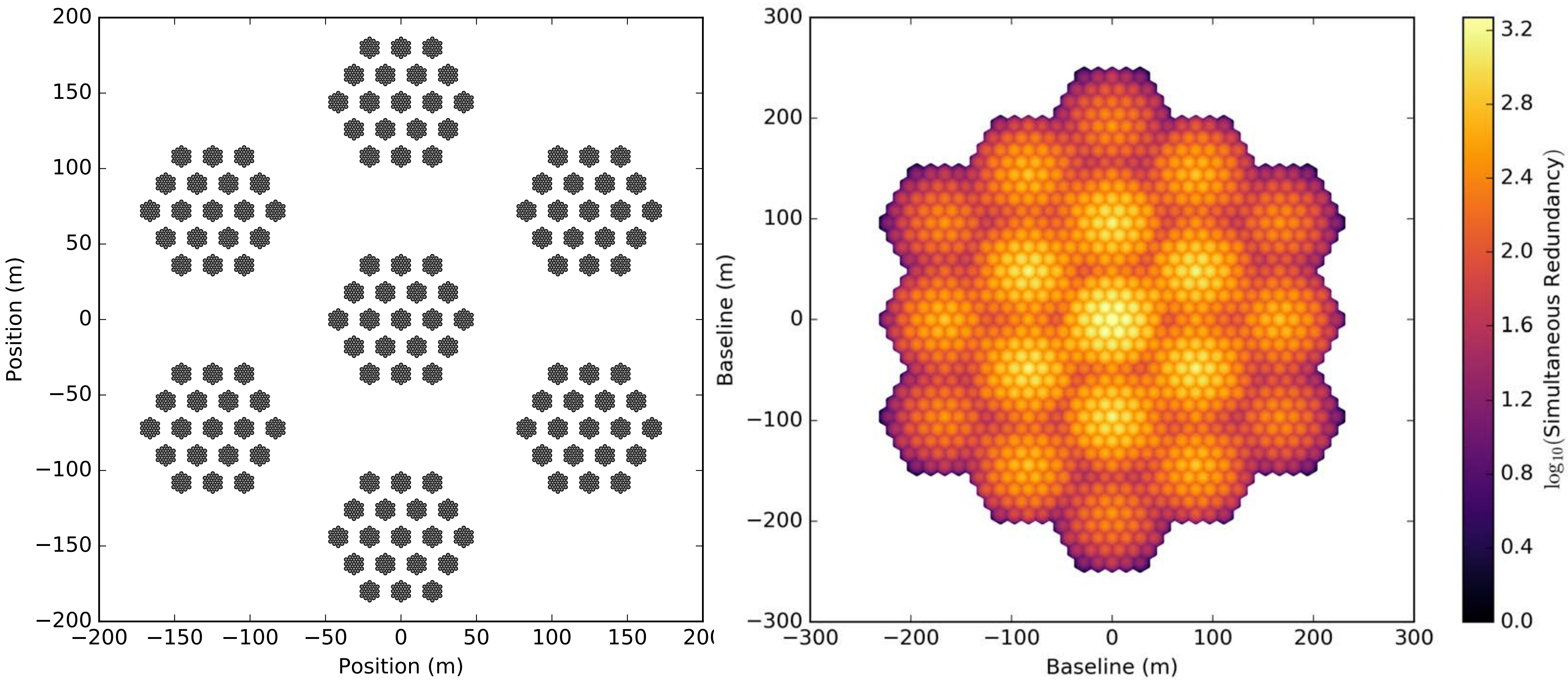
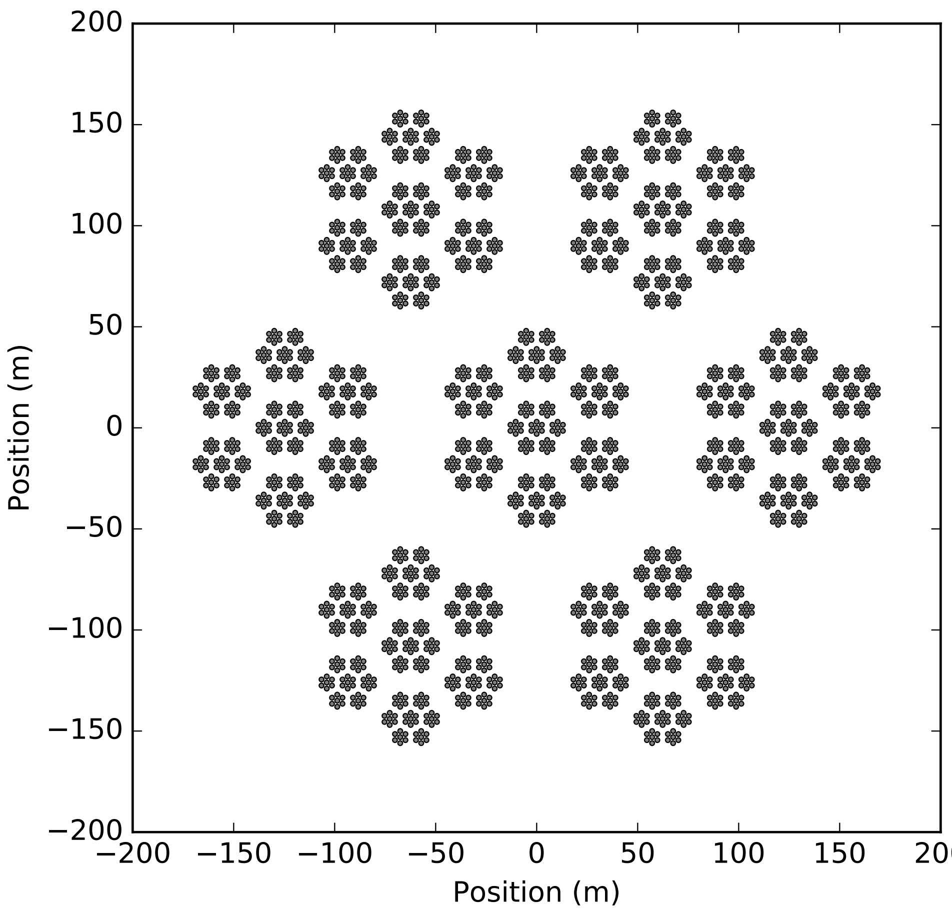
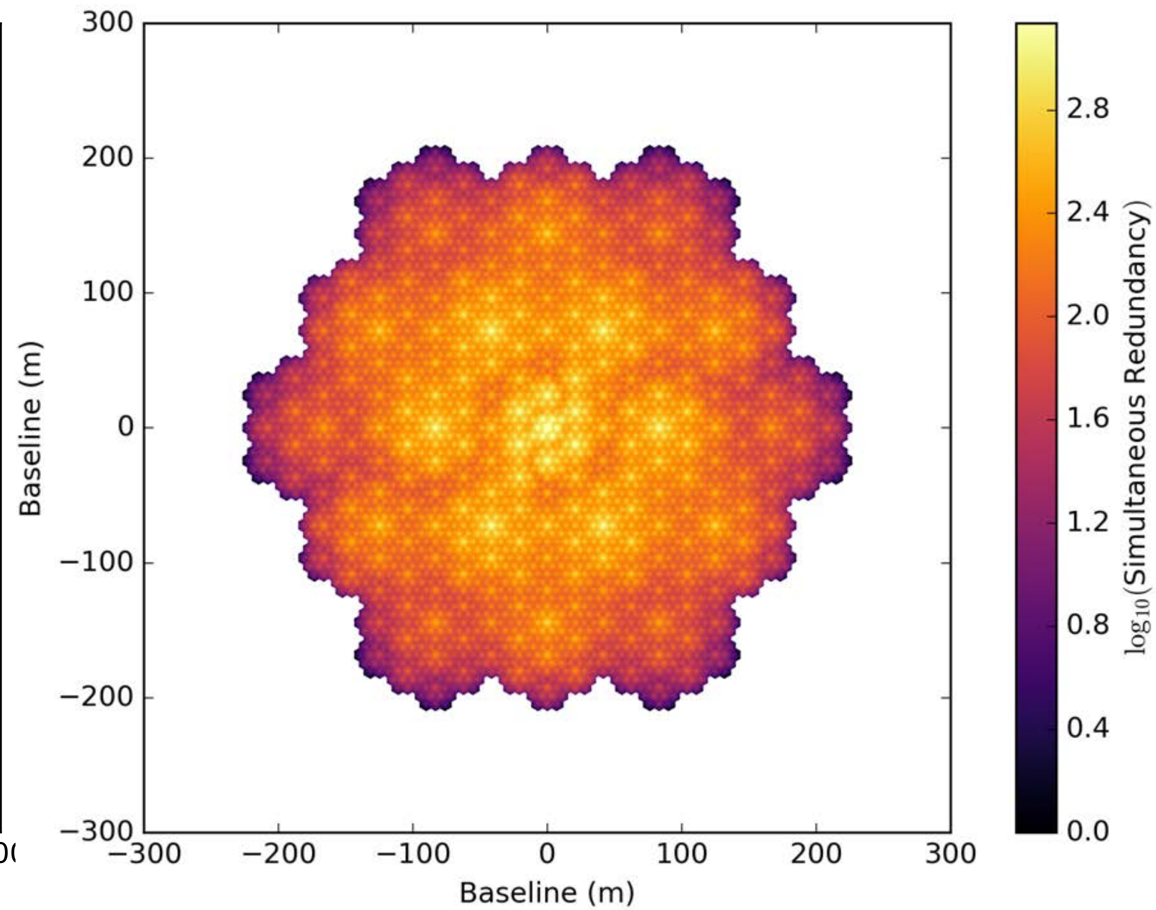


We go can go to three levels of hierarchy.



We go can go to four (or more) levels of hierarchy.





There are many different ways to take advantage of redundant arrays.

- Adjusting element size (and thus FoV)
- Adjusting antenna separation
- Packing: Square vs. Hex vs. ???
- Splitting cores to subsample the baseline grid
- Adding outriggers
- Multiple levels of hierarchy