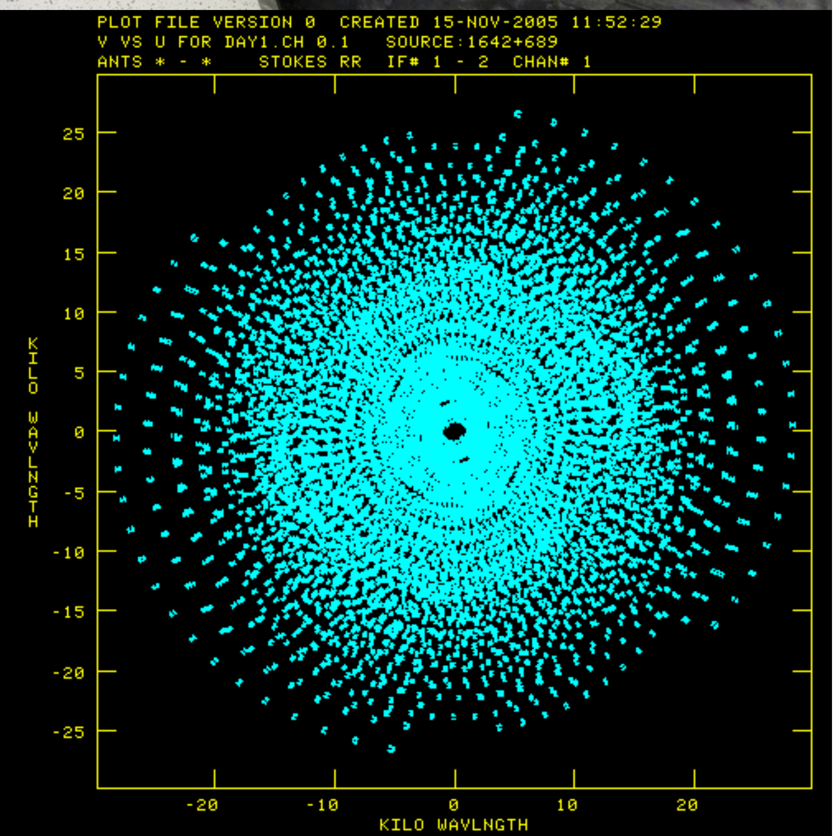

Cross correlators

for radio astronomy

Adam Deller

May 16, 2018

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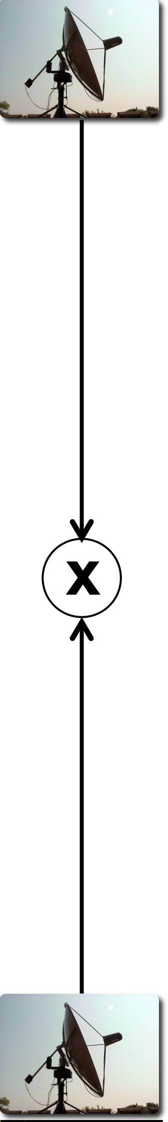
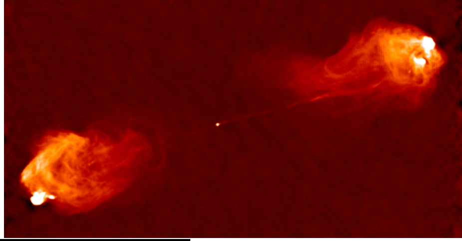
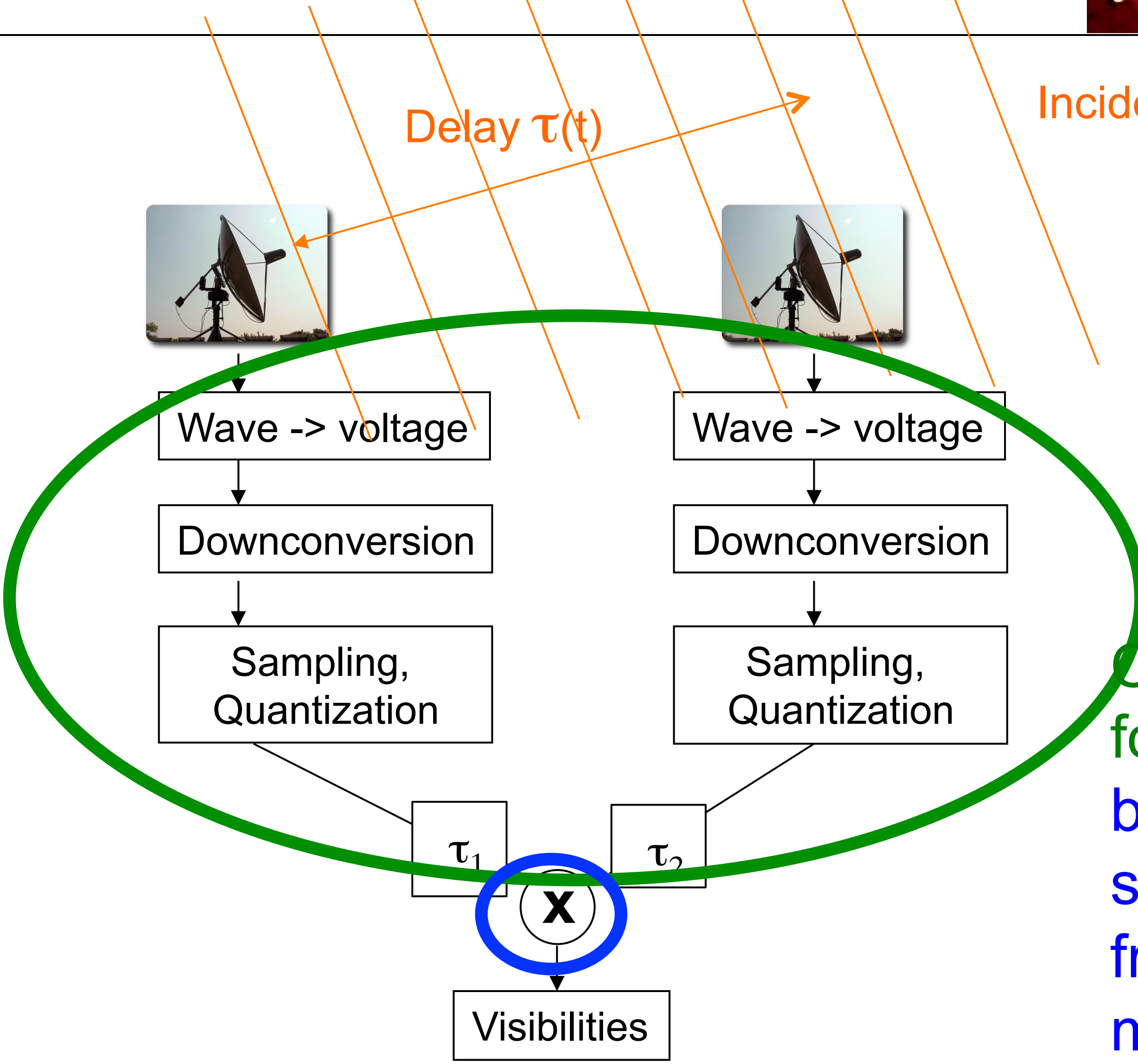




What is a correlator?



Compensate for this… before/while splitting by frequency and multiplying

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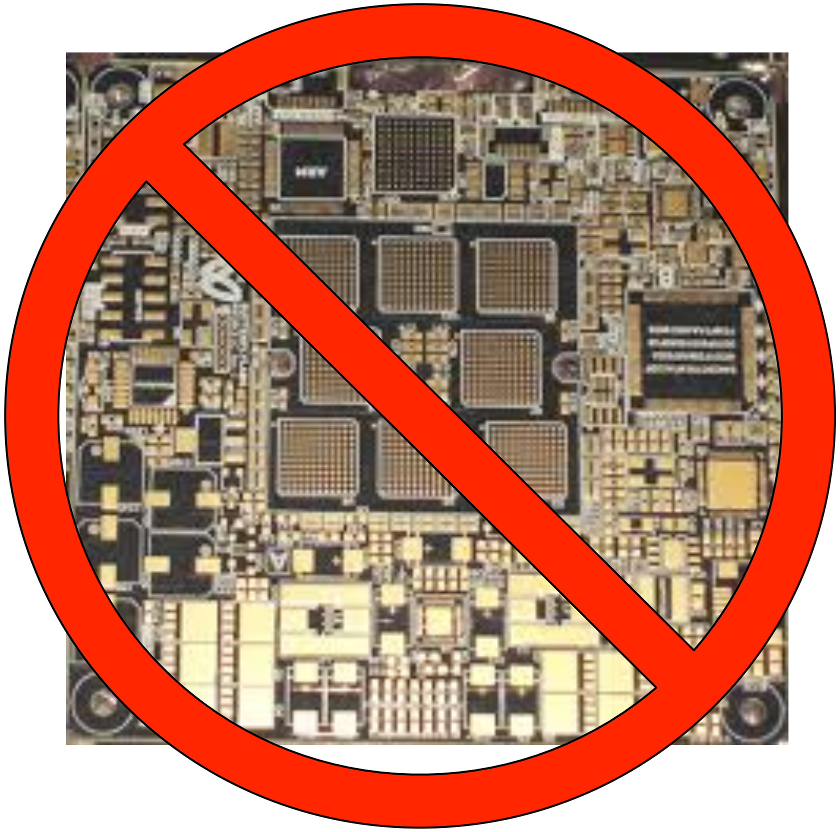
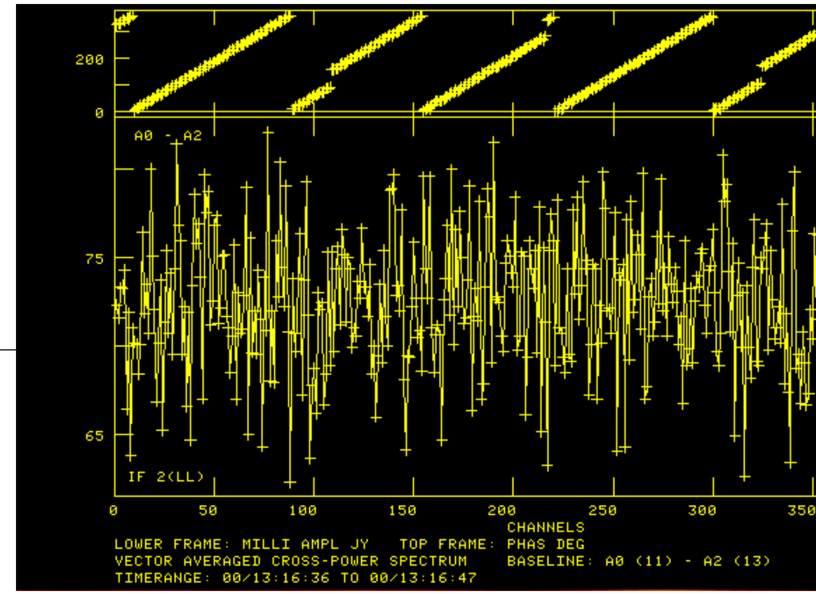
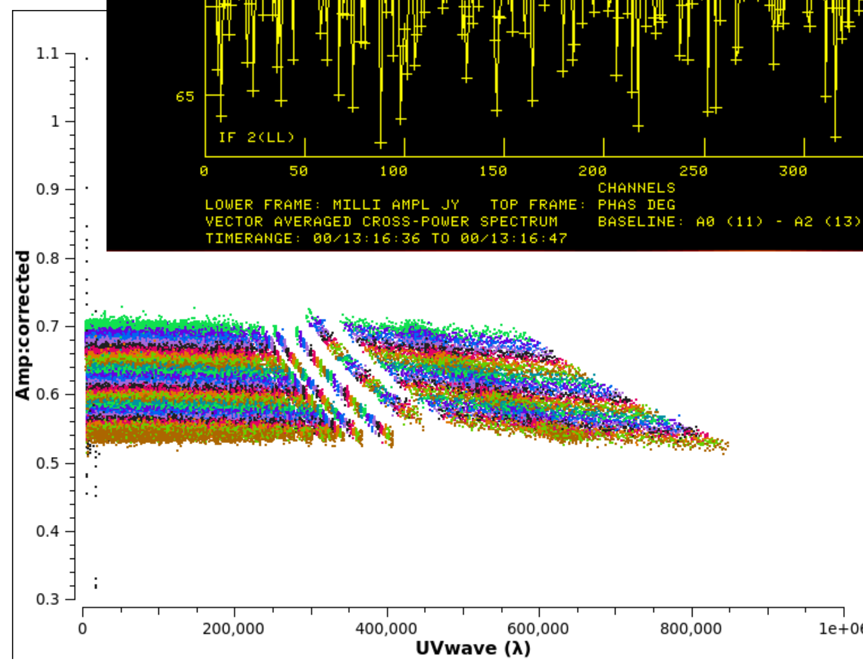
Why correlators matter to YOU







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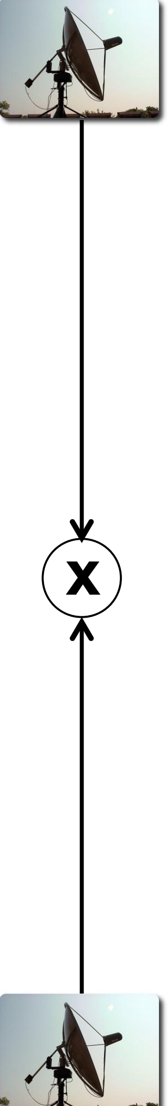
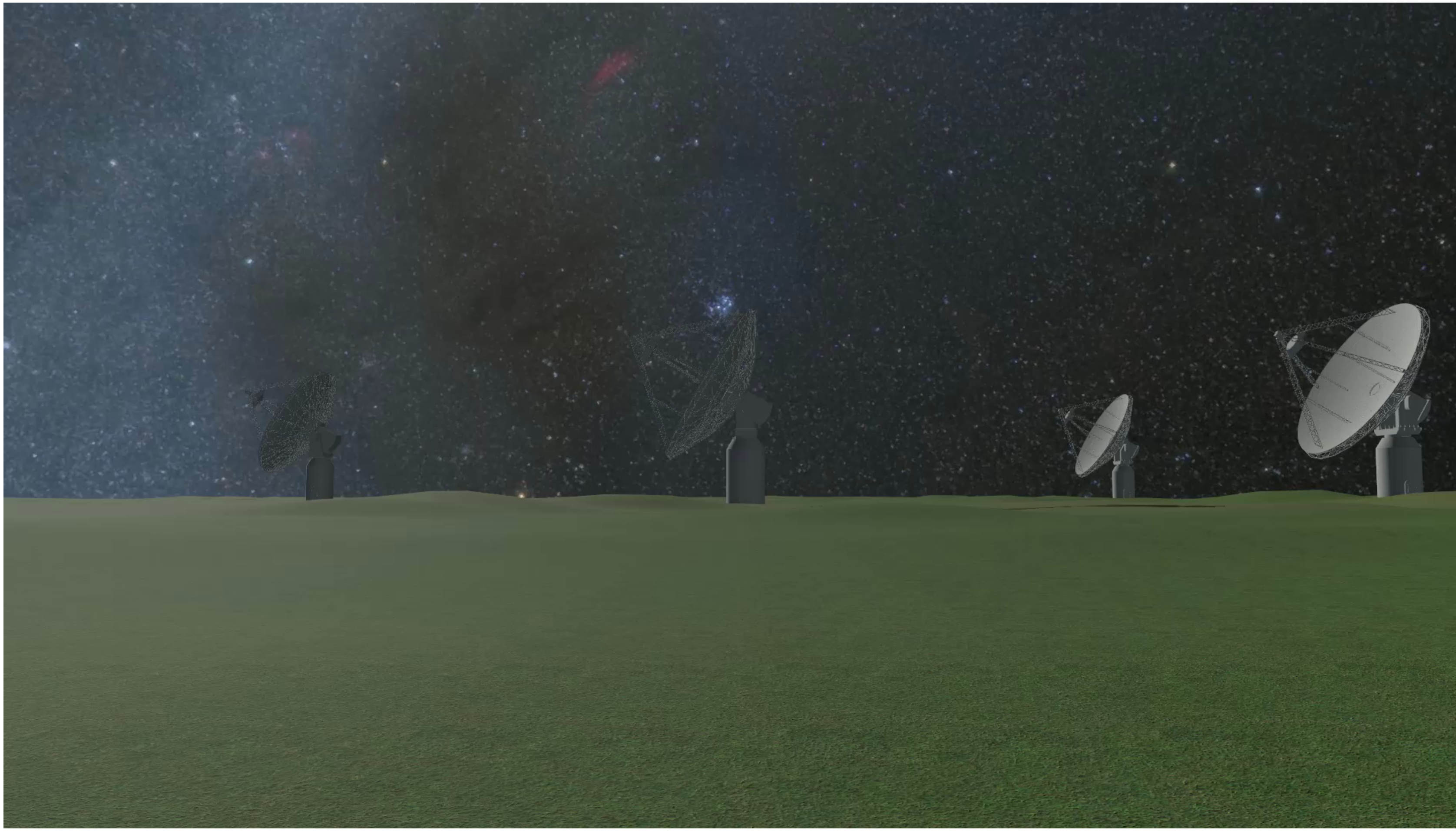
Correlators and interferometry





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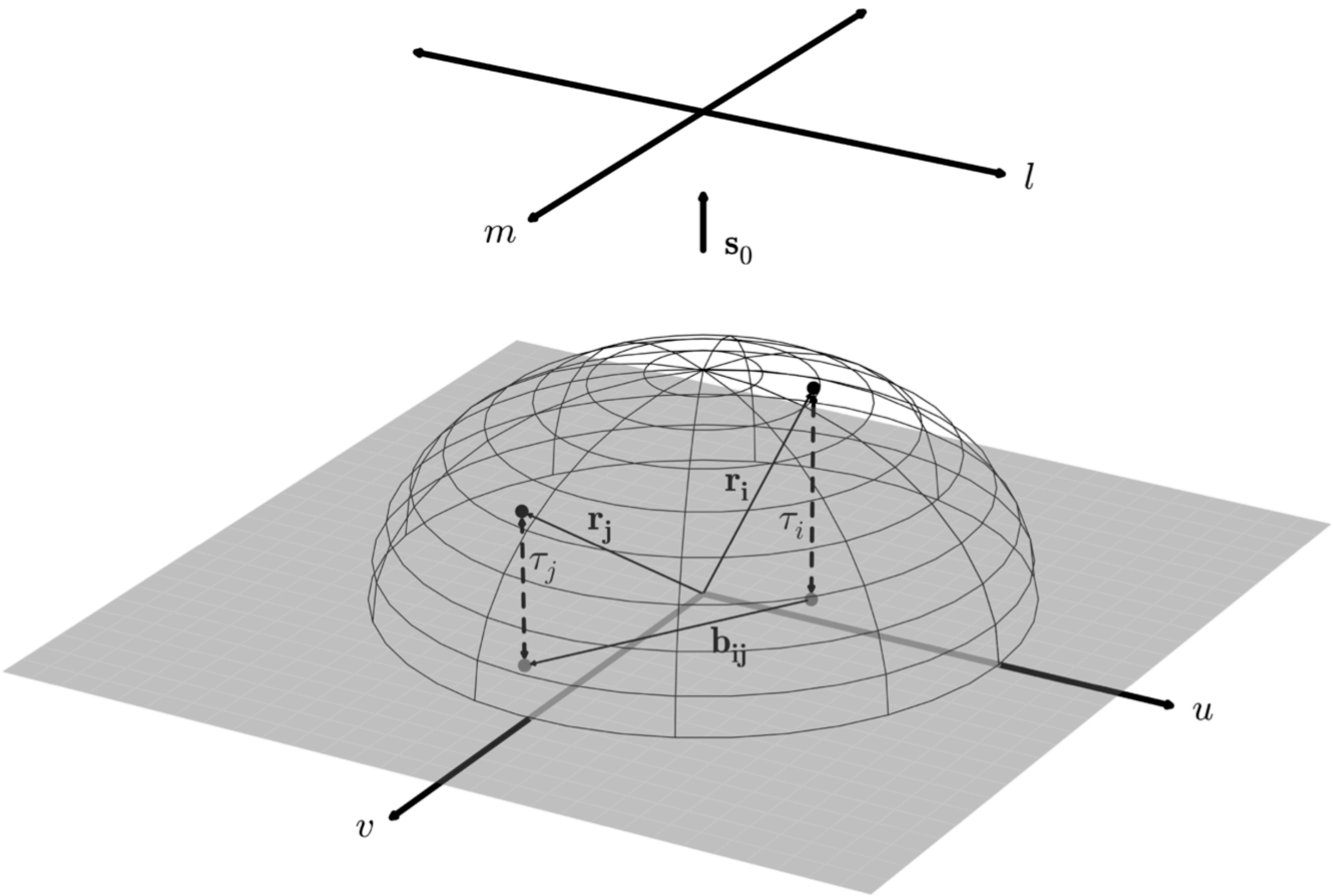
Correlators and Interferometry



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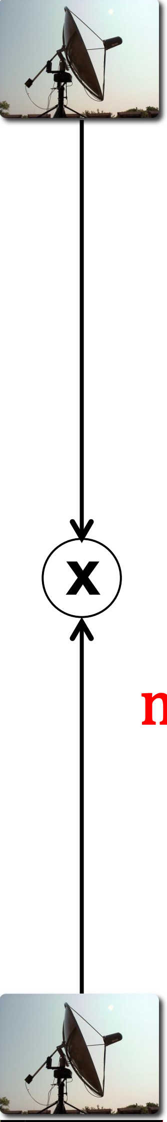
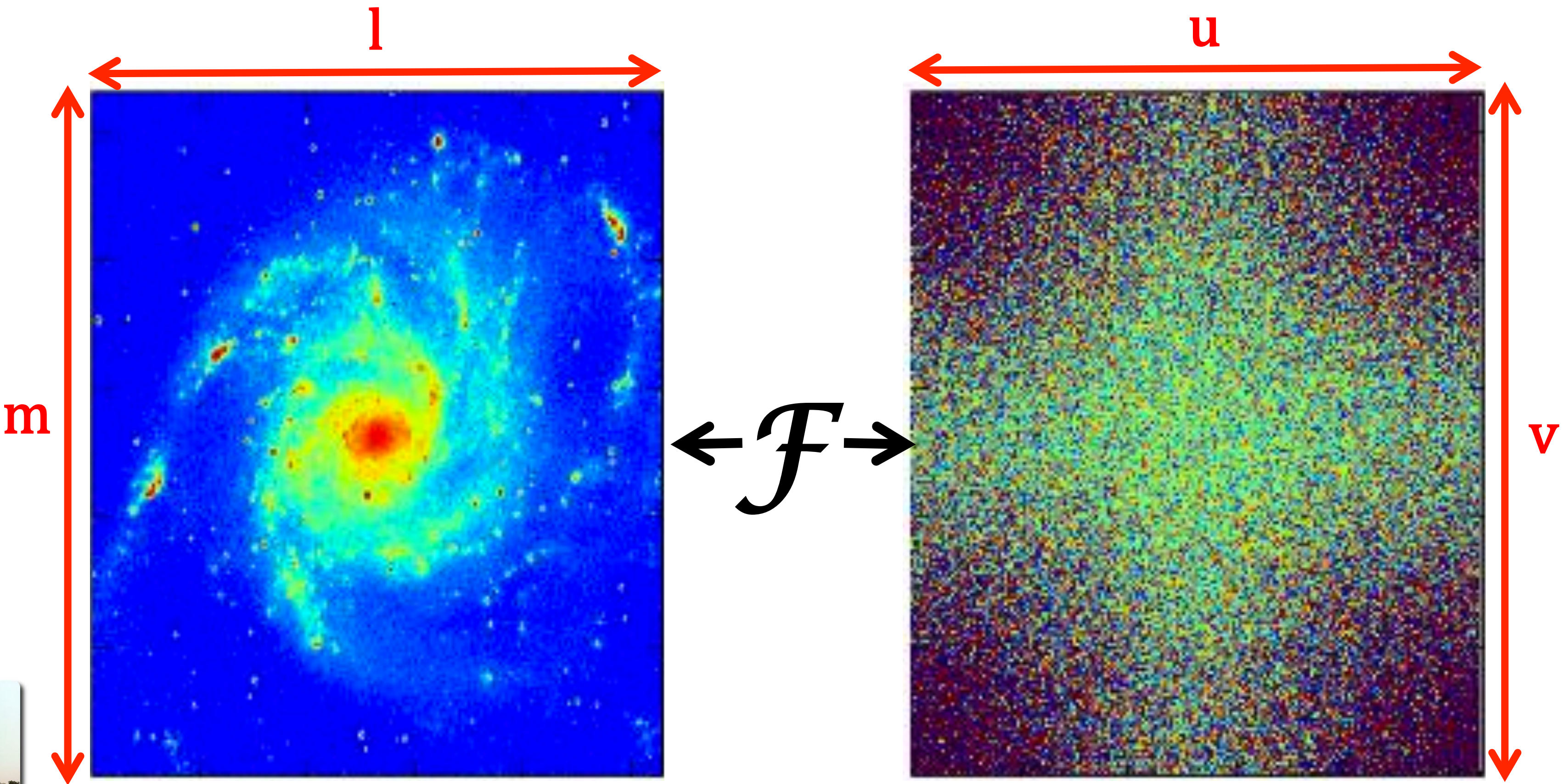
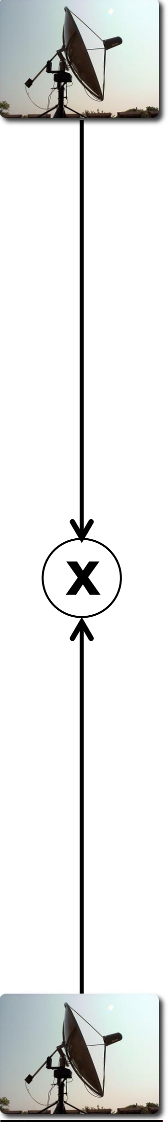


Correlators and Interferometry



Sky brightness at frequency ν_0

Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)



Monochromatic == problematic

- Mathematically: $u \times l + v \times m$ is supposed be constant, but both u and v are $\propto \nu$
- No truly monochromatic radiation!
- Fortunately, "fairly narrow" band of $\Delta\nu$ (*quasi-monochromatic*) can suffice:
  - Real world viewpoint: different frequency components stay "in phase" as wavefront propagates from one antenna to the next

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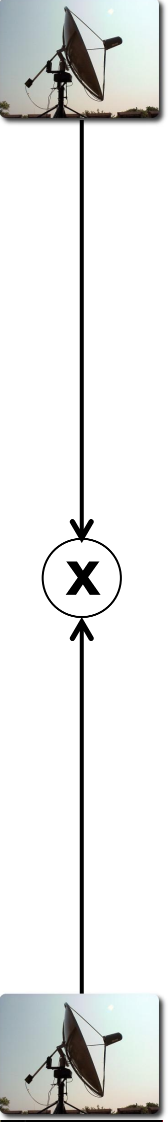
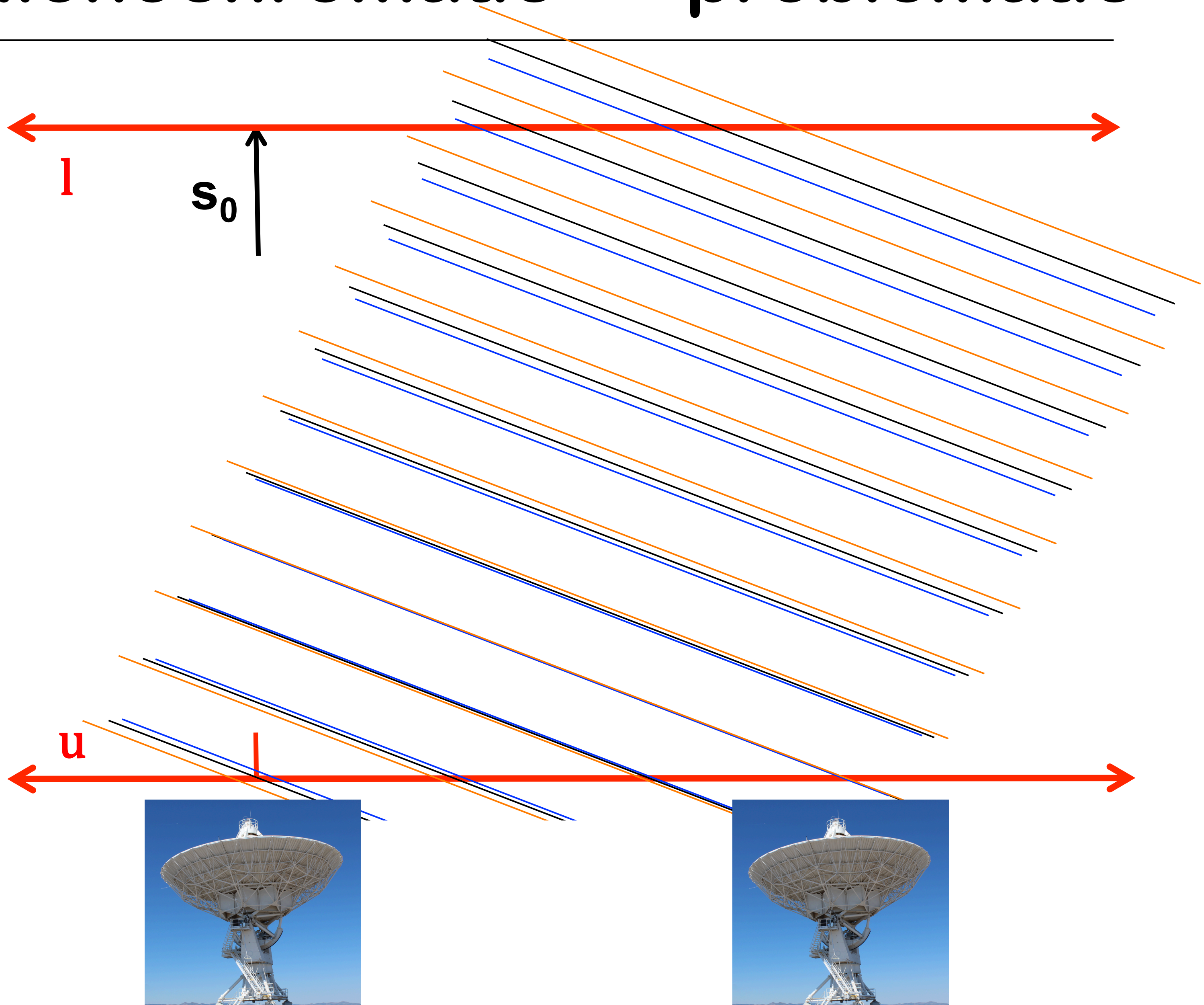
Monochromatic == problematic





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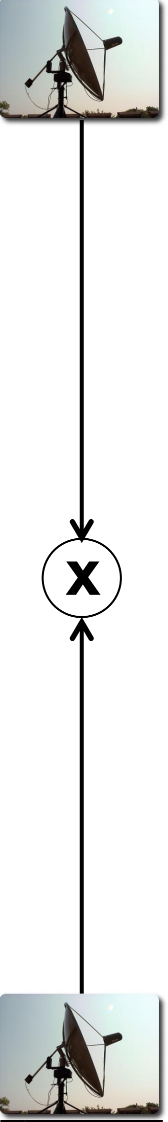

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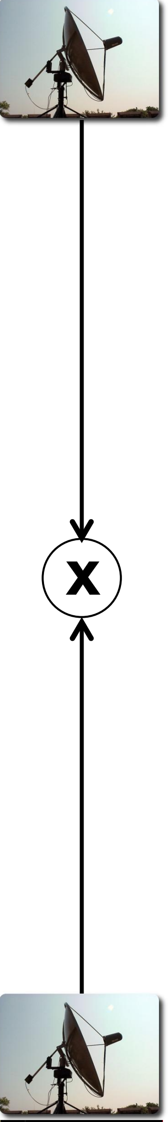
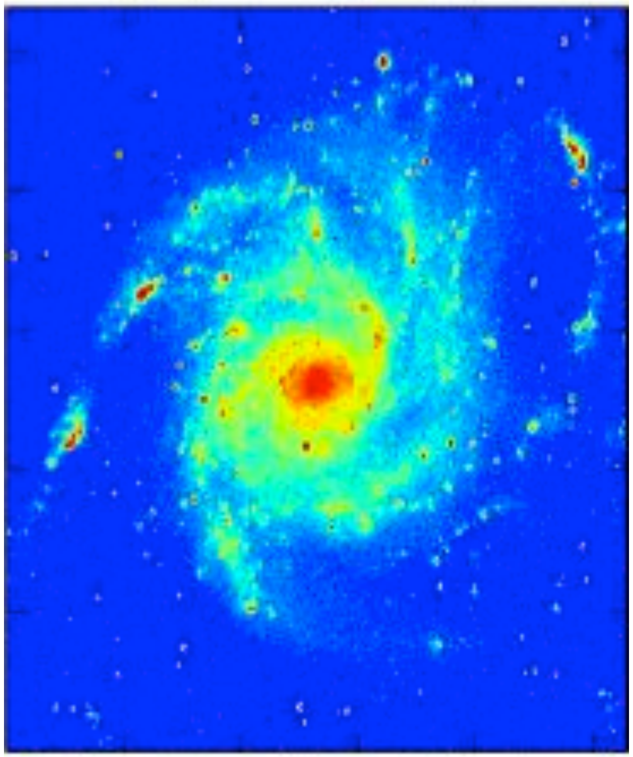
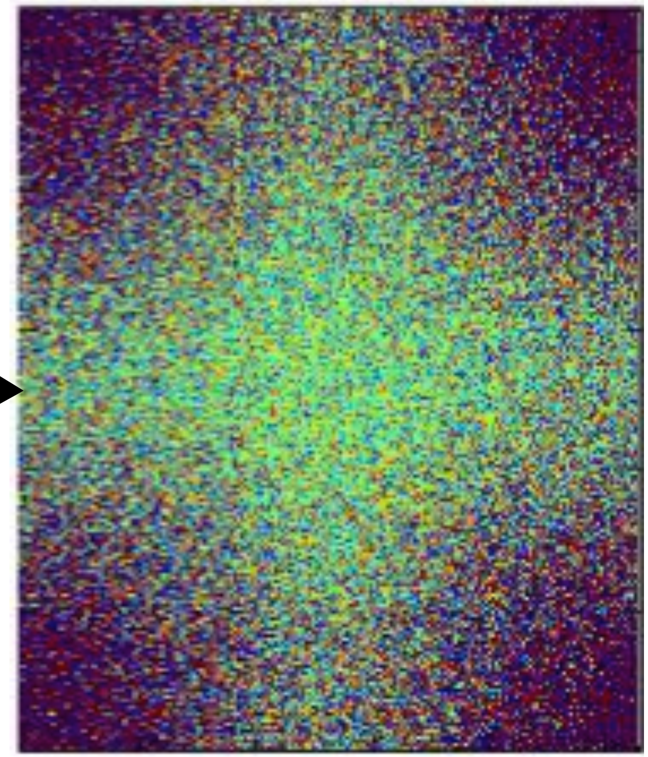
- No truly monochromatic radiation!
- Fortunately, "fairly narrow" band of $\Delta\nu$ (*quasi-monochromatic*) can suffice:
  - if $\Delta u \times l \ll 1$ and $\Delta v \times m \ll 1$ then the different frequency components stay in phase and we're ok
  - Correlator needs to slice at least this finely

Correlators and Interferometry





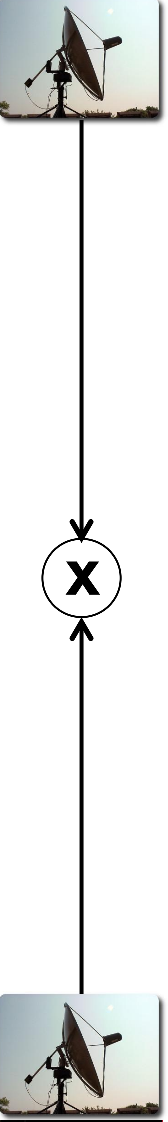
$\leftarrow \mathcal{F} \rightarrow$

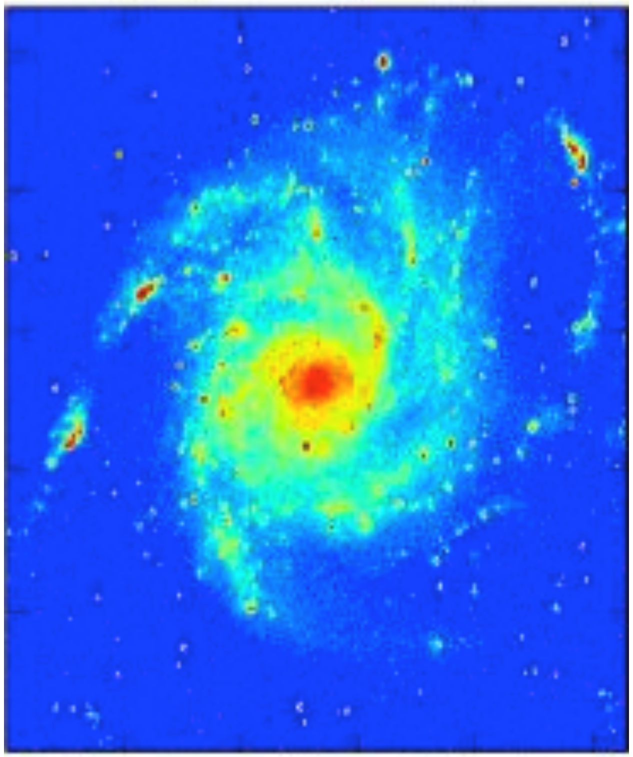


Sky brightness at frequency ν_0

Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)

Correlators and Interferometry





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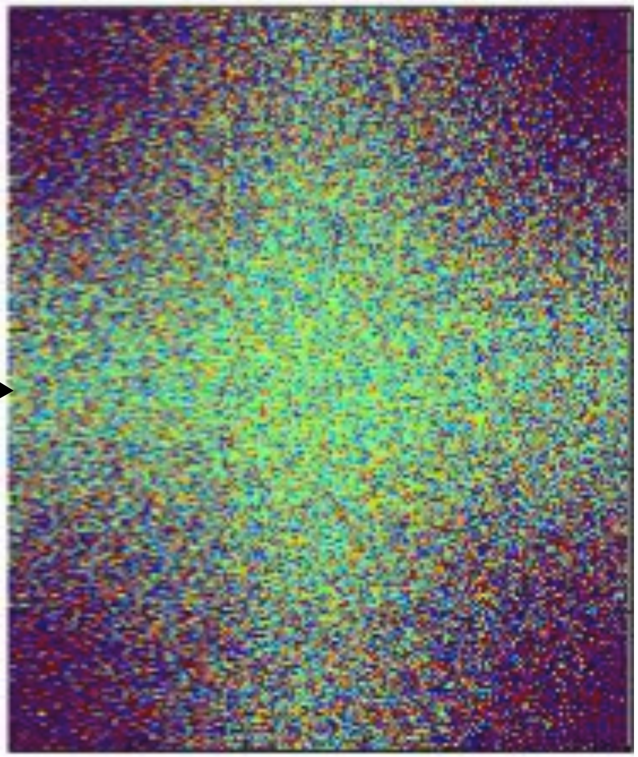

Sky brightness at frequency $\nu' = \nu_0 + \delta\nu$

Visibilities (real component shown, unit is $\lambda' = c / \nu'$)

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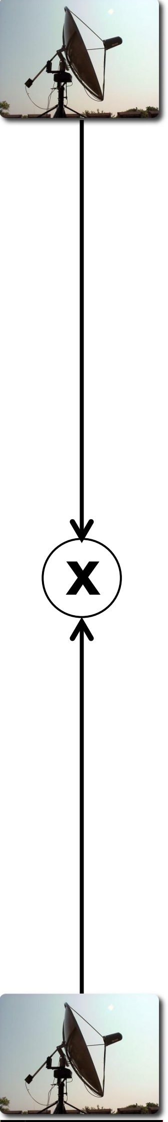
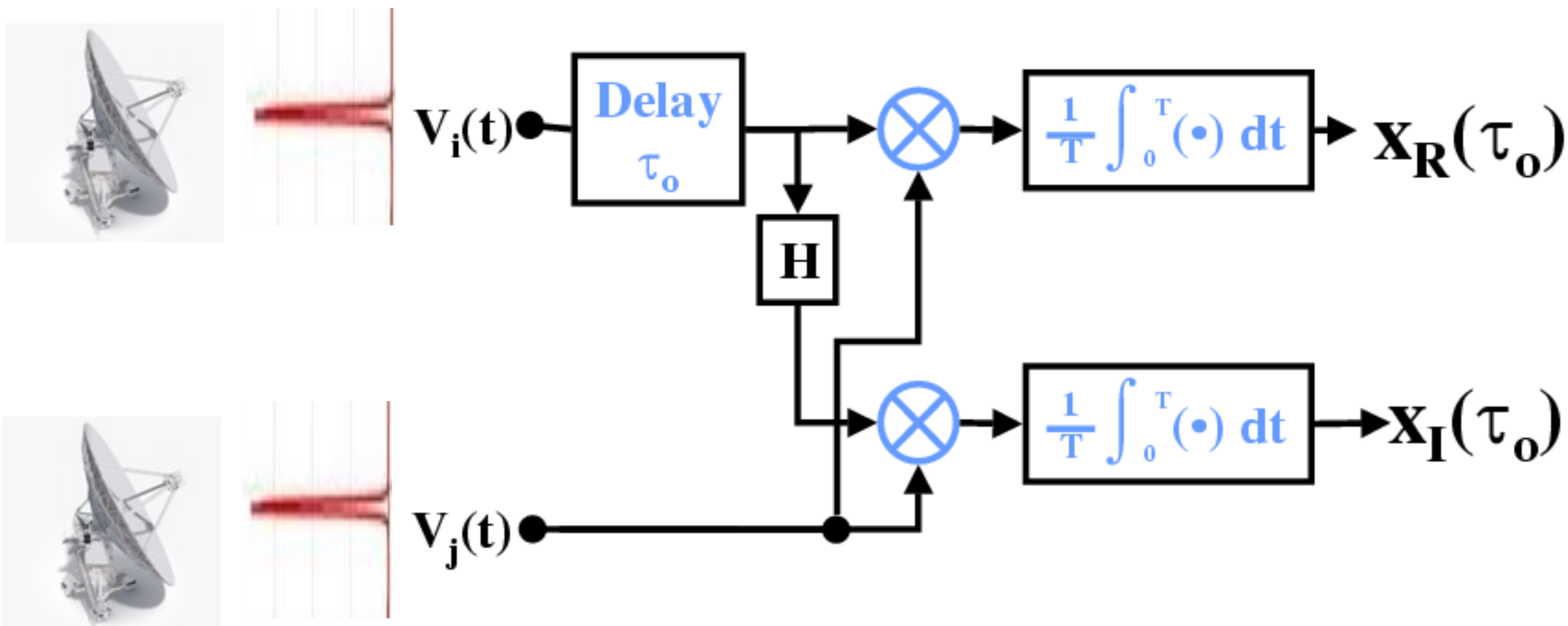


A "dumb" correlator

- Use many analog filters to make many narrow channels; correlate each one separately with a standard complex correlator:



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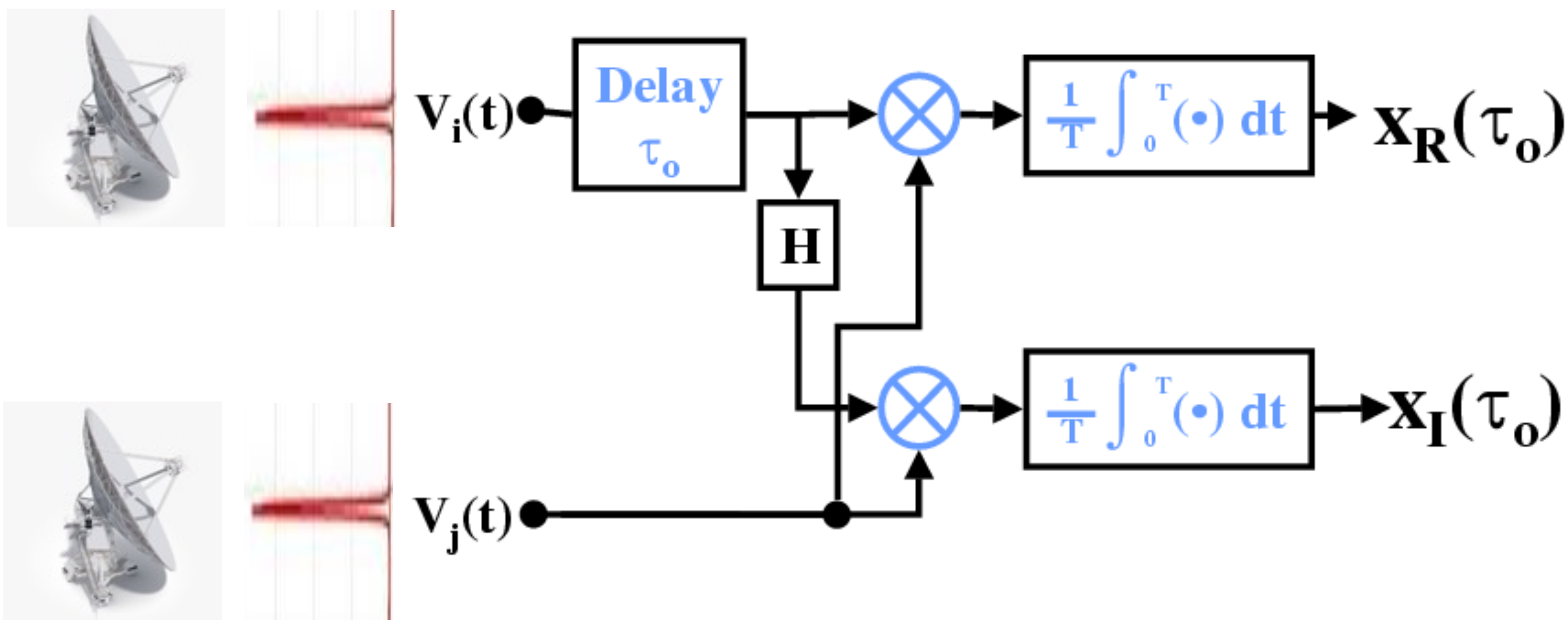
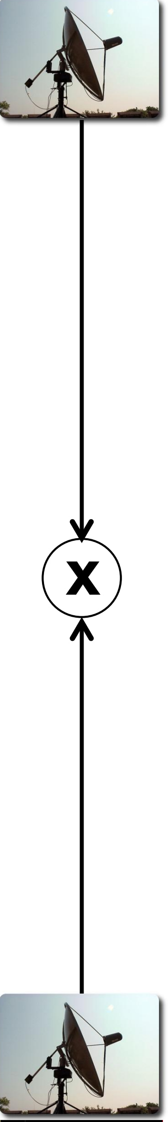
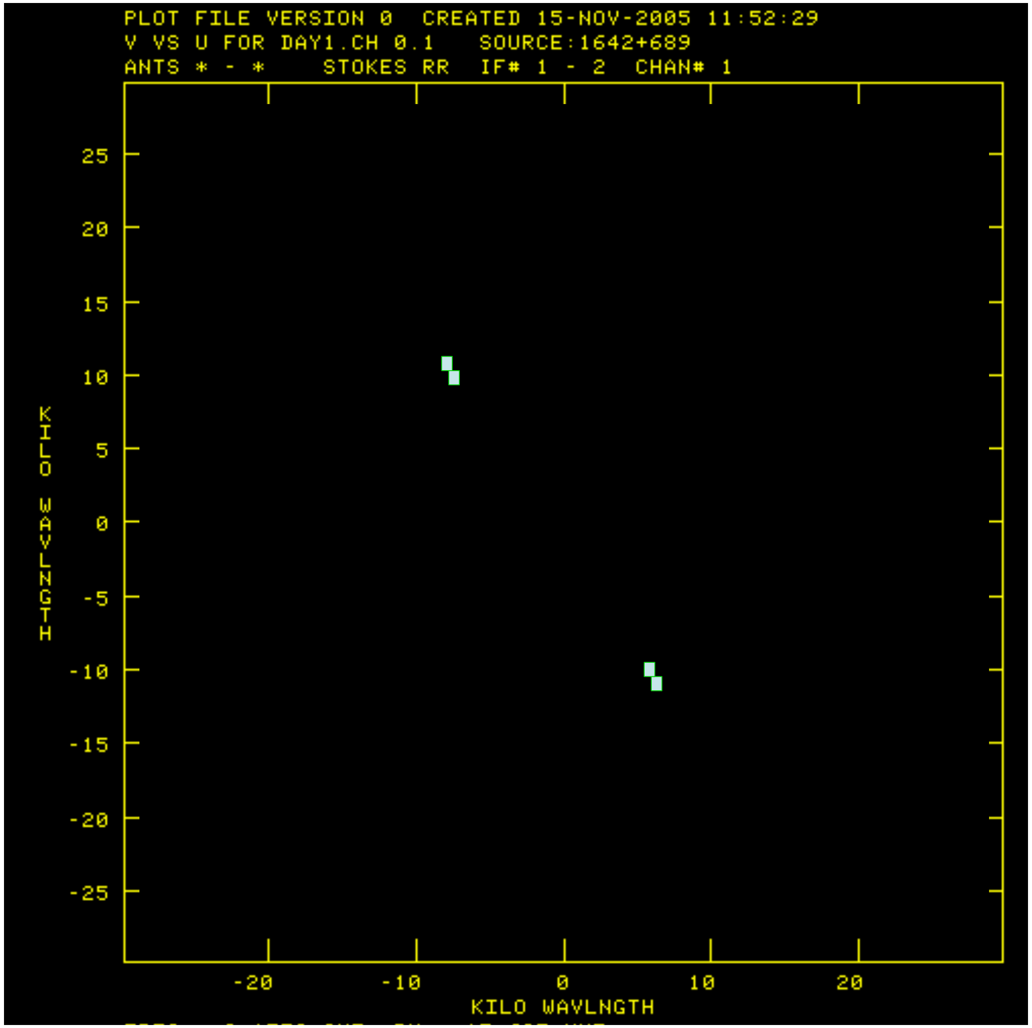
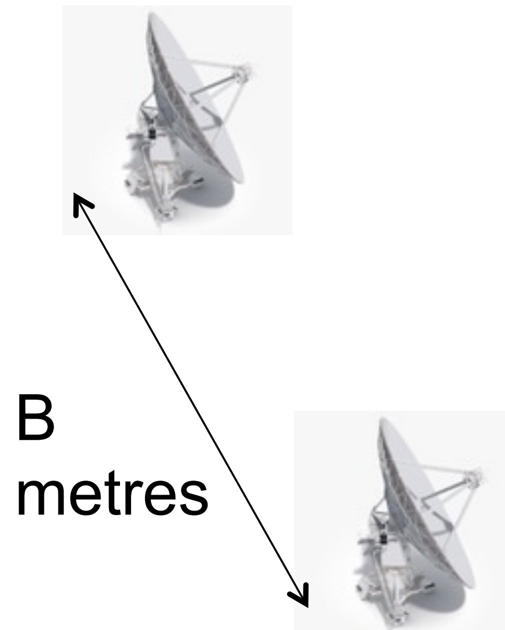


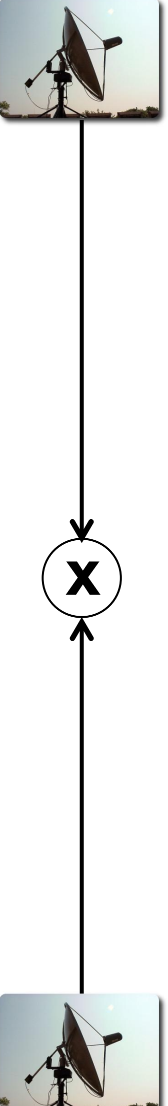
The output

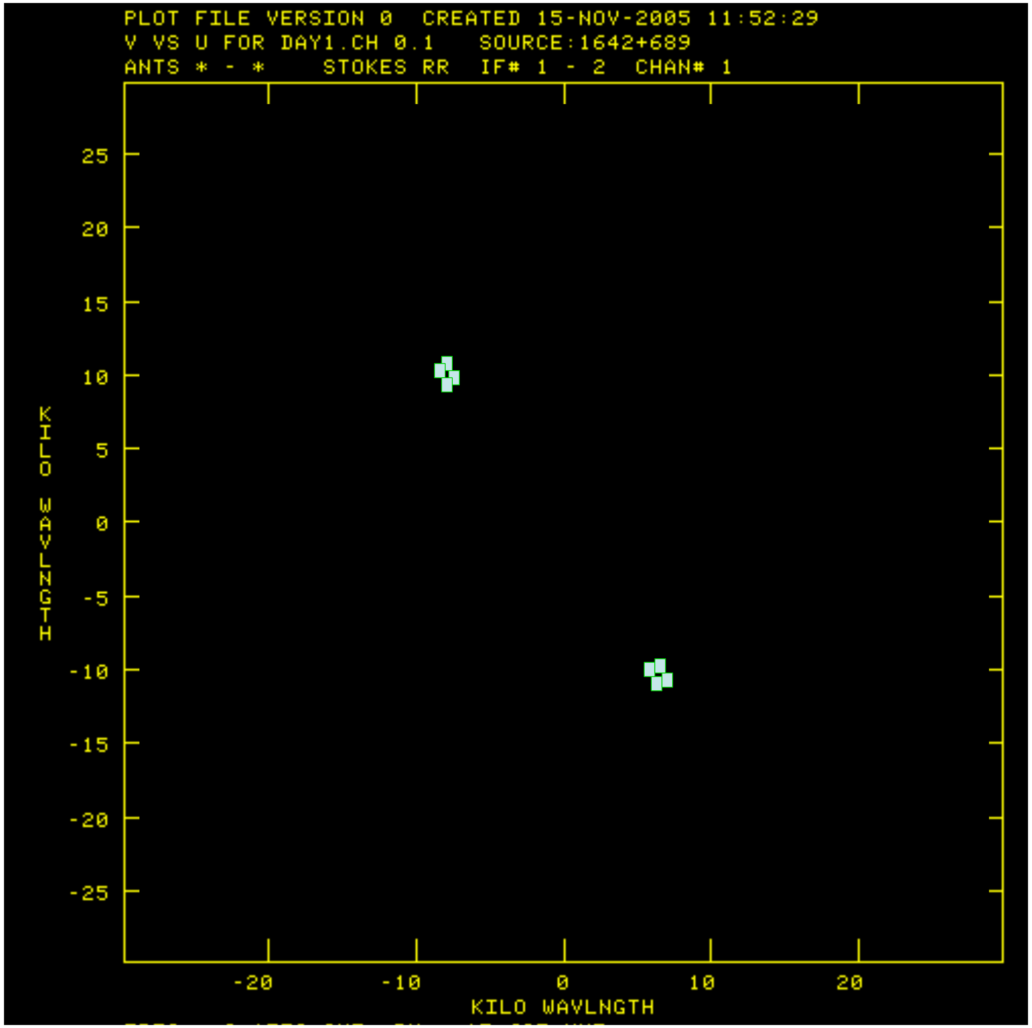

X

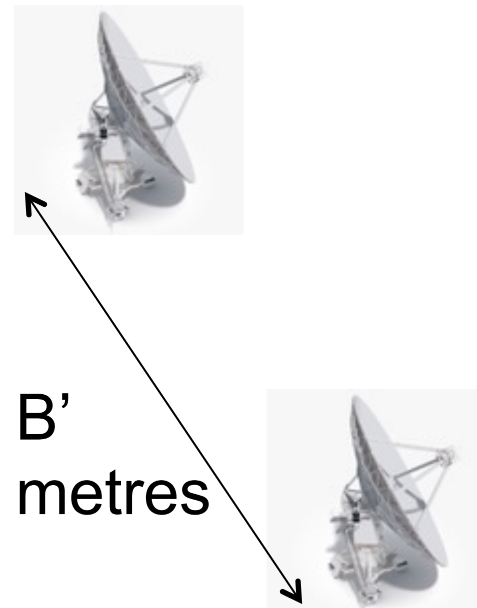

PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1   SOURCE:1642+689
ANTS * - *   STOKES RR   IF# 1 - 2   CHAN# 1
KILO WAVLNGTH
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
20


B
metres

The output

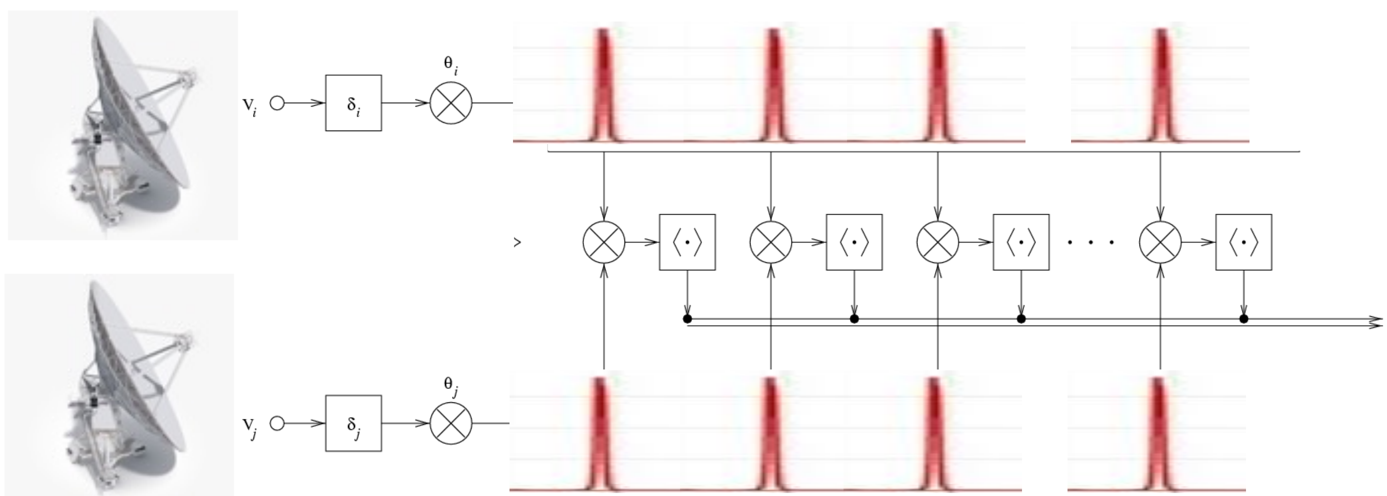

X


PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1 SOURCE:1642+689
ANTS * - * STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
20
KILO WAVLNGTH


B'
metres

Making it feasible

- Analog filters are costly & finnicky; this would be expensive and temperamental



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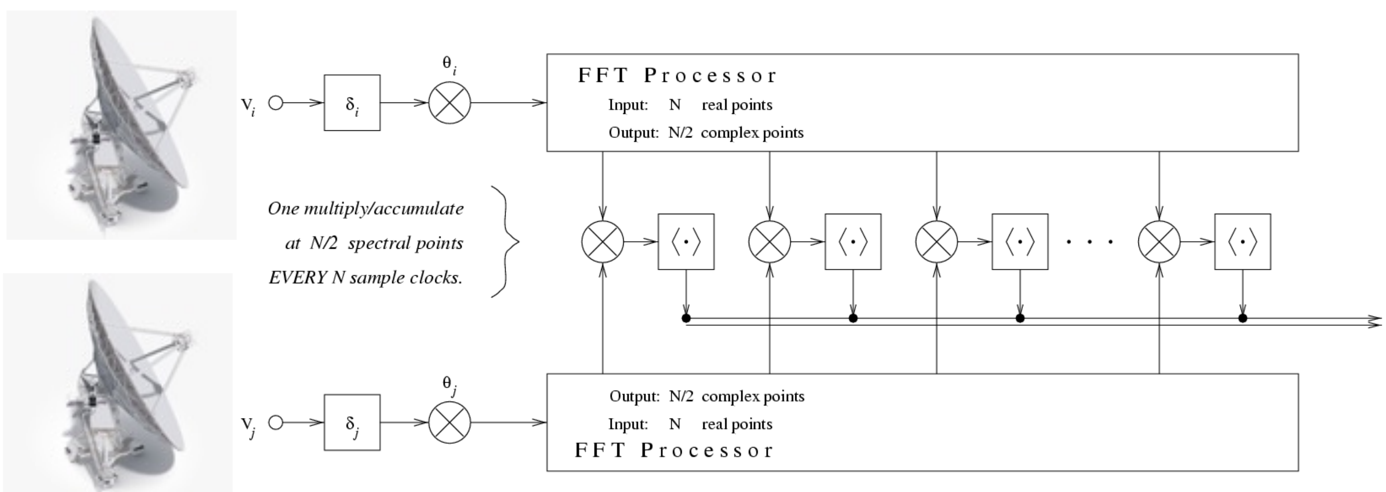
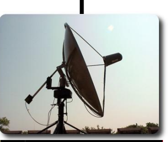
Making it feasible

- Analog filters are costly & finnicky; this would be expensive and temperamental
- Fortunately, we can (and do) digitize the signal – meaning we can use a digital substitute: **digital filterbank**





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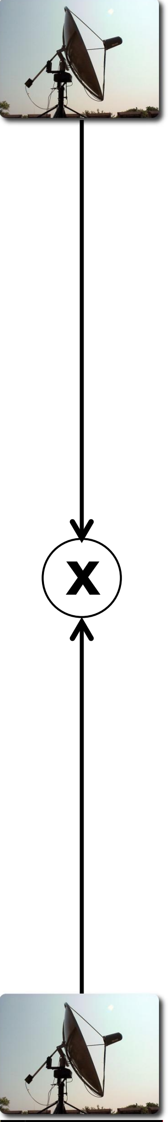
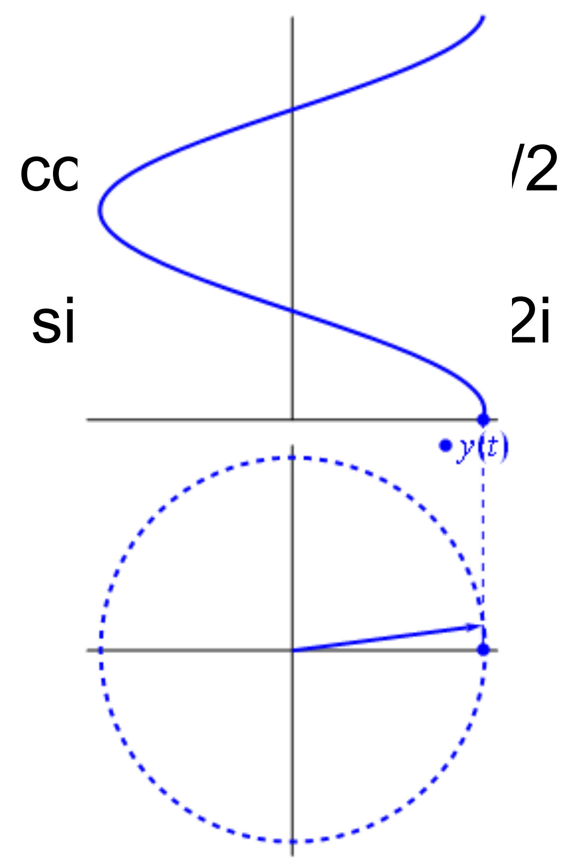


The advantage of going digital

- Stable, cheap filters
- Produces complex output: when cross-multiplying, use 1 complex multiplier rather than 2 real multipliers and a phase shift

$e^{i\phi} = \cos\phi + i \sin\phi$



Animation from http://en.wikipedia.org/wiki/File:Unfasor.gif

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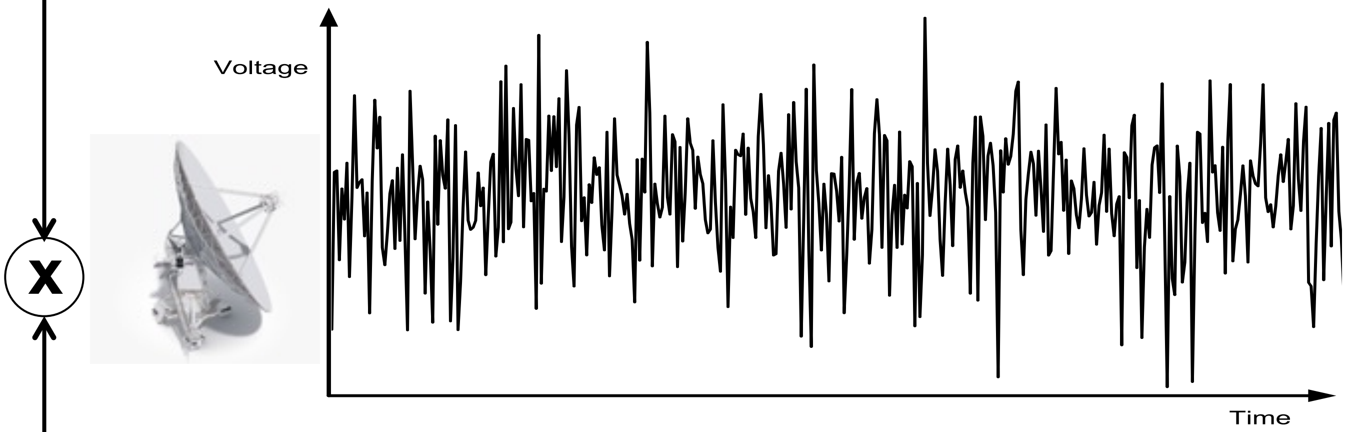
The “FX” correlator



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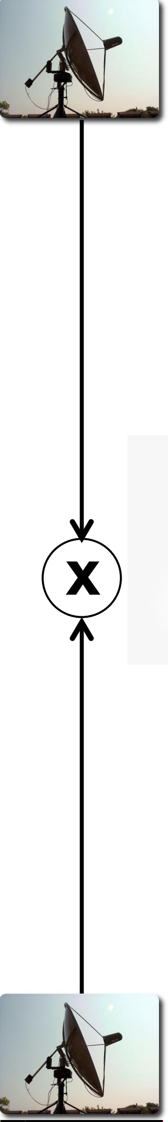

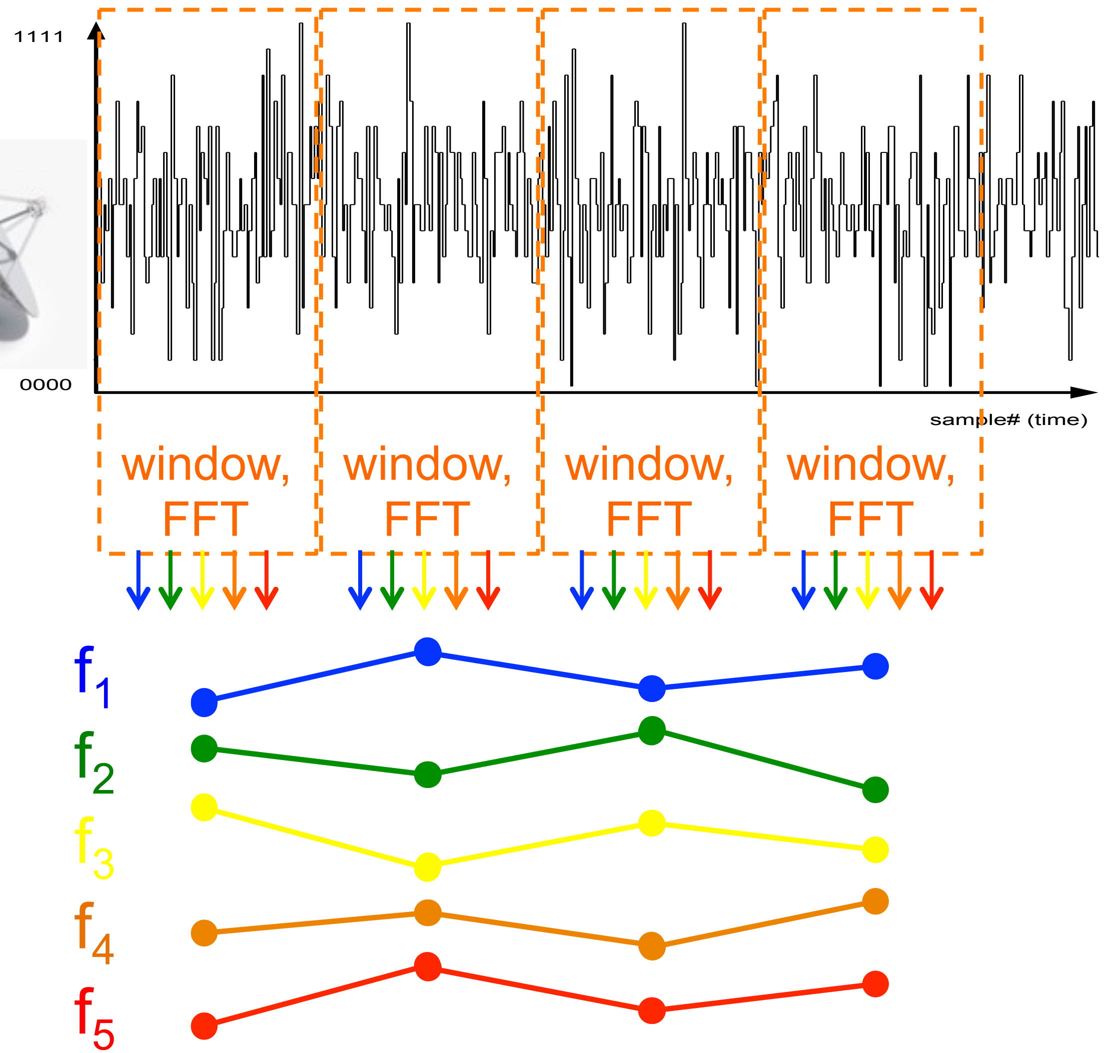

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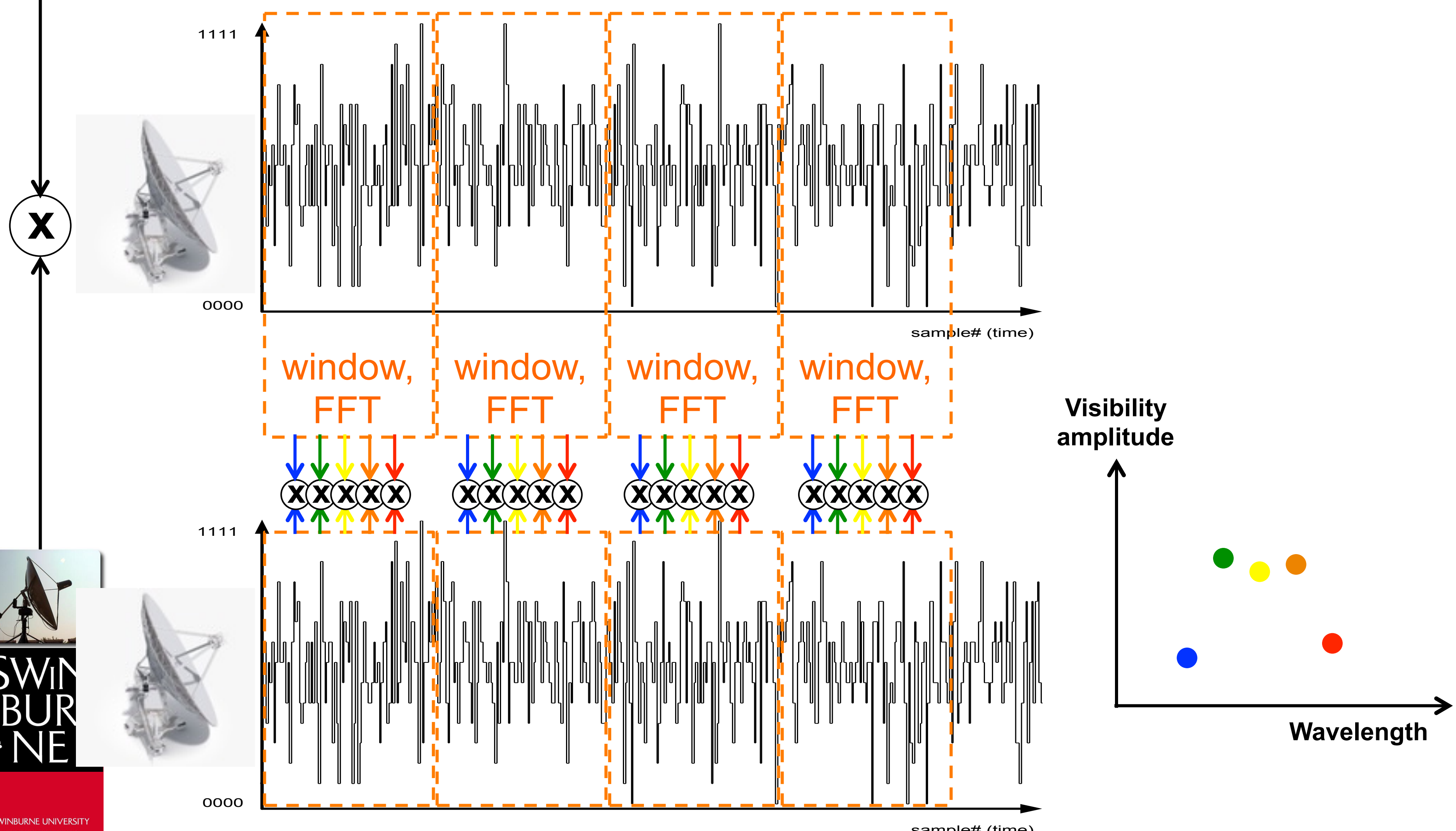

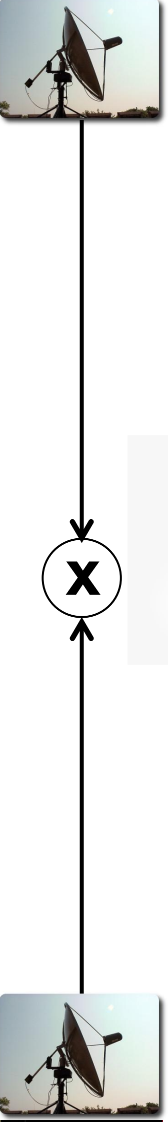

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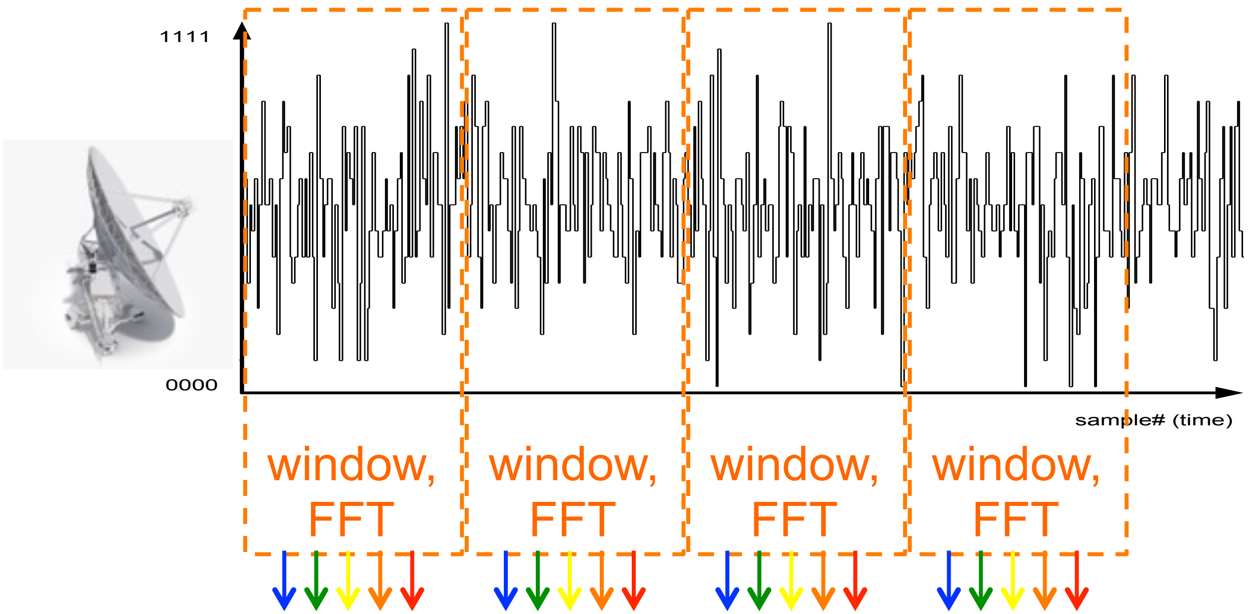

- Since this architecture consists of a Fourier transform (F) followed by cross-multiplication (X), we dub this the “FX” correlator

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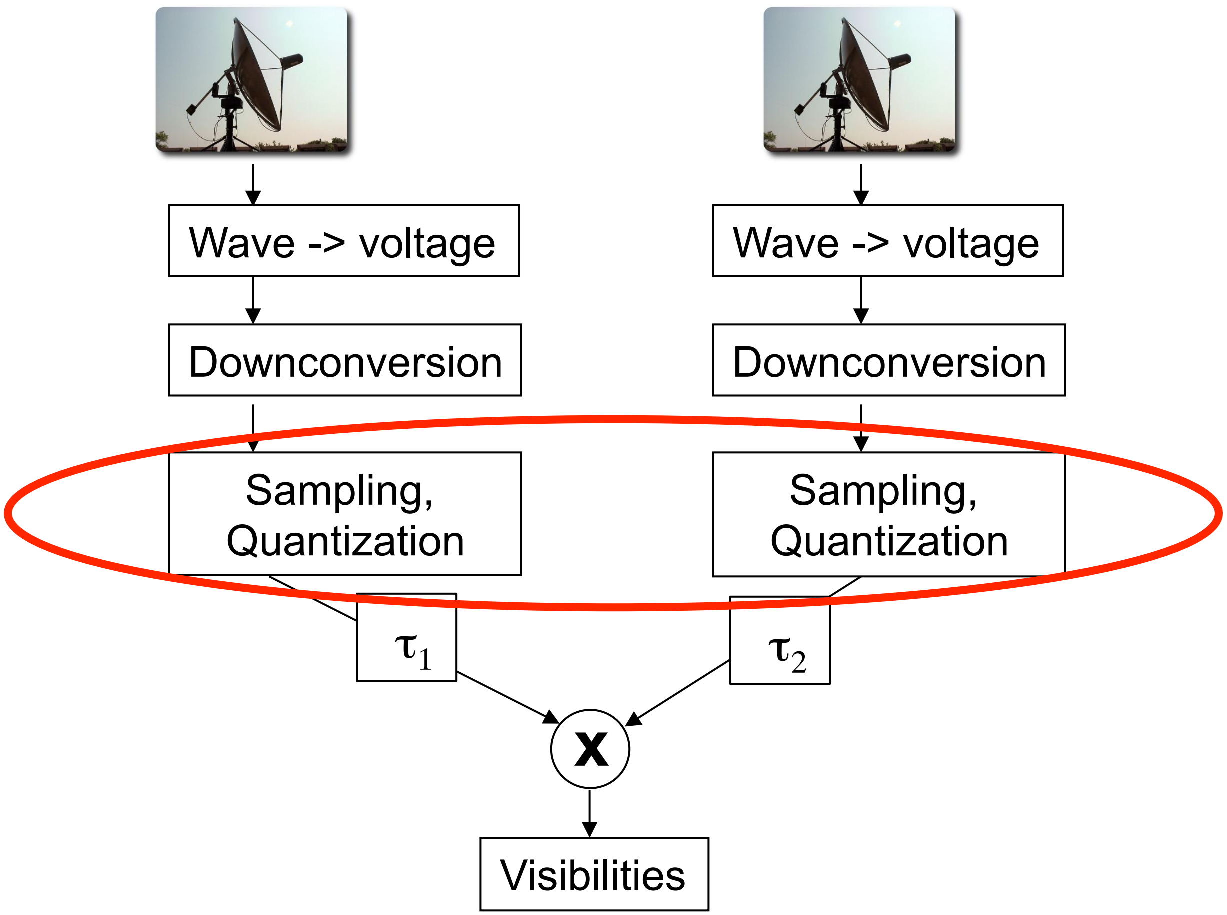
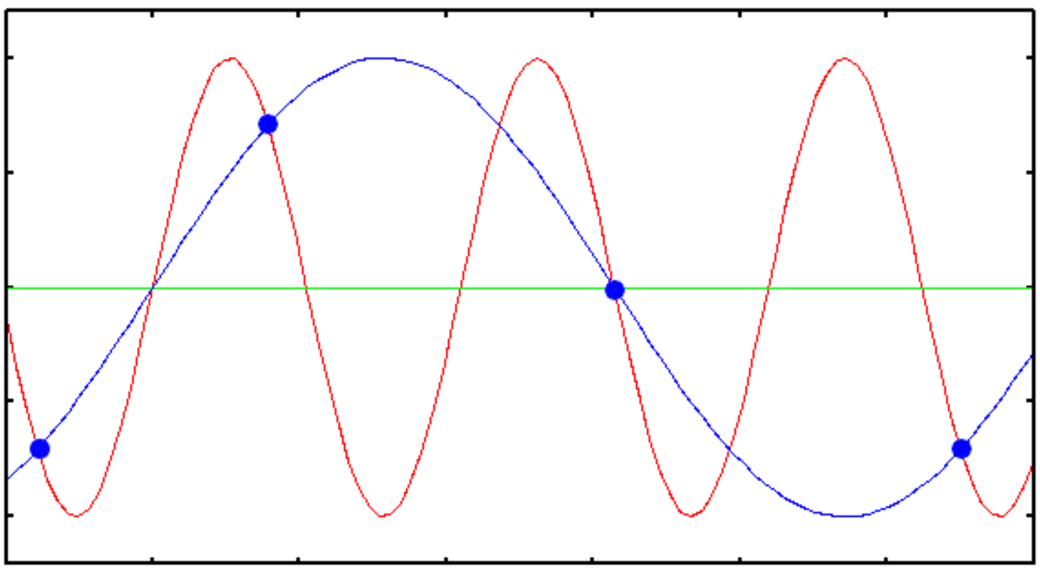
Righting the wrongs



Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every Δt sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies ≥ 1 / (2 Δt) Hz (*band-limited*)



Adequately sampled

Undersampled, cannot be reconstructed

Sampling

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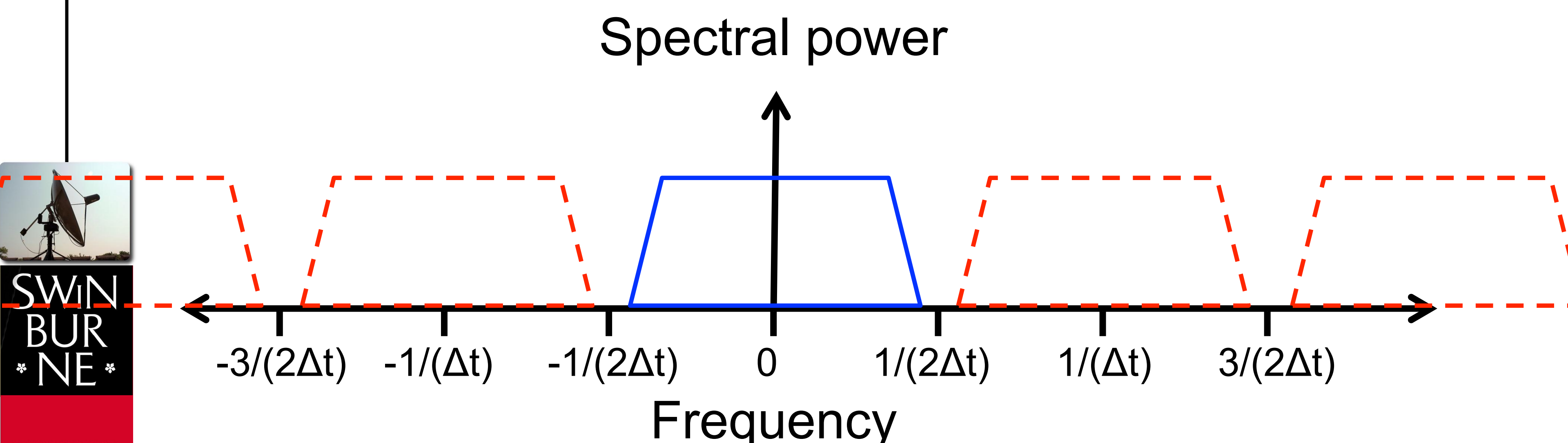

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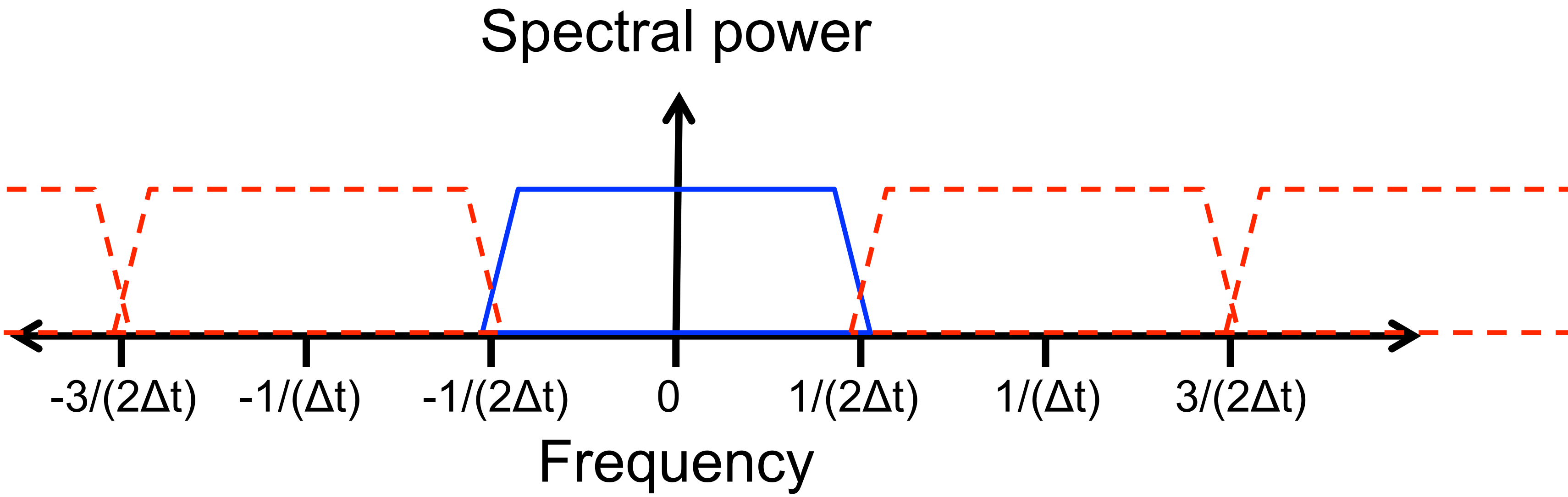
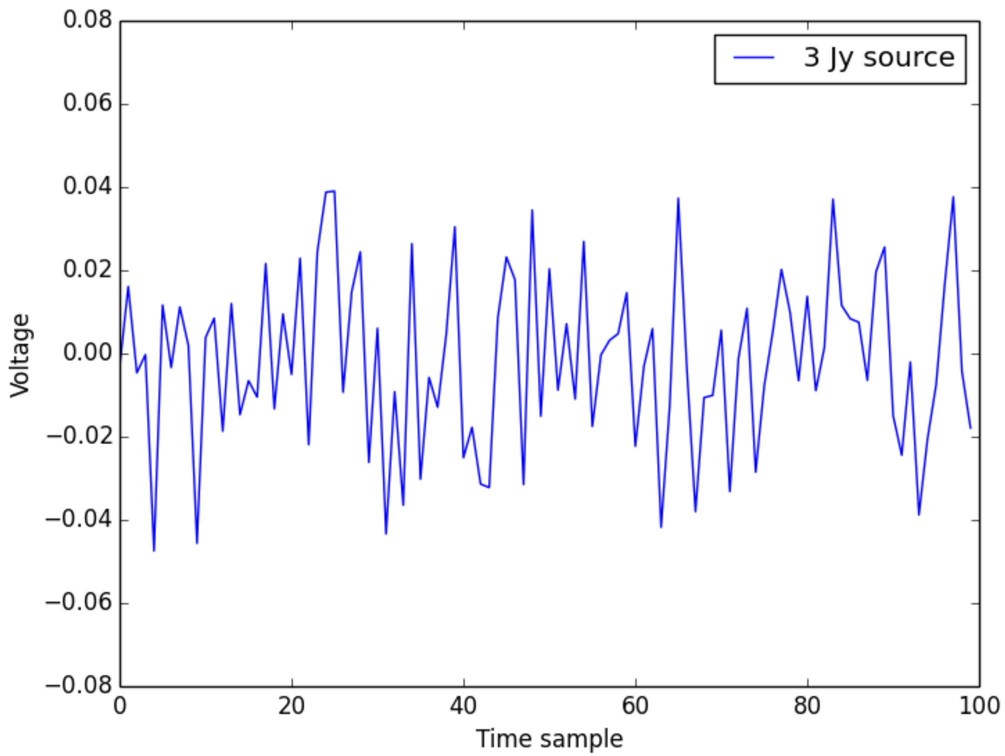


Quantization

- When correlation is low (almost always) even very coarse quantization is ok!



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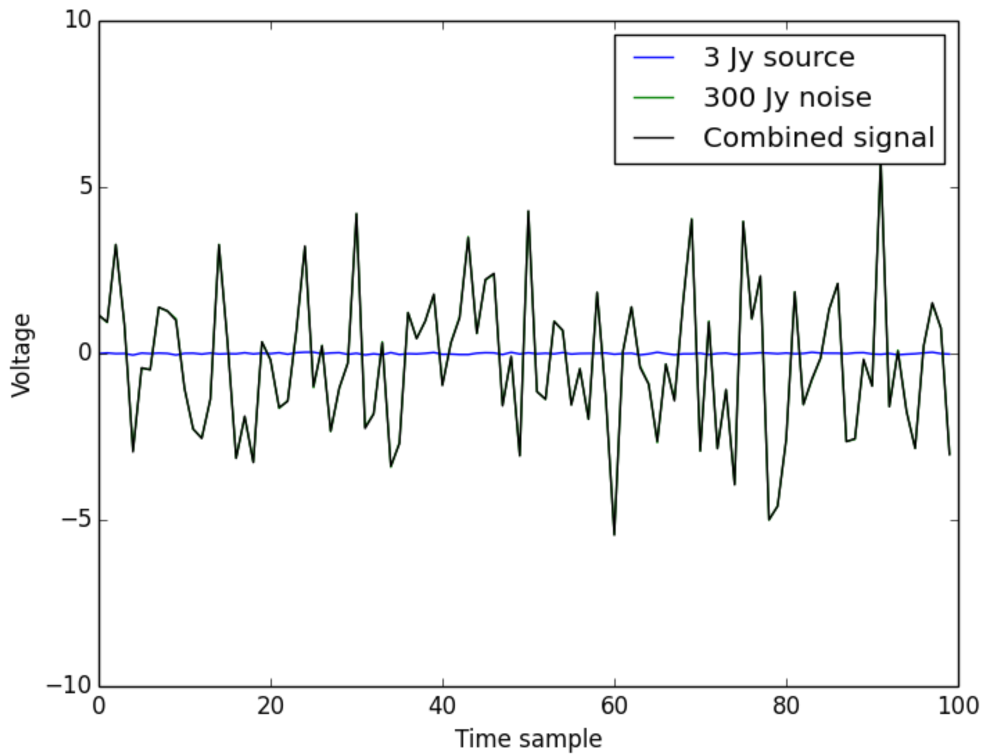

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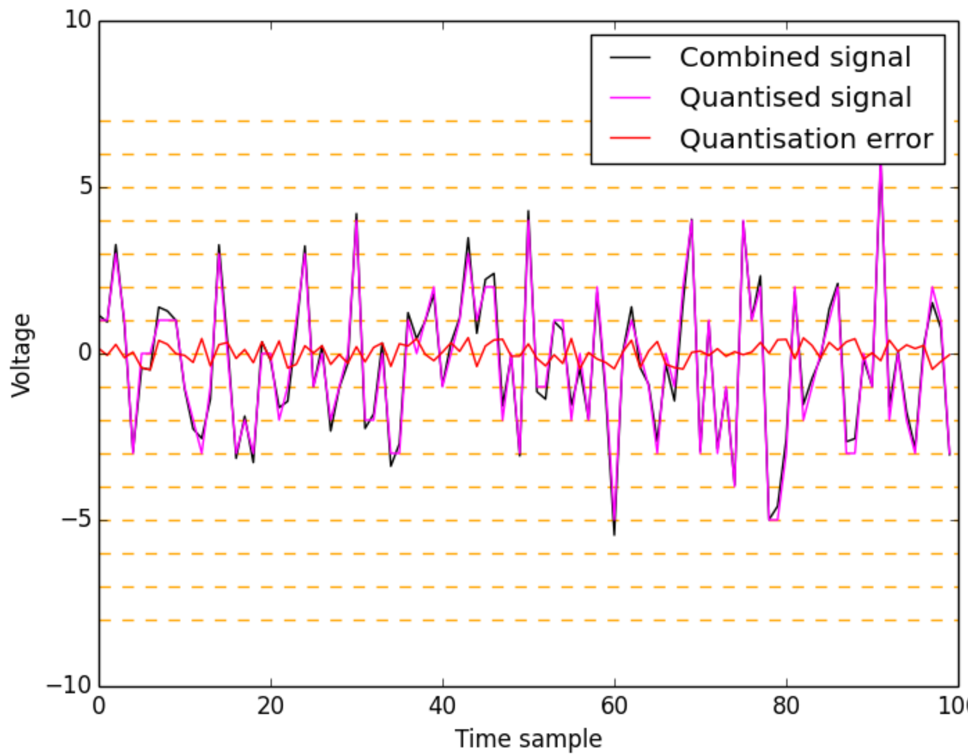

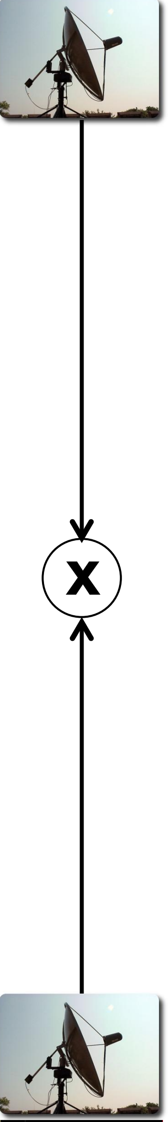

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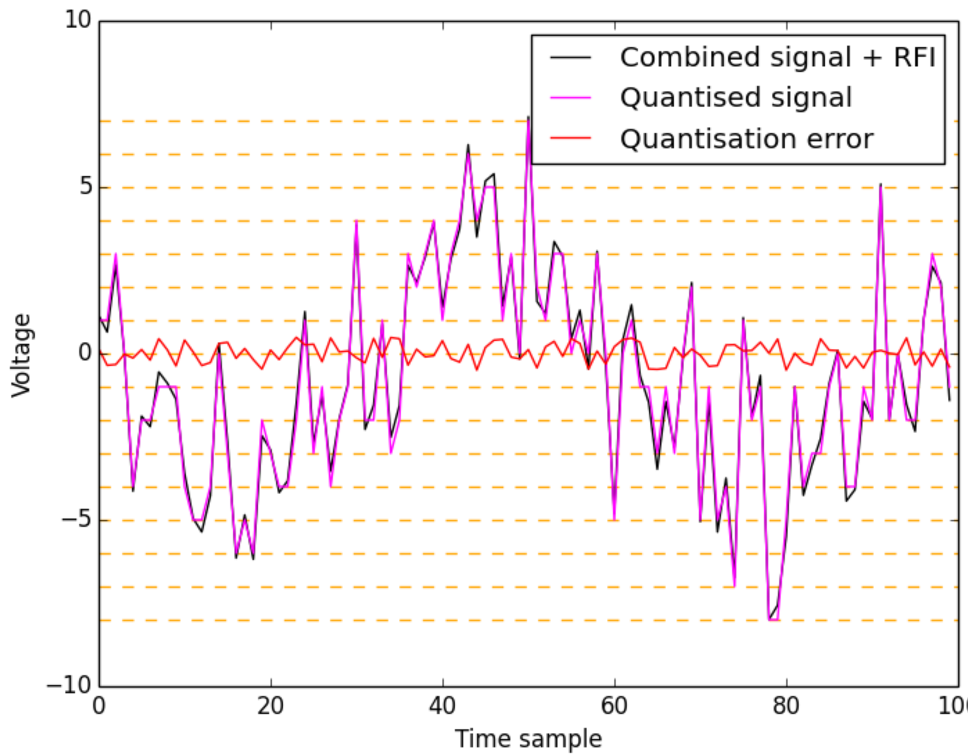

Quantization

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until the headroom runs out…

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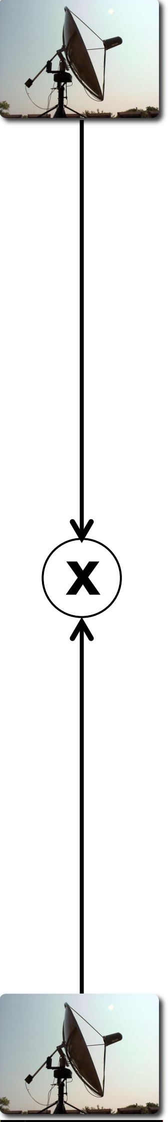

Quantization

- When correlation is low (almost always) even very coarse quantization is ok!
- Sensitivity loss due to quantisation:
  - 8 bit: 0.1%
  - 4 bit: 1.3%
  - 2 bit: 12%
  - 1 bit: 36%
- Correct visibility amplitudes for this sensitivity loss

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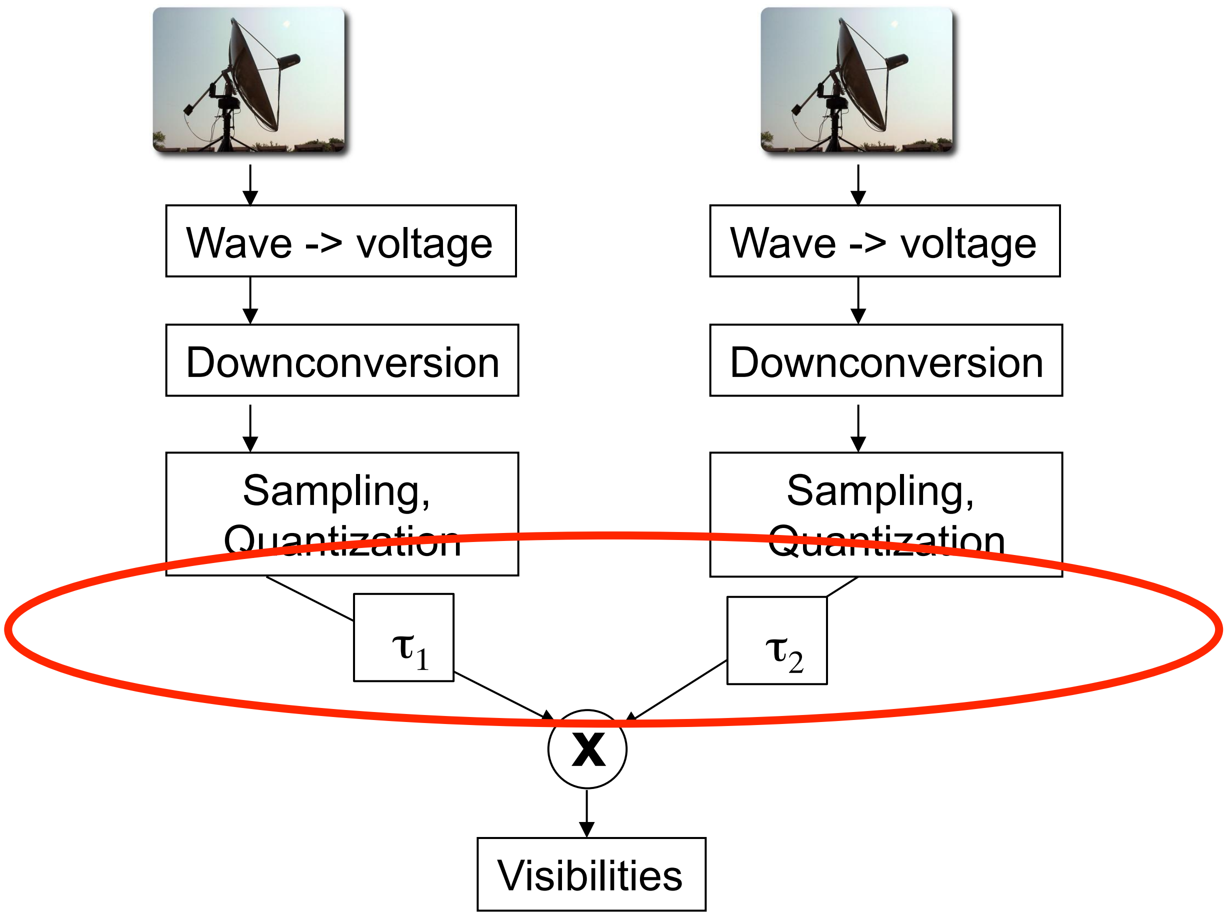
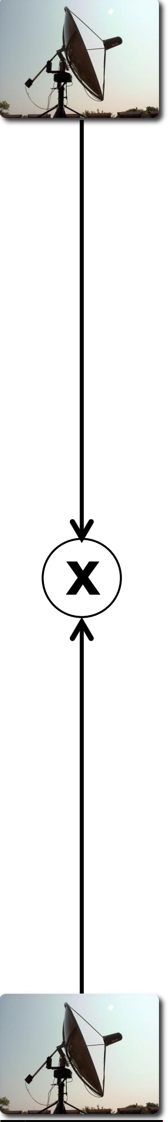
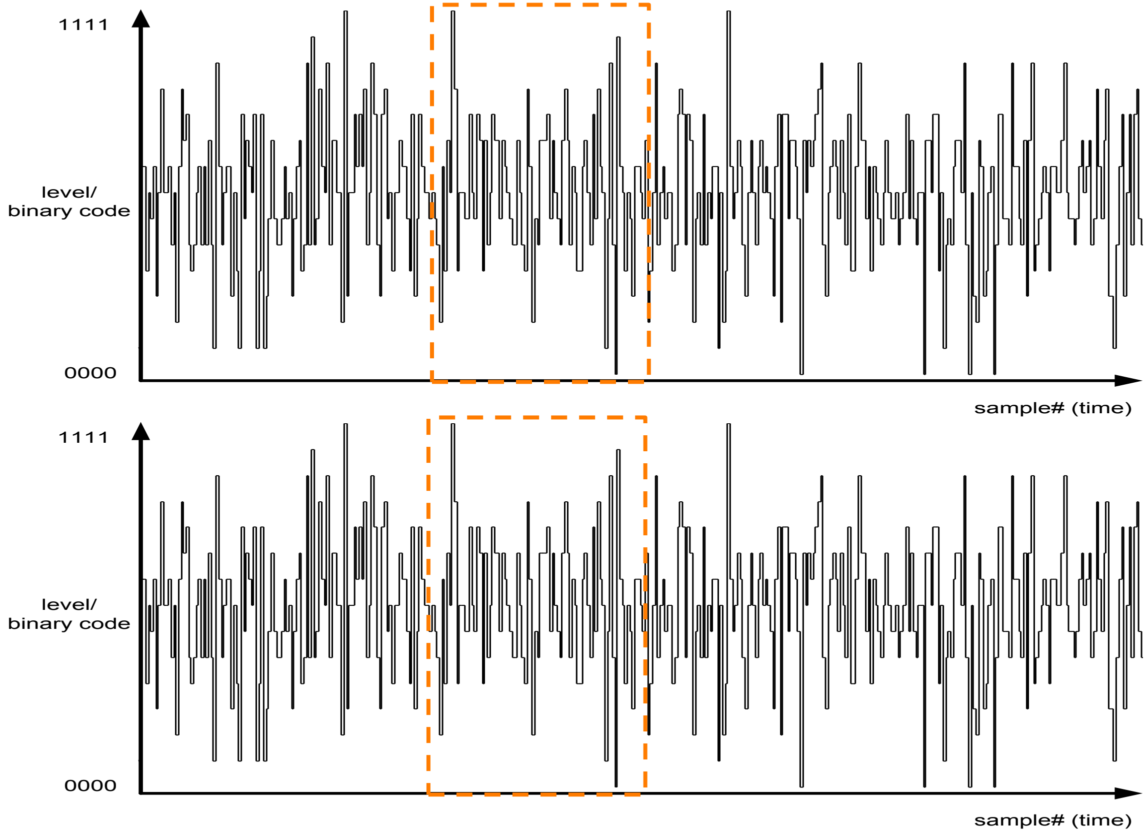
Righting the wrongs

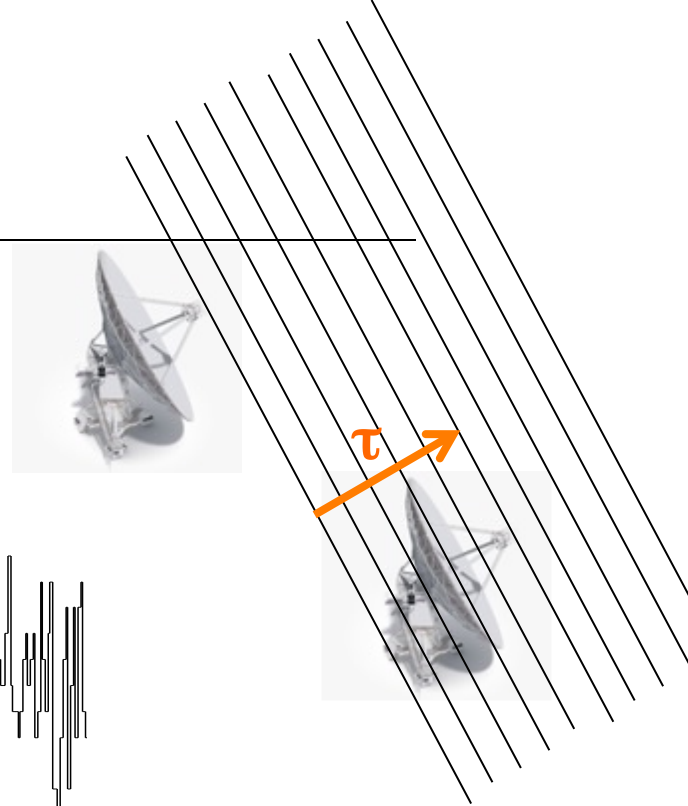


Delay compensation

- Delay to the nearest sample is easy:







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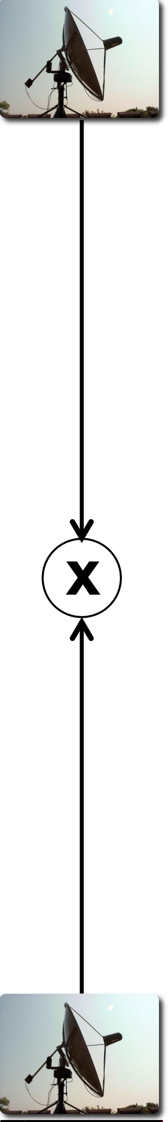

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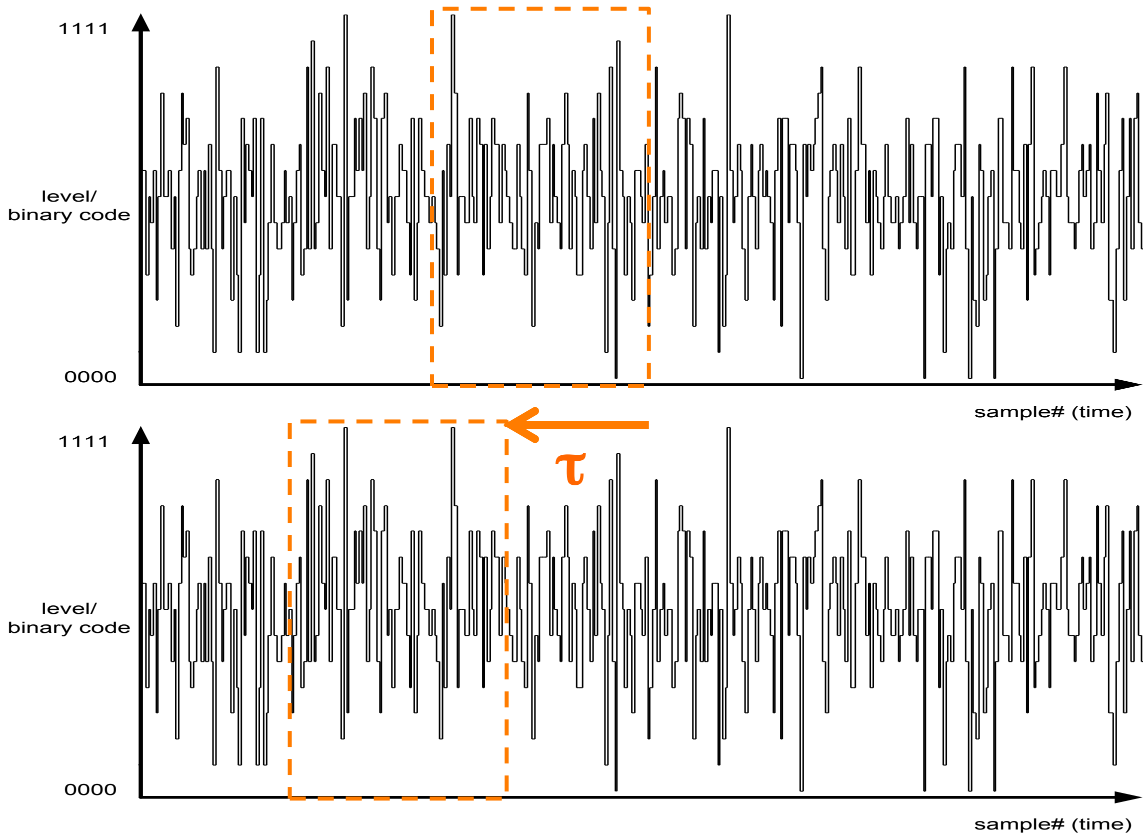

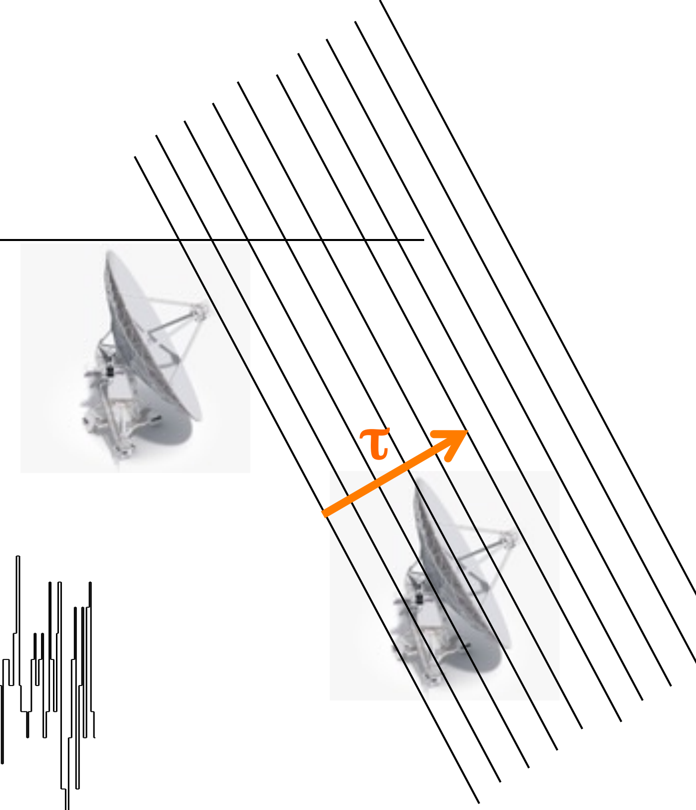

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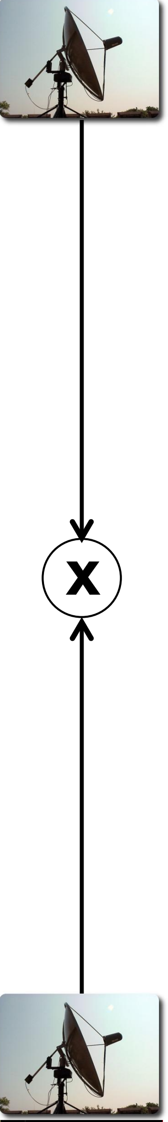
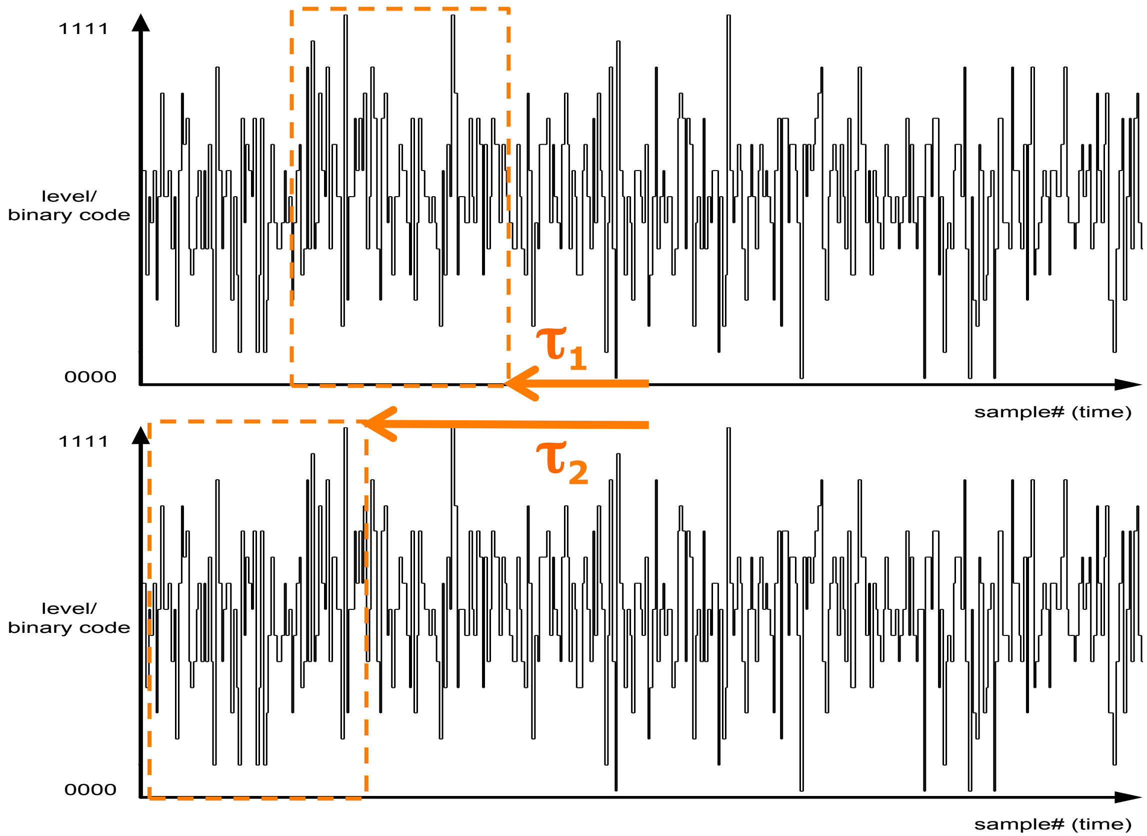
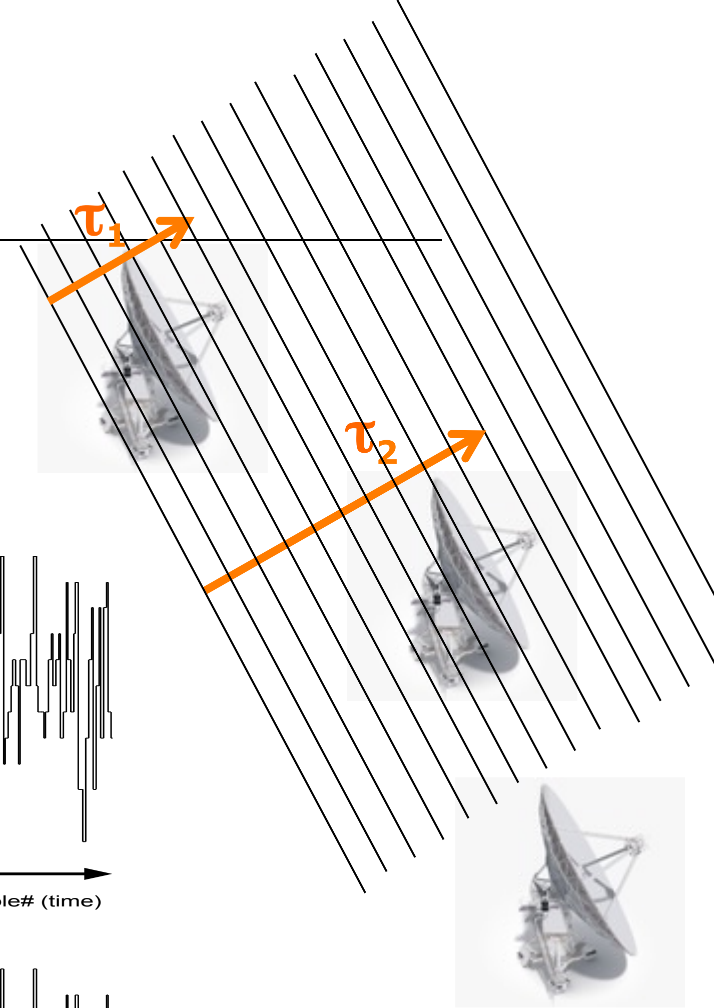
Delay compensation

- In practise, delay all to common reference







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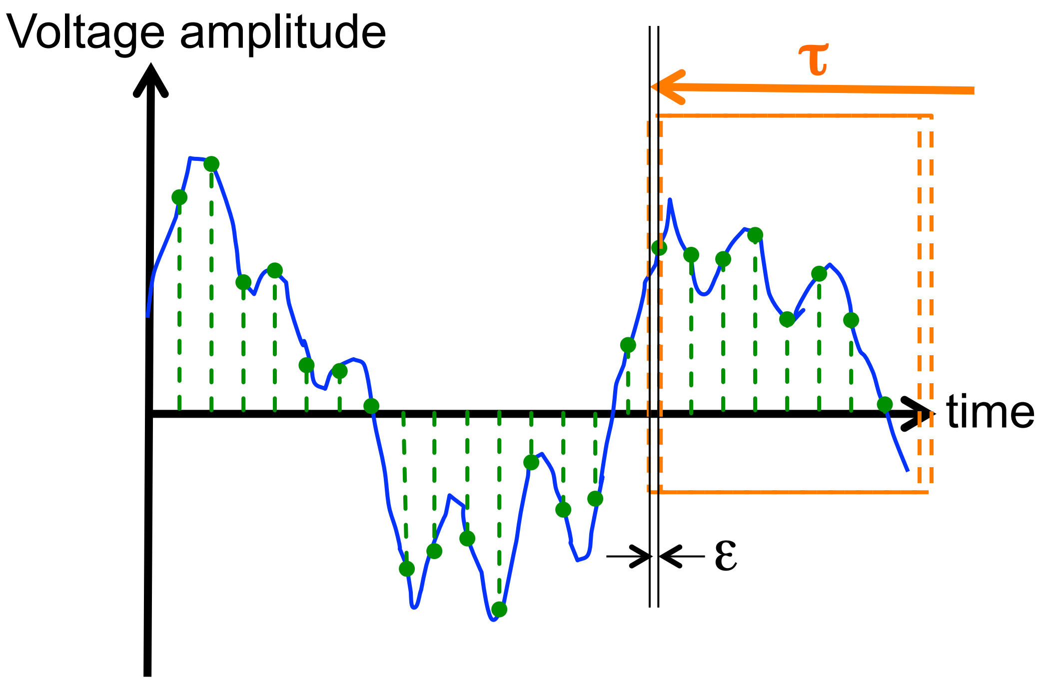
Fractional-sample correction

- Sampling prevents perfect alignment of datastreams; always a small error



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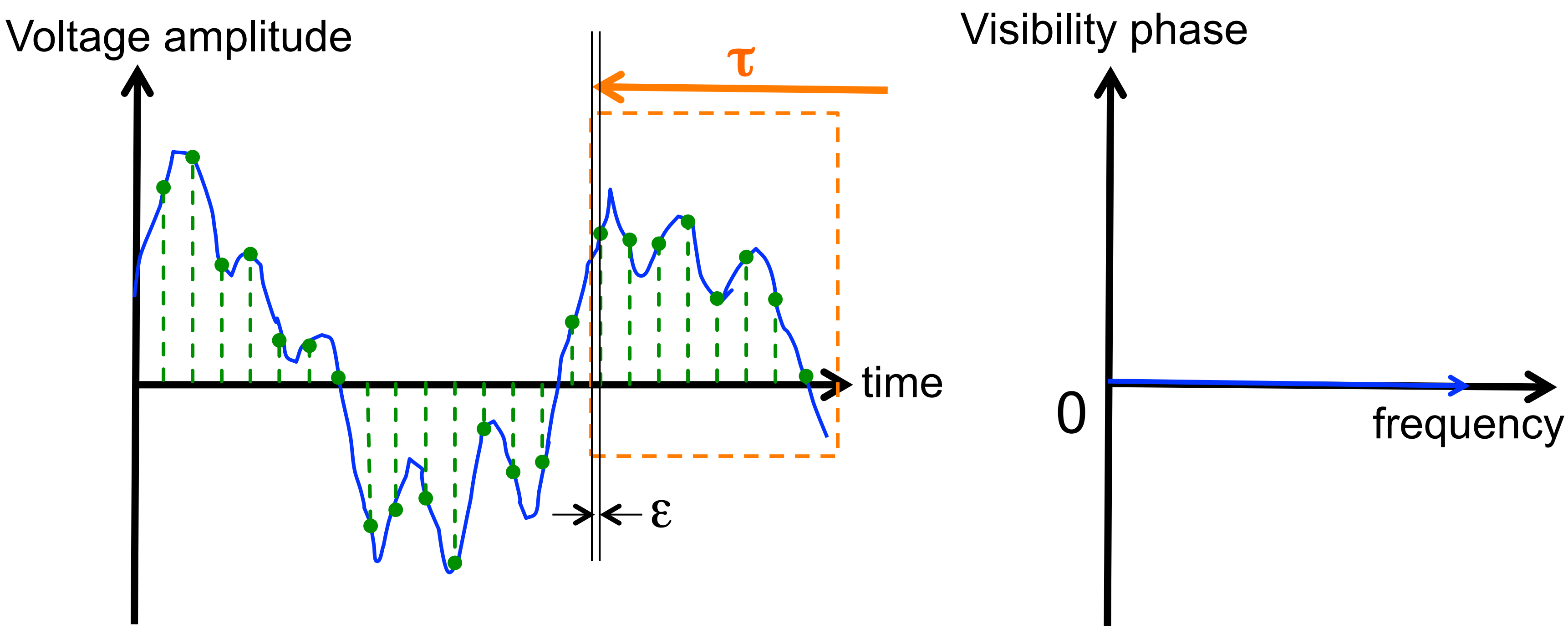

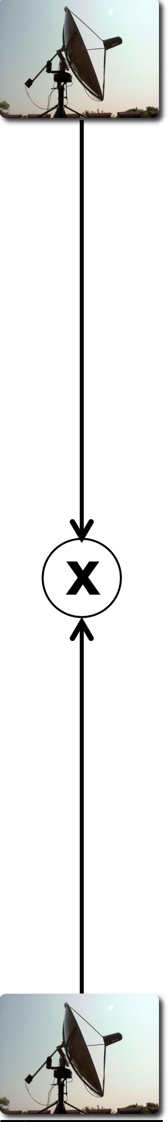

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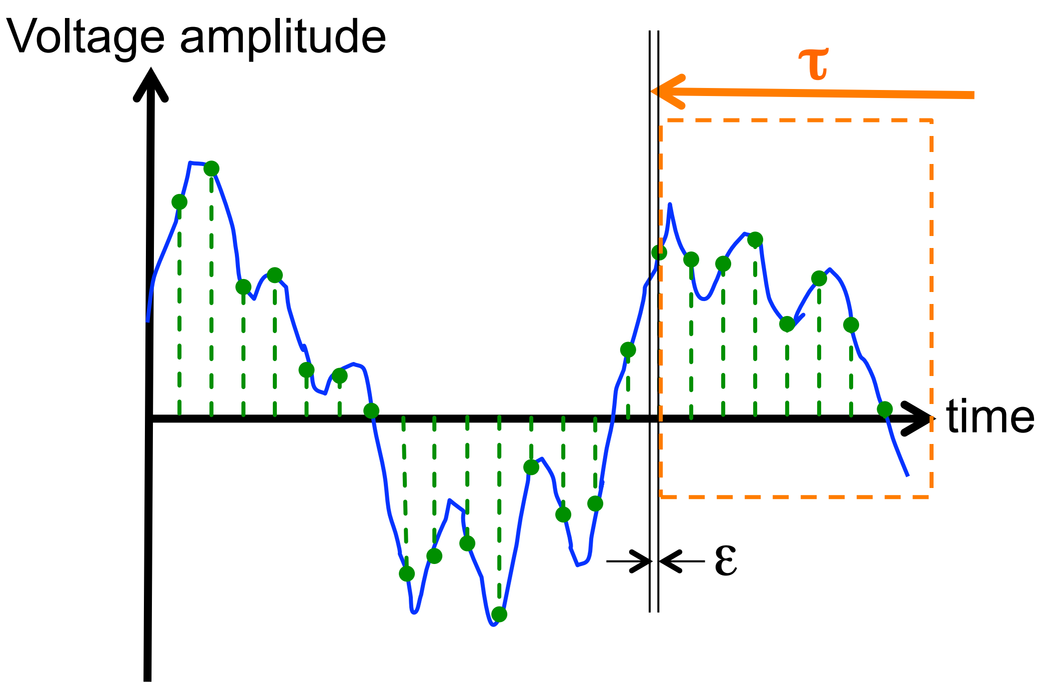

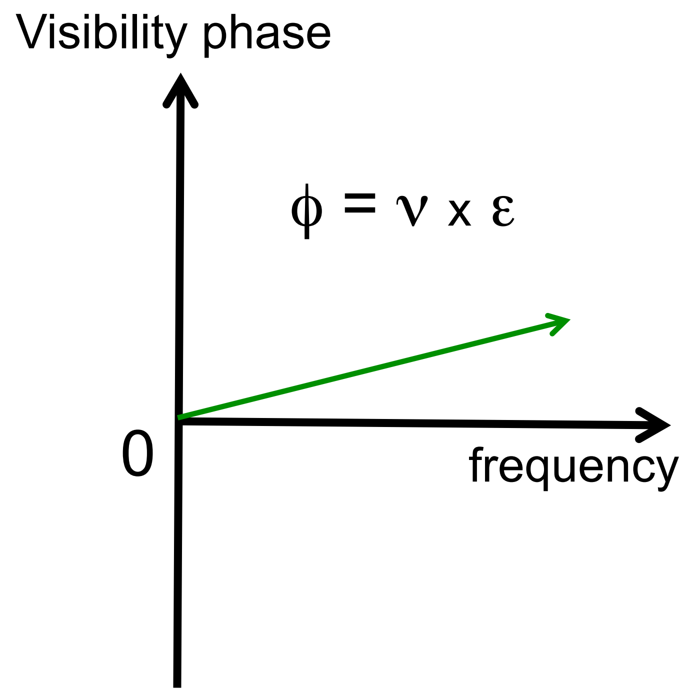

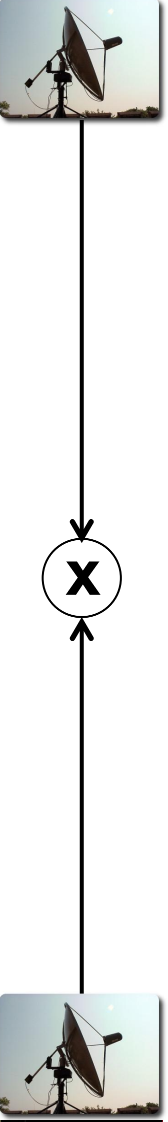

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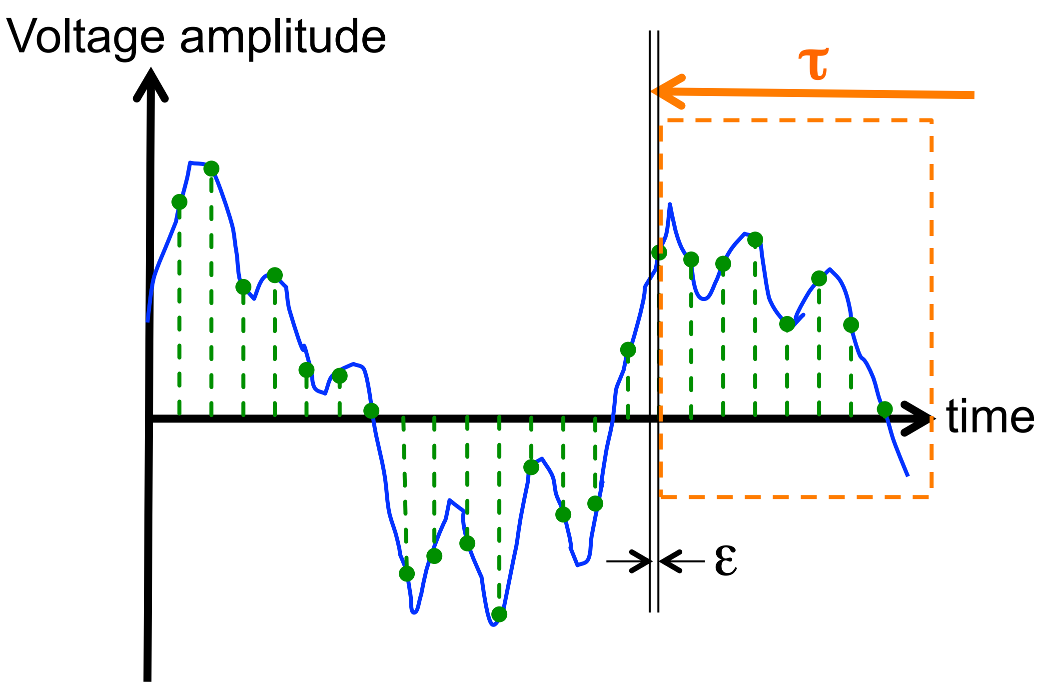
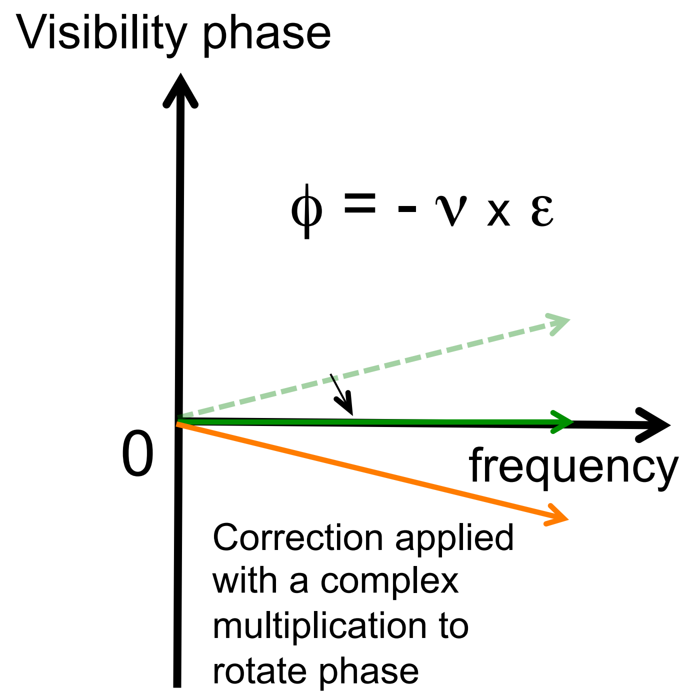
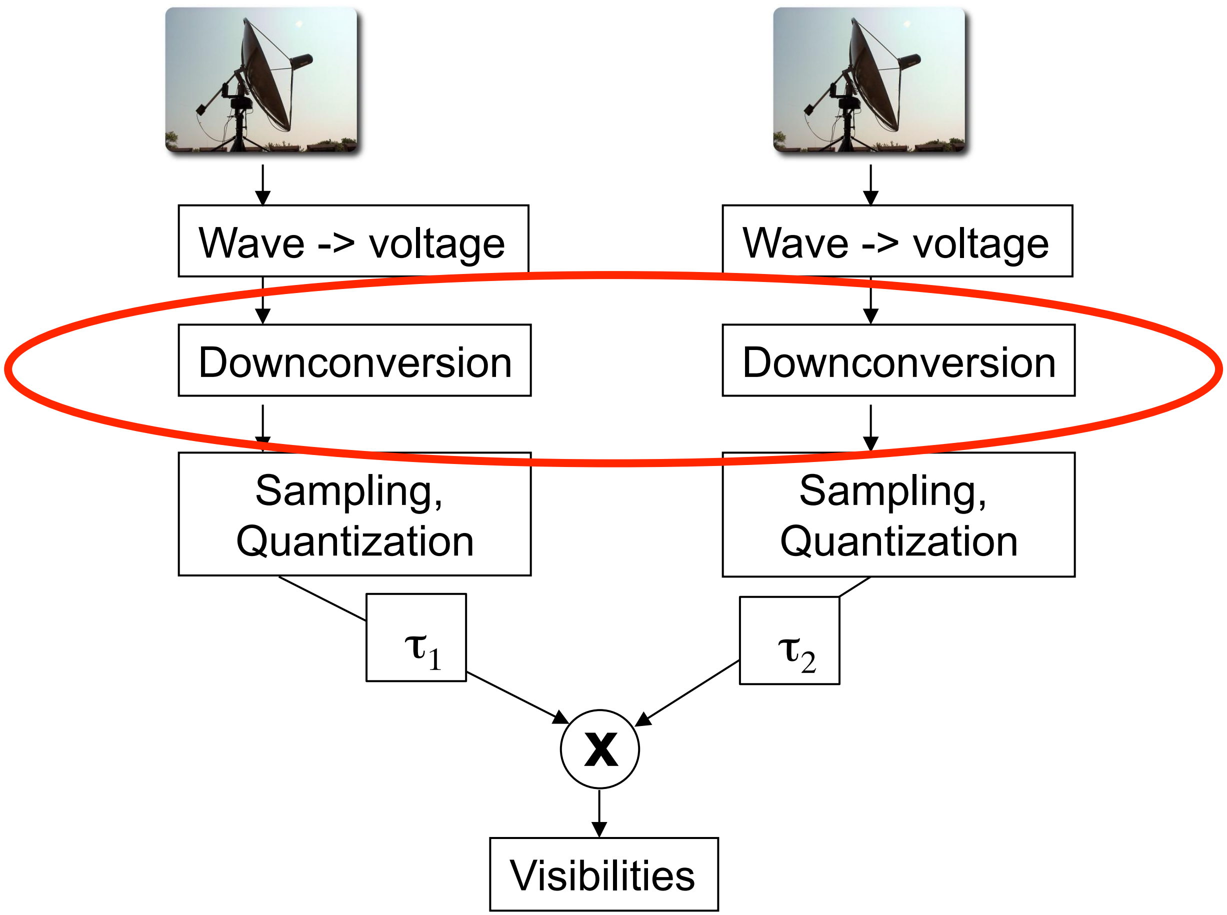
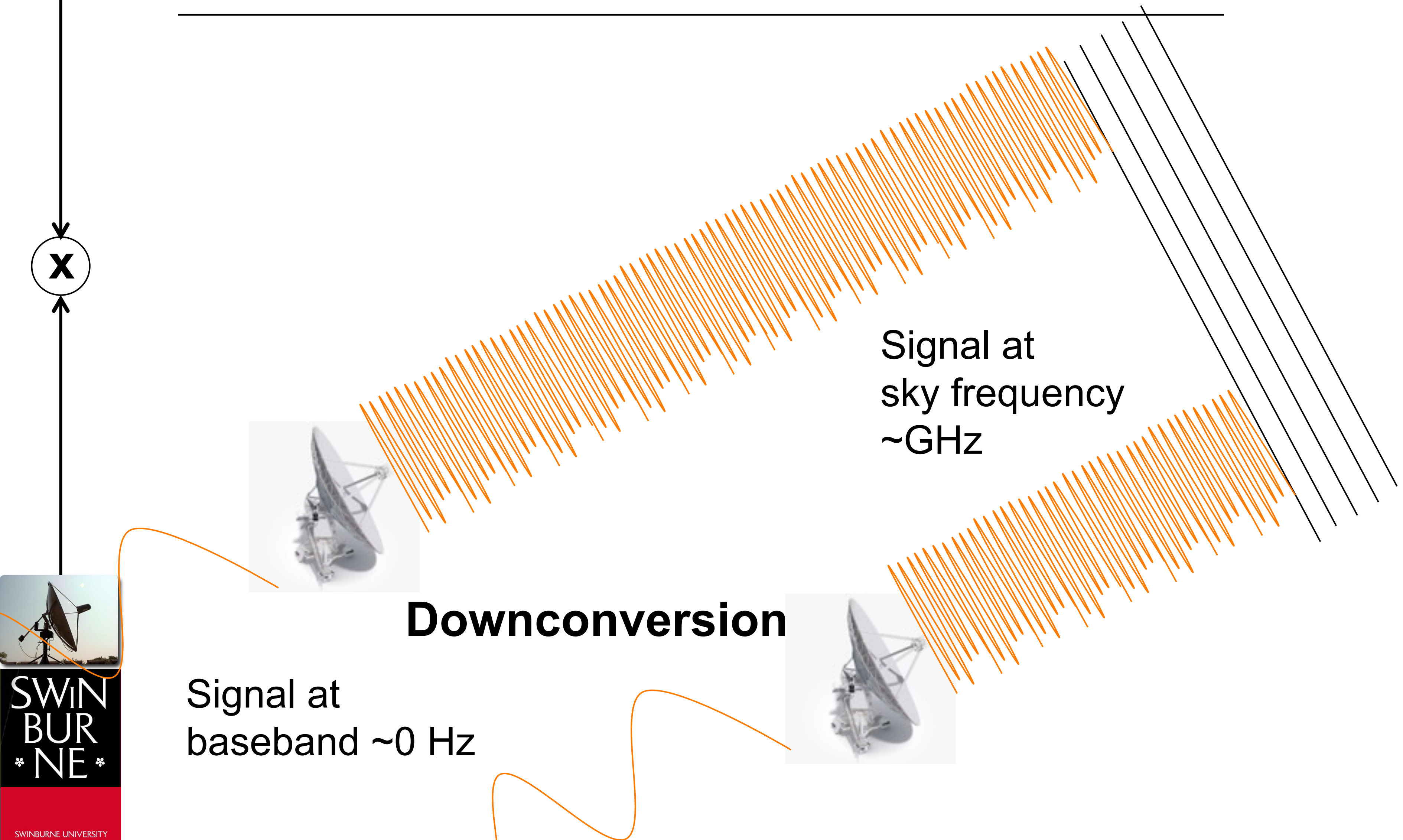


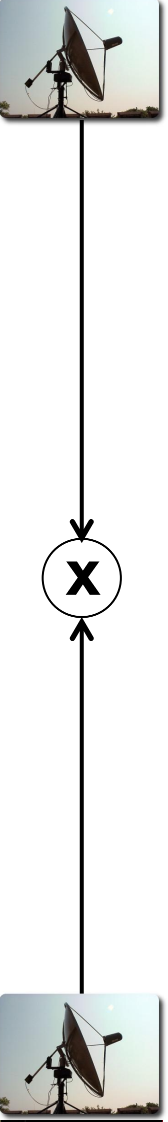


Righting the wrongs



Fringe rotation





Fringe rotation

- Implementation: rotate phase using complex multiplier
- $\Delta\phi = 2\pi \nu_{lo} \tau_g$ ν_{lo} = local oscillator frequency, τ_g = applied delay
- Update rate of $\Delta\phi$ depends on how fast τ_g changes:
  - If τ_g is changing fast, update every sample in the time domain
  - For shorter baseline / low frequency instruments, can do post-channelisation or even post-accumulation

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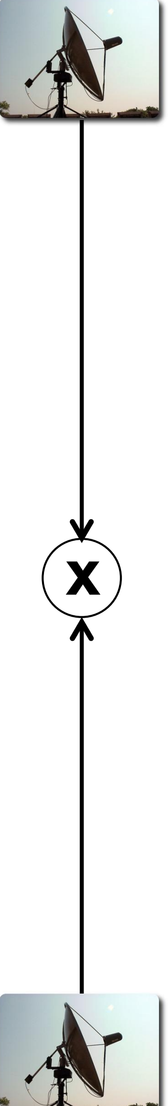
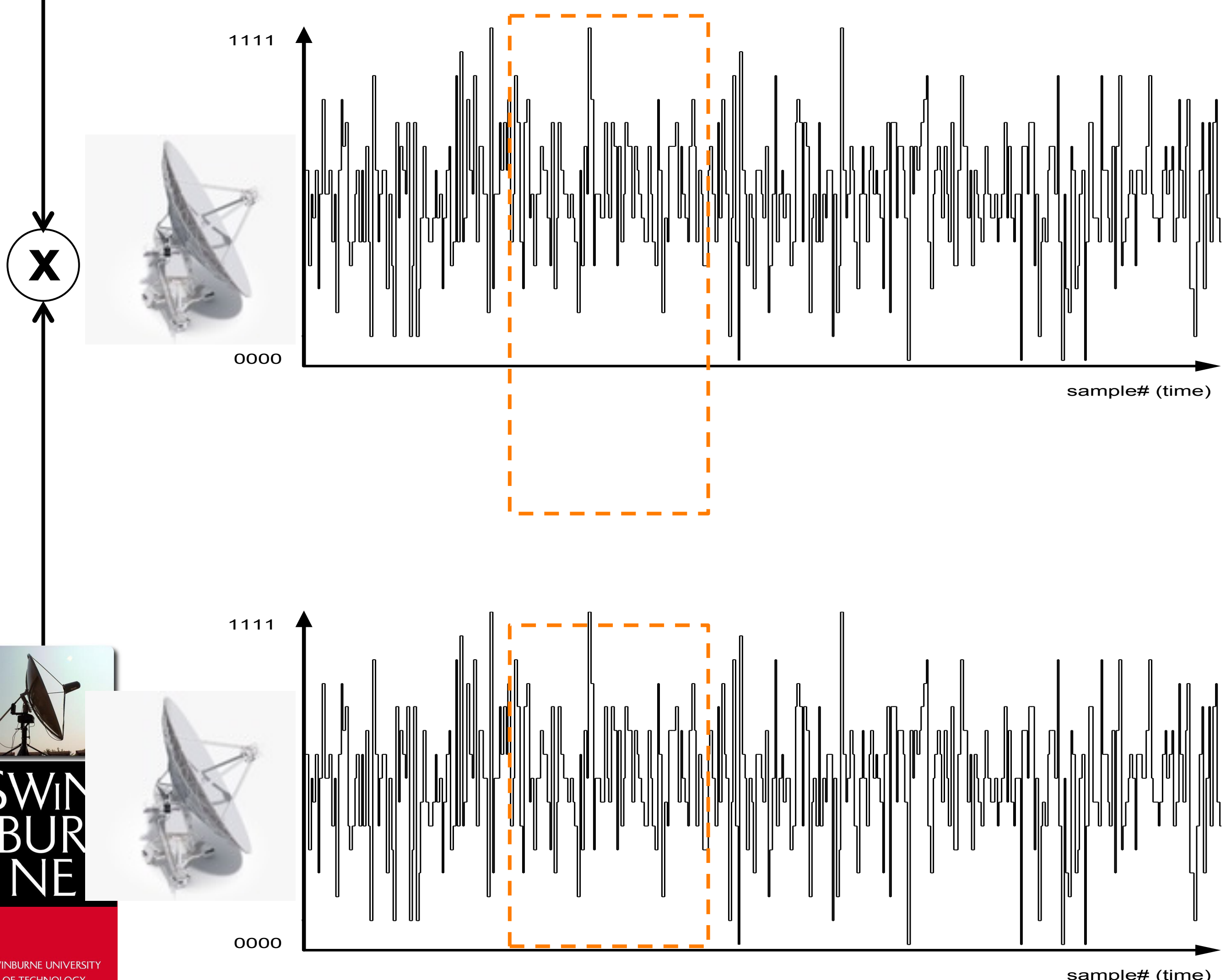


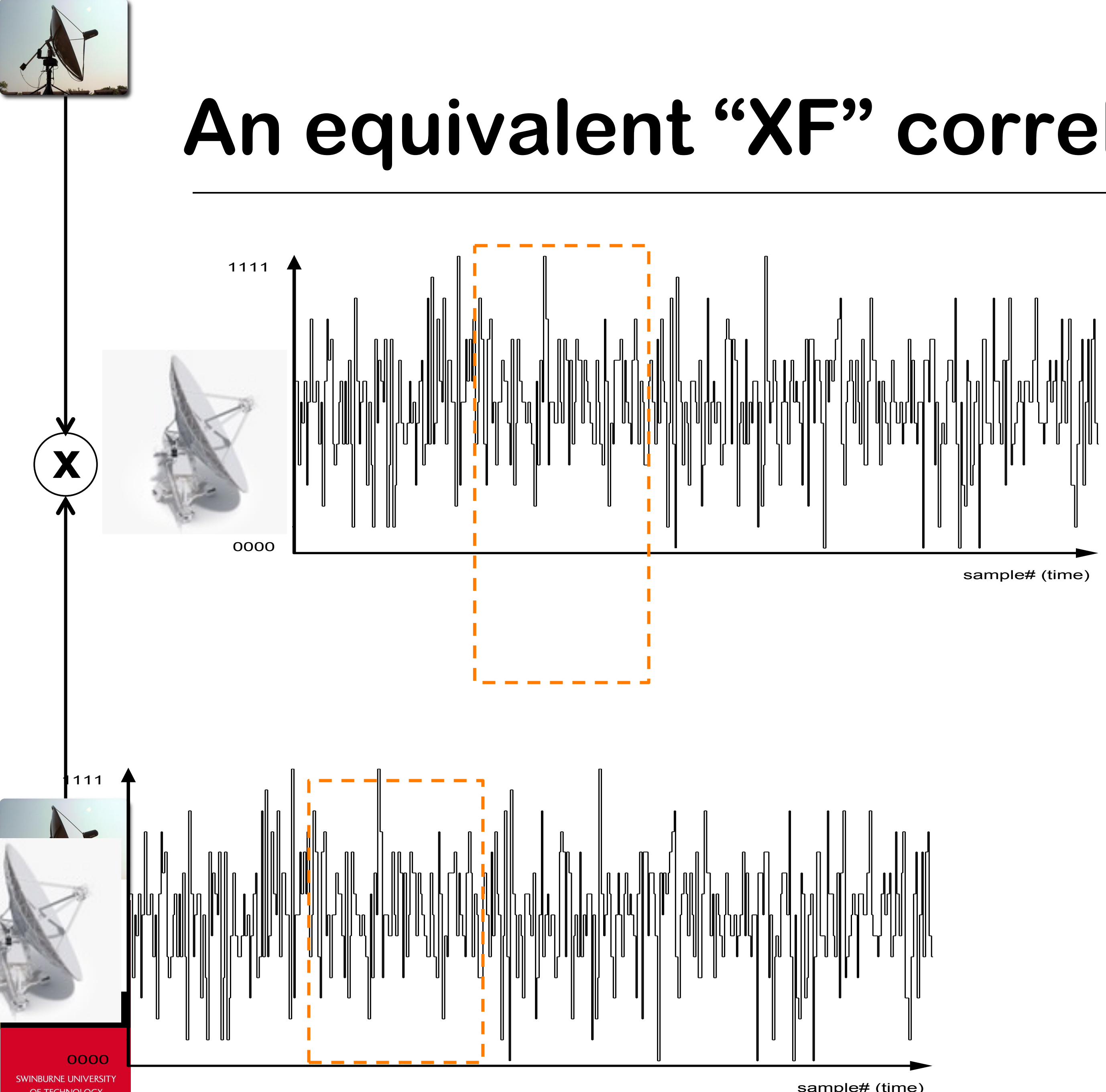
Alternate implementation

- We have shown how to build a practical FX correlator, which first Fourier transforms and then multiplies
- Convolution theorem: **Multiplication** in the frequency domain is equivalent to **convolution** in the time domain
- It is mathematically equivalent to convolve the two signals in the time domain and then Fourier transform

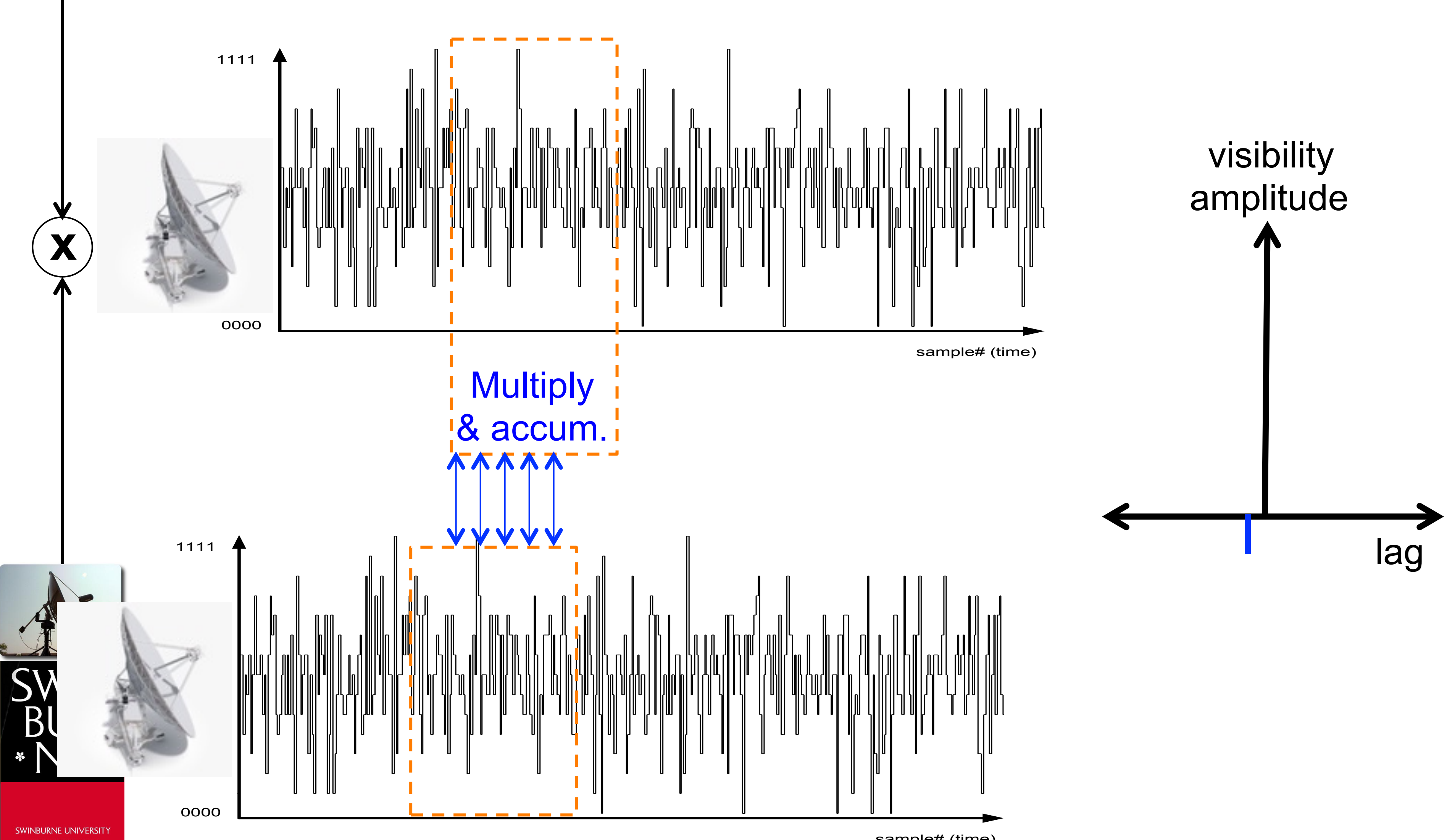
An equivalent "XF" correlator



An equivalent "XF" correlator



An equivalent “XF” correlator



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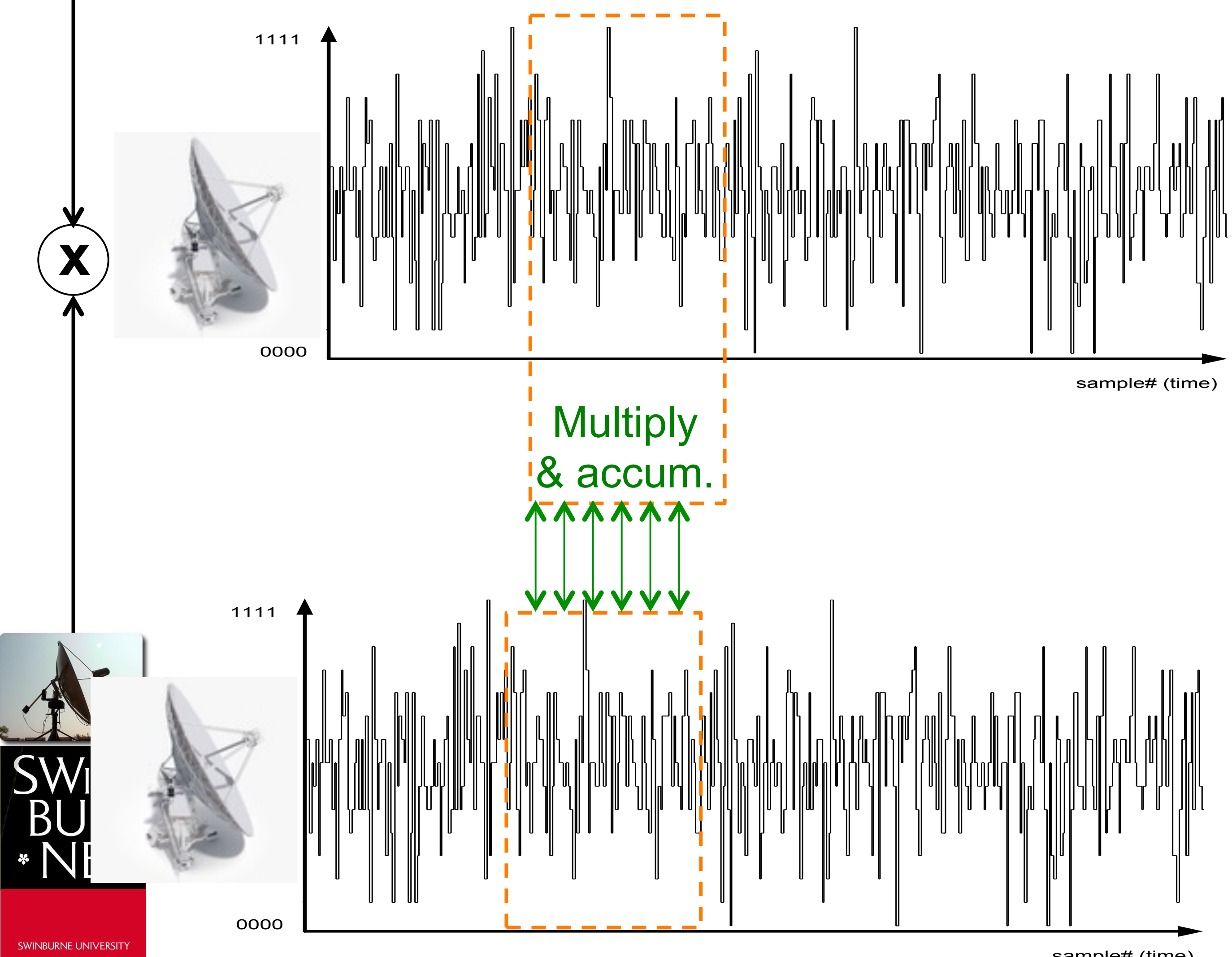

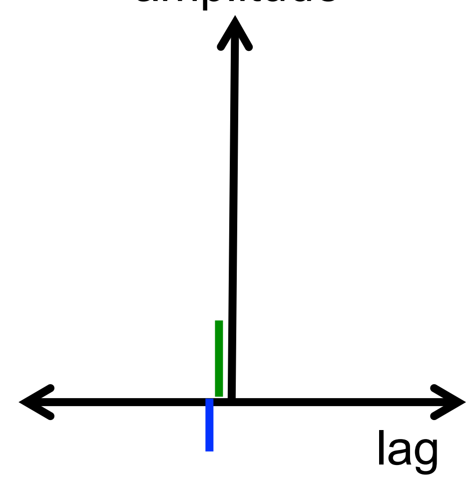

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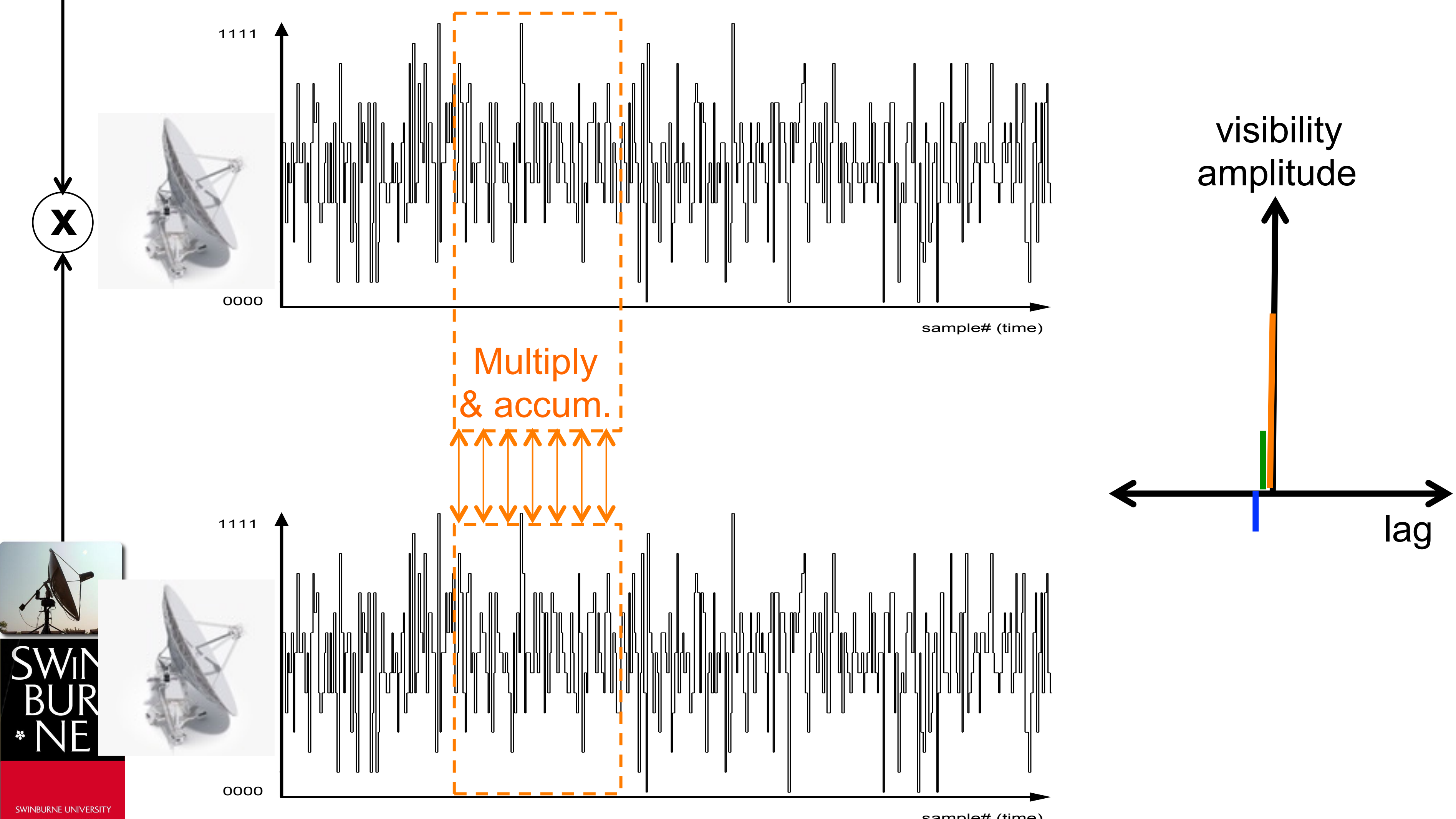
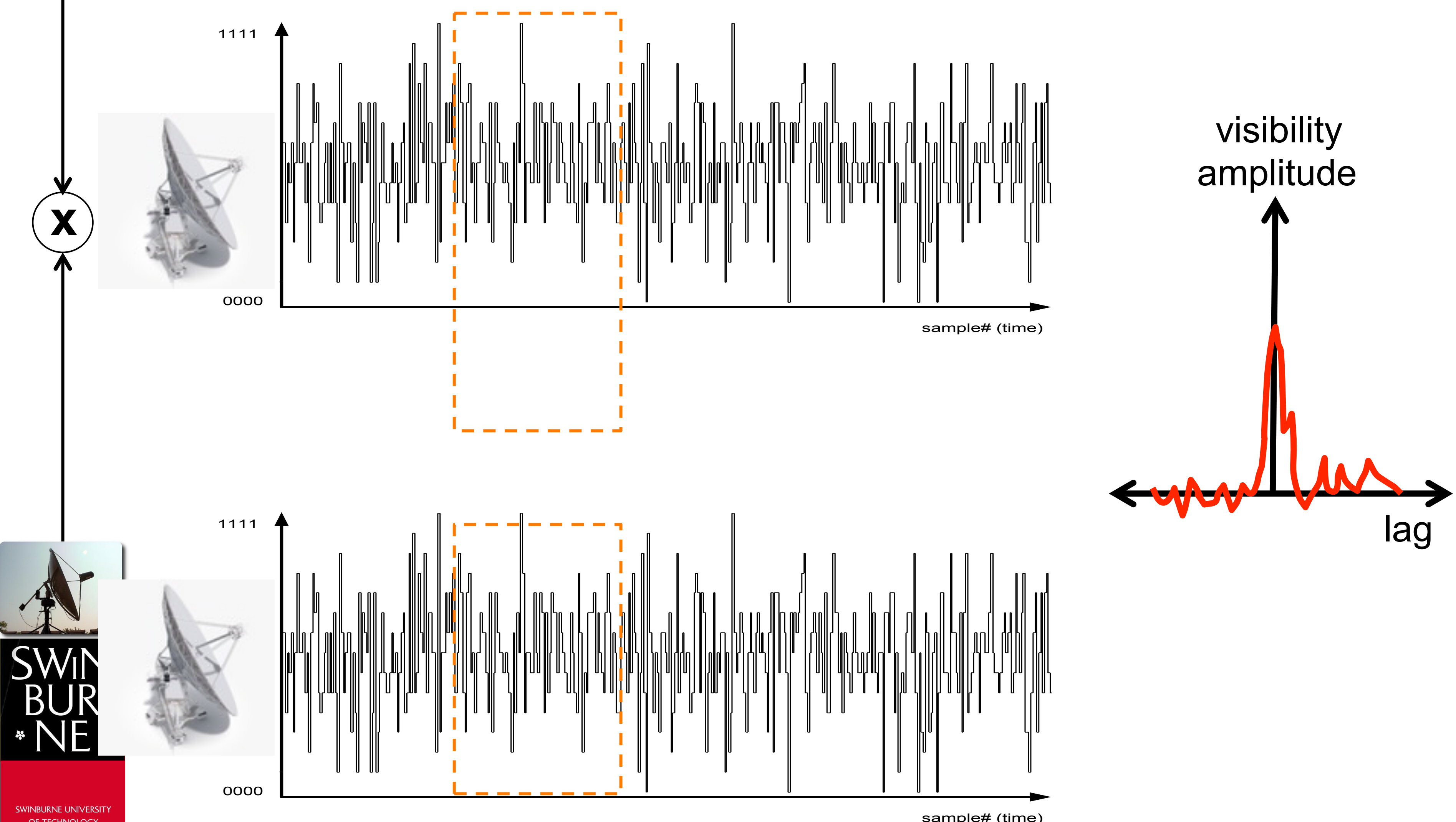
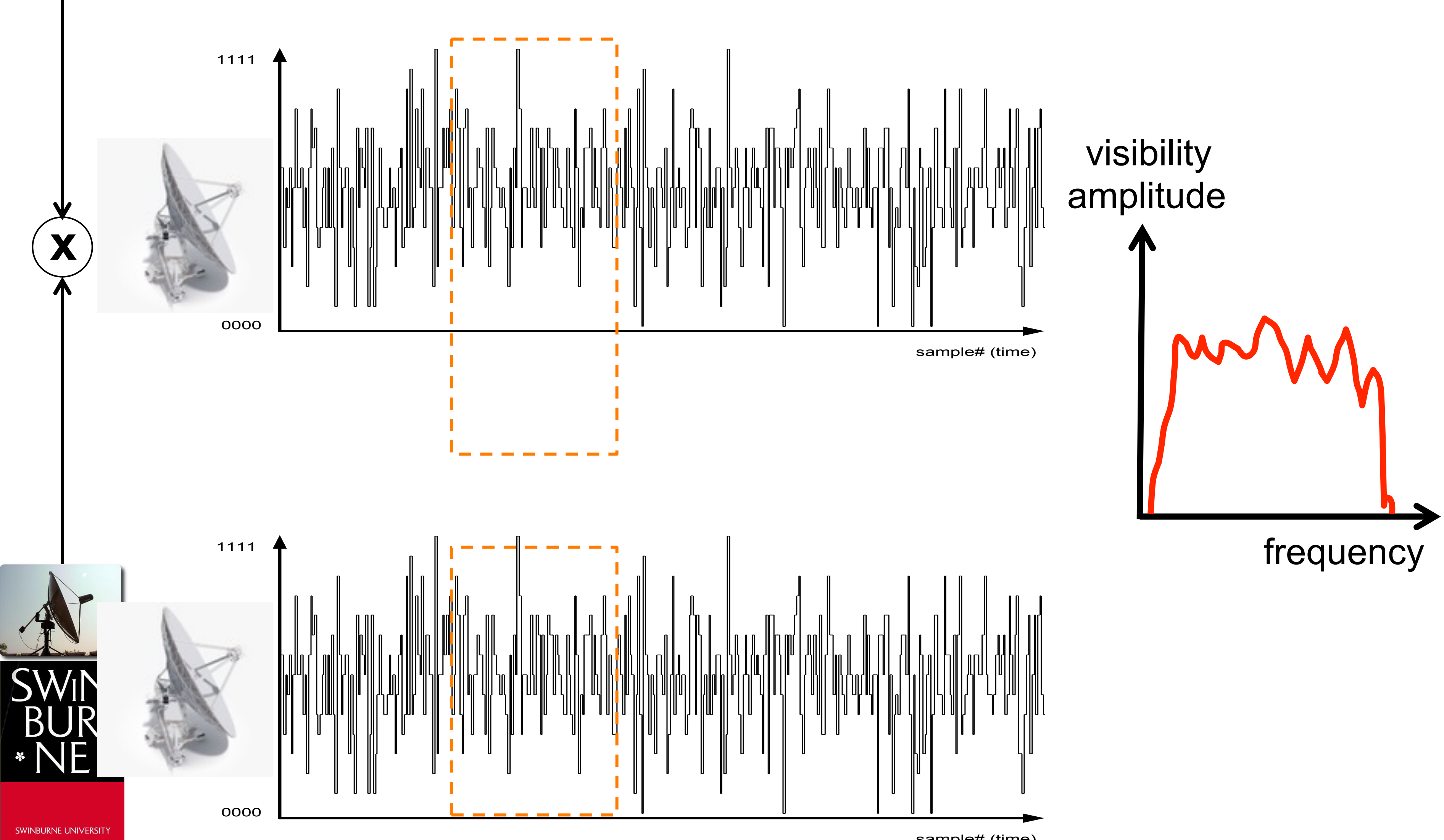


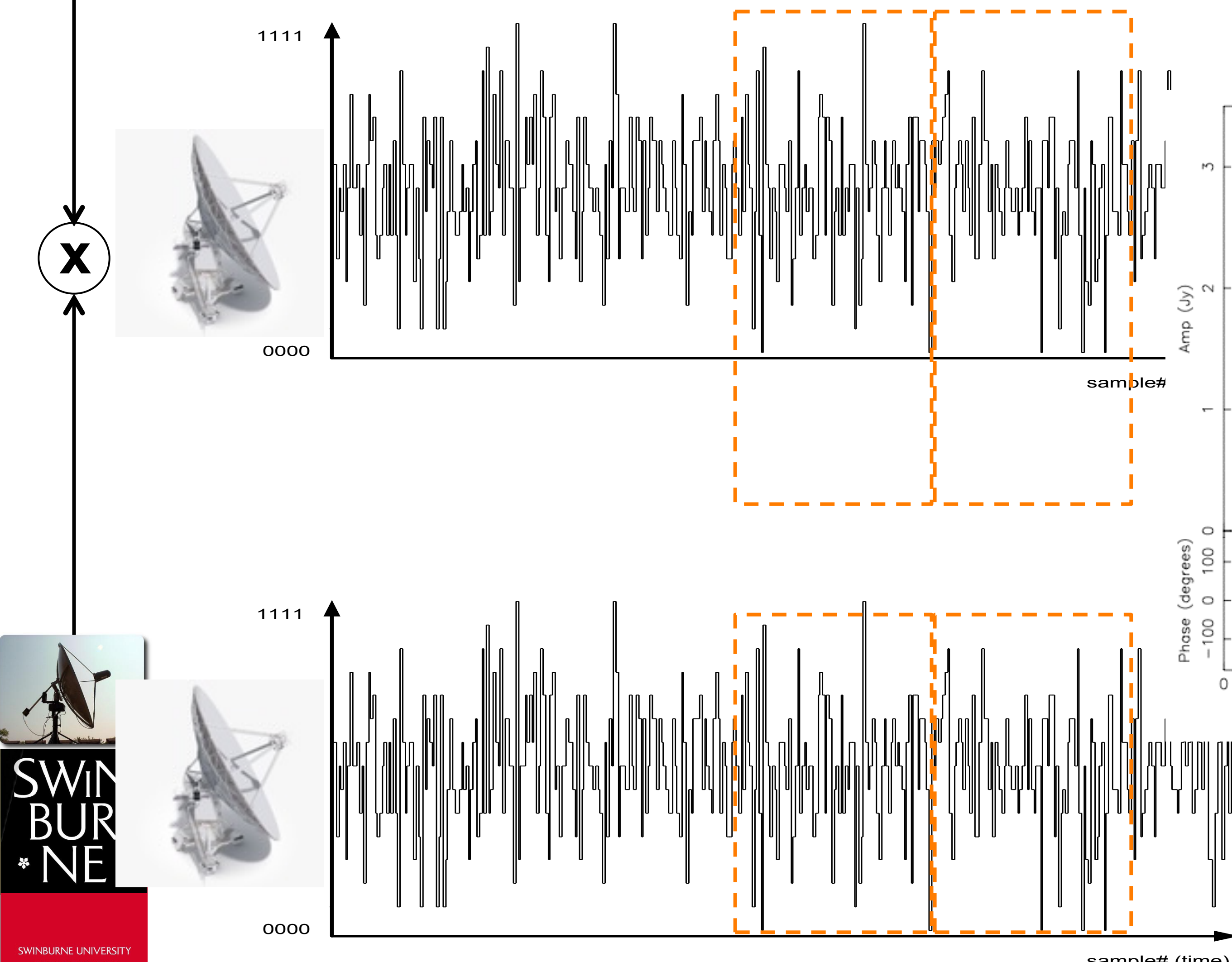
An equivalent "XF" correlator

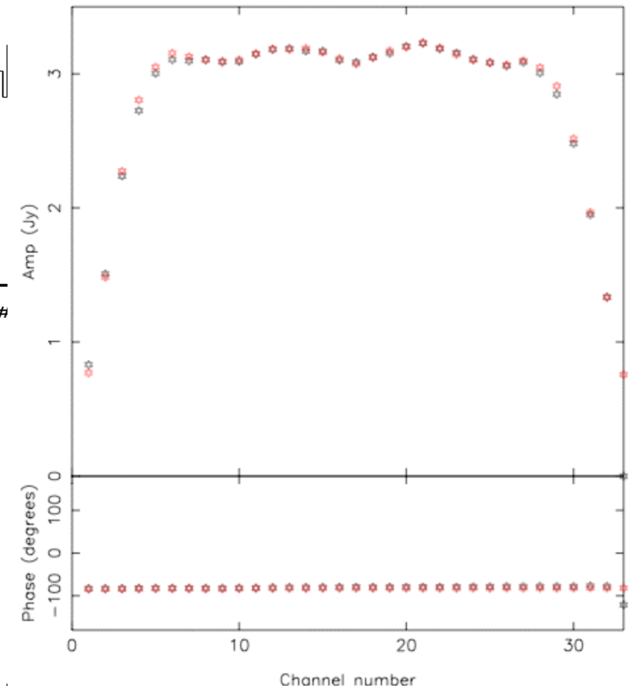


An equivalent "XF" correlator

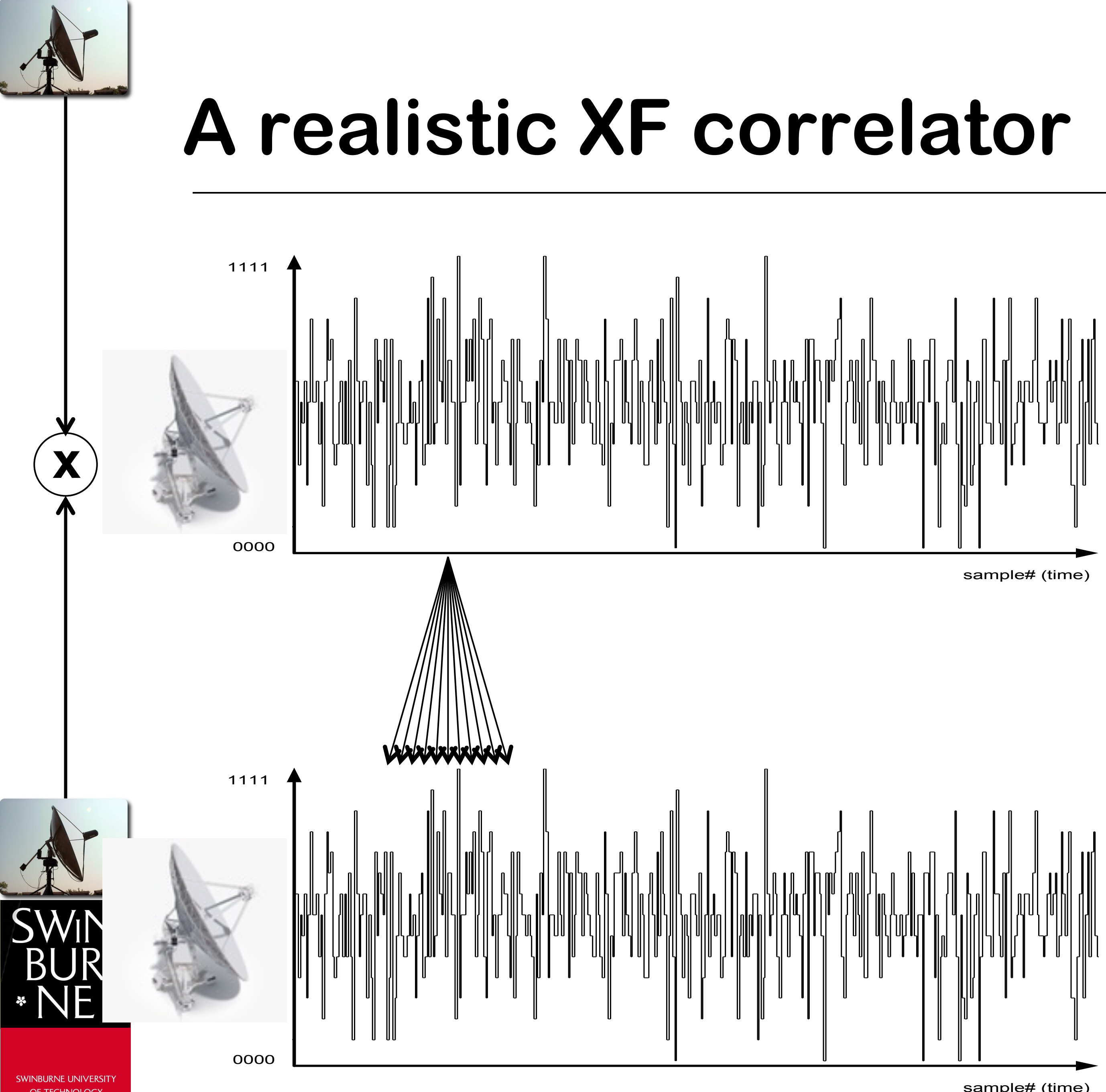


An equivalent "XF" correlator

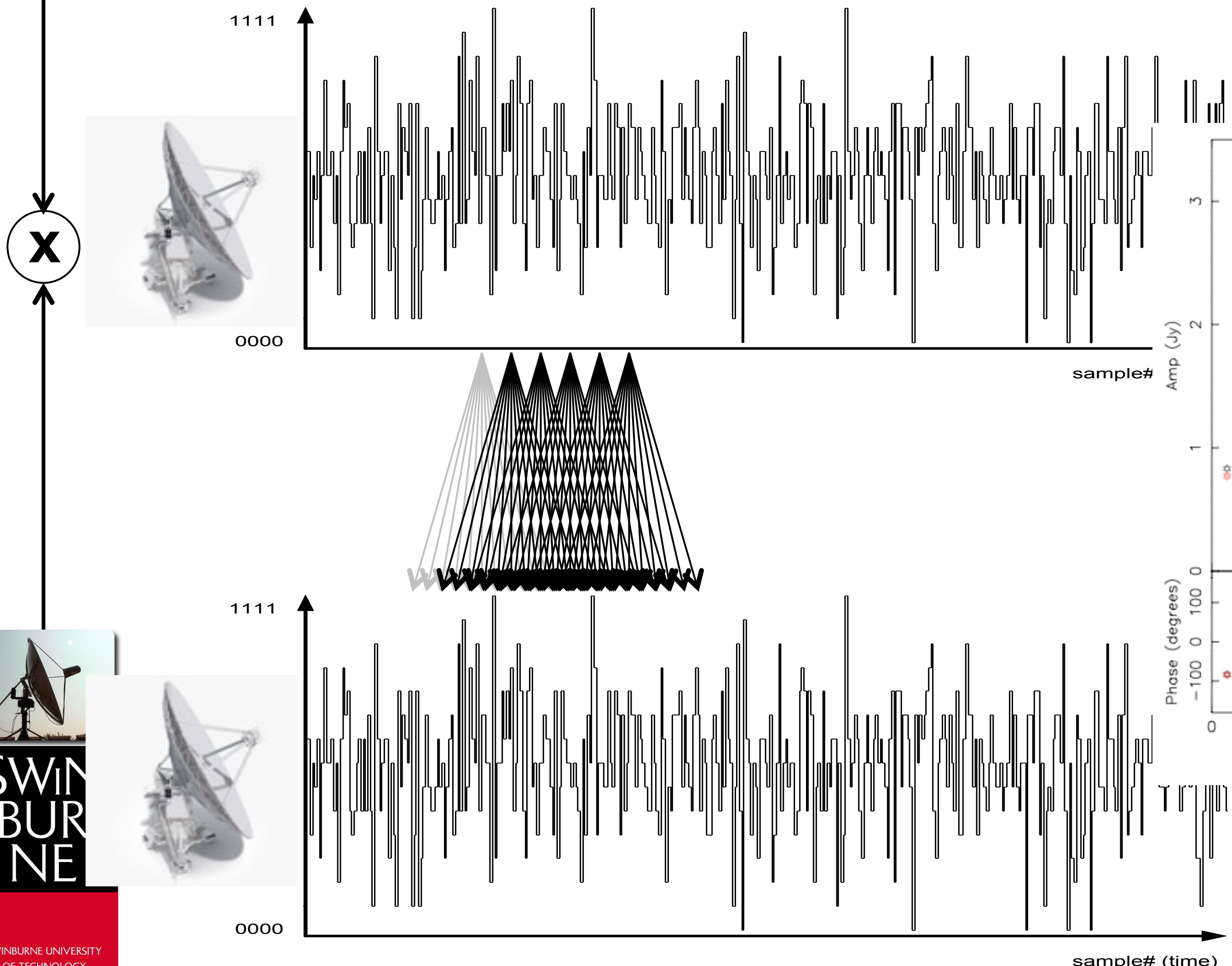


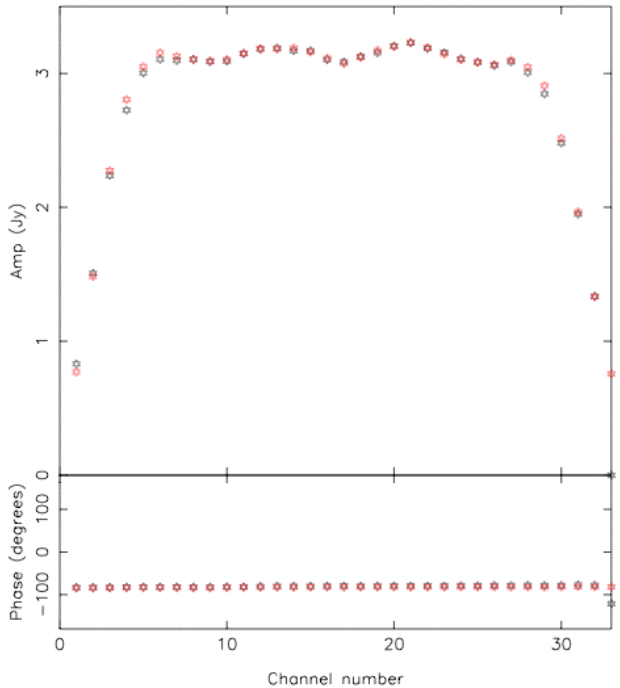


A realistic XF correlator



A realistic XF correlator





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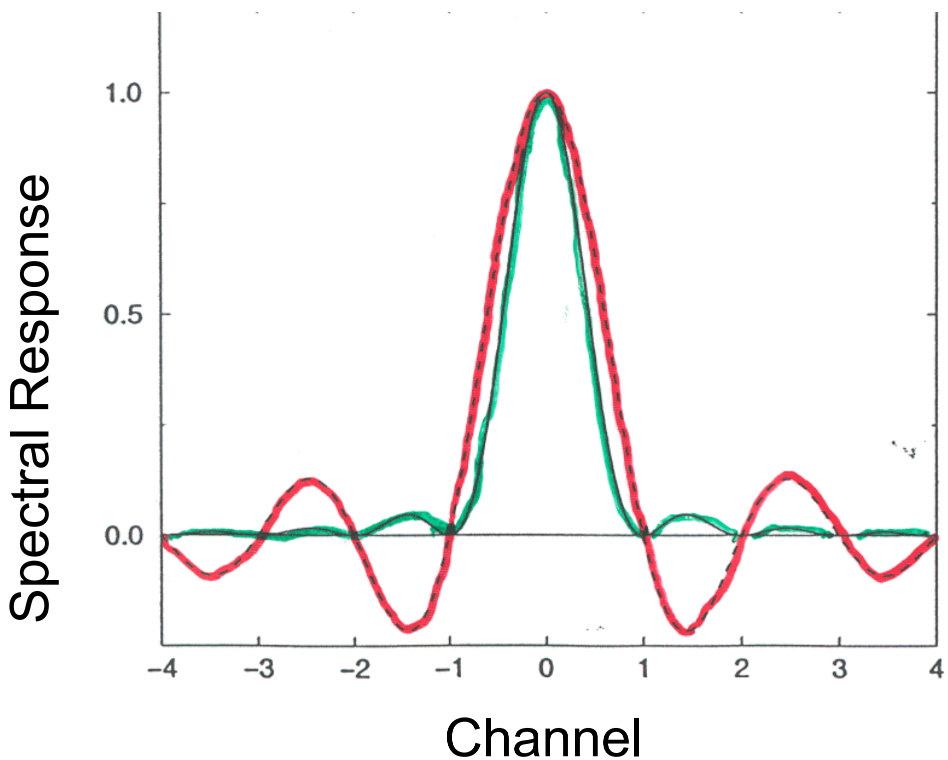
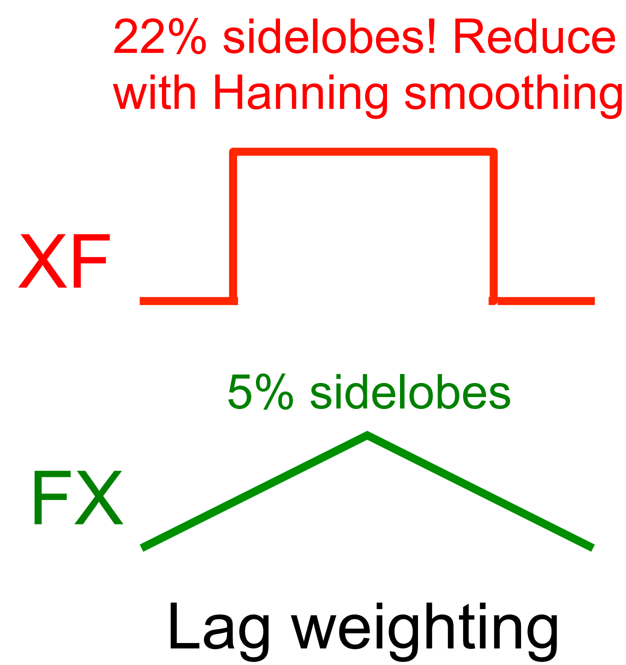
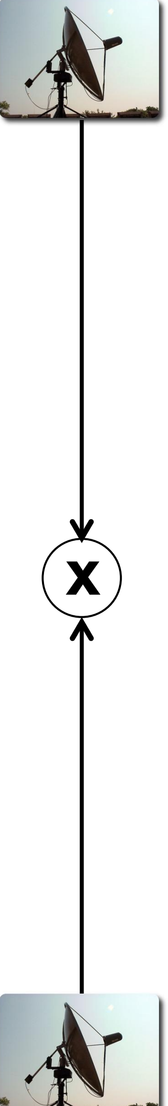
XF vs FX

- Different windowing in time domain gives different spectral response



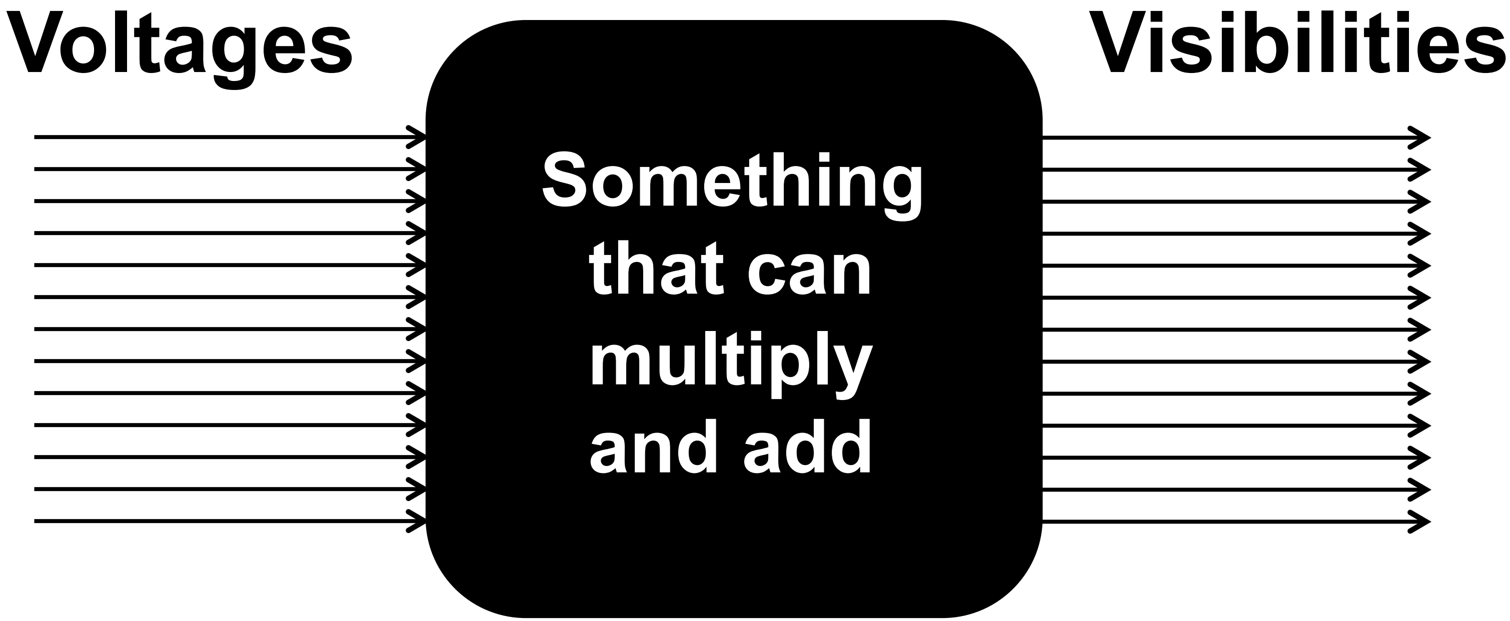




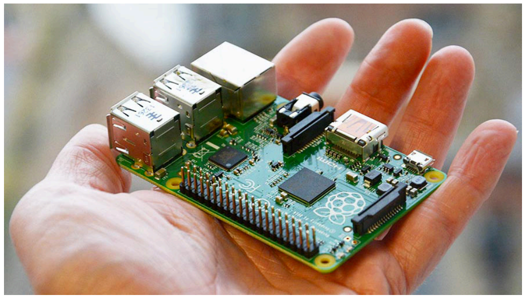
XF vs FX: which is better?

- Advantages and disadvantages to both
  - FX many fewer operations overall
  - XF can make use of very efficient low-precision integer multipliers up-front (but need special-purpose hardware)
  - FX: access to frequency domain at short timescale allows neat tricks and higher precision correction of delay effects
  - Modern correlators mostly FX-style, but use digital filterbank to channelise rather than FFT (shape channel response, contain RFI)

Correlator platforms



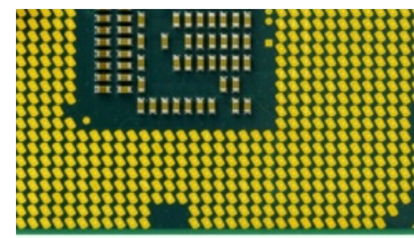
Correlators on CPUs




```
status = vectorFFT_CtoC_cf32(complexunpacked, fftd, pFFTSpecC, fftbuffer);
if(status != vecNoErr)
  csevere << startl << "Error doing the FFT!!!" << endl;

     ...

status = vectorAddProduct_cf32(vis1, vis2, &(scratchspace->threadcrosscorrs[resul
```



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Correlators on CPUs

- Many positive points:
  - Can implement in "normal" code (e.g., C++); maintainable, many skilled coders
  - Development effort transferrable across generations of hardware
  - Incremental development is trivial
  - Natively good at floating point (good for FX), no cost to do high precision
- One major disadvantage:
  - CPUs not optimised for correlation; big system like JVLA would take **many** CPUs.

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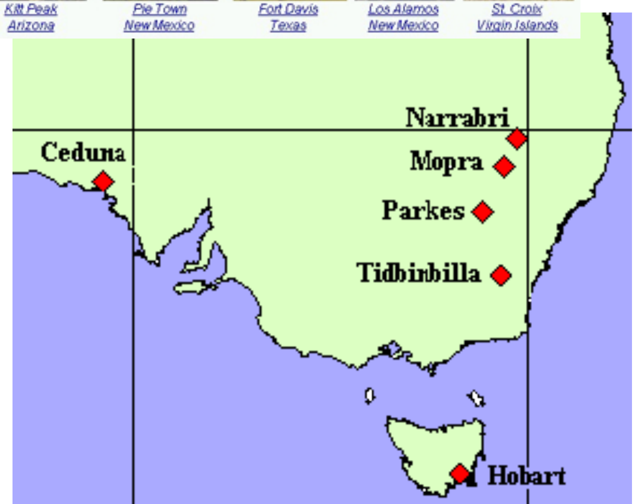
Correlators on CPUs

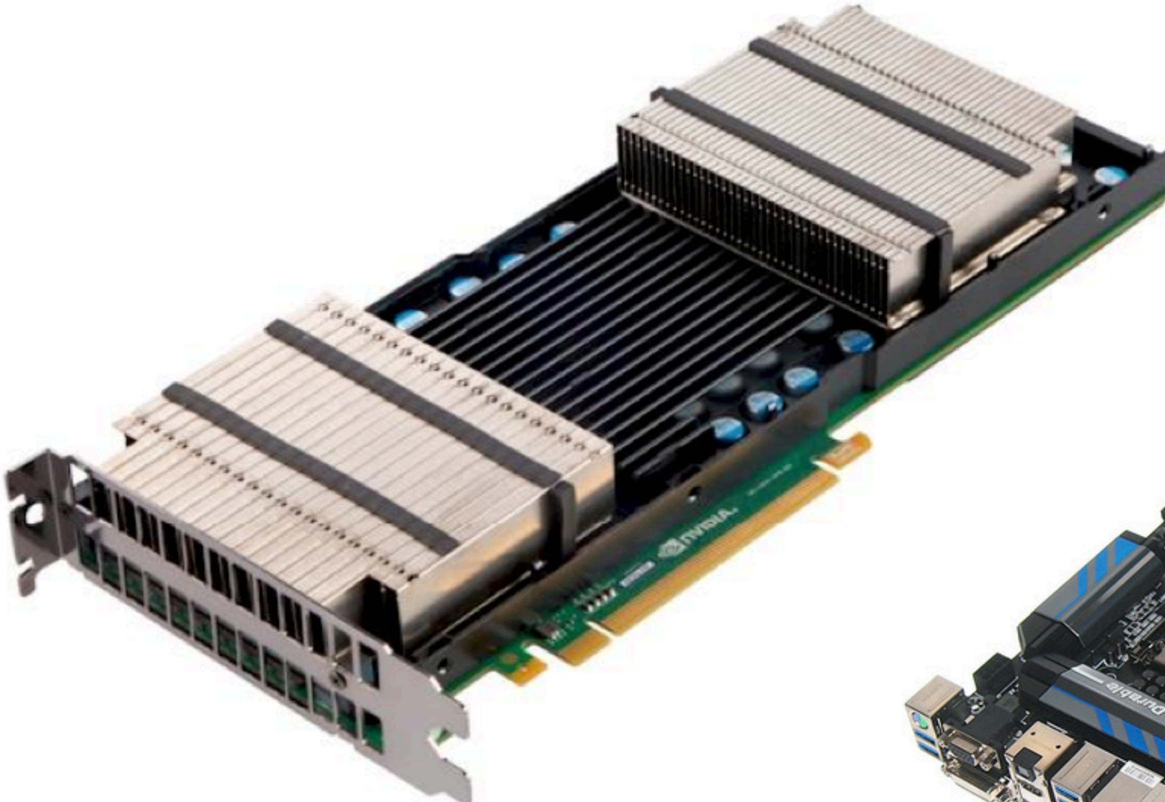
The Very Long Baseline Array, 10 stations

The European VLBI Network, ~20 stations



The Long Baseline Array, Australia, ~6 stations

Correlators on GPUs

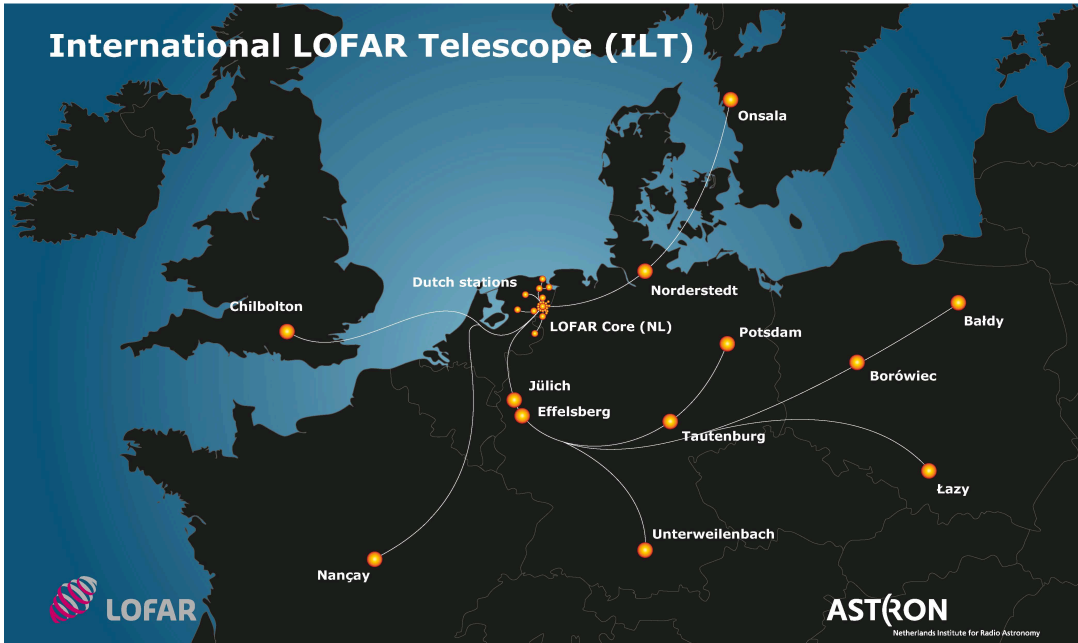


Like CPUs, GPUs are mounted on a standard motherboard

Correlators on GPUs

- Advantages:
  - More powerful and more efficient than CPUs
  - Also good at floating point
- Disadvantages:
  - Writing code is more difficult (GPUs are more specialized, less flexible: need to carefully manage data transfers)
  - Fewer trained GPU programmers available
  - Transfer-ability of code across hardware generations harder (capabilities change faster, need new code to use)

Correlators on GPUs



The Low Frequency Array (LOFAR), 73 stations

GMRT, India, 30 stations

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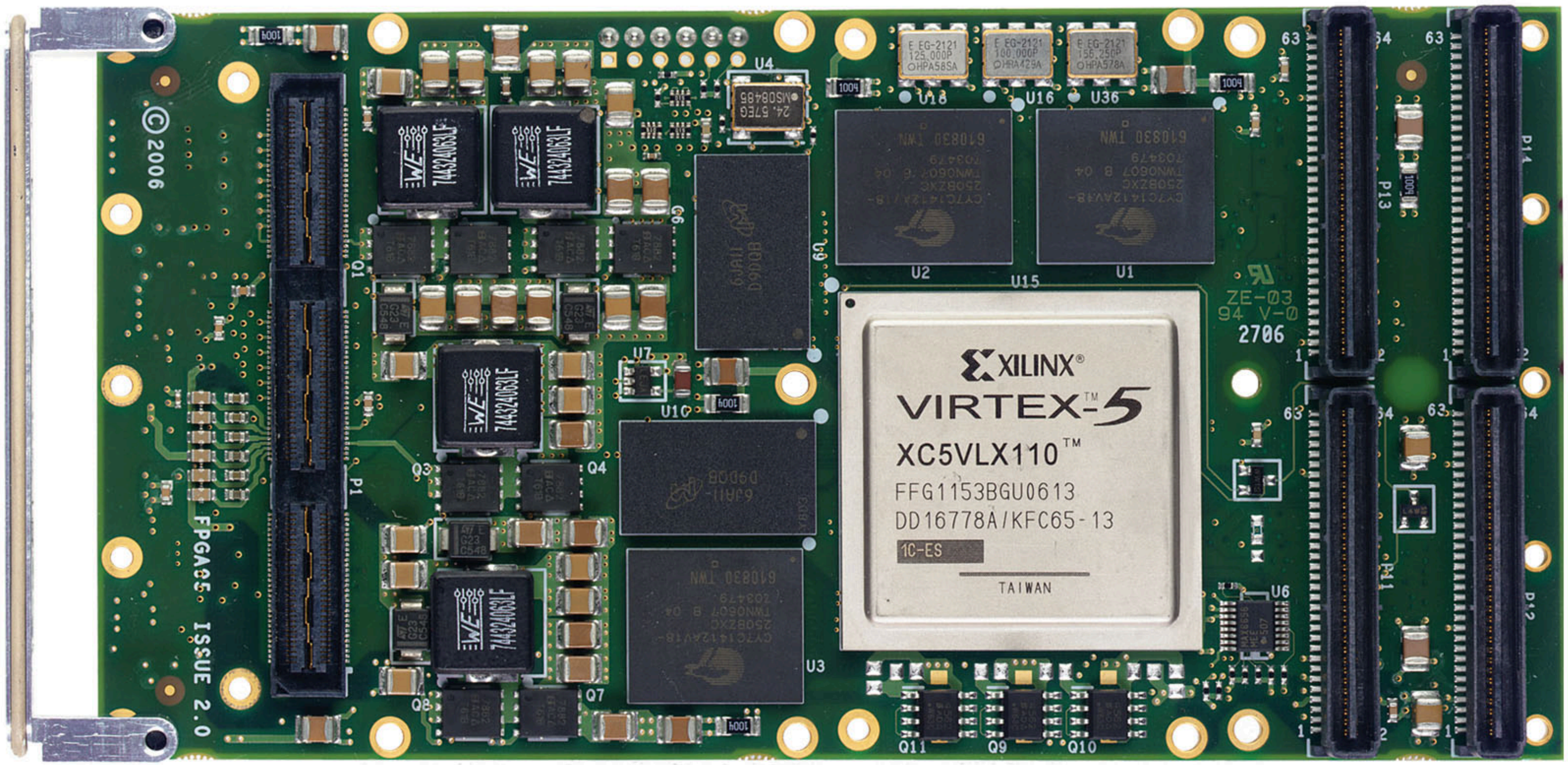
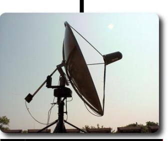
Correlators on FPGAs



Correlators on FPGAs

- Advantages:
  - More efficient than CPUs or GPUs, particularly for integer multiplication
- Disadvantages:
  - Programming is harder again (especially debugging), yet fewer trained people
  - Transfer-ability across hardware generations even more limited
  - Synchronous (clocked) system, less robust to perturbations c.f. CPUs/GPUs



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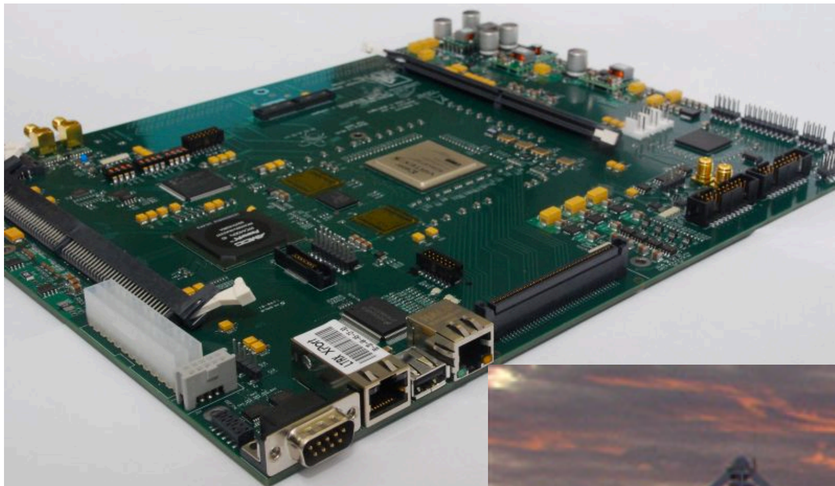
Correlators on FPGAs



“Roach” reconfigurable FPGA board used for correlation

MeerKAT, 64 dishes, under construction

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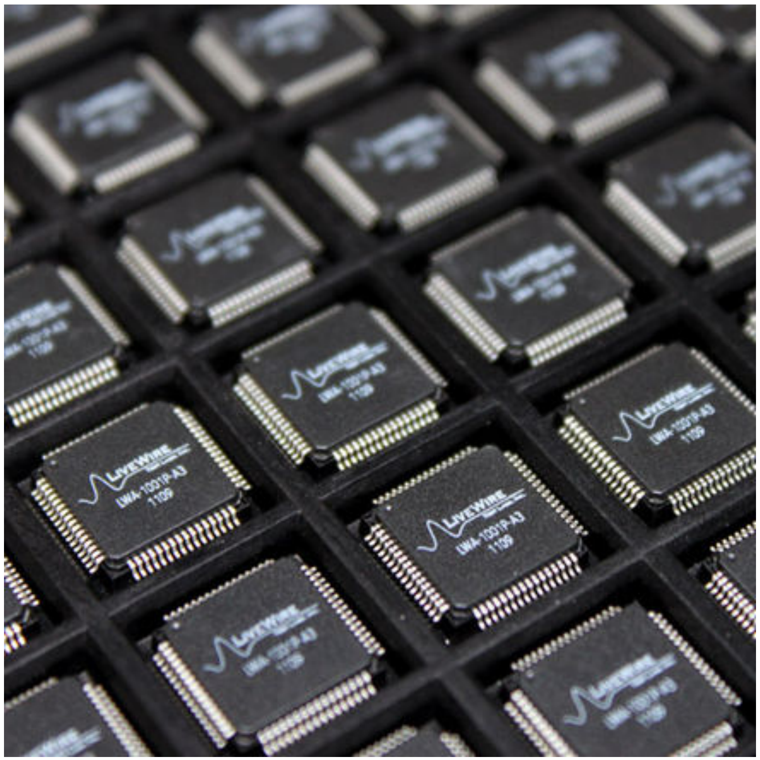
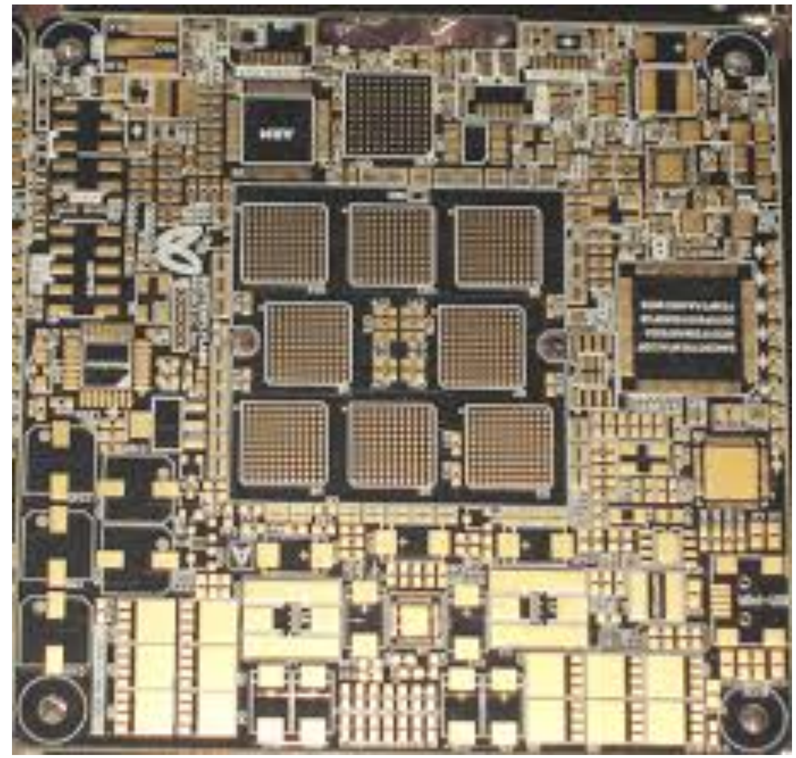
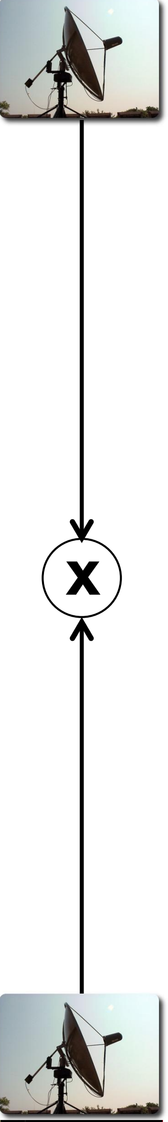
Correlators on ASICs





As with FPGAs, ASICs are mounted on boards



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Correlators on ASICs

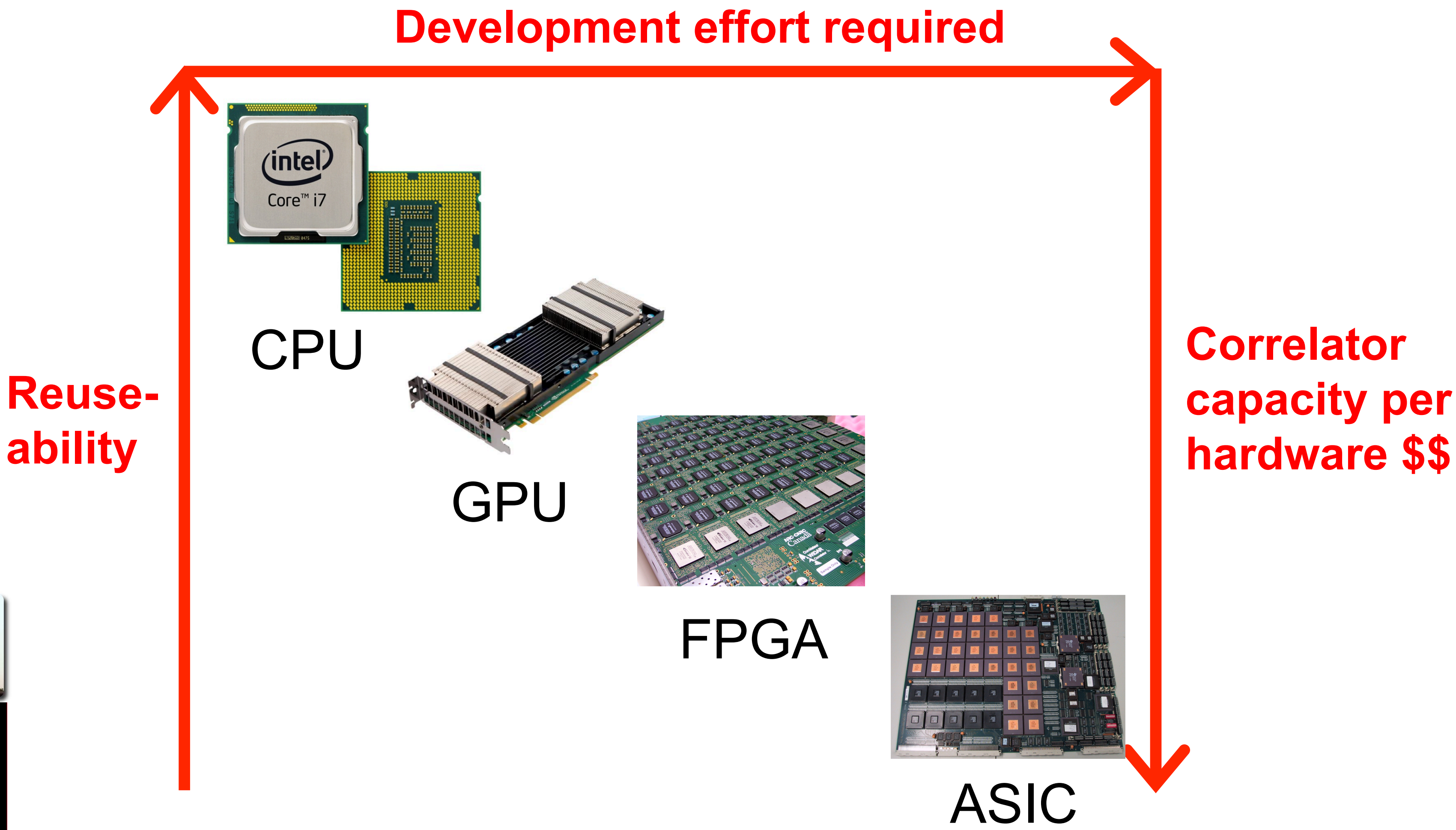
- Advantages:
  - Highest possible efficiency, low per-unit cost
- Disadvantages:
  - Highest development cost (time and manufacturing setup)
  - Specialized knowledge required
  - Can’t be changed / very difficult to upgrade during lifetime

Correlators on ASICs

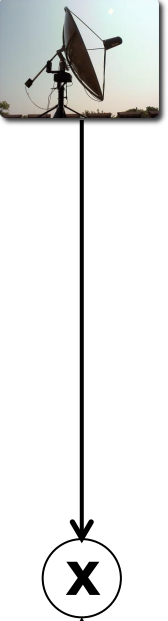
The Westerbork Synthesis Radio Telescope, Netherlands

The Very Large Array, New Mexico

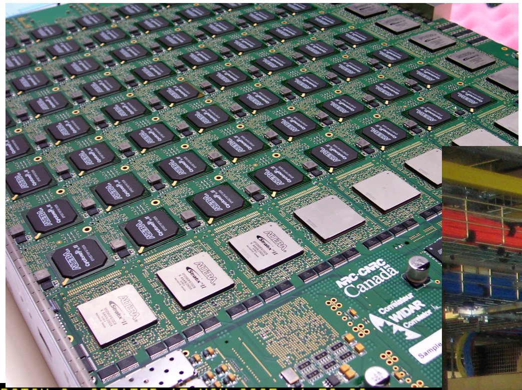
Correlator platform overview



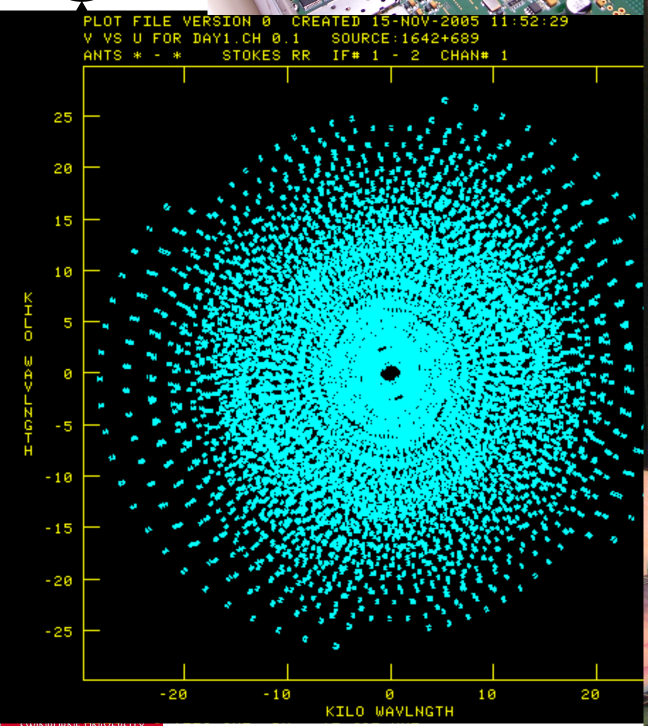
The end



X



NRC-CNRC Canada



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KILO WAVLNGTH