

Polarization Calibration

(in 30 min or less?!...)

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References

- Interferometry and Synthesis in Radio Astronomy 2nd Ed. (Thompson, Moran, & Swenson) (TMS): Chapters 4, 10, 13
- Synthesis Imaging in Radio Astronomy II (Taylor, Carilli, Perley, eds.): Lectures 5, 6, 25, 29
- The Measurement Equation:
 - Hamaker, Bregman, & Sault 1996, A&AS 117, 137.
 - Sault, Hamaker, & Bregman 1996, A&AS 117, 149.
- Online EVLA Tutorials:
 - http://casaguides.nrao.edu/index.php?title=EVLA_Tutorials
 - Polarimetry: 3C391

Correlations and Stokes Parameters

- Measured *correlations* or *visibilities* are 4-vectors (circular feed basis):

$$\begin{aligned}V_{RR} &= \mathcal{I} + \mathcal{V} \\V_{RL} &= Q + i\mathcal{U} \\V_{LR} &= Q - i\mathcal{U} \\V_{LL} &= \mathcal{I} - \mathcal{V}\end{aligned}$$

- Calibrated* correlation visibilities are combined to form the Stokes visibilities:

$$\begin{aligned}\mathcal{I} &= (V_{RR} + V_{LL})/2 \\Q &= (V_{RL} + V_{LR})/2 \\U &= (V_{RL} - V_{LR})/2i \\V &= (V_{RR} - V_{LL})/2\end{aligned}$$

- All four correlations must be *consistently* calibrated, so the combinations can be formed correctly
- Stokes *visibilities* may then be inverted to form Stokes *images*
 - Linear polarization: $\mathcal{P} = (Q^2 + U^2)^{1/2}$ $\chi = 0.5 \tan^{-1}(U/Q)$

Generic Matrix Calibration

(antenna-based ‘voltage’ domain)

- Scalar: $x_i = J_i s_i$
- Vector:
$$\begin{bmatrix} x_R \\ x_L \end{bmatrix}_i = \begin{bmatrix} J_{R \leftarrow R} & J_{R \leftarrow L} \\ J_{L \leftarrow R} & J_{L \leftarrow L} \end{bmatrix} \begin{bmatrix} s_R \\ s_L \end{bmatrix}_i$$
 - Polarization may mix!
 - $J_{R \leftarrow L}, J_{L \leftarrow R}$ generally small, by design, but not strictly zero (in fact larger in EVLA than in VLA...)
- Correlation: yields a 4-vector

Factored Calibration

$$\mathbf{J}_i \mathbf{s}_i = \mathbf{B}_i \mathbf{G}_i \mathbf{D}_i \mathbf{P}_i \mathbf{T}_i \mathbf{F}_i \mathbf{s}_i$$

\mathbf{B}_i : bandpass

\mathbf{P}_i : parallactic angle

\mathbf{G}_i : (electronic) gain

\mathbf{D}_i : instrumental polarization

\mathbf{T}_i : troposphere

\mathbf{F}_i : ionosphere

- Approximately (and adequately!) “physical”
 - OK to imagine each antenna’s received signal, \mathbf{s}_i , corrupted by terms in order from right to left, though actual effects are not strictly quite so discrete
- Really a convenient factorization of effects that organizes the matrix algebra
 - Consistent with physics and hardware design (e.g., polarizer in front of amplifiers, etc.)
 - Order is important!
- Notation in this talk:
 - Upper case: matrix or operator
 - Lower case: vector or matrix element
 - One subscript: antenna-based
 - Two subscripts: baseline-based
 - Zero subscripts: mnemonic or ‘operator’ notation

The Measurement Equation

$$\mathbf{V}^{obs} = \mathbf{B} \ \mathbf{G} \ \mathbf{D} \ \mathbf{P} \ \mathbf{V}^{true}$$

$$\mathbf{V}^{corr} = \mathbf{P}^{-1} \ \mathbf{D}^{-1} \ \mathbf{G}^{-1} \ \mathbf{B}^{-1} \ \mathbf{V}^{obs}$$

- Where:
 - \mathbf{V}^{obs} = observed visibility 4-vector (voltage correlations)
 - \mathbf{V}^{true} = true visibility 4-vector; estimated by \mathbf{V}^{mod} when solving
 - \mathbf{V}^{corr} = corrected visibility 4-vector
- $\mathbf{B}, \mathbf{G}, \mathbf{D}, \mathbf{P}$ = baseline-based operators each representing pairwise combinations of antenna-based terms; order IS important
 - (other terms suppressed for clarity)
- We solve for relevant calibration terms with appropriate calibrator observations and heuristics, and bootstrap
- Each solve has the form:

$$(\mathbf{V}^{obs}) = \underline{\mathbf{J}} (\mathbf{V}^{mod})$$

- () indicates partial correction/corruption by already-known terms

Parallactic Angle, P

- Sky orientation (and thus source linear polarization) rotates in the field of view of an alt-az telescope:

$$\psi(t) = \frac{\cos b \sin H(t)}{\sin b \cos \delta - \cos b \sin \delta \cos H(t)}$$

b = *latitude*; $H(t)$ = *Hour Angle*; δ = *declination*

$$P_i^{circ} = \begin{pmatrix} e^{-i\psi} & 0 \\ 0 & e^{i\psi} \end{pmatrix}$$

P in the Circular Basis

- In general, parallel- and cross-hand correlations see parallactic angle differences and sums, respectively
- If $\psi_i = \psi$ for all antennas (geographically small array), only sums in cross-hands survive
 - Uniform rotation of linear polarization:

$$V_{RR} = I + V$$

$$V_{RL} = (Q + iU) e^{-i2\psi}$$

$$V_{LR} = (Q - iU) e^{+i2\psi}$$

$$V_{LL} = I - V$$

Instrumental Polarization, D

- Each polarized receptor sees some of the other polarization (“leakage”):

$$D_i = \begin{pmatrix} 1 & d_R(\nu) \\ d_L(\nu) & 1 \end{pmatrix}$$

- General matrix
- Note frequency-dependence
- Assumed time-stable (cf bandpass)
- Notation: “ d_R ” is “the fraction of L polarization spuriously sensed by R ”
- N.B.: On-diagonal effects factored into \mathbf{G} , \mathbf{B}
- Origins:
 - Finite impurities in polarizers
 - Reflections that return in opposite polarization: standing waves
 - Asymmetry in optics induces spurious polarization

D in the Circular Basis - I

$V = D P V^{\text{true}}$:

$$V_{RR} = (\mathcal{I} + \mathcal{V}) + (Q + iU)e^{-i2\psi} d_{Rj}^* + d_{Ri}(Q - iU)e^{+i2\psi} + d_{Ri}(\mathcal{I} - \mathcal{V})d_{Rj}^*$$

$$V_{RL} = (\mathcal{I} + \mathcal{V})d_{Lj}^* + (Q + iU)e^{-i2\psi} + d_{Ri}(Q - iU)e^{+i2\psi}d_{Lj}^* + d_{Ri}(\mathcal{I} - \mathcal{V})$$

$$V_{LR} = d_{Li}(\mathcal{I} + \mathcal{V}) + d_{Li}(Q + iU)e^{-i2\psi}d_{Rj}^* + (Q - iU)e^{+i2\psi} + (\mathcal{I} - \mathcal{V})d_{Rj}^*$$

$$V_{LL} = d_{Li}(\mathcal{I} + \mathcal{V})d_{Lj}^* + d_{Li}(Q + iU)e^{-i2\psi} + (Q - iU)e^{+i2\psi}d_{Lj}^* + (\mathcal{I} - \mathcal{V})$$

– Symmetries:

- d multiplies pure cross-/parallel-hands in parallel-/cross-hands
- d^2 multiplies other pure parallel-/cross-hand

D in the Circular Basis - II

- Linearized, sorted, $dV \sim 0$:

$$V_{RR} = (\mathcal{I} + V) + (Q + iU)e^{-i2\psi} d_{Rj}^* + d_{Ri}(Q - iU)e^{+i2\psi}$$

$$V_{RL} = (Q + iU)e^{-i2\psi} + \mathcal{I}(d_{Lj}^* + d_{Ri})$$

$$V_{LR} = (Q - iU)e^{+i2\psi} + \mathcal{I}(d_{Li} + d_{Rj}^*)$$

$$V_{LL} = (\mathcal{I} - V) + d_{Li}(Q + iU)e^{-i2\psi} + (Q - iU)e^{+i2\psi} d_{Lj}^*$$

- Traditionally, VLA also has ignored Q,U terms in V_{RR} , V_{LL}
 - Degenerate D solution: $(d_{Lj} - a)^* + (d_{Ri} + a^*) = (d_{Lj}^* + d_{Ri})$, so one d_R arbitrarily set to zero (refant)
 - Time-dependent closure errors in V_{RR} , V_{LL} for polarized sources: limits \mathcal{I} dynamic range to 10-100K...

D : Properties

- Orthogonality condition: $(d_{Ri} + d_{Li}^*) = 0$
 - Easier to achieve than purity in both hardware engineering and data post-processing
 - Clear symmetries typically evident in real and imag parts of d spectra, e.g.,

$$\mathcal{Re}(d_{Ri}) \approx -\mathcal{Re}(d_{Li})$$

$$\mathcal{Im}(d_{Ri}) \approx \mathcal{Im}(d_{Li})$$

- Orthogonality estimator: $(d_{Ri} + d_{Li}^*)/2$
- Mutual purity estimator: $(d_{Ri} - d_{Li}^*)/2$
 - (when already \sim orthogonal)
 - Beware *absolute* purity reference...

Gain-like terms: \mathbf{G} , \mathbf{B}

- “Electronic” gain: $\mathbf{G}_i = \begin{pmatrix} g_R & 0 \\ 0 & g_L \end{pmatrix}$
 - Diagonal
 - The ubiquitous term: typically describes many instrumental effects, and also atmosphere
 - Bandpass: $\mathbf{B}_i = \begin{pmatrix} b_R(\nu) & 0 \\ 0 & b_L(\nu) \end{pmatrix}$
 - Diagonal
 - Frequency-dependent version of \mathbf{G} , usually assumed time-stable
- $$V_{RR} = b_{Ri} b_{Rj}^* g_{Ri} g_{Rj}^* \{ (\mathcal{I} + \mathcal{V}) + (Q + i\mathcal{U}) e^{-i2\psi} d_{Rj}^* + d_{Ri} (Q - i\mathcal{U}) e^{+i2\psi} \}$$
- $$V_{RL} = b_{Ri} b_{Lj}^* g_{Ri} g_{Lj}^* \{ (Q + i\mathcal{U}) e^{-i2\psi} + \mathcal{I} (d_{Lj}^* + d_{Ri}) \}$$
- $$V_{LR} = b_{Li} b_{Rj}^* g_{Li} g_{Rj}^* \{ (Q - i\mathcal{U}) e^{+i2\psi} + \mathcal{I} (d_{Li} + d_{Rj}^*) \}$$
- $$V_{LL} = b_{Li} b_{Lj}^* g_{Li} g_{Lj}^* \{ (\mathcal{I} - \mathcal{V}) + d_{Li} (Q + i\mathcal{U}) e^{-i2\psi} + (Q - i\mathcal{U}) e^{+i2\psi} d_{Lj}^* \}$$
- \mathbf{G} & \mathbf{B} both solved from parallel-hands only, independently in each polarization (phase refant)...

Cross-hand phase spectrum

- An artifact of gain calibration reference antenna (refant)
- We do not measure absolute \mathbf{G}_i and \mathbf{B}_i ,
- Instead, we measure \mathbf{G}'_i and \mathbf{B}'_i , wherein a reference antenna's phase is fixed to zero in *both* polarizations, yielding relative phases for all other antennas
 - Phase differences among antennas in each polarization (*separately*) are preserved: no effect on parallel-hand calibration
- The refant's cross-hand bandpass phase remains undetected:

$$\mathbf{B}_i \mathbf{G}_i = \mathbf{B}'_i \mathbf{G}'_i \mathbf{X}'_k$$

$$\mathbf{X}'_k = \begin{bmatrix} e^{i\rho} & 0 \\ 0 & 1 \end{bmatrix}$$

- NB: \mathbf{X}'_k does *not* vary with antenna ($k = \text{refant}$)
- ...and uncorrected on *all* antennas/baselines:
$$\mathbf{G}'_i^{-1} \mathbf{B}'_i^{-1} \mathbf{B}_i \mathbf{G}_i = \mathbf{X}'_k$$
- \mathbf{X}'_k is as interesting as any bandpass phase spectrum in the system
 - Equivalently, an effective position angle offset spectrum that requires calibration (circular feeds)

Cross-hand Delay

- X_k^r may include a significant cross-hand delay, which we can (optionally) factor out:

$$X_k^r \rightarrow K_{crs} X_k^r$$

- X_k^r is now just the non-linear part of the cross-hand phase bandpass
- K_{crs} is also specific to the refant...
- Enables ~coherent channel-averaged views of cross-hands prior to considering instrumental polarization...
 - c.f. factoring parallel-hand delays from bandpass

Revised factorization

- We therefore re-write the calibration operator equation:

$$\begin{aligned}\mathbf{V}^{obs} &= \mathbf{B} \mathbf{G} \mathbf{D} \mathbf{P} \mathbf{V}^{mod} \\ &= \mathbf{B}^r \mathbf{G}^r (\mathbf{K}_{crs} \mathbf{X}^r) \mathbf{D} \mathbf{P} \mathbf{V}^{mod}\end{aligned}$$

- In the circular basis, \mathbf{X}^r is just a polarization position angle offset, which can be deferred for later external calibration
 - It is convenient to move \mathbf{X}_r upstream of \mathbf{D} :
$$= \mathbf{B}^r \mathbf{G}^r \mathbf{K}_{crs} \mathbf{D}^r \mathbf{X}^r \mathbf{P} \mathbf{V}^{mod} \\ (\mathbf{D}^r = \mathbf{X}^r \mathbf{D} \mathbf{X}^{r-1})$$
 - \mathbf{D}^r is the instrumental polarization measured in the *cross-hand* phase frame of the gain & bandpass calibration reference antenna
 - Important: \mathbf{X}^r must be time-stable, else \mathbf{D}^r will not be

Polarization Calibration Bootstrapping

- Calibration Model:

$$\mathbf{V}^{obs} = \mathbf{B}^r \mathbf{G}^r \mathbf{K}_{crs} \mathbf{D}^r \mathbf{X}^r \mathbf{P} \mathbf{V}^{mod}$$

- Basic Solve sequence:

- Normal \mathbf{B}^r and \mathbf{G}^r (parallel-hands; details suppressed):

$$\mathbf{V}^{obs} = \underline{\mathbf{B}^r} \mathbf{V}^I$$

$$(\mathbf{B}^{r-1} \mathbf{V}^{obs}) = \underline{\mathbf{G}^r} \mathbf{V}^I$$

- Cross-hand delay, \mathbf{K}_{crs} ($\langle \rangle$ is a baseline average of cross-hands):

$$\langle (\mathbf{G}^{r-1} \mathbf{B}^{r-1} \mathbf{V}^{obs}) \rangle = \underline{\mathbf{K}_{crs}} \langle \mathbf{V}^{QU} \rangle$$

- Apparent QU , \mathbf{D}^r (cross-hands):

$$(\mathbf{K}_{crs}^{-1} \mathbf{G}^{r-1} \mathbf{B}^{r-1} \mathbf{V}^{obs}) = \underline{\mathbf{D}^r} (\mathbf{P} \mathbf{V}^{QU})$$

- \mathbf{X}^r ($\langle \rangle$ is a baseline average of cross-hands):

$$\langle (\mathbf{D}^{r-1} \mathbf{K}_{crs}^{-1} \mathbf{G}^{r-1} \mathbf{B}^{r-1} \mathbf{V}^{obs}) \rangle = \underline{\mathbf{X}^r} \langle (\mathbf{P} \mathbf{V}^{QU}) \rangle$$

- Correction:

$$\mathbf{V}^{corr} = (\mathbf{P}^{-1} \mathbf{X}^{r-1} \mathbf{D}^{r-1} \mathbf{K}_{crs}^{-1} \mathbf{G}^{r-1} \mathbf{B}^{r-1} \mathbf{V}^{obs})$$

Practical Polarization Processing in CASA

- Basic Steps:
 - Cross-hand delay
 - Instrumental Polarization (and calibrator Q, U)
 - Position angle (cross-hand phase)
- Additional considerations
 - Ionosphere, F
 - Linear feed basis (ALMA, EVLA < 1 GHz)
 - Beam polarization
 - Etc...

Cross-hand delay: gaincal

- `gaintype='KCROSS'`
- Solver estimates delay from baseline-averaged cross-hands
- Required for later per-spw source polarization estimation (chan-dep ~coherence)
- Optional if instrumental polarization calibrator is UN-polarized
- Use strongly polarized, ~point-like calibrator (position angle calibrator):
 - `smodel=[1,1,0,0]`
 - (true model not required, but cross-hand model must be non-zero)
- Pre-apply all standard calibration

Instrumental Polarization solve: polcal

- UN-polarized calibrator
 - `poltype='Df'`
 - Solver estimates channelized d's
 - Single scan on ~point-like calibrator is sufficient
- Polarized calibrator
 - `poltype='Df+QU'`
 - Solver estimates per-spw (not per-channel) source Q,U, then estimates channelized d's
 - Need 3+ scans at range of parallactic angle (to separate source from instr pol) on ~point-like calibrator
- Also...
 - Pre-apply all standard calibrations, plus cross-hand delay, model Stokes I should be consistent with pre-applied net gain calibration
 - `refant` must be specified for the VLA (same as gain calibrations)
 - Resulting d spectra is in cross-hand phase frame of the net gain reference antenna.
 - NB: if cross-hand delay pre-apply is omitted, it will be evident in the effective cross-hand phase frame (to be estimated in the next step)

Solving for D^r

- Unpolarized source

$$V_{RL} = \mathcal{I}(d_{Lj}^* + d_{Ri}) = \mathcal{I}[(d_{Li} - a)^* + (d_{Ri} + a^*)]$$

- Simple, but d 's are degenerate, to first order:
 - for any complex number a
- Therefore, one d remains effectively unconstrained
- Standard convention is to apply a reference antenna that effectively enforces $a = -d_{Rref}^*$:

$$d_{Ri} \rightarrow (d_{Ri} + a^*) \quad d_{Li} \rightarrow (d_{Li} - a) \quad (\text{for all } i)$$

- These referenced d 's correct the data to some *orthogonal* (to first order) basis that is defined by the refant's true d_R -- but not a *pure* one

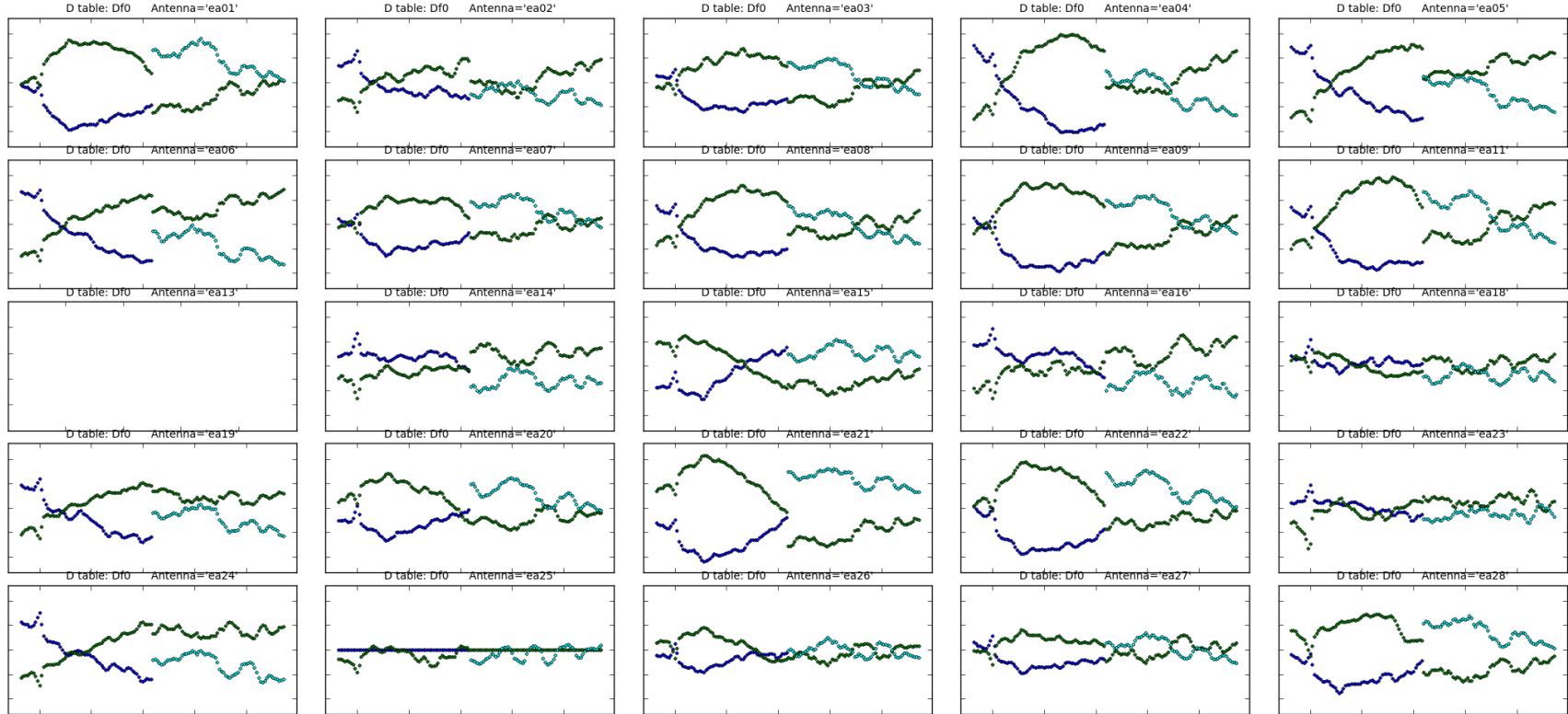
Solving for D^r

- Polarized calibrator:

$$V_{RL} = (Q + iU)e^{-i2\psi} + I(d_{Lj}^* + d_{Ri})$$

- single scan ($\psi = \text{const}$)?
 - Unknown source polarization inseparable
 - If source polarization known a priori, unknown cross-hand phase offset distorts phase of source term...
- 2+ scans formally required (point source)
 - At distinct parallactic angles vis-à-vis available SNR
 - Source polarization (up to position angle offset) separable
- NB: d 's themselves are *still* degenerate (relative)
 - Need linear terms in parallel hands... (c.f. ALMA)

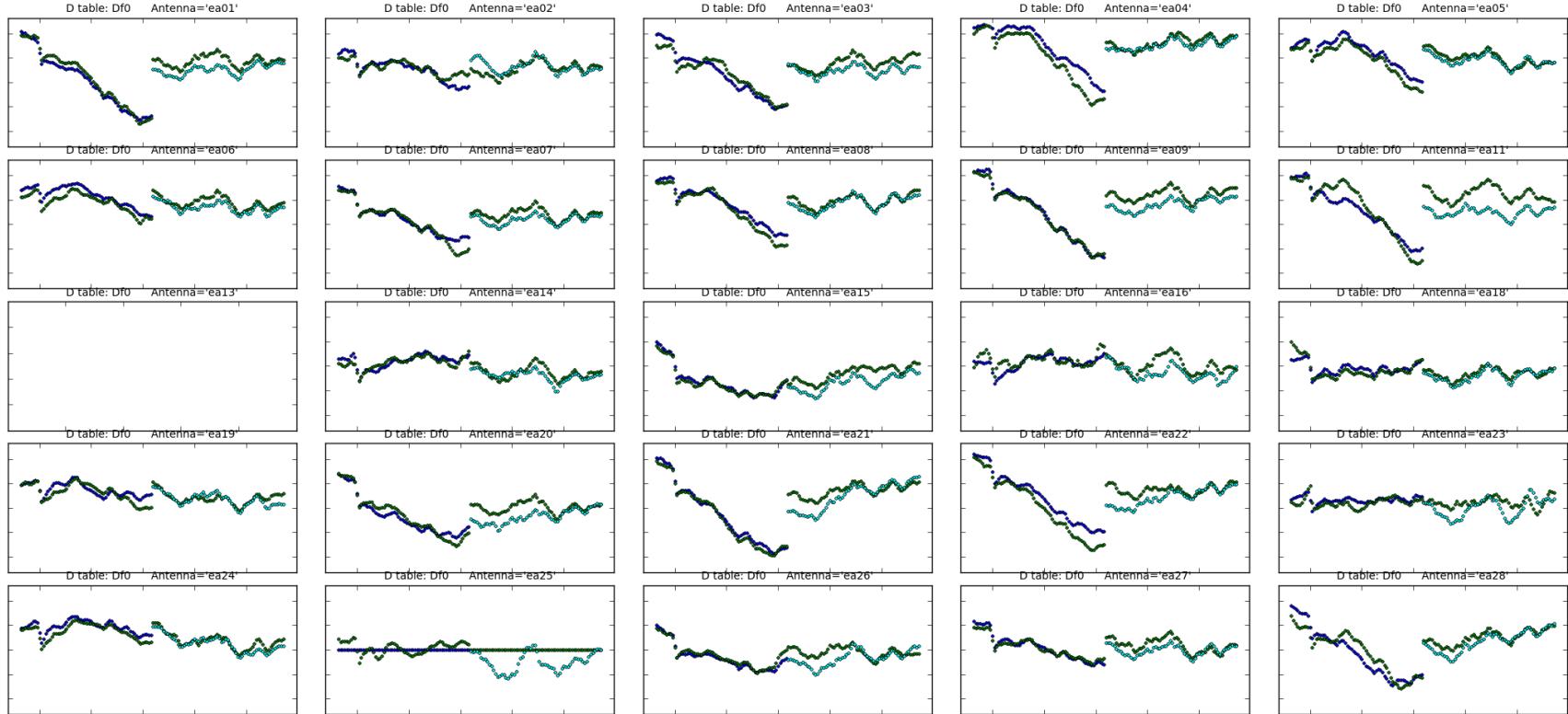
D' spectra (real part)



Vertical ticks: 5% (centered at 0.0)

Horizontal ticks: 50 MHz (centered at 4958 MHz; 2 subbands = 256 MHz)

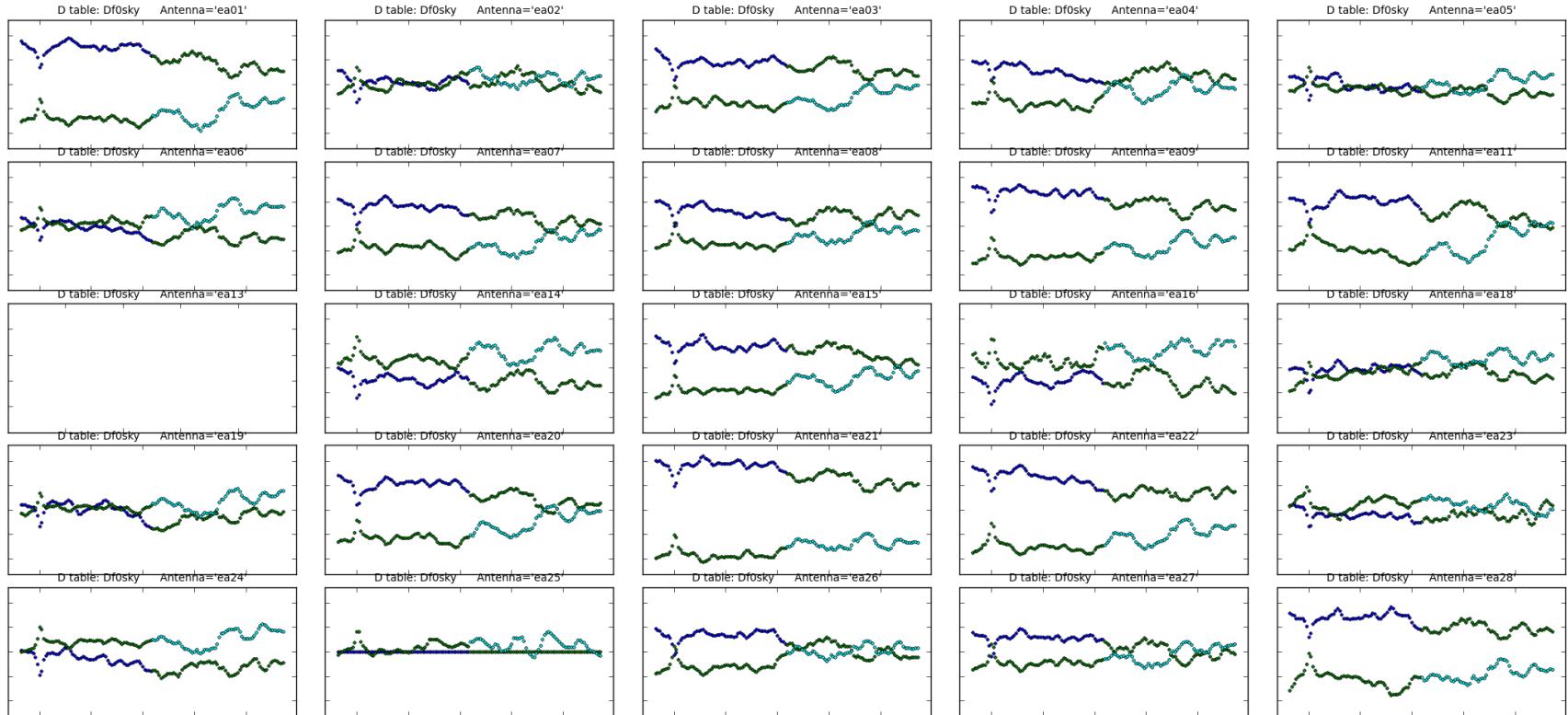
D^r spectra (imag part)



Vertical ticks: 5% (centered at 0.0)

Horizontal ticks: 50 MHz (centered at 4958 MHz; 2 subbands = 256 MHz)

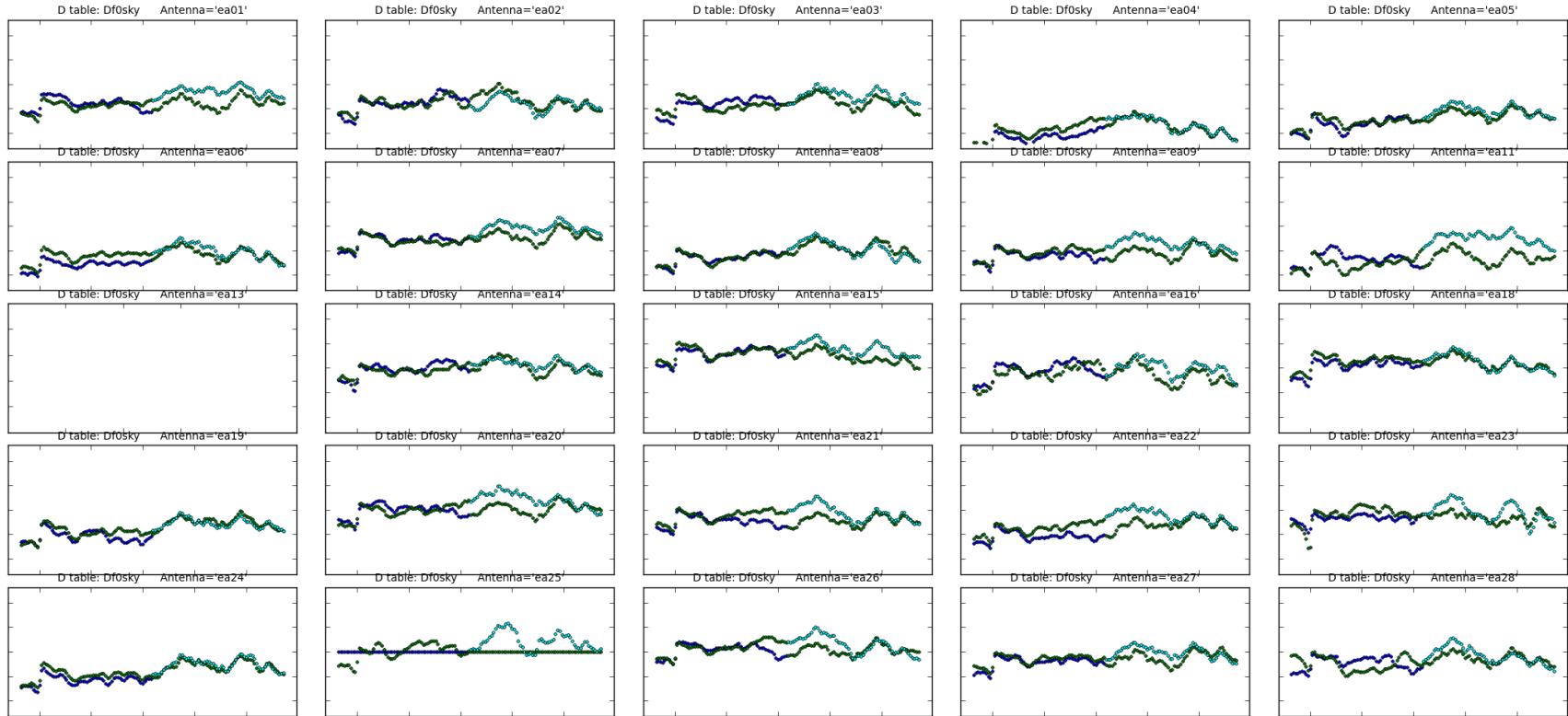
D spectra (real part)



Vertical ticks: 5% (centered at 0.0)

Horizontal ticks: 50 MHz (centered at 4958 MHz; 2 subbands = 256 MHz)

D spectra (imag part)



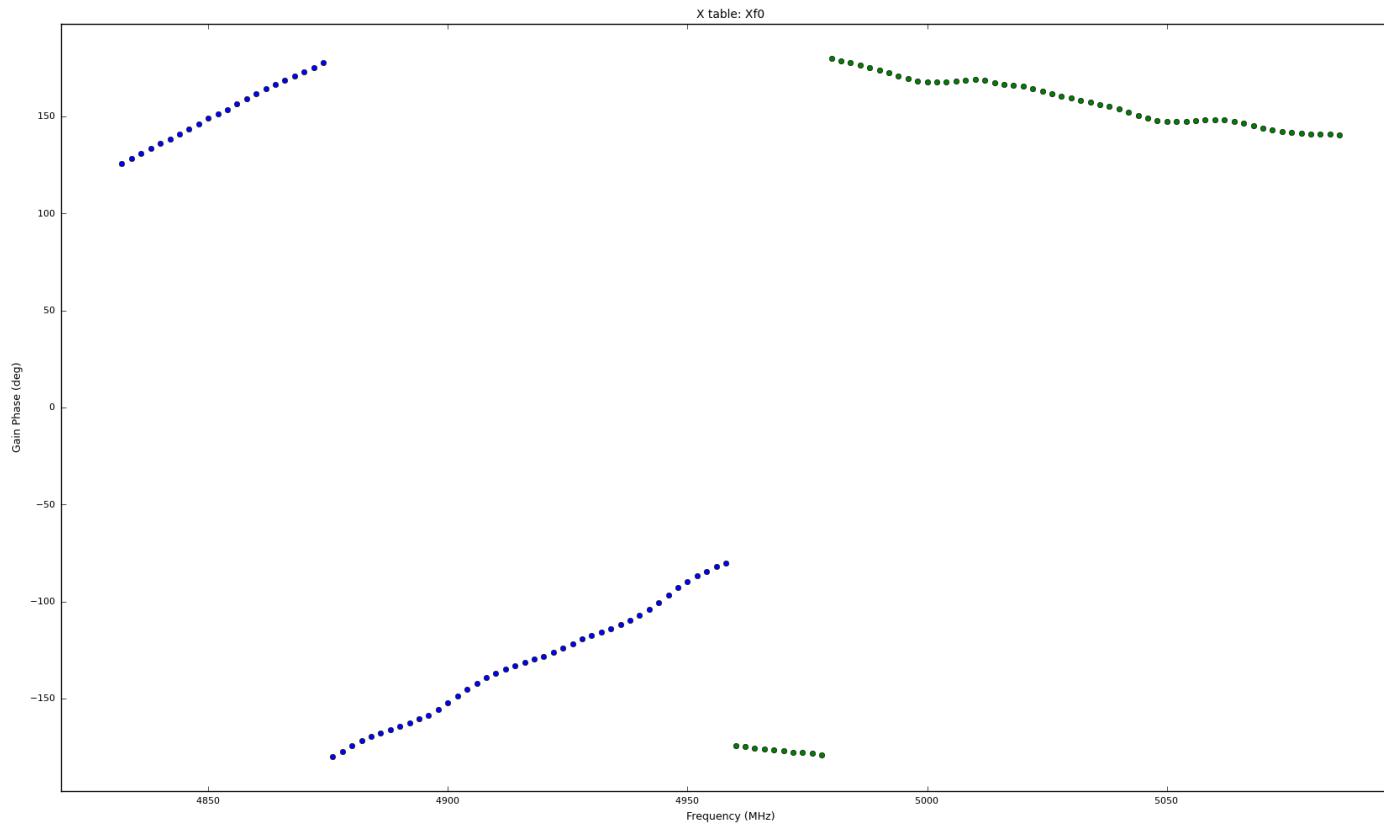
Vertical ticks: 5% (centered at 0.0)

Horizontal ticks: 50 MHz (centered at 4958 MHz; 2 subbands = 256 MHz)

Position angle: polcal

- `poltype='xf'`
- Solver estimates cross-hand phase spectrum of the net (gain) refant (which appears uniformly on all unresolved baselines)
- Use strongly polarized, ~point-like calibrator with known position angle, specified via correct Stokes parameters in `smodel` or prior run of `setjy`
- Pre-apply all standard calibration, plus leakage and cross-hand delay (if used previously)

X^r phase spectrum (including cross-hand delay)



Final apply: `applycal`

- Don't forget `parang=T`!
- NB: By default, the current leakage correction is just the linearized one (cross-hands only)
 - ALMA-related scripts available to trigger general matrix correction, but not yet advised for EVLA since *relative* d's effect on 2nd order terms not yet well-understood
- In future, if 'canned' leakage solutions are available, care must be used to distinguish D and D^r in the heuristics described above
 - ALMA-related scripts are available to convert $D^r \rightarrow D$ if X^r is known; useful for *physical* analysis of instrumental polarization, e.g., continuity at baseband boundaries, etc.

Additional Considerations

- Ionospheric corrections
 - Dispersive delay and Faraday rotation (time-dependent)
 - Modeled from ionospheric TEC measurements and B-field
 - Calibration model: $\mathbf{V}^{obs} = \mathbf{B}^r \mathbf{G}^r \mathbf{K}_{crs} \mathbf{D}^r \mathbf{X}^r \mathbf{P} \mathbf{F} \mathbf{V}^{mod}$
 - Available, but still under test
- Linear feeds (VLA < 1 GHz)
 - See ALMA casaguide (3C286_Polarization)
- Beam polarization (wide-field)
- Circular polarization: where's $\mathcal{V}=0$?
 - Purity difficult to constrain at interesting level ($\sim 0.1\%$)
 - Squint
 - Specific line profile (e.g., Zeeman) \sim easier
- Pipeline heuristics under development
- Iteration (generalized selfcal)