# Imaging and Deconvolution

David J. Wilner (Harvard-Smithsonian Center for Astrophysics)

Sixteenth Synthesis Imaging Workshop

16-23 May 2018

thanks to
M. MacGregor
L. Matra

NRAO ALMA AUI NSF New Mexico Tech SCIENCE · ENGINEERING · RESEARCH · UNIVERSITY The University of New Mexico New Mexico CONSORTIUM

# Overview

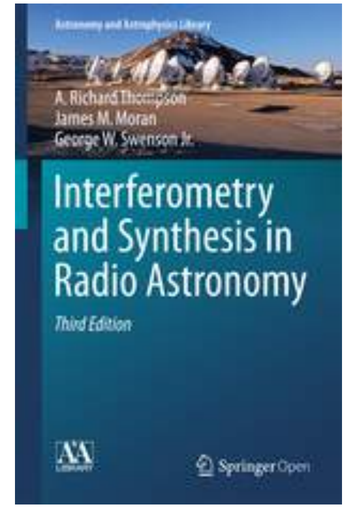
- gain intuition about interferometric imaging
- understand the need for deconvolution

topics

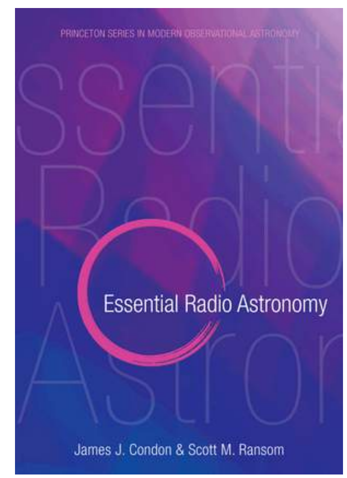
- get comfortable with Fourier Transforms
- review "visibility" concept and sampling the u,v plane
- formal description of imaging
- imaging in practice
- deconvolution and the clean algorithm

# References

- Thompson, A.R., Moran, J.M. & Swensen, G.W. 2017, "Interferometry and Synthesis in Radio Astronomy" 3rd edition
  - open access: download pdf on link.springer.com (free!)

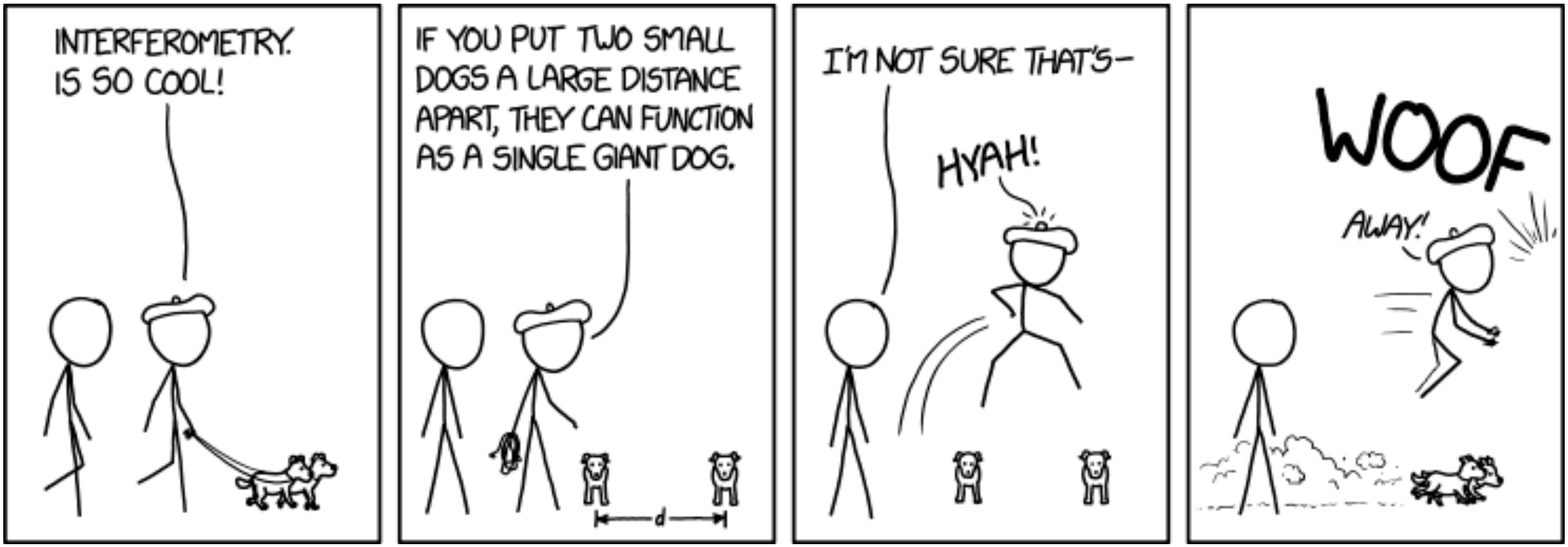



- past NRAO Synthesis Imaging Workshop proceedings
  - Perley, R.A., Schwab, F.R., Bridle, A.H., eds. 1989, ASP Conference Series 6, "Synthesis Imaging in Radio Astronomy"
  - lecture slides: *www.aoc.nrao.edu/events/synthesis*
- IRAM 2000 Interferometry School proceedings
  - *www.iram.fr/IRAMFR/IS/IS2008/archive.html*
- Condon, JJ & Ransom, S.M. 2016, "Essential Radio Astronomy", a complete one semester course, on-line at
  - *science.nrao.edu/opportunities/courses/era*




plus many other useful pedagogical presentations, e.g. ALMA Primer

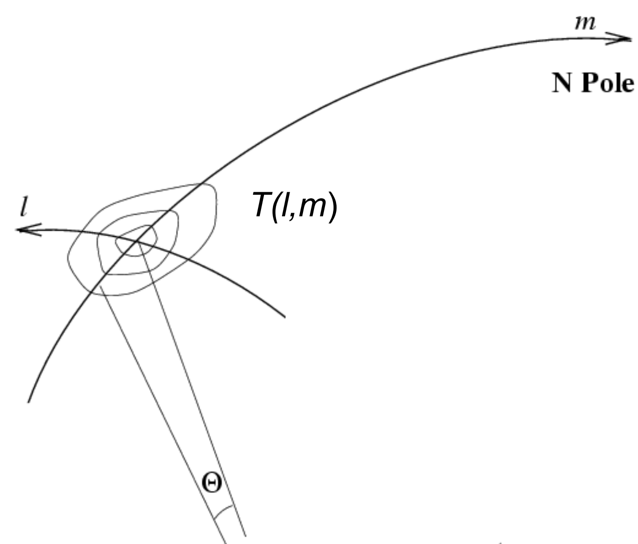
# xkcd.com/1922/




It is important to note that while the size of dog can be arbitrarily large, it's not any more of a good dog than the two original dogs.

# Visibility and Sky Brightness

*V(u,v), the complex visibility function, is the 2D Fourier transform of T(l,m), the sky brightness distribution (for an incoherent source, small field of view, far field, etc.)* [see TMS for derivation]




mathematically

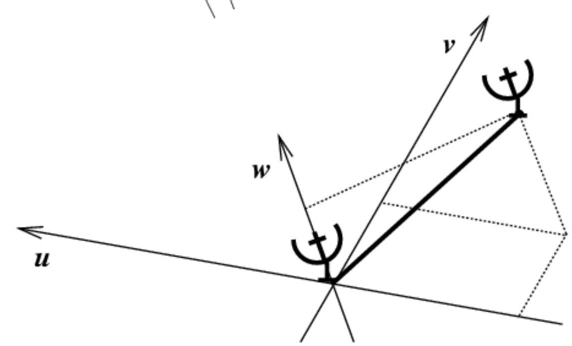
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

$$T(l,m) = \int\int V(u,v)e^{i2\pi(ul+vm)}dudv$$

*u,v* are E-W, N-S spatial frequencies [wavelengths]

*l,m* are E-W, N-S angles in the tangent plane [radians]

(recall $e^{ix} = \cos x + i \sin x$)

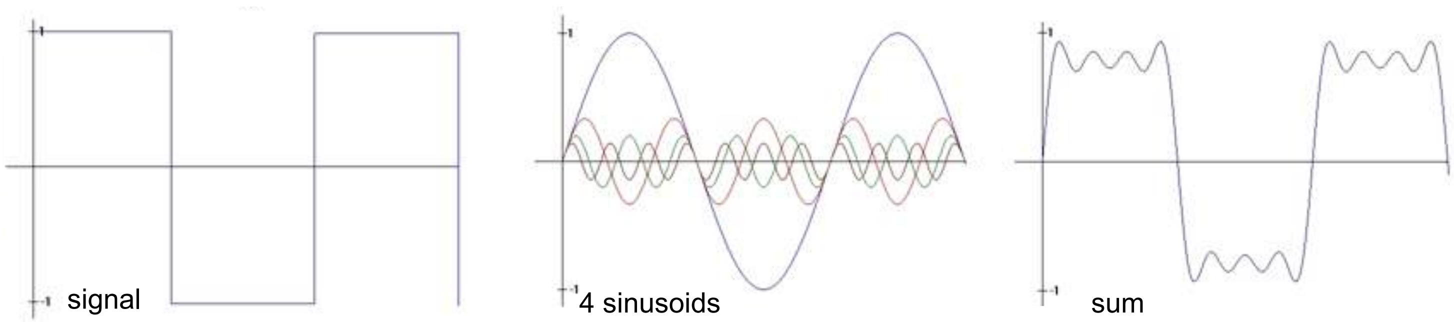



$$V(u,v) \xrightarrow{\mathcal{F}} T(l,m)$$

# The Fourier Transform

Jean Baptiste
Joseph Fourier
1768-1830

- Fourier theory: any well behaved signal (including images) can be expressed as the sum of sinusoids




$$x(t) = \frac{4}{\pi}\left(\sin(2\pi f t) + \frac{1}{3}\sin(6\pi f t) + \frac{1}{5}\sin(10\pi f t) + \cdots\right)$$

- the Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components
- the Fourier transform contains *all* of the information of the original signal

# The Fourier Domain

- acquire some comfort with the Fourier domain
  - in older texts, functions and their Fourier transforms occupy *upper* and *lower* domains, as if "functions circulated at ground level and their transforms in the underworld" (Bracewell 1965)

- some properties of the Fourier transform $g(x) \xrightarrow{\mathcal{F}} G(s)$

  adding $g(x) + h(x) = G(s) + H(s)$

  scaling $g(\alpha x) = \alpha^{-1} G(s/\alpha)$

  shifting $g(x - x_0) = G(s) e^{i 2\pi x_0 s}$

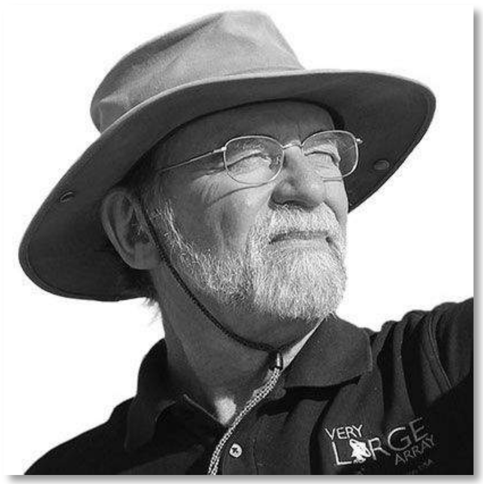
  convolution/multiplication $g(x) = h(x) * k(x) \quad G(s) = H(s)K(s)$

  Nyquist-Shannon sampling theorem $g(x) \subset \Theta$ completely determined if $G(s)$ sampled at $\leq 1/\Theta$

# Visibilities

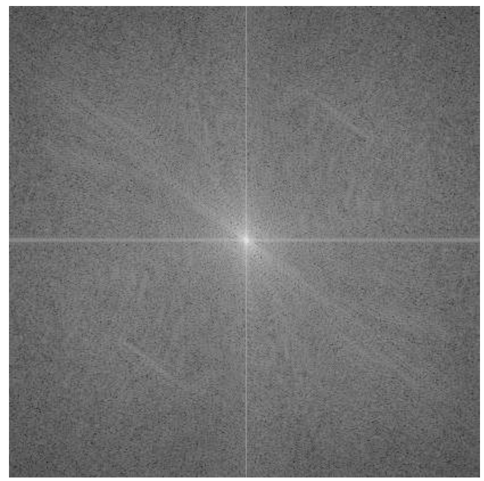
- each *V(u,v)* is a complex quantity
  - expressed as *(real, imaginary)* or *(amplitude, phase)*

*T(l,m)*

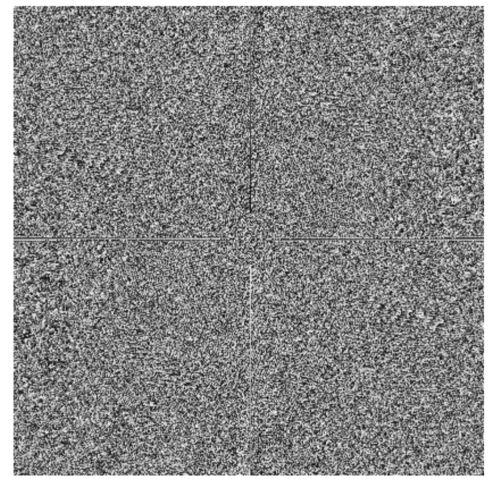


$\xrightarrow{\mathcal{F}}$

*V(u,v) amplitude*

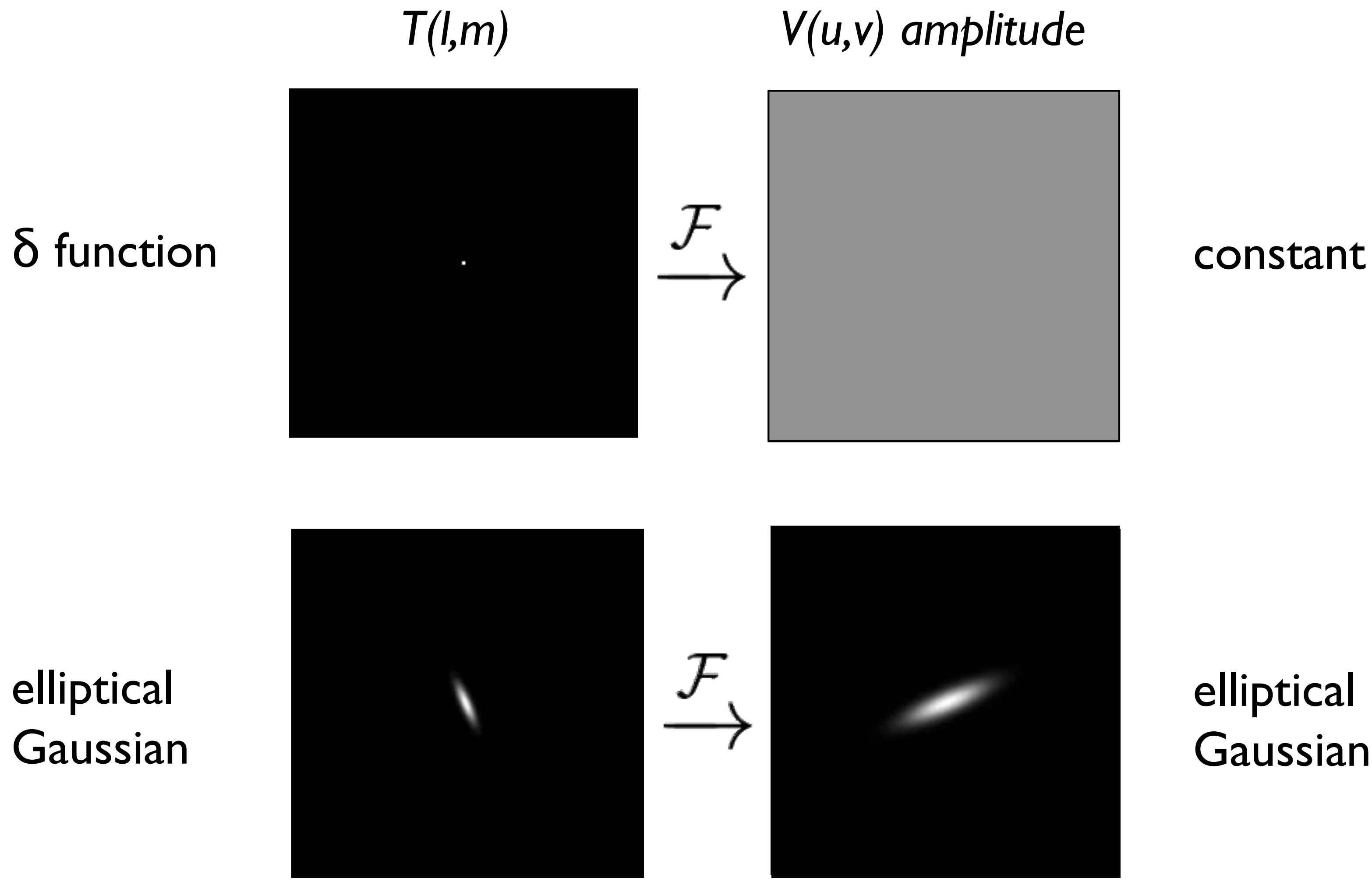


*V(u,v) phase*



- each *V(u,v)* contains information on *T(l,m) everywhere*, not just at a given *(l,m)* coordinate or within a particular subregion

# Example 2D Fourier Transforms

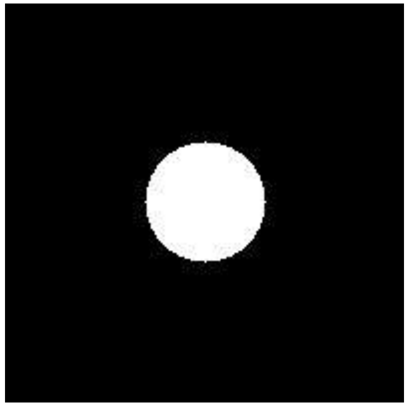



*narrow features transform into wide features (and vice-versa)*

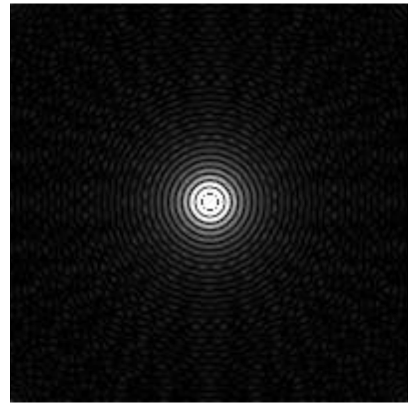
# Example 2D Fourier Transforms

*T(l,m)* *V(u,v) amplitude*

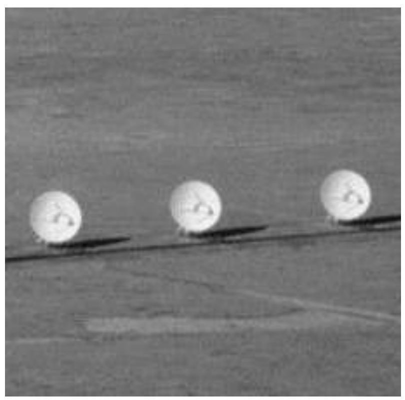
uniform disk



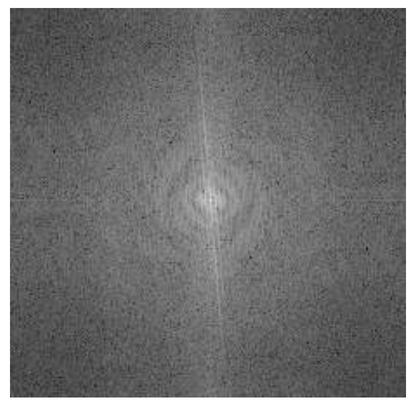
$\xrightarrow{\mathcal{F}}$



Bessel function



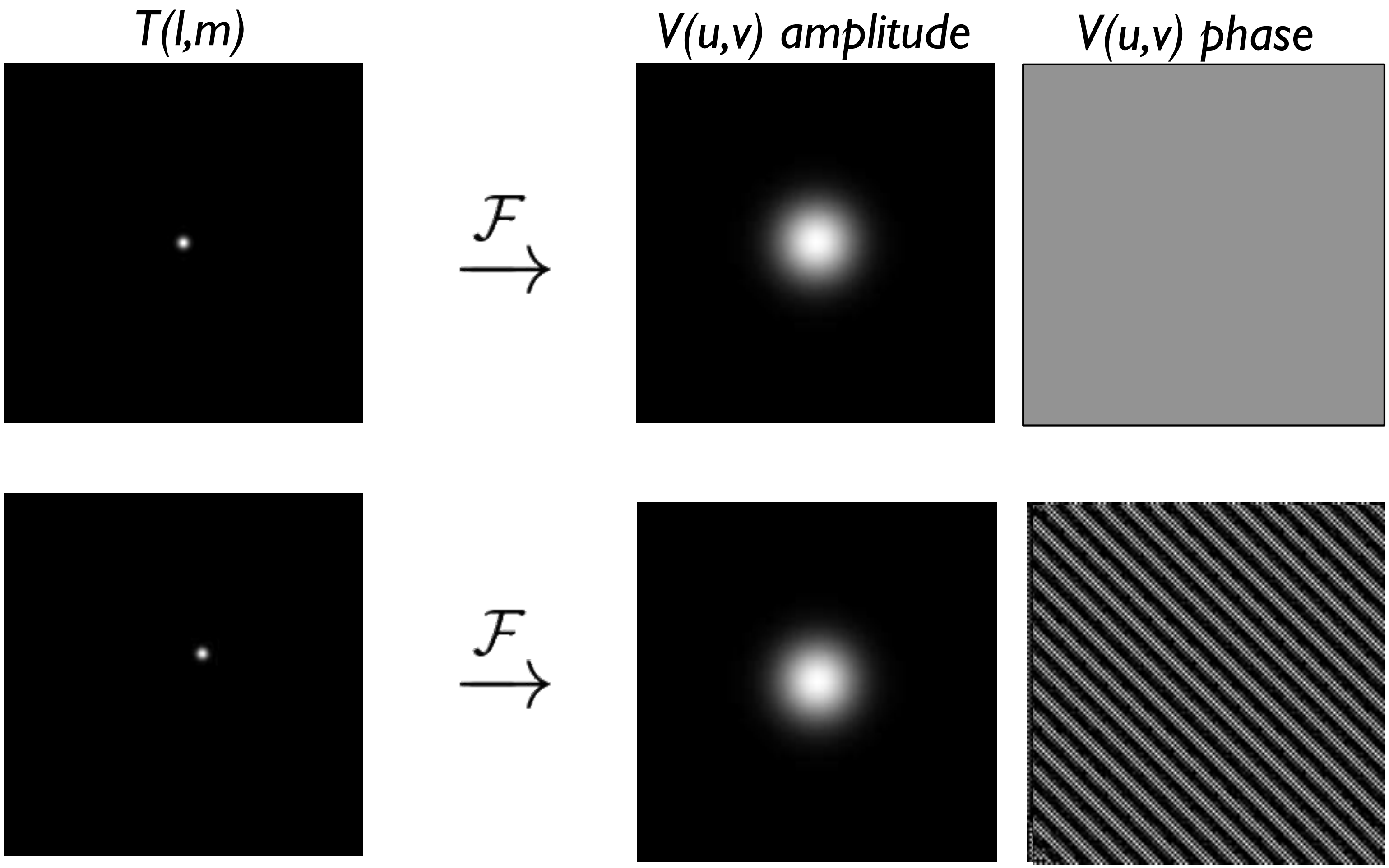
$\xrightarrow{\mathcal{F}}$



*sharp edges result in many high spatial frequencies*

# Amplitude and Phase

- amplitude tells "how much" of a certain spatial frequency
- phase tells "where" this spatial frequency component is located



# The Visibility Concept

$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

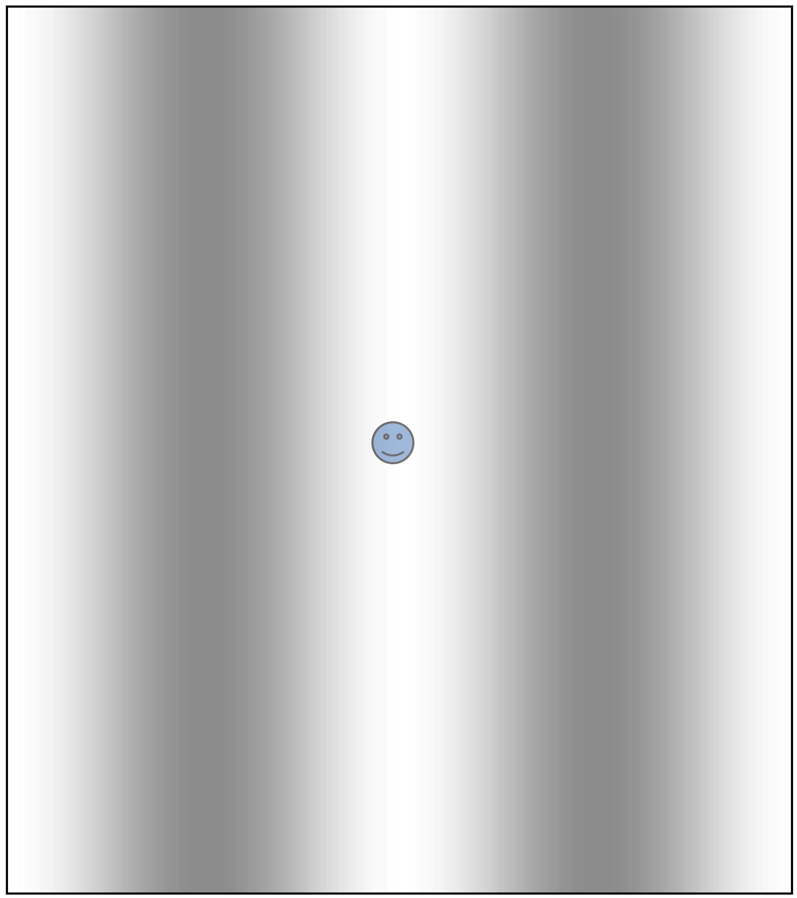
- visibility as a function of baseline coordinates *(u,v)* is the Fourier transform of the sky brightness distribution as a function of the sky coordinates *(l,m)*

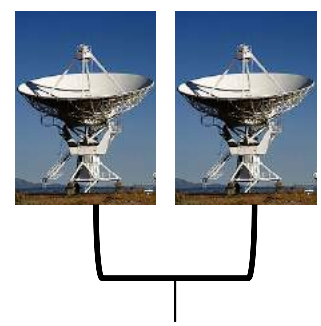
- since $T(l,m)$ is real, $V(u,v)$ is Hermitian and $V(-u,-v) = V^*(u,v)$ (get two visibilities for each *(u,v)* measurement)

- $V(u=0,v=0)$ is the integral of $T(l,m)dldm$ = total flux density

# Small Source, Short Baseline

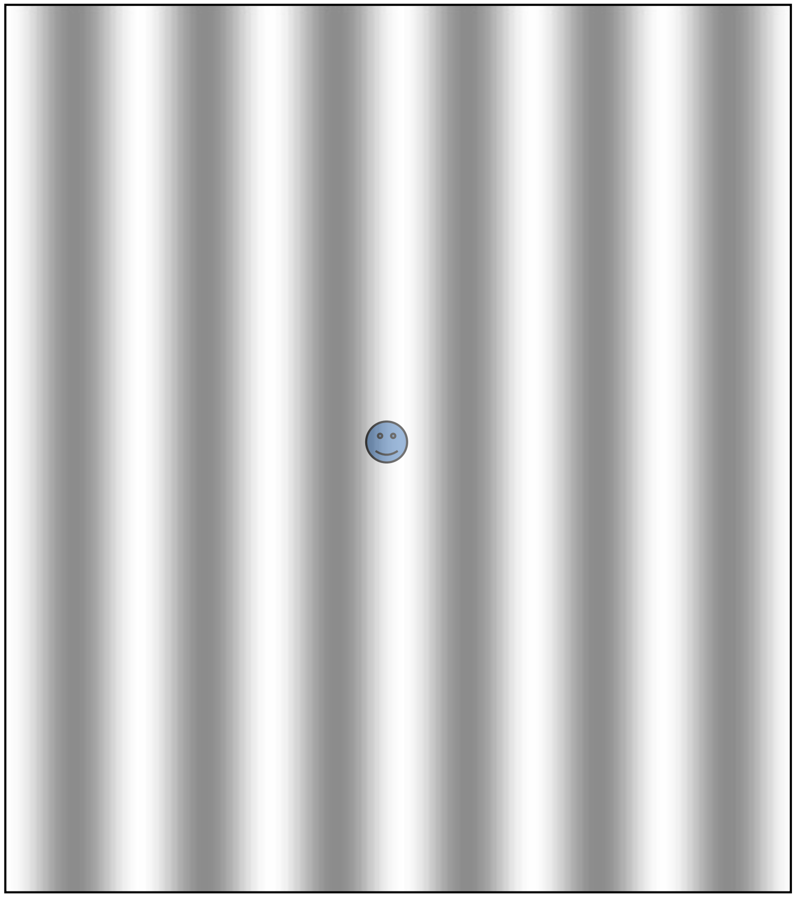
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

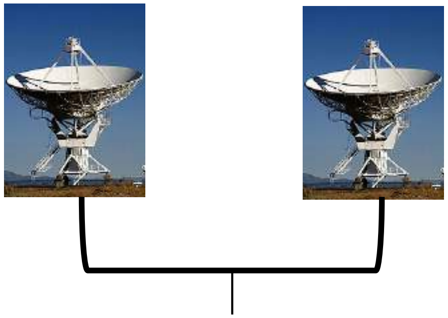




# Small Source, Long Baseline

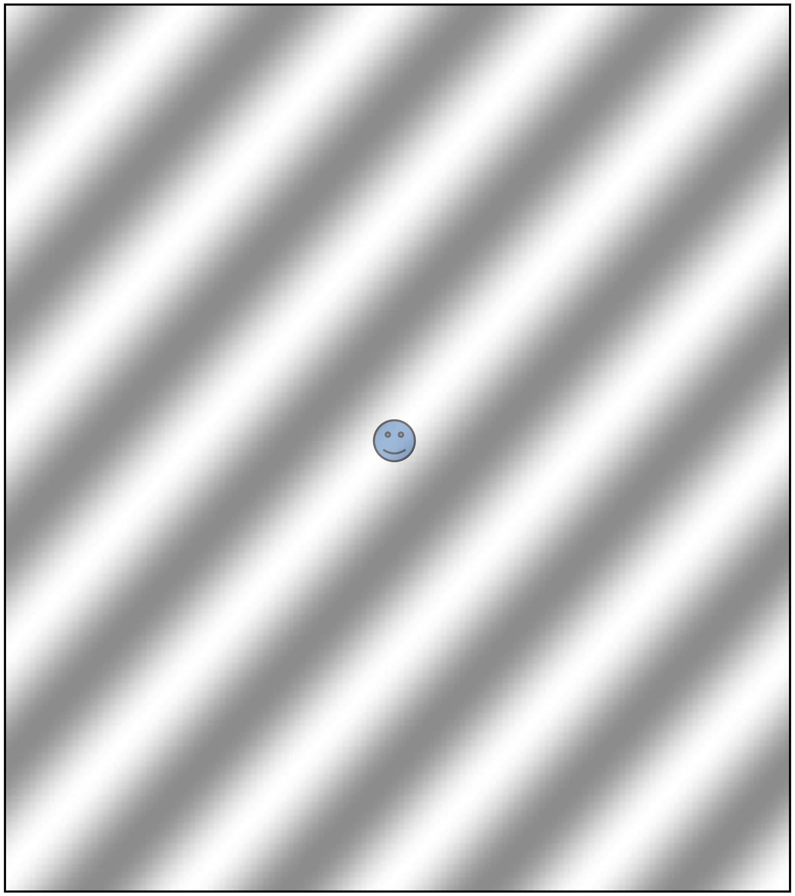
$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$

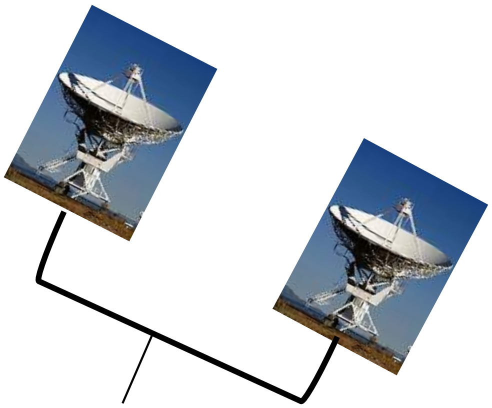




# Small Source, Long Baseline

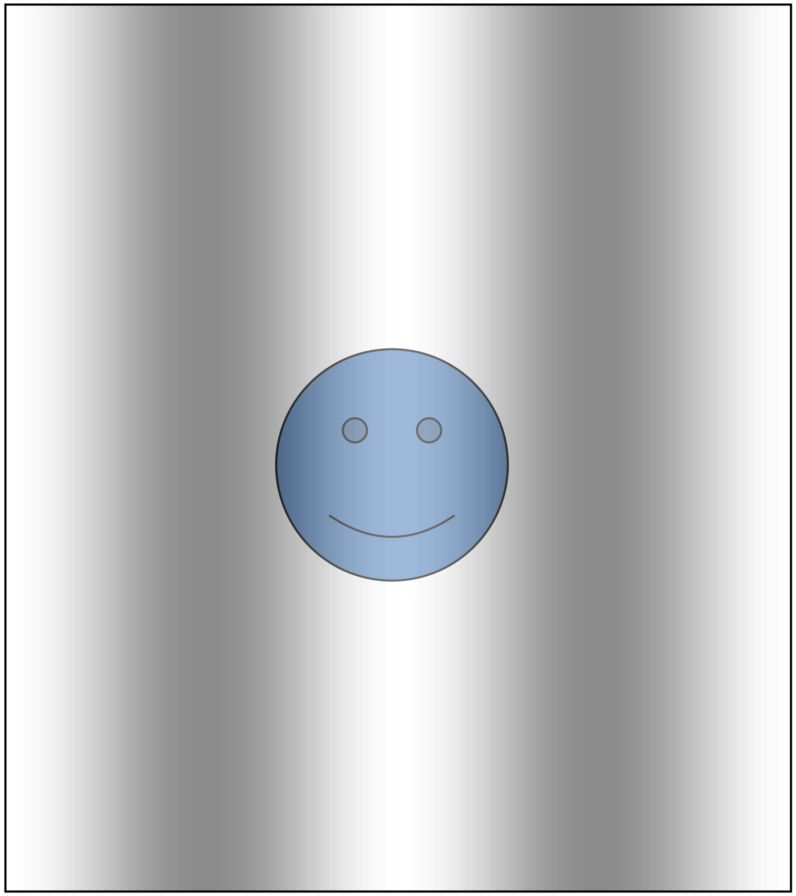
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

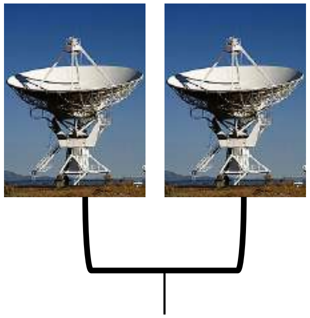




# Extended Source, Short Baseline

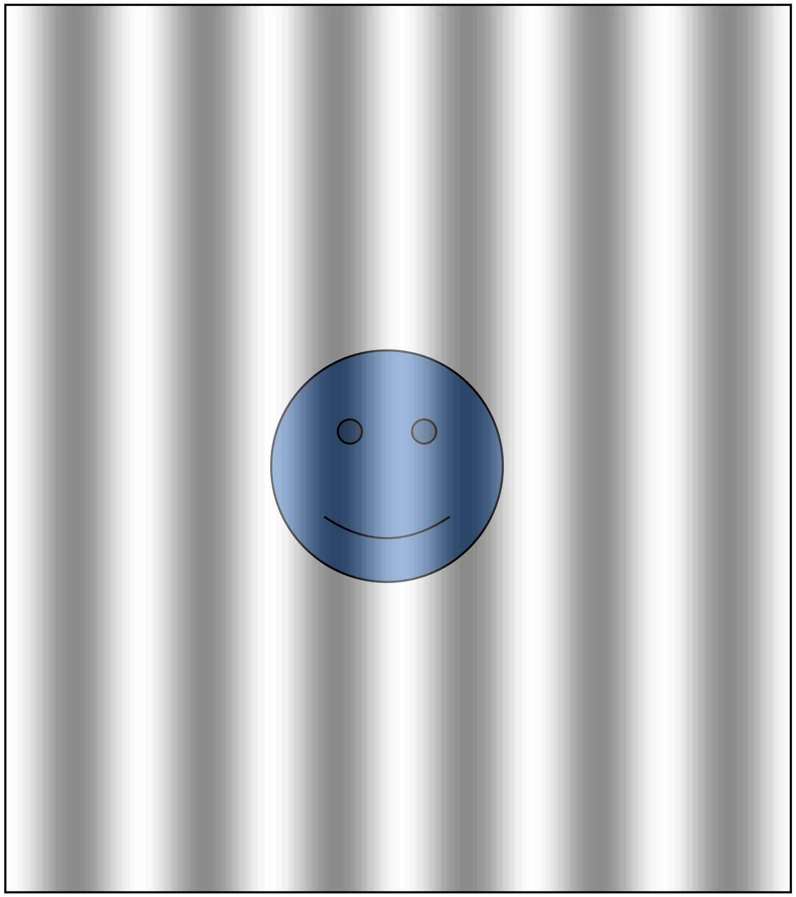
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

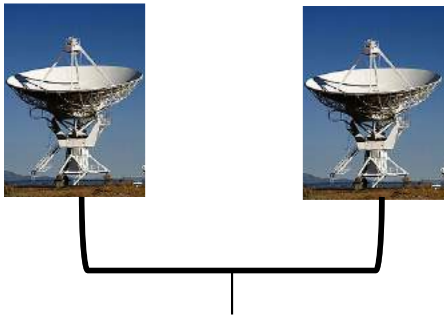




# Extended Source, Long Baseline

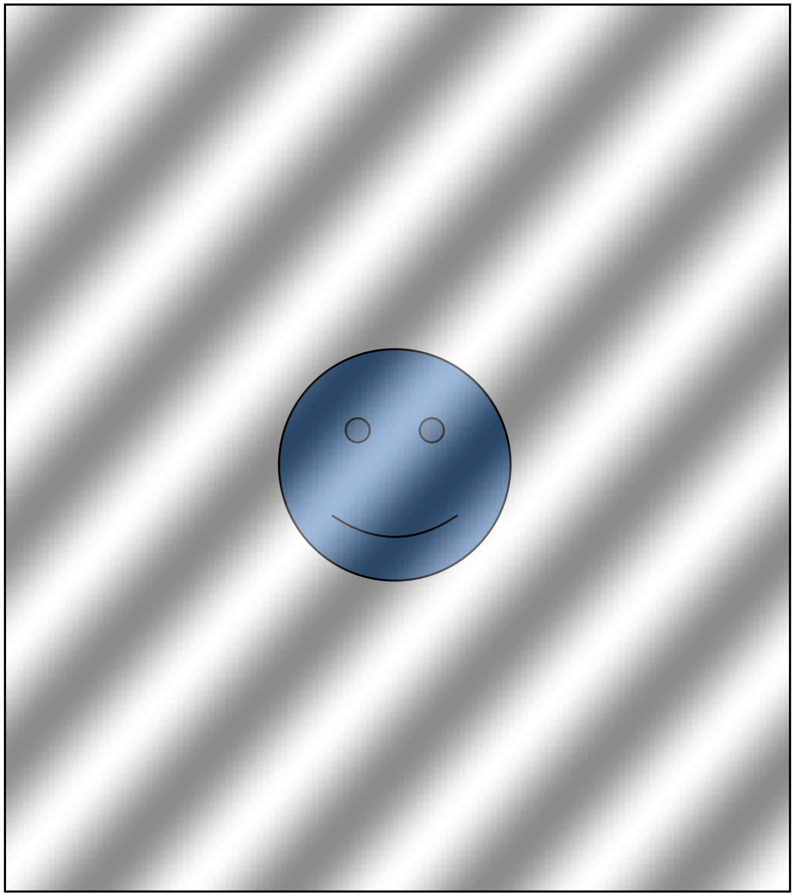
$$V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm$$

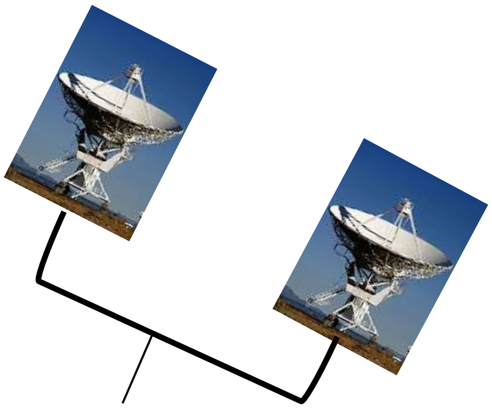




# Extended Source, Long Baseline

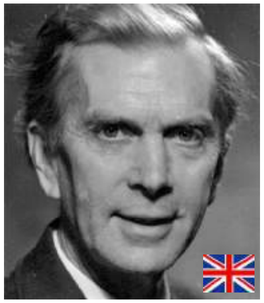
$$V(u,v) = \int\int T(l,m) e^{-i2\pi(ul+vm)} dl dm$$





# Aperture Synthesis

- basic idea: sample $V(u,v)$ at enough $(u,v)$ points using distributed small aperture antennas to synthesize a large aperture antenna of size $(u_{max},v_{max})$
- use more antennas for more samples
  - one pair of antennas = two $(u,v)$ samples at a time
  - N antennas = N(N-1) samples at a time
  - reconfigure physical layout of N antennas for more
- use Earth rotation for more samples
  - fill in (u,v) plane over time
- use more wavelengths for more samples
  - need to determine source structure at some wavelength and the change with wavelength, e.g. Taylor expansion
  - "multi-frequency synthesis" for continuum imaging [Urvashi Rao,Monday]

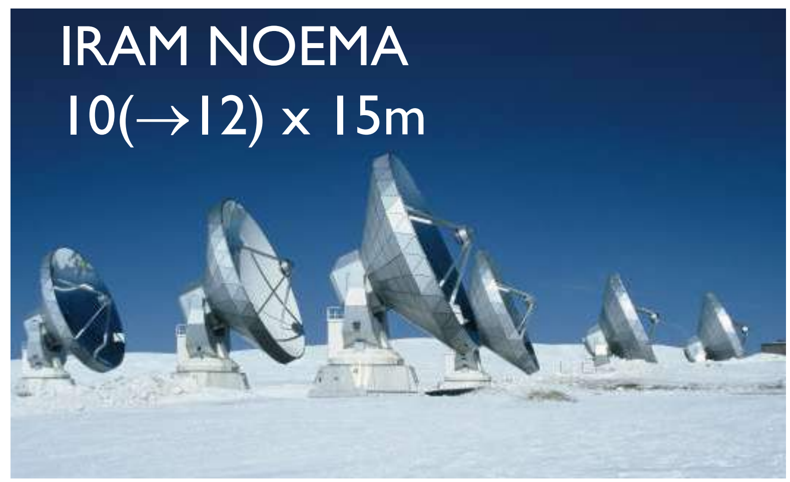


**Sir Martin Ryle**
**1918-1984**

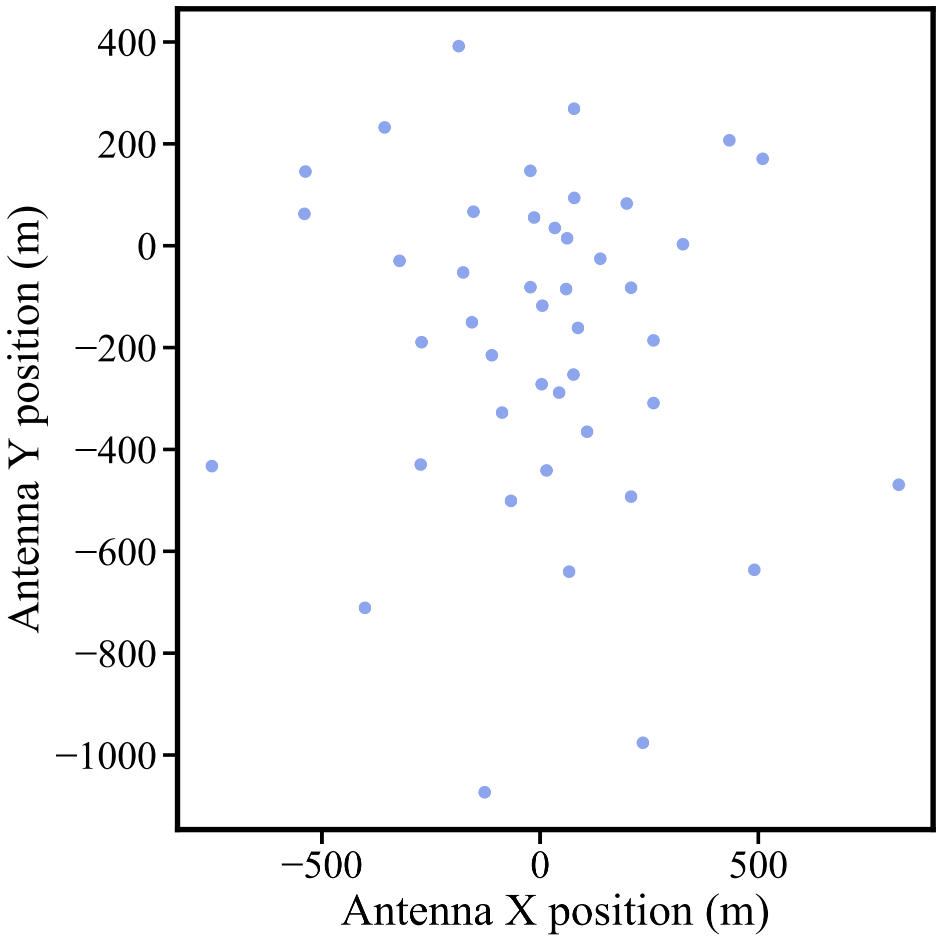
**1974 Nobel**
**Prize in Physics**

# A few Aperture Synthesis Telescopes for Observations at Millimeter Wavelengths





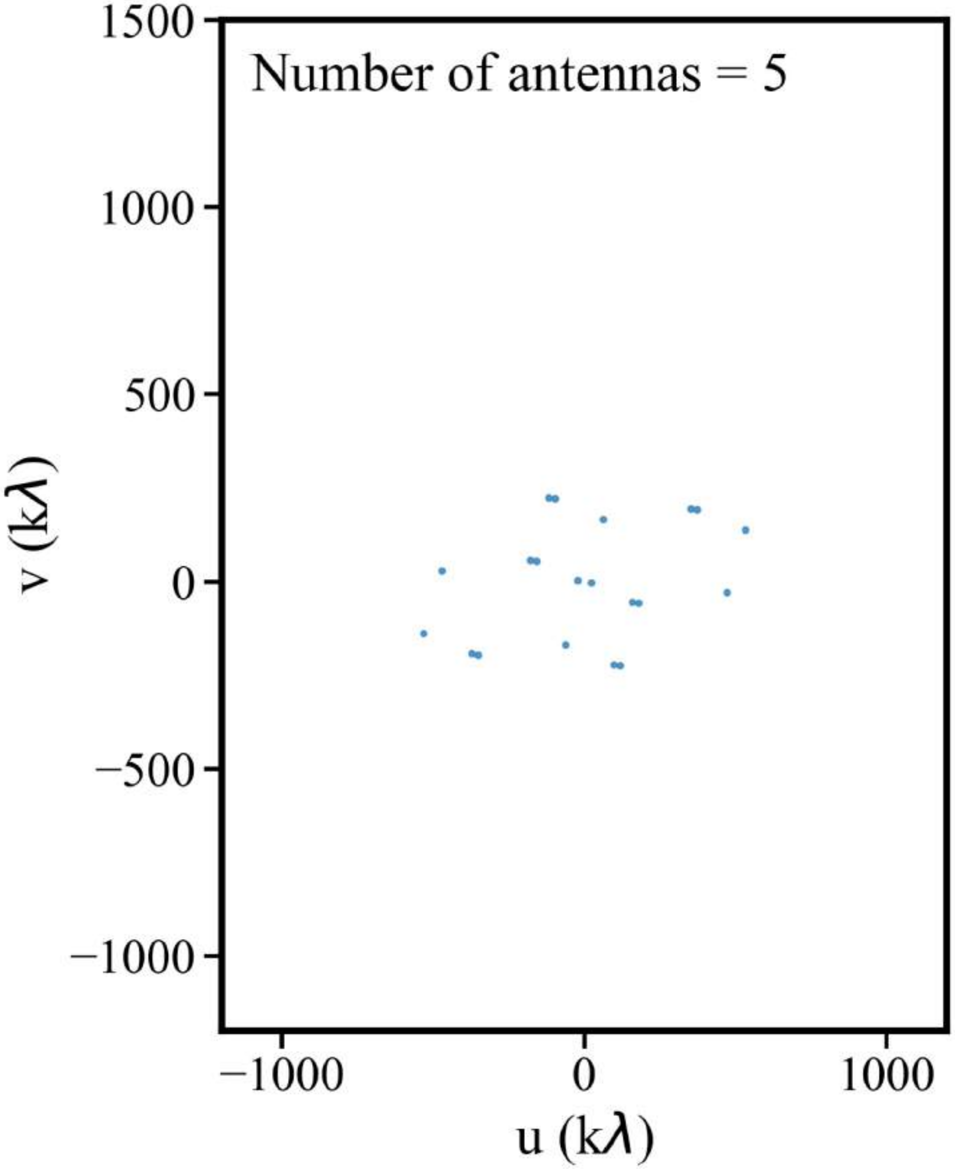
# Example of (u,v) Plane Sampling

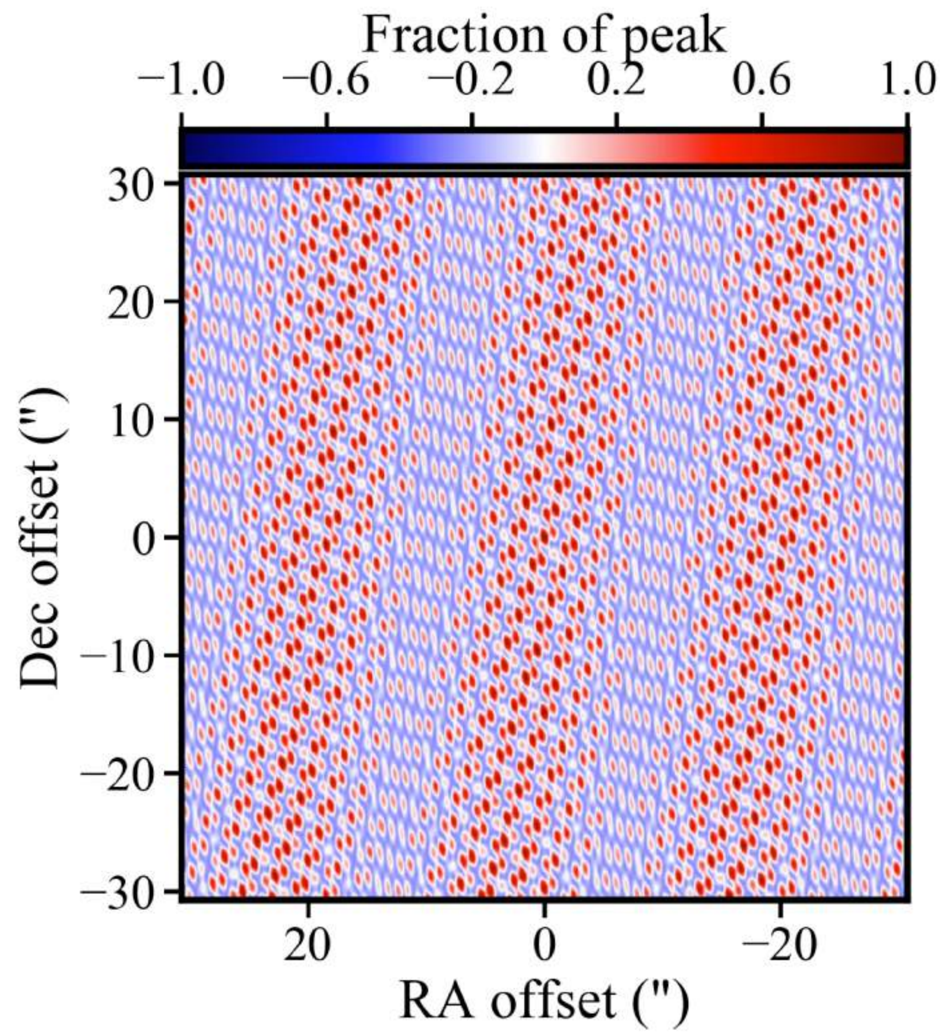



ALMA 12m antenna locations on August 8, 2015

[Array Design: Craig Walker, Monday]

# (u,v) Plane Sampling: more Antennas

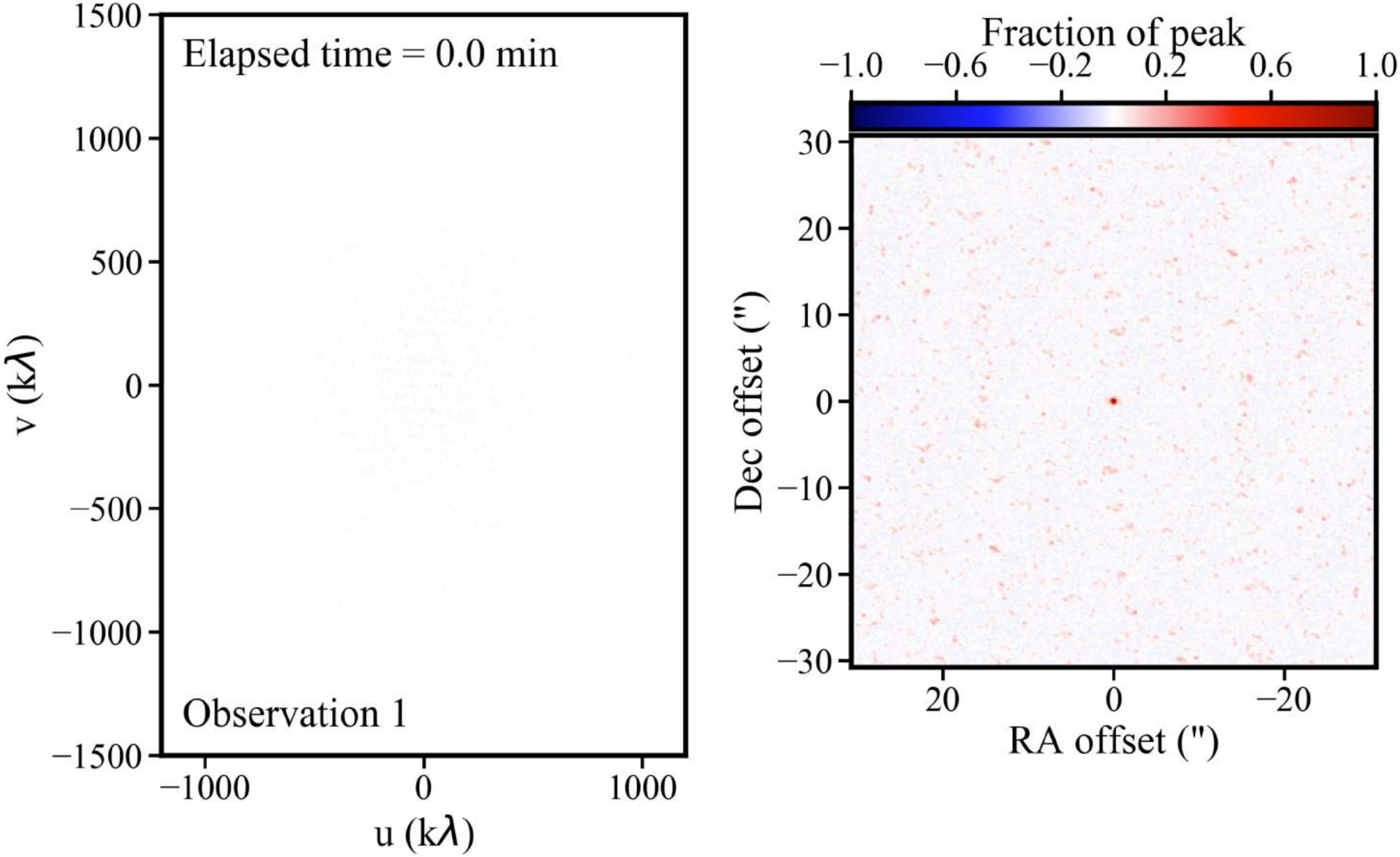


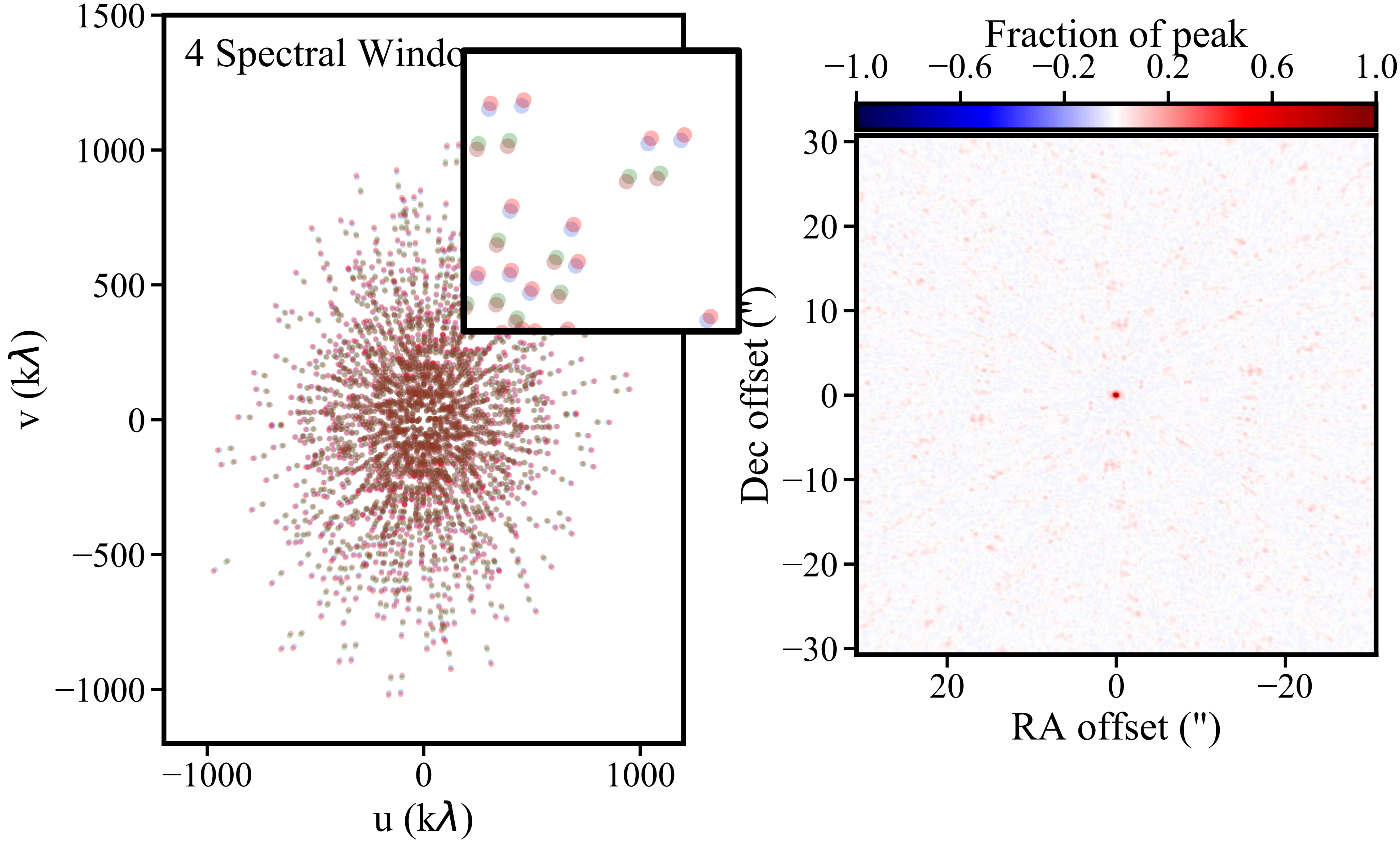

source dec -51 deg

ν=224.6 GHz (λ=1.3mm)

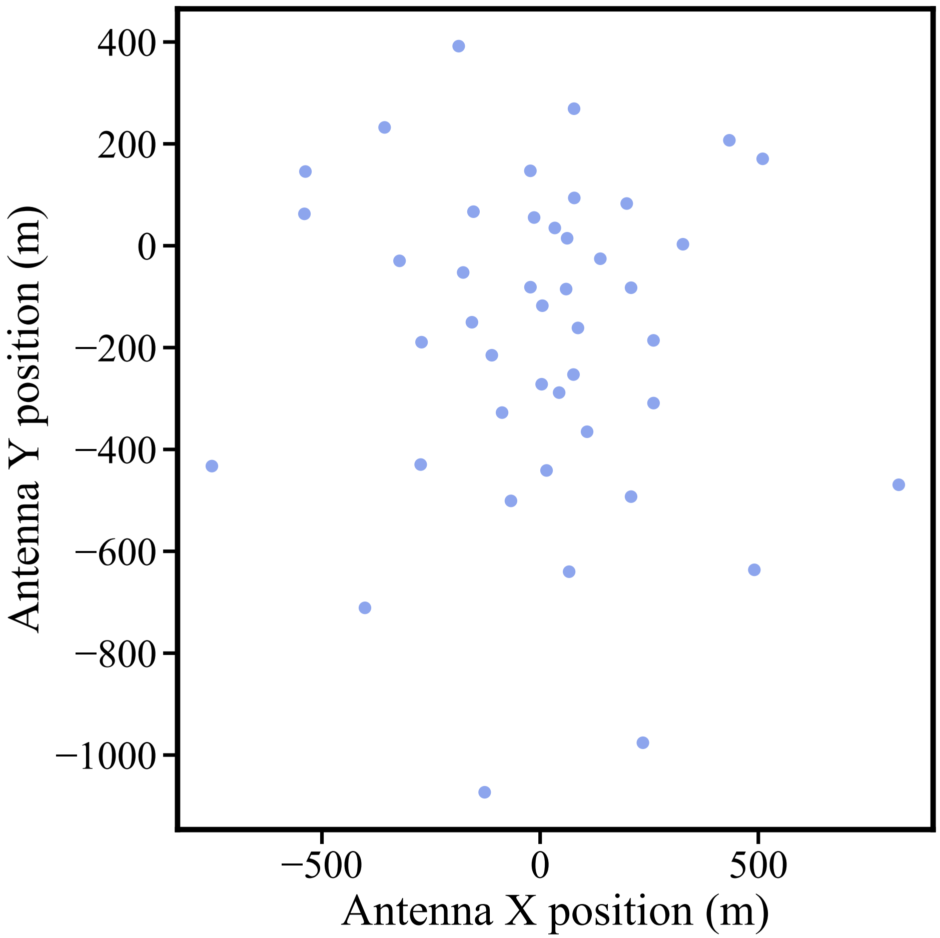
# (u,v) Plane Sampling: Earth rotation

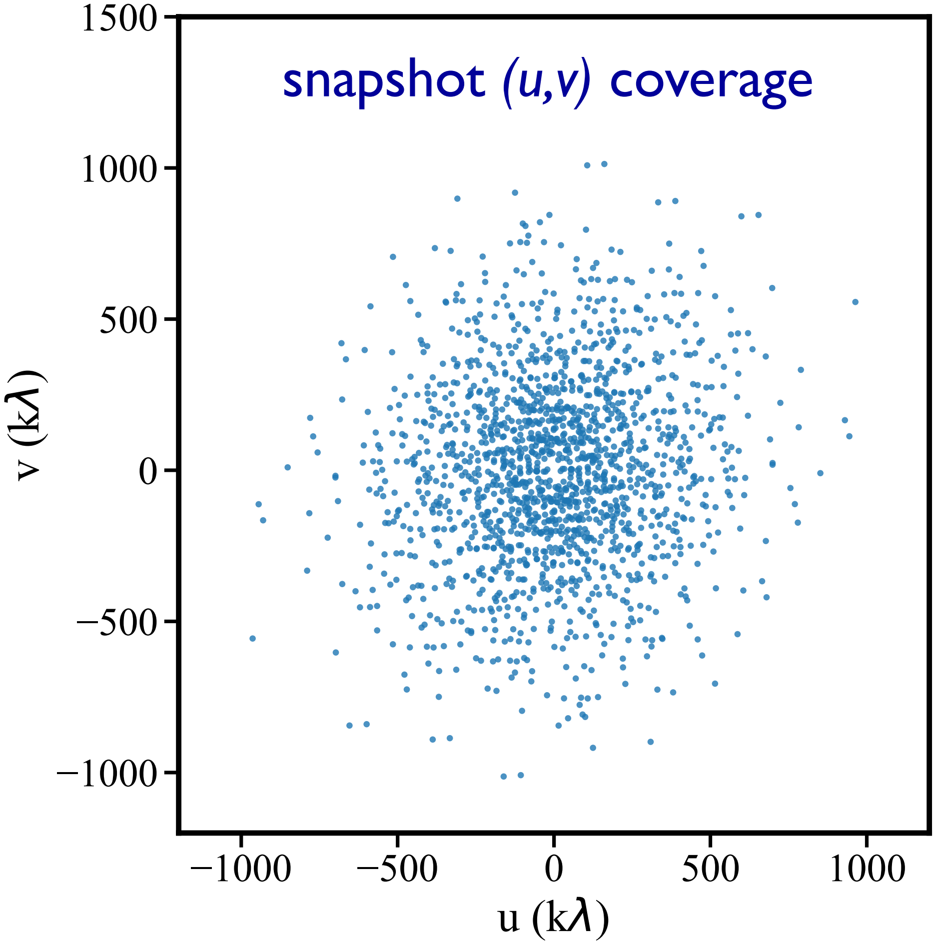


# (u,v) Plane Sampling: more wavelengths

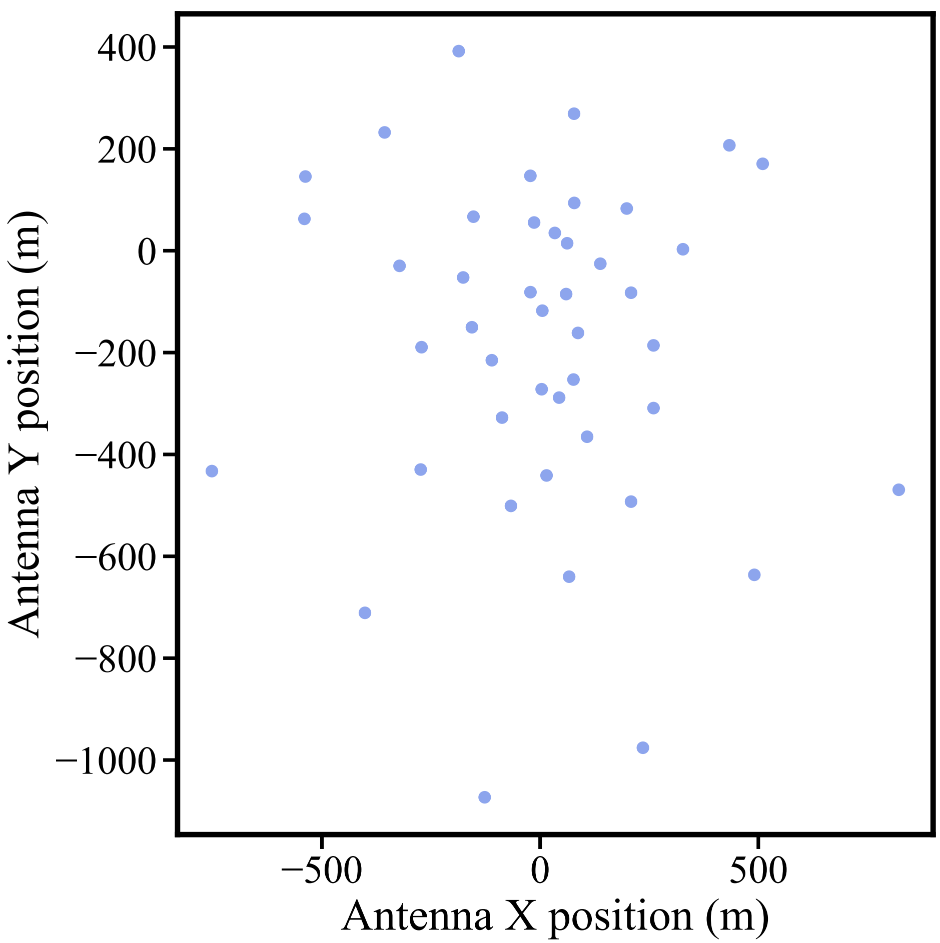


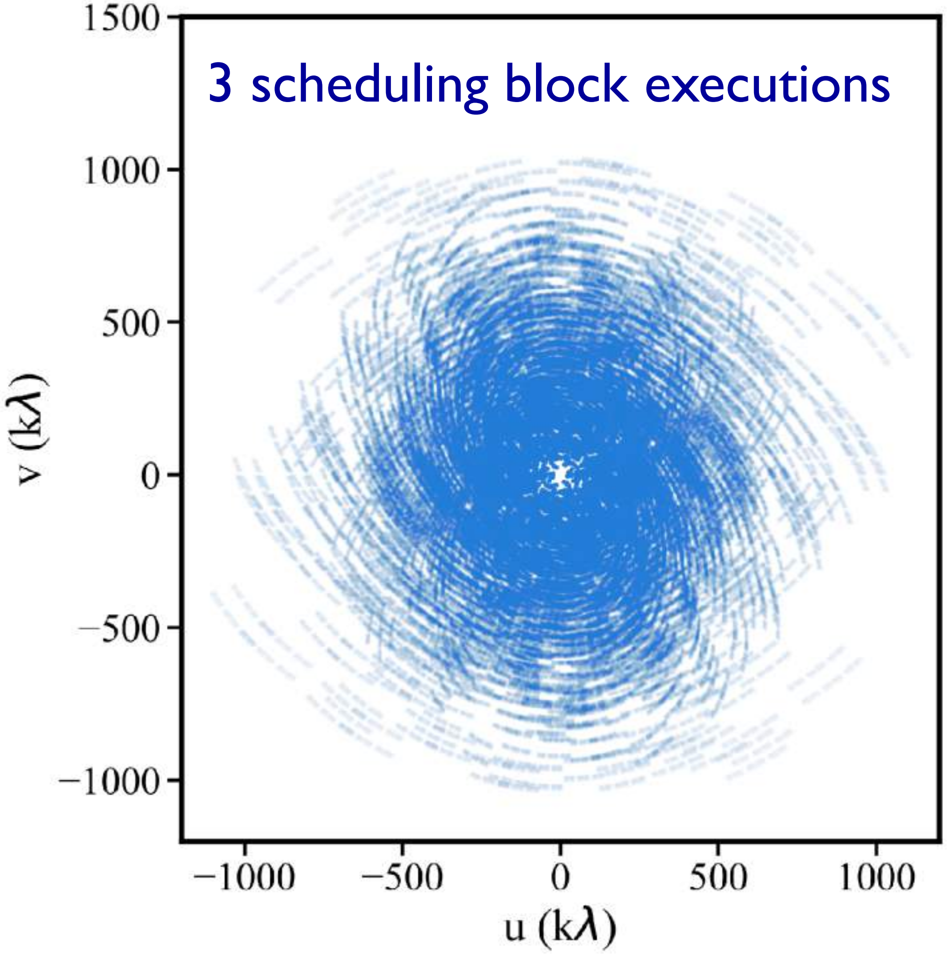
# Example of (u,v) Plane Sampling



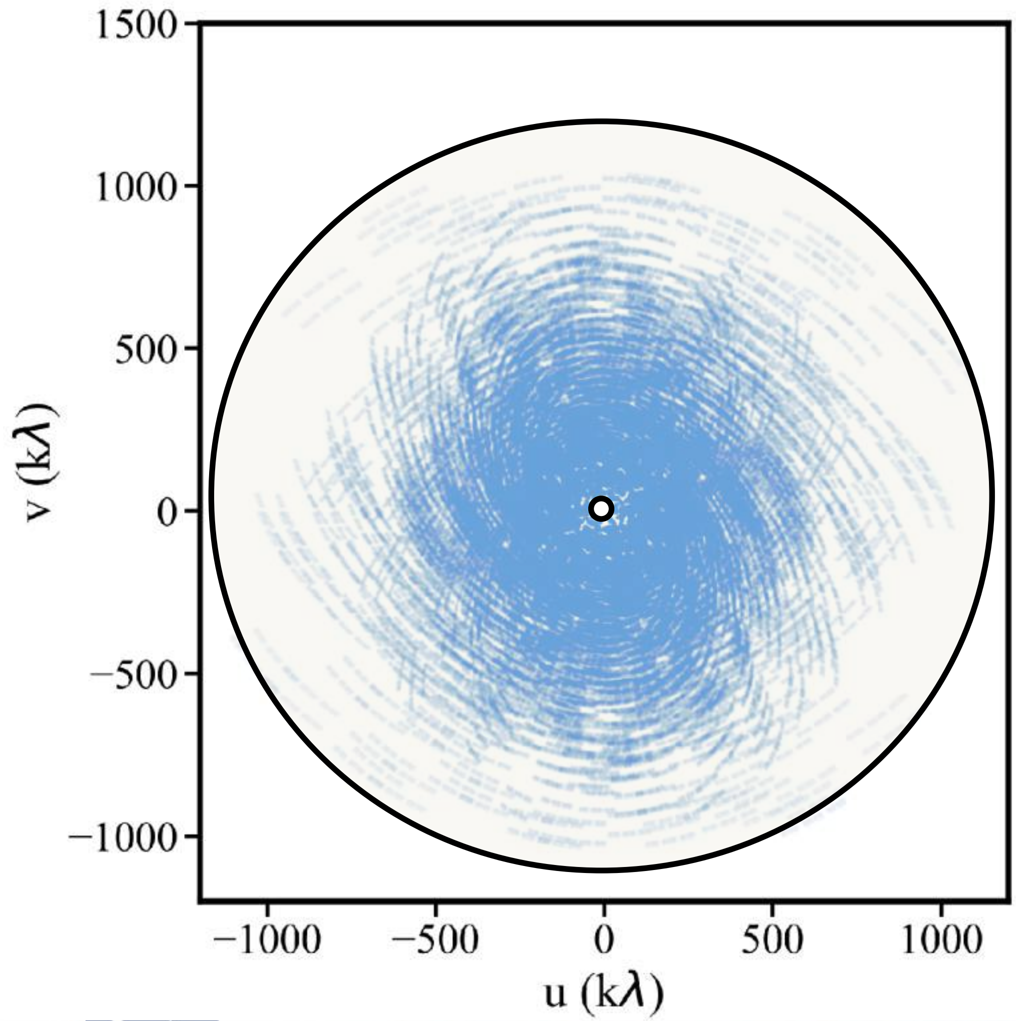
# Example of (u,v) Plane Sampling



# Implications of (u,v) Plane Sampling

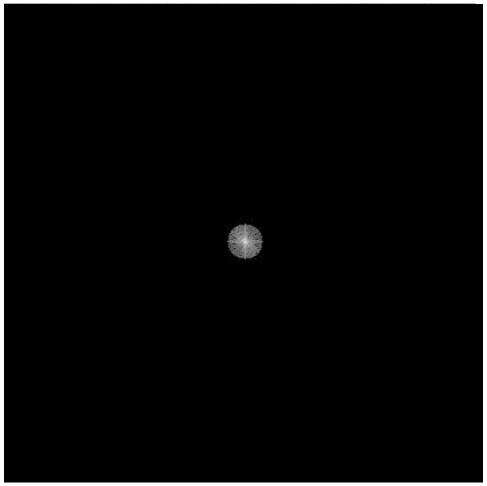
*V(u,v)* samples are limited by # antennas and Earth-sky geometry



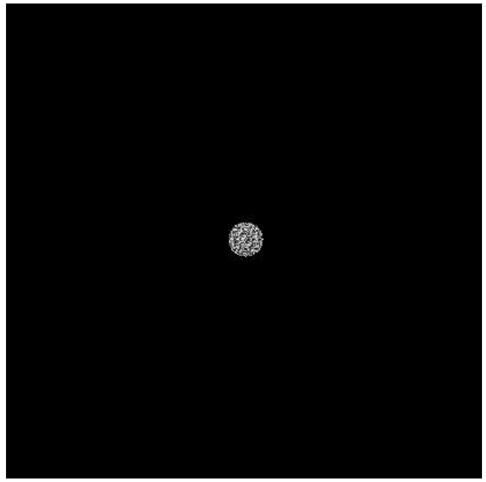

- *outer boundary*
  - no info on smaller scales
  - resolution limit
- *inner hole*
  - no info on larger scales
  - extended structures invisible
- *irregular sampling in between*
  - sampling theorem violated
  - information missing

# Inner and Outer (u,v) Boundaries

*V(u,v) amplitude*



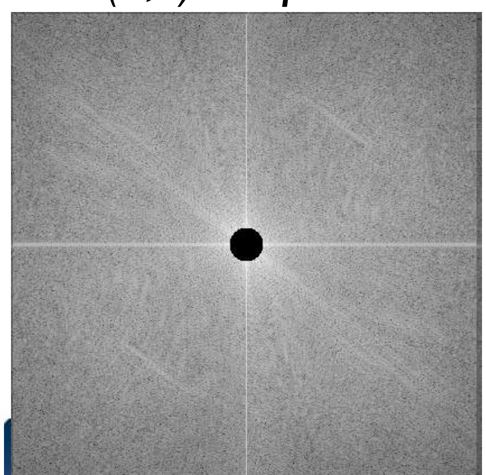
*V(u,v) phase*



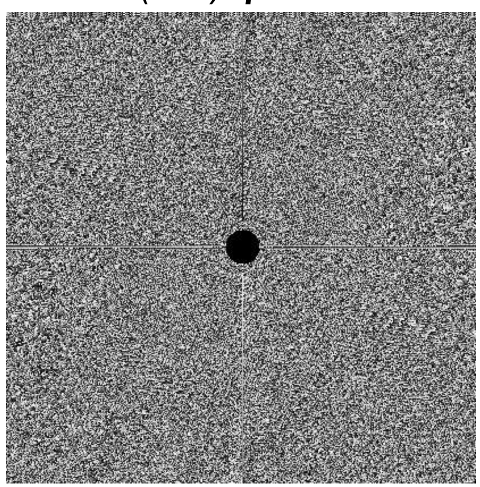
$\xrightarrow{\mathcal{F}}$

*T(l,m)*

*V(u,v) amplitude*



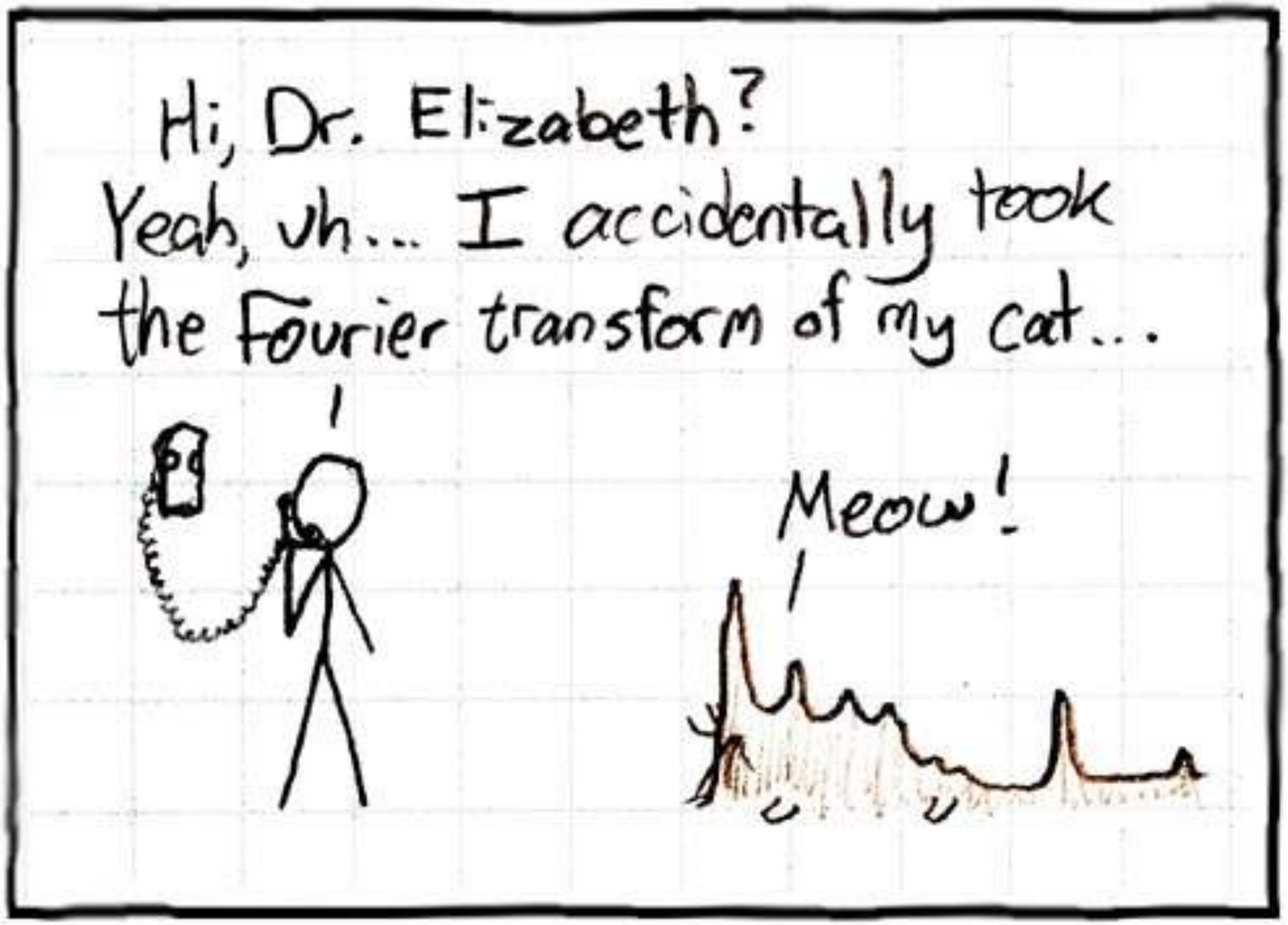
*V(u,v) phase*



$\xrightarrow{\mathcal{F}}$

*T(l,m)*

# xkcd.com/26/




That cat has some serious periodic components.

# Calibrated Visibilities… What's Next?

- analyze directly $V(u,v)$ samples by model fitting
  - good for simple structures, e.g. point sources, symmetric disks
  - for a purely statistical description of sky brightness (e.g. fluctuations)
  - visibilities have well defined noise properties

[Greg Taylor, Tuesday]

- recover an image from the observed incomplete and noisy samples of its Fourier transform for analysis
  - Fourier transform $V(u,v)$ to get $T^D(l,m)$,
    but difficult to do science with the dirty image $T^D(l,m)$
  - deconvolve $s(l,m)$ from $T^D(l,m)$ to determine a model of $T(l,m)$
  - work with the model of $T(l,m)$

CASA
`tclean`

# Formal Description of Imaging

$$V(u,v) \xrightarrow{\mathcal{F}} T(l,m)$$

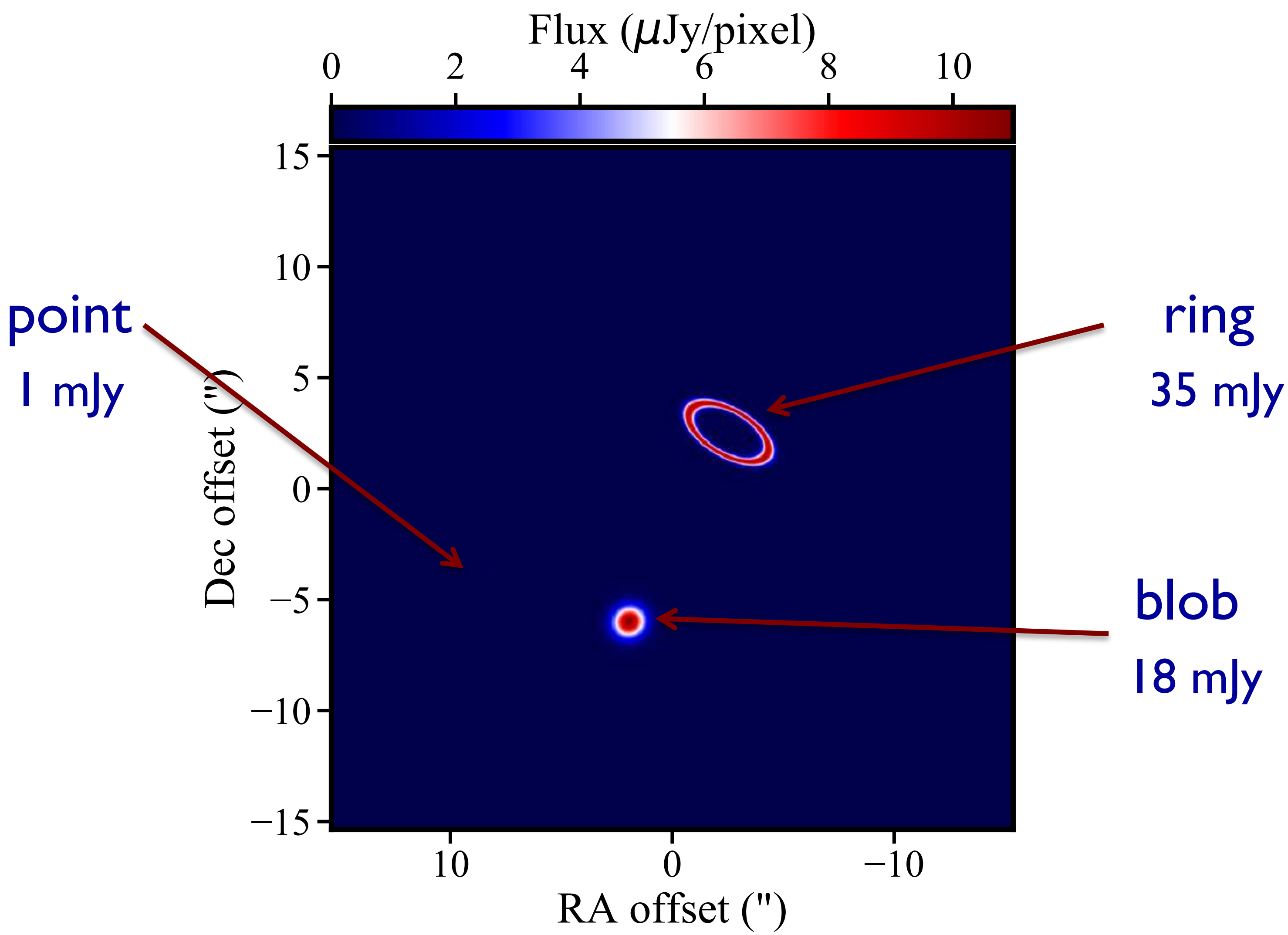
- sample Fourier domain at discrete points $S(u,v) = \sum_{k=1}^{M} \delta(u-u_k, v-v_k)$
- Fourier transform the sampled visibilities $V(u,v)S(u,v) \xrightarrow{\mathcal{F}} T^D(l,m)$
- apply the convolution theorem $T(l,m) * s(l,m) = T^D(l,m)$

  where the Fourier transform of the sampling pattern $s(l,m) \xrightarrow{\mathcal{F}} S(u,v)$ is the point spread function

*the Fourier transform of the sampled visibilities yields the true sky brightness convolved with the point spread function*

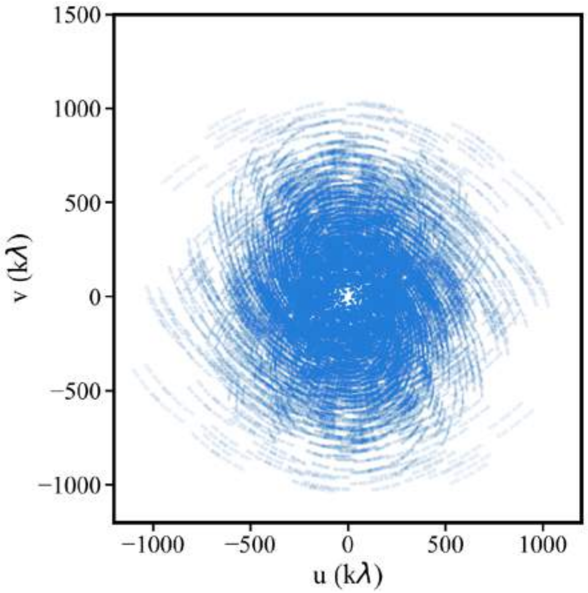
(radio jargon: "dirty image" is true image convolved with "dirty beam")

# A model *T(l,m)* sky brightness distribution

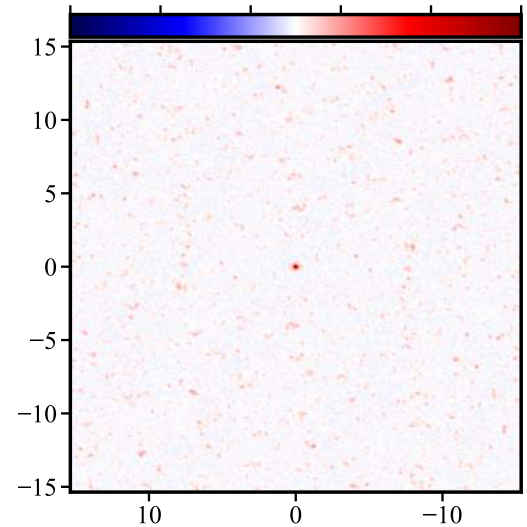


# Dirty Beam and Dirty Image

*S(u,v)*



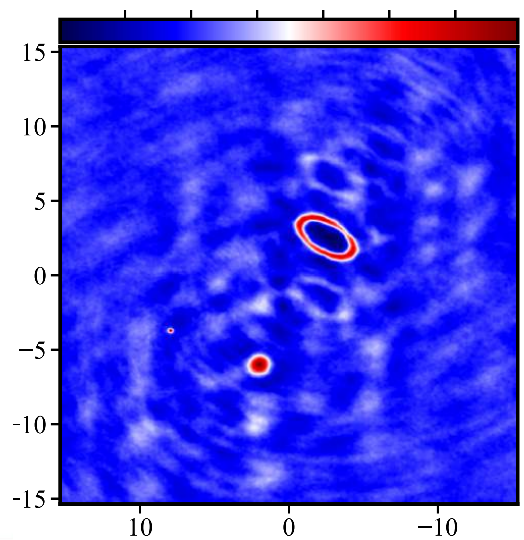

$\xrightarrow{\mathcal{F}}$



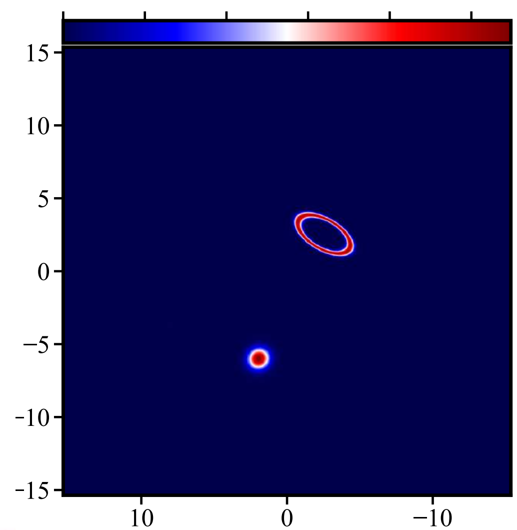

*s(l,m)*
*"dirty beam"*

$*$

*$T^D$(l,m)*
*"dirty image"*

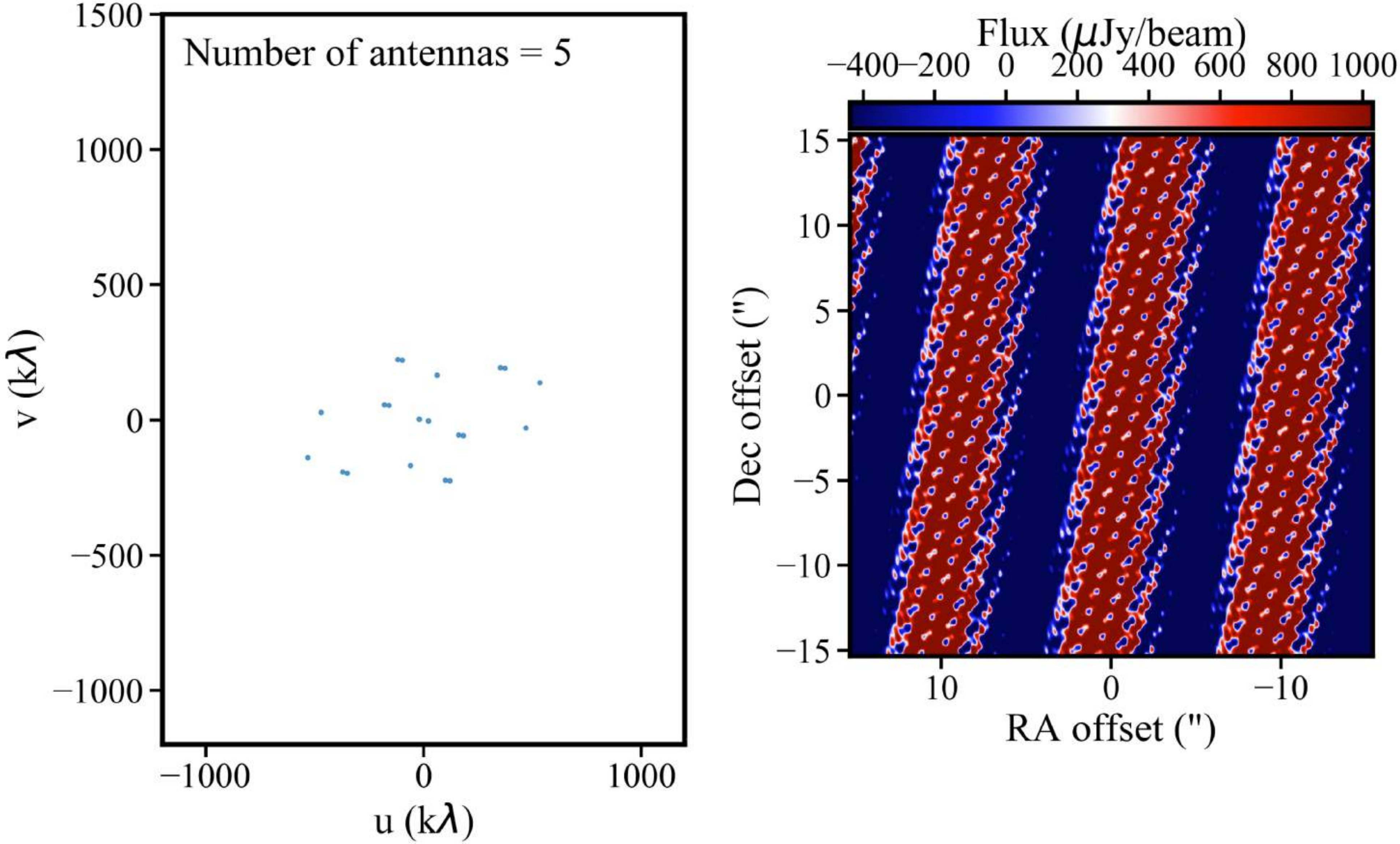



$\leftarrow$

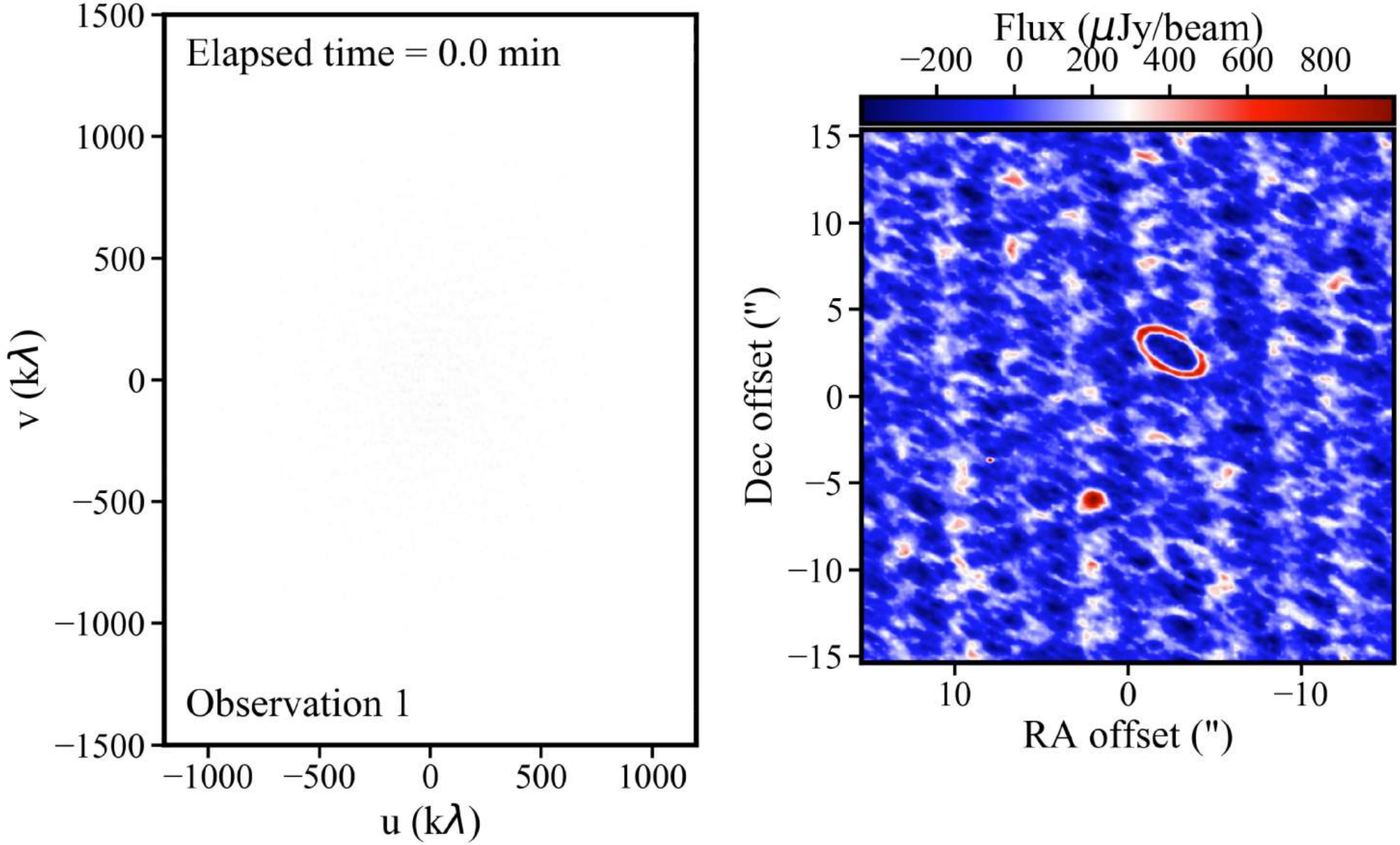



*T(l,m)*

# (u,v) Plane Sampling: more antennas



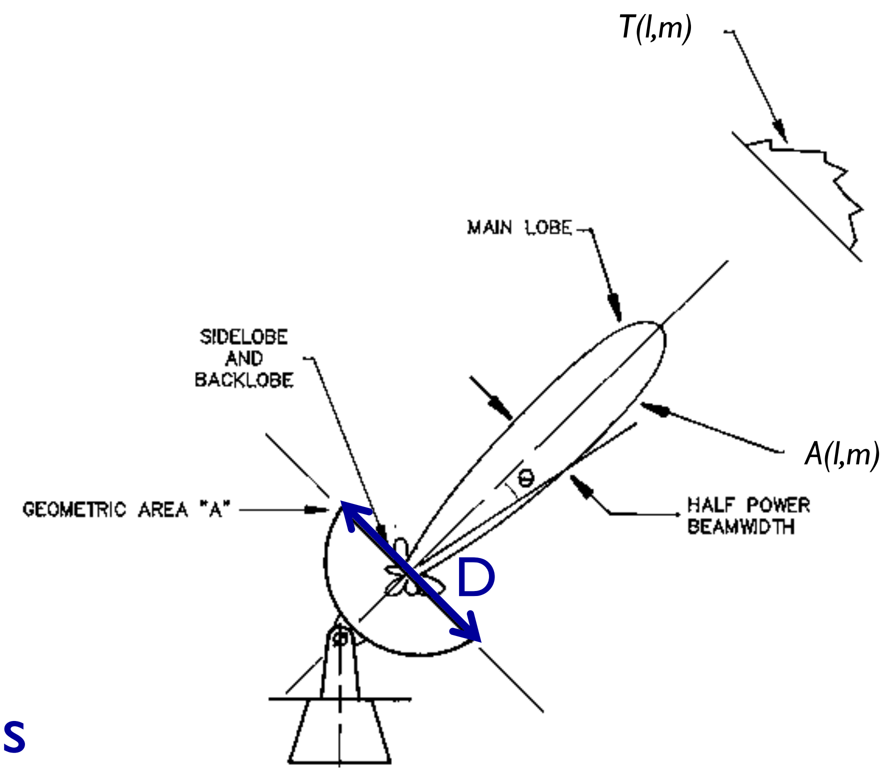
# (u,v) Plane Sampling: Earth rotation

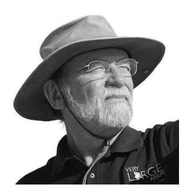


# Field of View

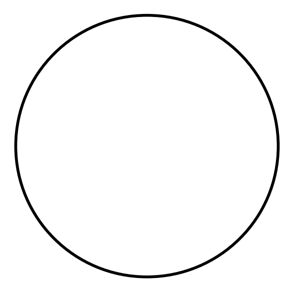
- antenna response *A(l,m)* is not uniform across the entire sky
  - “primary beam” fwhm ~ $\lambda$/D
  - response beyond primary beam can be important (“sidelobes”)

- antenna response *A(l,m)* modifies the sky brightness distribution
  - $T(l,m) \rightarrow\ T(l,m)A(l,m)$
  - can correct with division by *A(l,m)* in the image plane
  - large source extents require multiple pointings of antennas = mosaicking [Brian Mason, Friday]

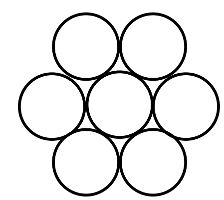


*T(l,m)*



ALMA 12 m
115 GHz

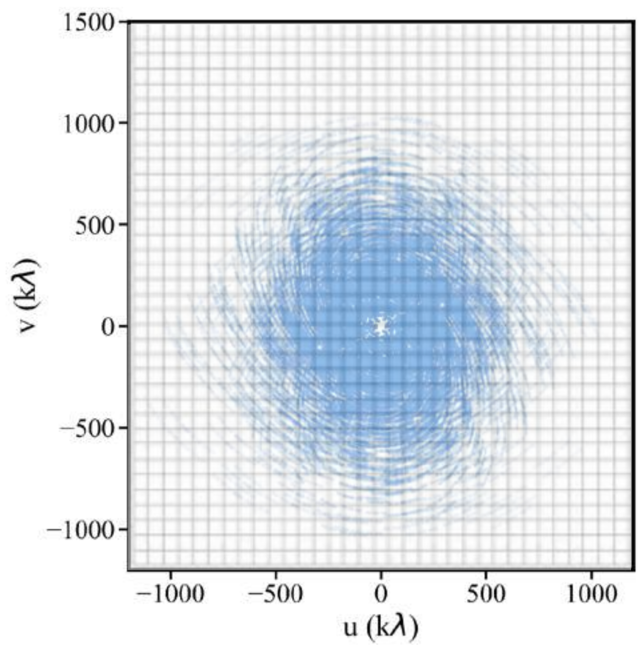


ALMA 12 m
460 GHz

# FFTs and Gridding

- "Fourier transform"
  - Fast Fourier Transform (FFT) algorithm is much faster than simple Fourier summation, *O*(n*log*n)
  - FFT requires data on a regularly spaced grid
  - aperture synthesis does not provide *V(u,v)* on a regularly spaced grid, so…
- "gridding" used to resample *V(u,v)* for FFT
  - customary to use a convolution method
  - *(u,v)* cell ≈ 0.5D, where D = antenna diameter
  - special ("spheroidal") functions

    that minimize smoothing and aliasing




$$V^G(u,v) = V(u,v)S(u,v) * G(u,v)$$
$$\xrightarrow{F} T^D(l,m)g(l,m)$$

CASA `tclean` "gridder"

# Pixel and Image Size

- pixel size: satisfy sampling theorem for longest baselines

$$\Delta l < \frac{1}{2u_{max}} \qquad \Delta m < \frac{1}{2v_{max}}$$

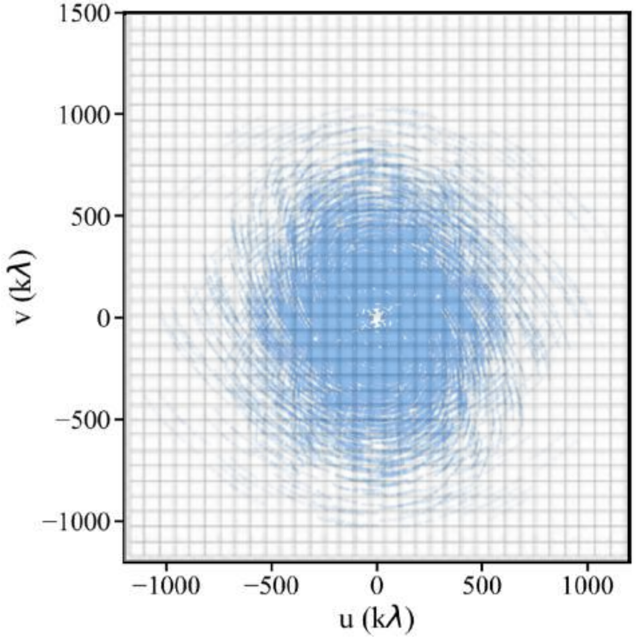
  - in practice, 3 to 5 pixels across dirty beam main lobe to aid deconvolution
  - e.g. ALMA at 1.3 mm, baselines to 1 km $\rightarrow$ pixel size < 0.13 arcsec
  - CASA `tclean` "cell"

- image size: natural choice often full primary beam *A(l,m)*
  - e.g. ALMA at 1.3 mm, 12 meter antennas $\rightarrow$ image size 2 x 27 arcsec
  - if there are bright sources in *A(l,m)* sidelobes, then the FFT will alias them into the image $\rightarrow$ make a larger image (or image outlier fields)
  - CASA `tclean` "imsize"

# Visibility Weighting Schemes

- introduce weighting function *W(u,v)*
- modifies sampling function
- $S(u,v) \rightarrow S(u,v)W(u,v)$
- changes *s(l,m)*, the dirty beam
- *W(u,v)* is gridded for FFT, too

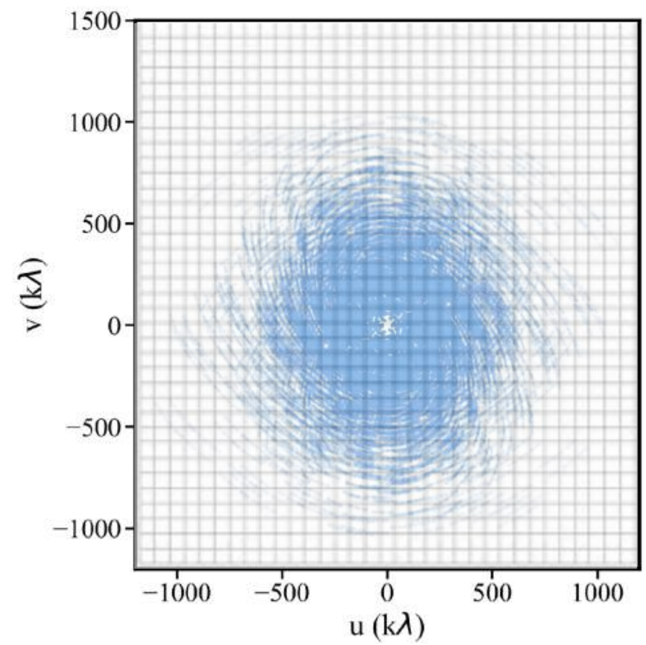
CASA `tclean` "weighting"

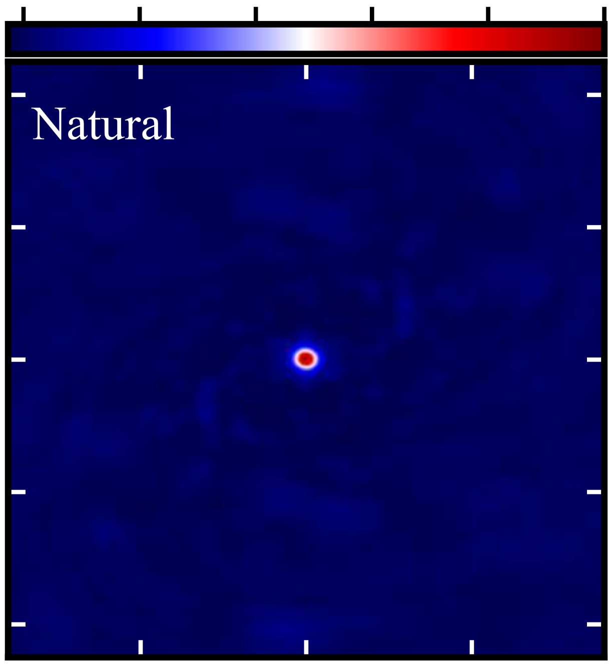


# Natural Weighting

- $W(u,v) = 1/\sigma^2$ in occupied cells, where $\sigma^2$ is the noise variance

- generally gives more weight to short baselines, so the angular resolution is degraded
- maximizes point source sensitivity
- lowest rms in image

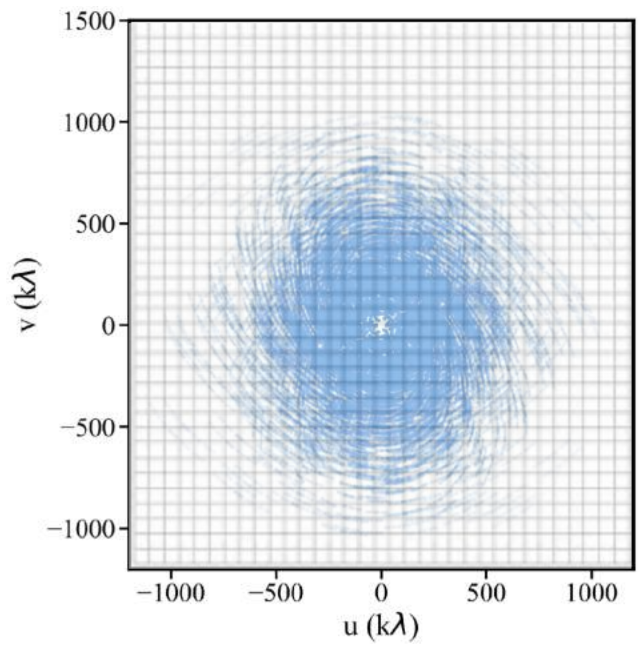


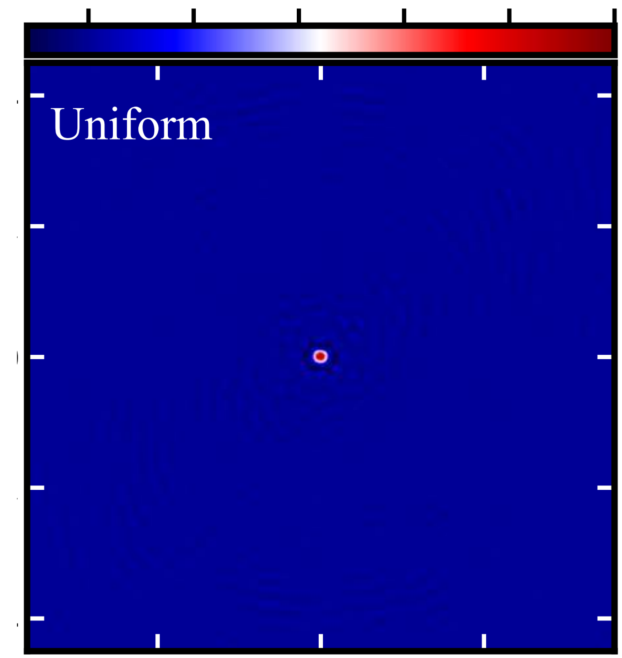



# Uniform Weighting

- *W(u,v)* inversely proportional to local density of *(u,v)* samples
- weight for occupied cell = const

- fills *(u,v)* plane more uniformly so dirty beam sidelobes are lower
- gives more weight to long baselines, so angular resolution is enhanced
- downweights some data, so point source sensitivity is degraded
- n.b. can be trouble with sparse *(u,v)* coverage: cells with few samples have same weight as cells with many



# Robust (Briggs) Weighting

- variant of uniform weighting that avoids giving too much weight to cells with low natural weight
- software implementations differ
- e.g. $$W(u,v) = \frac{1}{\sqrt{1 + S_N^2 / S_{thresh}^2}}$$

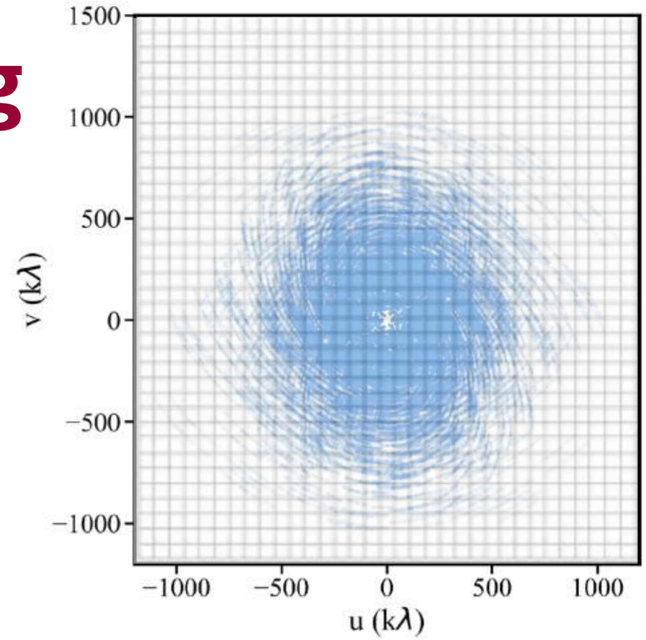
$S_N$ is cell natural weight

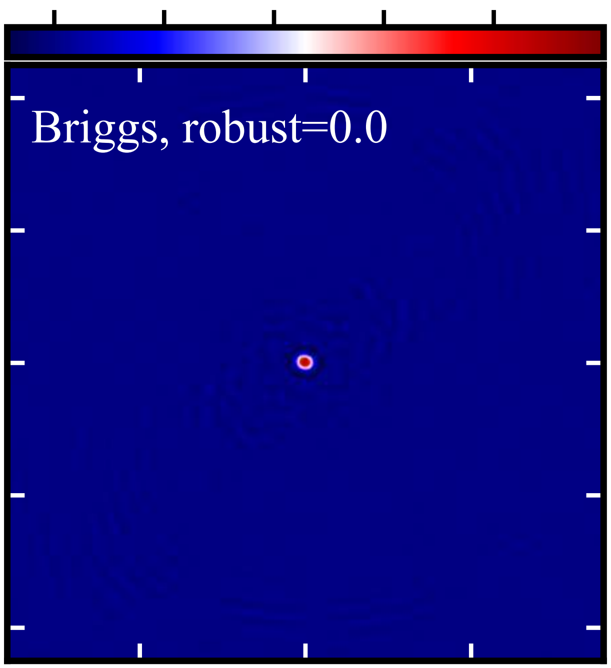
$S_{thresh}$ is a threshold

high threshold $\rightarrow$ natural weight

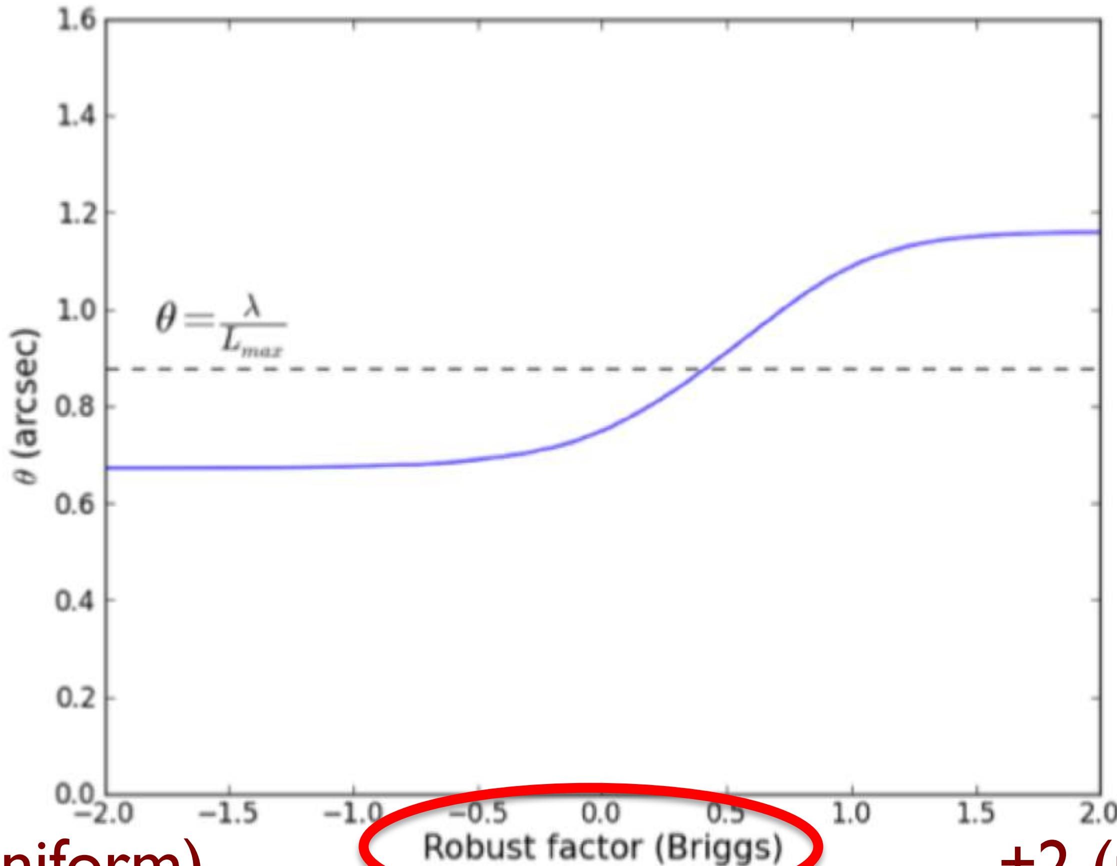
low threshold $\rightarrow$ uniform weight

*an adjustable parameter that allows for continuous variation between maximum point source sensitivity and resolution*



# ALMA C43-4 Configuration Resolution




CASA tclean

-2 (uniform)

+2 (natural)

Figure 7.6: Angular resolution achieved using different values of the CASA *robust* parameter for a 1-hour observation at 100 GHz and a declination of -23 deg in the C43-4 configuration. Note that *robust* $= -2$ is close to uniform weighting and *robust* $= 2$ is close to natural weighting. The dotted line corresponds to $k = 1$.

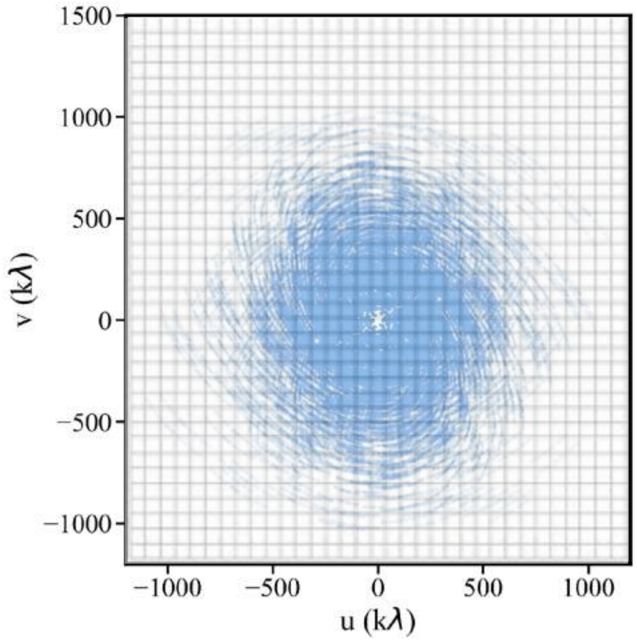
ALMA Cycle 6 Technical Handbook

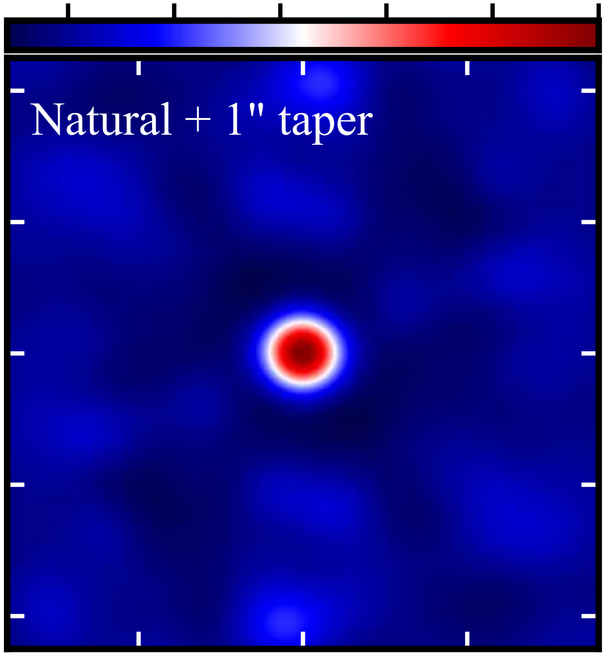
# Tapering

- apodize *(u,v)* sampling by a Gaussian

$$W(u,v) = \exp\left(-\frac{(u^2+v^2)}{t^2}\right)$$

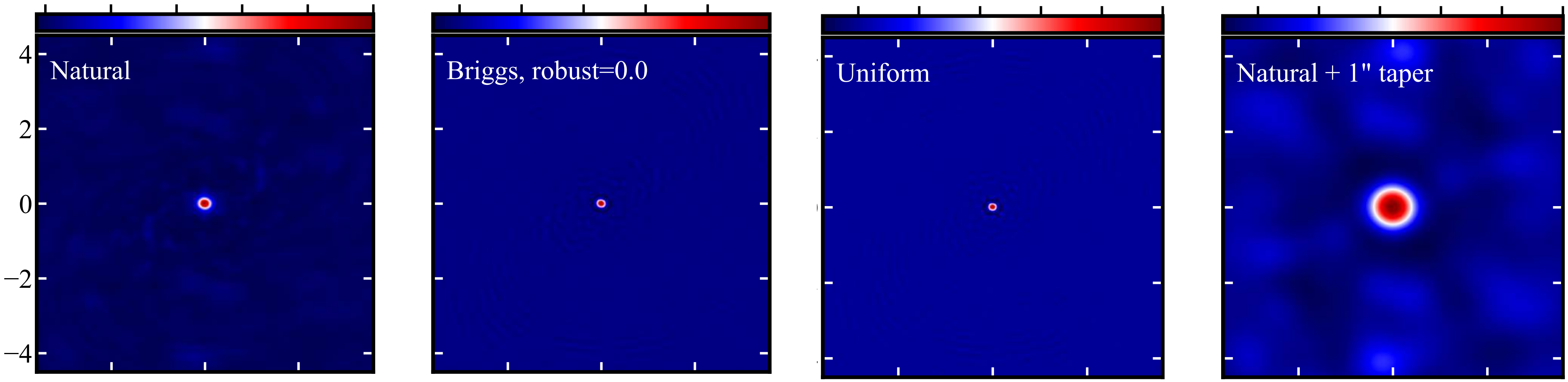
  *t* = adjustable tapering parameter
- like convolving image by a Gaussian
- downweights data at long baselines, so point source sensitivity degraded and angular resolution degraded
- may improve sensitivity to extended structure sampled by short baselines
- limits to usefulness



# Weighting Schemes and Noise

- natural = equal weight for all visibilities  [lowest noise]
- uniform = equal weight for filled *(u,v)* cells [highest noise]
- robust/Briggs = continuous variation between natural and uniform
- taper = decrease resolution, increase surface brightness sensitivity




| natural | robust=0 | uniform | natural + 1” taper |
|---|---|---|---|
| (rms 10 μJy/beam) | (rms 16 μJy/beam) | (rms 28 μJy/beam) | (rms 23 μJy/beam) |

# Summary of Visibility Weighting Schemes

- imaging parameters provide a lot of freedom
- appropriate choices depend on science goals, e.g.
  - point source detection: natural weight
  - fine detail of strong source: uniform weight
  - complicated emission distribution: robust 0 to 1
  - weak and extended source: taper

| | Robust/Uniform | Natural | Taper |
|---|---|---|---|
| resolution | higher | medium | lower |
| sidelobes | lower | higher | depends |
| point source sensitivity | lower | maximum | lower |
| extended source sensitivity | lower | medium | higher |

# Beyond the Dirty Image

- to keep you awake at night…
- $\exists$ an infinite number of $T(l,m)$ compatible with sampled $V(u,v)$, with "invisible" distributions $R(l,m)$ where $s(l,m) * R(l,m) = 0$
  - no data beyond $u_{max}$,$v_{max}$ $\rightarrow$ unresolved structure
  - no data within $u_{min}$,$v_{min}$ $\rightarrow$ limit on largest size scale
  - holes in between $\rightarrow$ synthesized beam sidelobes
- also noise $\rightarrow$ undetected/corrupted structure in $T(l,m)$
- no unique prescription to extract optimum estimate of $T(l,m)$

# Deconvolution Philosophy

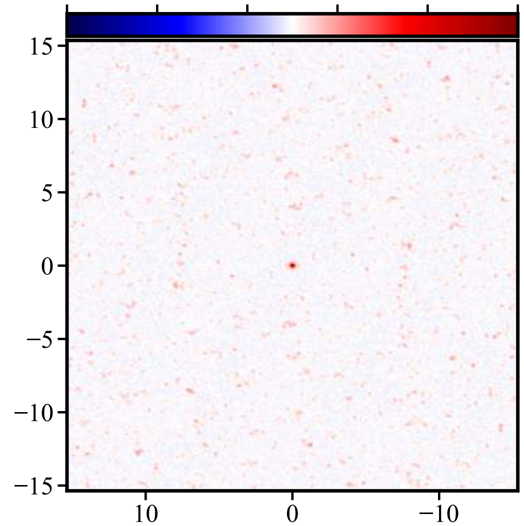
- use non-linear techniques to interpolate/extrapolate samples of *V(u,v)* into unsampled regions of the *(u,v)* plane

(remove sidelobes of the dirty beam from the image)

- aim to find a sensible model of *T(l,m)* compatible with data
- requires *a priori* assumptions about *T(l.m)* to pick plausible "invisible" distributions to fill unsampled parts of *(u,v)* plane
- main assumption: real sky does not look like typical dirty beam
- "clean" deconvolution algorithm (and its variants) by far dominant in radio astronomy, though there are others in use
- a very active research area, e.g. compressed sensing

# Classic Högbom (1974) clean Algorithm

- *a priori* assumption: *T(l,m)* is a collection of point sources

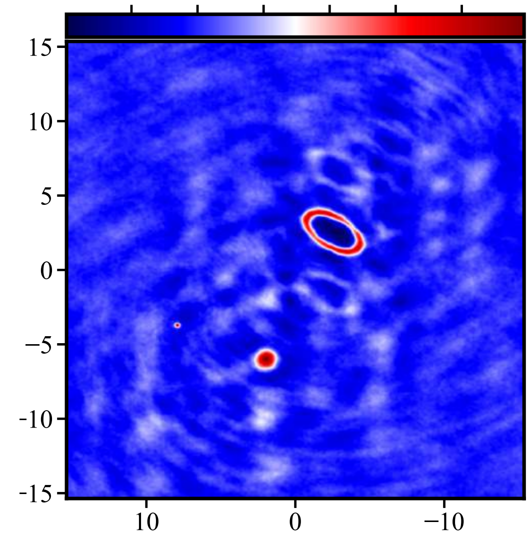



*s(l,m)*

initialize a *clean component* list

initialize a *residual image* = dirty image

1. identify the highest peak in the *residual image* as a point source
2. subtract a scaled dirty beam *s(l,m)* x "loop gain" from this peak
3. add this point source location and amplitude to the *clean component* list
4. goto step 1 (an iteration) unless stopping criterion reached




$T^D(l,m)$

# Classic Högbom (1974) clean Algorithm

- stopping criterion
  - *residual map* maximum < threshold = multiple of rms , e.g. 2 x rms (if noise limited)
  - *residual map* maximum < threshold = fraction of dirty map maximum (if dynamic range limited)

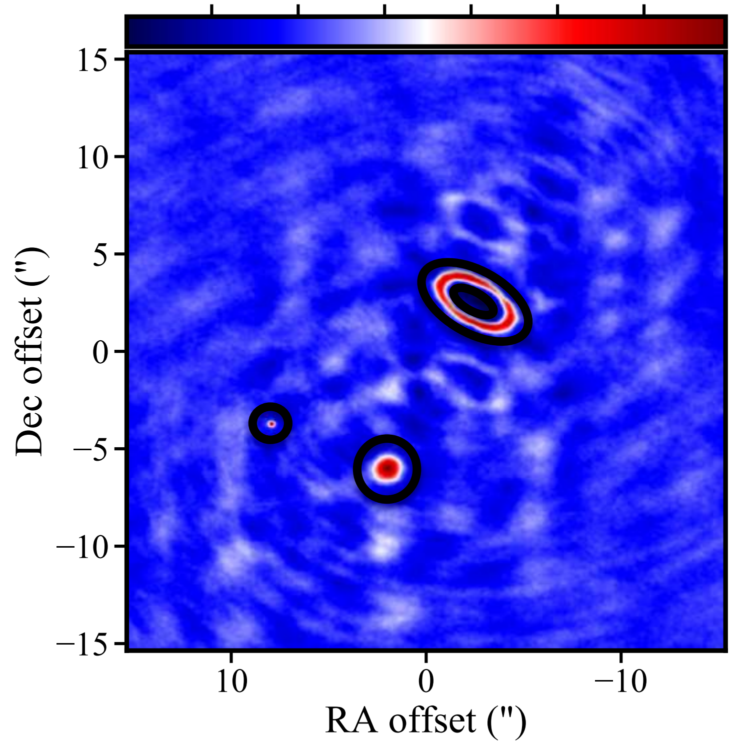
- loop gain parameter
  - good results for g=0.1 (CASA `tclean` default)
  - lower values can work better for smooth and extended emission

- don't "overclean" to artificially low noise level
  - generally a problem only when *(u,v)* coverage is sparse

# Classic Högbom (1974) clean Algorithm

- finite support
  - easy to include *a priori* information about where in the dirty map to search for *clean components*
  - implemented as image masks or clean boxes; CASA `tclean` "mask"
  - very useful, often essential for best results, but potentially dangerous
  - use with care

  - can be an arduous manual process; automasking under development

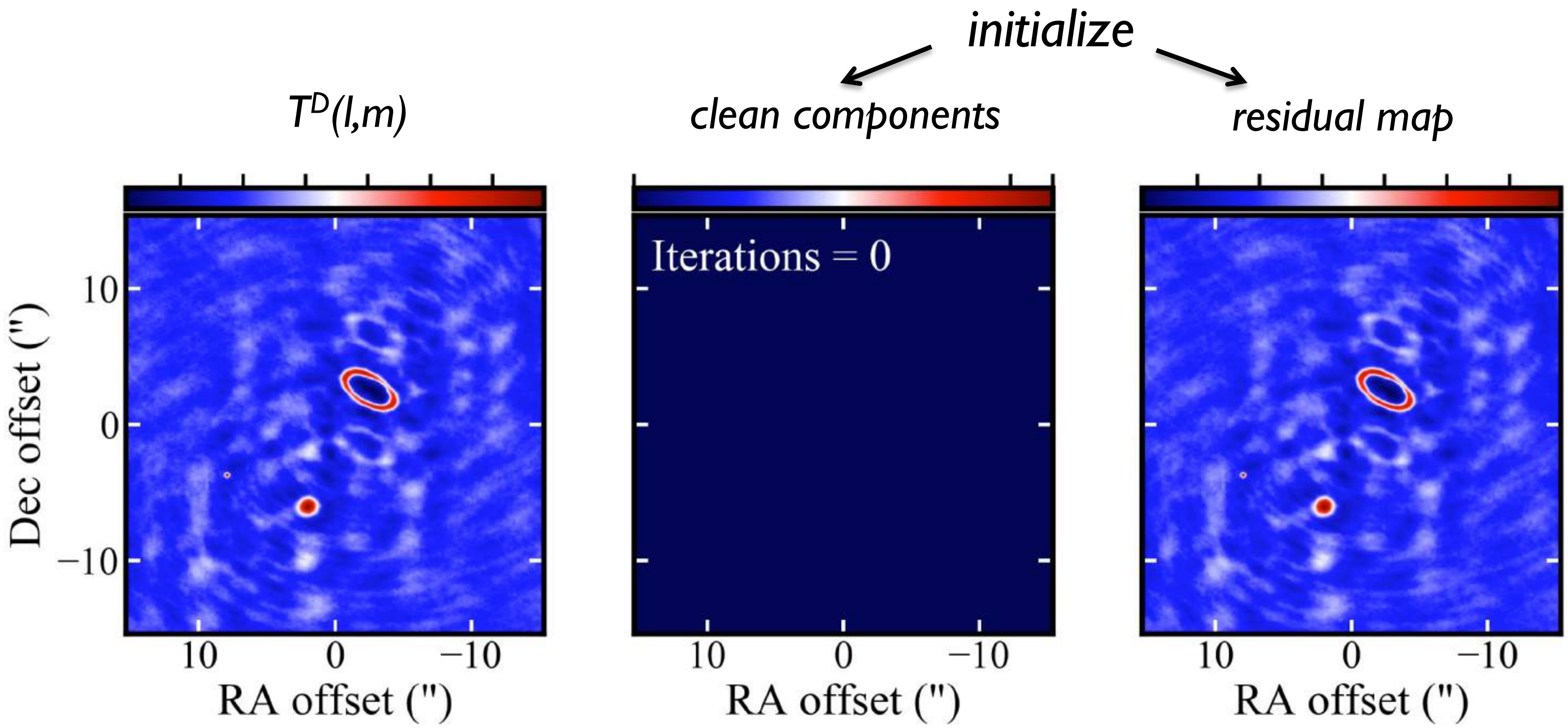


# Classic Högbom (1974) clean Algorithm

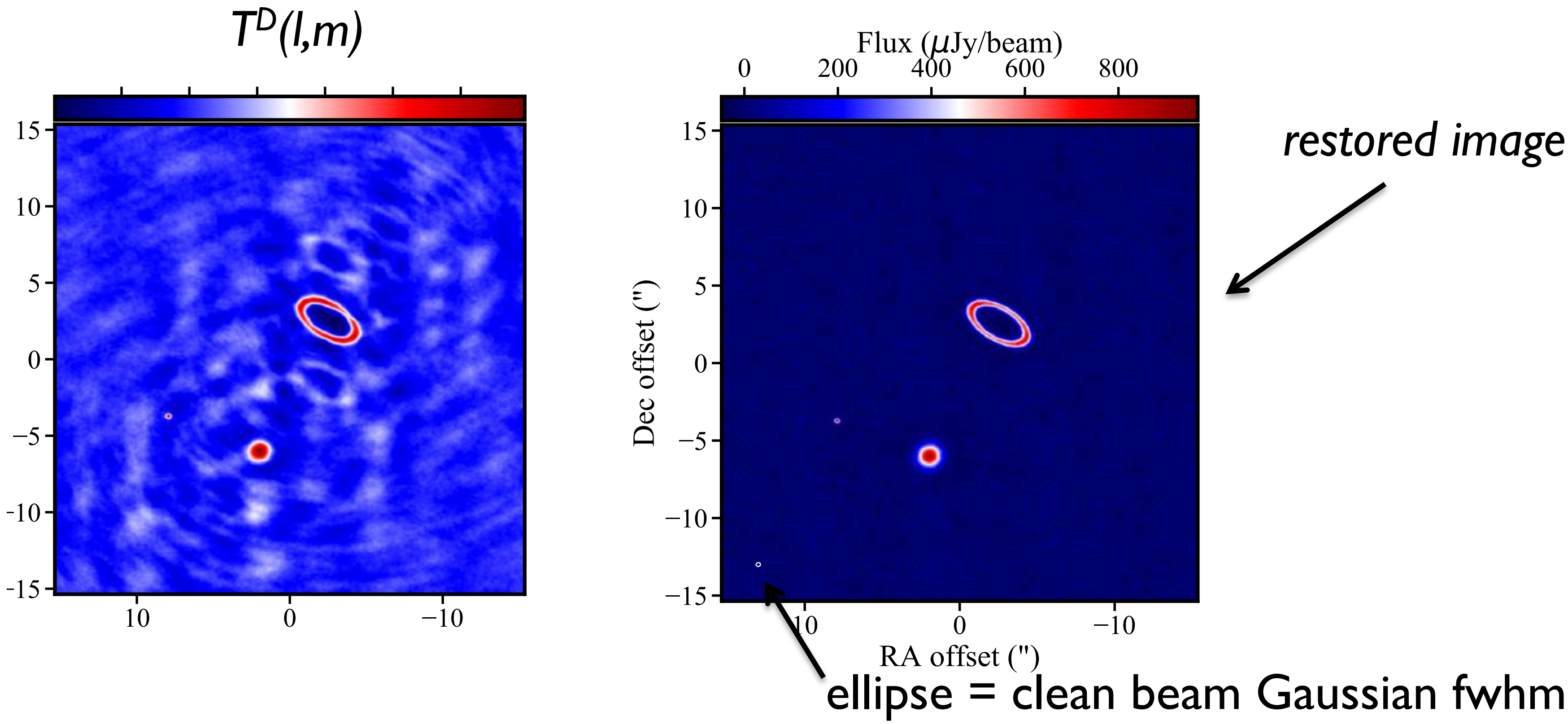
- last step is to create a final “restored” image
  - make a model image with all point source *clean components*
  - convolve point source model image with a “clean beam”, an elliptical Gaussian fit to the main lobe of the dirty beam (avoids super-resolution of the point source component model)
  - add back *residual map* with noise and structure below the threshold
- restored image is an estimate of the true sky brightness *T(l,m)*
  - units of the restored image are (mostly) Jy per clean beam area = intensity, or brightness temperature

- Schwarz (1978) showed that clean is equivalent to a least squares fit of sinusoids to visibilities in the case of no noise

# clean algorithm example



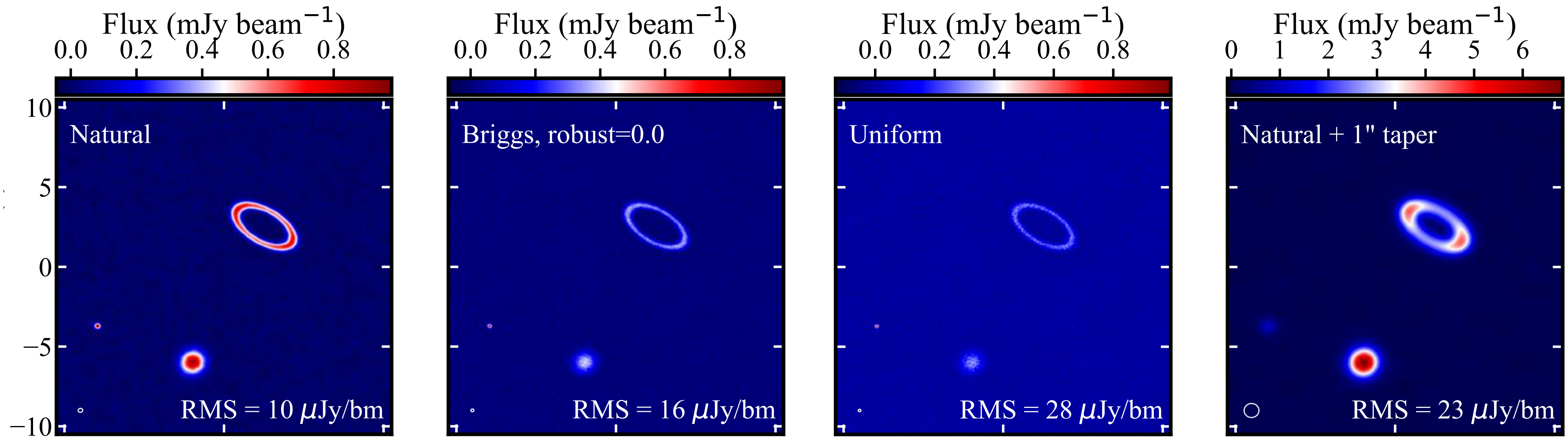
# clean algorithm example: restored image




*final image depends on*

*imaging parameters (pixel size, visibility weighting scheme, gridding) and deconvolution (algorithm, iterations, masks, stopping criteria)*

# Results from Different Weighting Schemes




| natural | robust=0 | uniform | natural + 1” taper |
|---|---|---|---|
| 0.29x0.25 p.a. -81 | 0.19x0.17 p.a. -78 | 0.17x0.15 p.a. -87 | 0.93x0.88 p.a. -86 |

# Tune Imaging Parameters to Science

THE ASTROPHYSICAL JOURNAL LETTERS, 820:L40 (5pp), 2016 April 1 doi:10.3847/2041-8205/820/2/L40

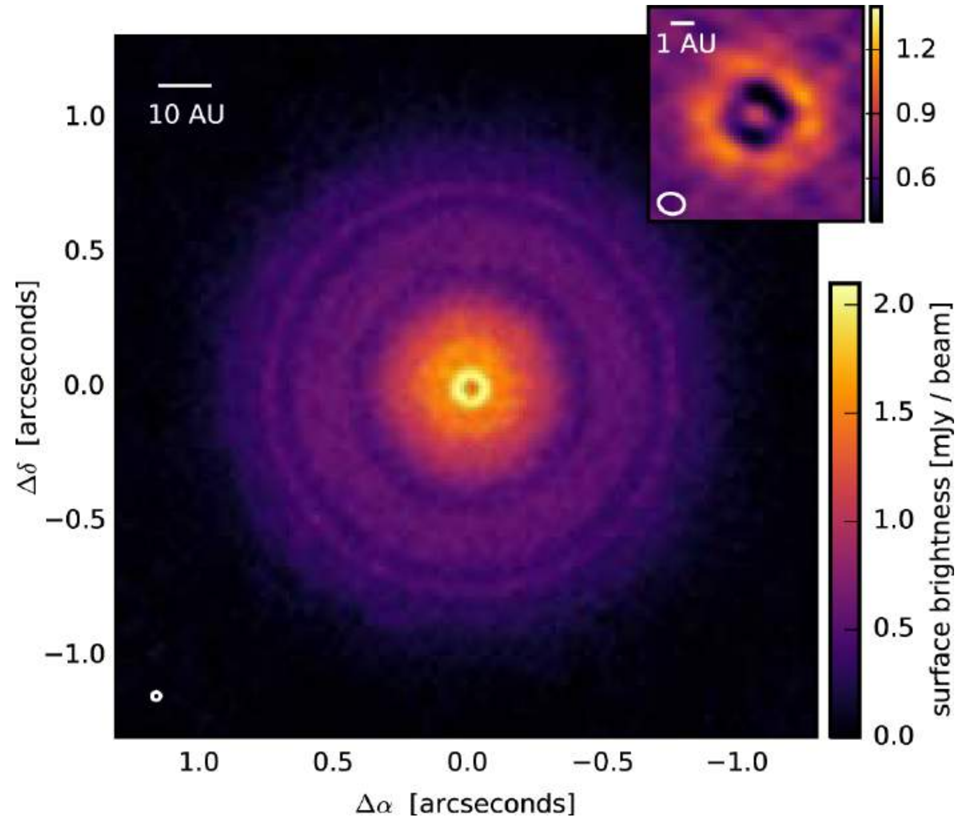

CrossMark

RINGED SUBSTRUCTURE AND A GAP AT 1 au IN THE NEAREST PROTOPLANETARY DISK

SEAN M. ANDREWS[1], DAVID J. WILNER[1], ZHAOHUAN ZHU[2], TILMAN BIRNSTIEL[3], JOHN M. CARPENTER[4], LAURA M. PÉREZ[5], XUE-NING BAI[1], KARIN I. ÖBERG[1], A. MEREDITH HUGHES[6], ANDREA ISELLA[7], AND LUCA RICCI[1]

presents two images of 340 GHz emission from TW Hya protoplanetary disk from the same ALMA visibilities

- robust=0.5 + taper, for a circular 30 mas beam, to show the large scale structure of the disk
- inset: robust=0 for higher resolution, 0.24 x 0.18 mas, to highlight the gap at 1 AU radius



# Variants on Basic clean Algorithm

- "Clark" clean
  - minor cycle

    Högbom clean with smaller beam patch, improves speed
  - major cycle

    *clean components* removed from *gridded* visibilities at once by FFT

- "Cotton-Schwab" clean
  - minor cycle

    Högbom clean with smaller beam patch, improves speed
  - major cycle

    *clean components* removed from *original* visibilities by FFT, then entire imaging process repeated to create residual image

see CASA `tclean` "deconvolver"

# Scale Sensitive Deconvolution

- basic clean is scale-free, treats each pixel as independent
- adjacent pixels in an image may not be independent
  - resolution limit
  - intrinsic source size: an extended source covering 1000 pixels might be better characterized by a few parameters than by 1000 parameters, e.g. 6 parameters for a Gaussian distribution

- scale sensitive deconvolution algorithms employ fewer degrees of freedom to model plausible sky brightness distributions

CASA `tclean` “deconvolver=multiscale” and “scales”

– user must input appropriate scales

– typically a few (delta function, synthesized beam size, few times that)

# CASA `tclean` filename extensions

- <*imagename*>.image
  - restored image
- <*imagename*>.psf
  - point spread function (= dirty beam)
- <*imagename*>.model
  - model image after deconvolution, e.g. clean components
- <*imagename*>.residual
  - residual image, e.g. after subtracting clean components
- <*imagename*>.mask
  - deconvolution mask
- <*imagename*>.pb
  - primary beam model
- <*imagename*>.sumwt
  - a single value sum of visibility weights [for natural weight, rms=$(\text{sumwt})^{-0.5}$ ]

# Maximum Entropy Algorithm

- *a priori* assumption: *T(l,m)* is smooth and positive

maximize a measure of smoothness (entropy)*

$$H = -\sum_k T_k \log\left(\frac{T_k}{M_k}\right)$$

subject to the constraints

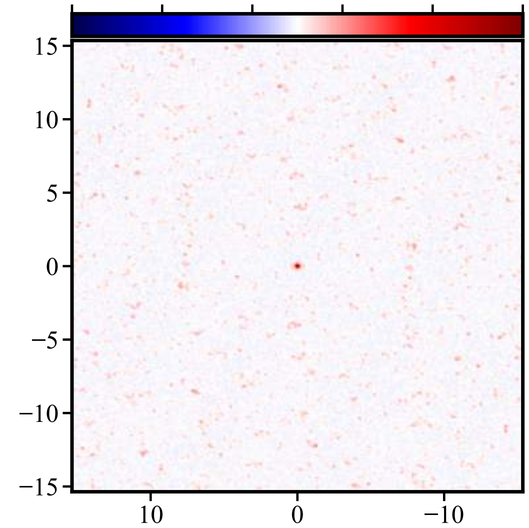
$$\chi^2 = \sum_k \frac{|V(u_k,v_k) - \mathrm{FT}\{T\}|^2}{\sigma_k^2}$$

$$F = \sum_k T_k$$

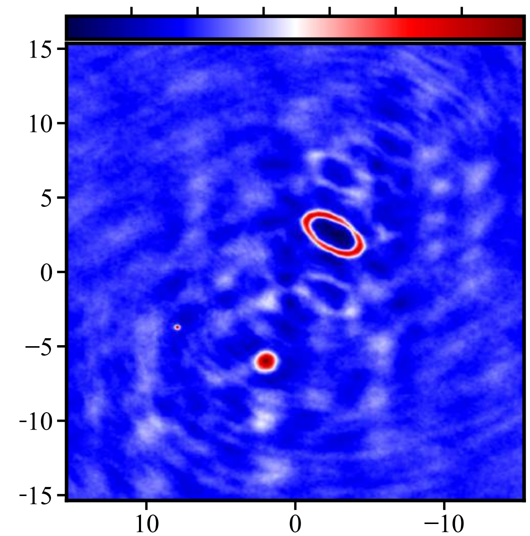
where M is the "default image"

fast (NlogN) solver, Cornwell & Evans (1983)

optional: convolve with Gaussian beam and add residual image to make final image




*s(l,m)*




*$T^D$(l,m)*

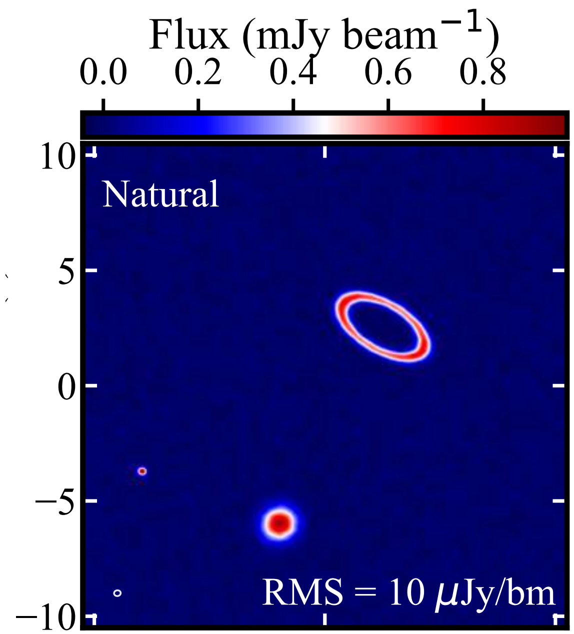
*vast literature about deep meaning of entropy as information content

# Maximum Entropy Algorithm

- easy to include *a priori* information with the default image
  - flat default is usual assumption if nothing known
  - a single dish image may be a good default
- straightforward to generalize $\chi^2$ to combine different observations/telescopes to obtain optimal image
- many measures of entropy available
  - e.g. replace log with cosh $\rightarrow$ "emptiness" (does not enforce positivity)

- less robust and harder to drive than clean
- works best on smooth, extended emission
- can have trouble with point source sidelobes (could remove the point sources first with clean)

# Measures of Image Quality




- *dynamic range*
  - ratio of peak brightness in image to rms noise in a region void of emission
  - easy way to calculate a lower limit to the error in brightness in a non-empty region
  - e.g. peak = 0.9 mJy/beam, rms = 10 μJy/beam

    $\rightarrow$ dynamic range = 90
- *fidelity*
  - difference between any produced image and the correct image
  - fidelity image = input model / difference

    = model * beam / abs(model * beam – reconstruction)

    = inverse of the relative error
  - need knowledge of the correct image to calculate
  - fidelity often much worse than dynamic range

# “Invisible” Large Scale Structure

- important structure missed in central hole of *(u,v)* plane
- to estimate if lack of short baselines will be problematic
  - simulate the observations with a source model
  - check simple expressions for a Gaussian source or uniform disk

*Homework Problem*

- Q: By what factor is the central brightness reduced as a function of source size due to missing short spacings for a Gaussian characterized by fwhm $\theta_{1/2}$
- A: a Gaussian source central brightness is reduced 50% when

$$\theta_{1/2} = 18'' \left(\frac{\nu}{100\ GHz}\right)^{-1} \left(\frac{B_{min}}{15\ meters}\right)^{-1}$$

where $B_{min}$ is the shortest baseline [meters], ν is the frequency [GHz] (derivation in appendix of Wilner & Welch 1994, ApJ, 427, 898)

# ALMA "Maximum Recoverable Scale"

- adopted to be 10% of the total flux density of a uniform disk (not much!)

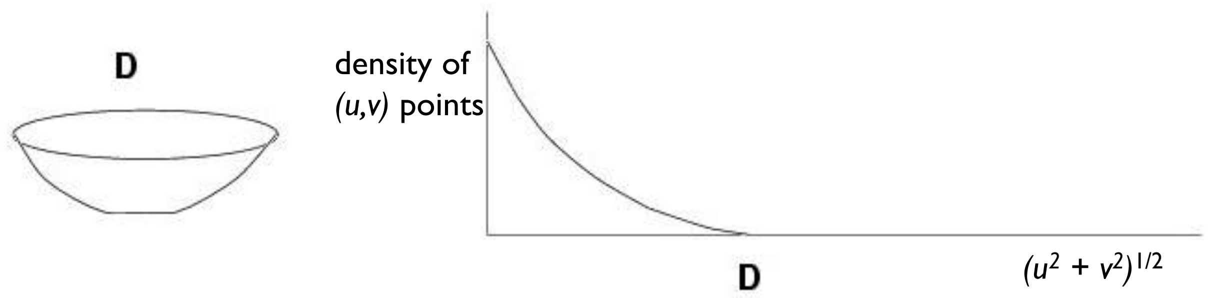
| | Band | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| | Frequency (GHz) | 100 | 150 | 185 | 230 | 345 | 460 | 650 | 870 |
| Configuration | | | | | | | | | |
| 7-m | $\theta_{res}$ (arcsec) | 12.5 | 8.35 | 6.77 | 5.45 | 3.63 | 2.72 | 1.93 | 1.44 |
| | $\theta_{MRS}$ (arcsec) | 66.7 | 44.5 | 36.1 | 29.0 | 19.3 | 14.5 | 10.3 | 7.67 |
| C43-1 | $\theta_{res}$ (arcsec) | 3.38 | 2.25 | 1.83 | 1.47 | 0.98 | 0.735 | 0.52 | 0.389 |
| | $\theta_{MRS}$ (arcsec) | 28.5 | 19.0 | 15.4 | 12.4 | 8.25 | 6.19 | 4.38 | 3.27 |
| C43-2 | $\theta_{res}$ (arcsec) | 2.3 | 1.53 | 1.24 | 0.999 | 0.666 | 0.499 | 0.353 | 0.264 |
| | $\theta_{MRS}$ (arcsec) | 22.6 | 15.0 | 12.2 | 9.81 | 6.54 | 4.9 | 3.47 | 2.59 |
| C43-3 | $\theta_{res}$ (arcsec) | 1.42 | 0.943 | 0.765 | 0.615 | 0.41 | 0.308 | 0.218 | 0.163 |
| | $\theta_{MRS}$ (arcsec) | 16.2 | 10.8 | 8.73 | 7.02 | 4.68 | 3.51 | 2.48 | 1.86 |
| C43-4 | $\theta_{res}$ (arcsec) | 0.918 | 0.612 | 0.496 | 0.399 | 0.266 | 0.2 | 0.141 | 0.106 |
| | $\theta_{MRS}$ (arcsec) | 11.2 | 7.5 | 6.08 | 4.89 | 3.26 | 2.44 | 1.73 | 1.29 |
| C43-5 | $\theta_{res}$ (arcsec) | 0.545 | 0.363 | 0.295 | 0.237 | 0.158 | 0.118 | 0.0838 | 0.0626 |
| | $\theta_{MRS}$ (arcsec) | 6.7 | 4.47 | 3.62 | 2.91 | 1.94 | 1.46 | 1.03 | 0.77 |
| C43-6 | $\theta_{res}$ (arcsec) | 0.306 | 0.204 | 0.165 | 0.133 | 0.0887 | 0.0665 | 0.0471 | 0.0352 |
| | $\theta_{MRS}$ (arcsec) | 4.11 | 2.74 | 2.22 | 1.78 | 1.19 | 0.892 | 0.632 | 0.472 |
| C43-7 | $\theta_{res}$ (arcsec) | 0.211 | 0.141 | 0.114 | 0.0917 | 0.0612 | 0.0459 | 0.0325 | 0.0243 |
| | $\theta_{MRS}$ (arcsec) | 2.58 | 1.72 | 1.4 | 1.12 | 0.749 | 0.562 | 0.398 | 0.297 |
| C43-8 | $\theta_{res}$ (arcsec) | 0.096 | 0.064 | 0.0519 | 0.0417 | 0.0278 | - | - | - |
| | $\theta_{MRS}$ (arcsec) | 1.42 | 0.947 | 0.768 | 0.618 | 0.412 | - | - | - |
| C43-9 | $\theta_{res}$ (arcsec) | 0.057 | 0.038 | 0.0308 | 0.0248 | - | - | - | - |
| | $\theta_{MRS}$ (arcsec) | 0.814 | 0.543 | 0.44 | 0.354 | - | - | - | - |
| C43-10 | $\theta_{res}$ (arcsec) | 0.042 | 0.028 | 0.0227 | 0.0183 | - | - | - | - |
| | $\theta_{MRS}$ (arcsec) | 0.496 | 0.331 | 0.268 | 0.216 | - | - | - | - |

Table 7.1: Resolution ($\theta_{res}$) and maximum recoverable scale ($\theta_{MRS}$) for the 7-m Array and 12-m Array configurations available during Cycle 6 as a function of a representative frequency in a band. The value of $\theta_{MRS}$ is computed using the $5^{th}$ percentile baseline (L05) from Table 7.2 and Equation 7.7. The value of $\theta_{res}$ is the mean size of the interferometric beam obtained through simulation with CASA, using Briggs $(u, v)$ plane weighting with *robust=0.5*. The computations were done for a source at zenith; for sources transiting at lower elevations, the North-South angular measures will increase proportional to 1/sin(ELEVATION).

ALMA Cycle 6 Technical Handbook

# Techniques to Obtain Short Spacings (1)

use a large single dish telescope




- *all* Fourier components from 0 to D sampled, where D is dish diameter (weighting depends on illumination)
- scan single dish across sky to make an image $T(l,m) * A(l,m)$
  where $A(l,m)$ is the single dish response pattern
- Fourier transform single dish image, $T(l,m) * A(l,m)$, to get $V(u,v)a(u,v)$
  and then divide by $a(u,v)$ to estimate $V(u,v)$ for baselines < D
- choose D large enough to overlap interferometer samples of $V(u,v)$
  and avoid using data where $a(u,v)$ becomes small
- example: VLA and GBT

# Techniques to Obtain Short Spacings (II)

use a separate array of smaller antennas

- small antennas can observe short baselines inaccessible to larger ones
- the larger antennas can be used as single dish telescopes to make images with Fourier components not accessible to the smaller antennas
- example: ALMA main array + ACA

main array

50x12m: 12m to 16km

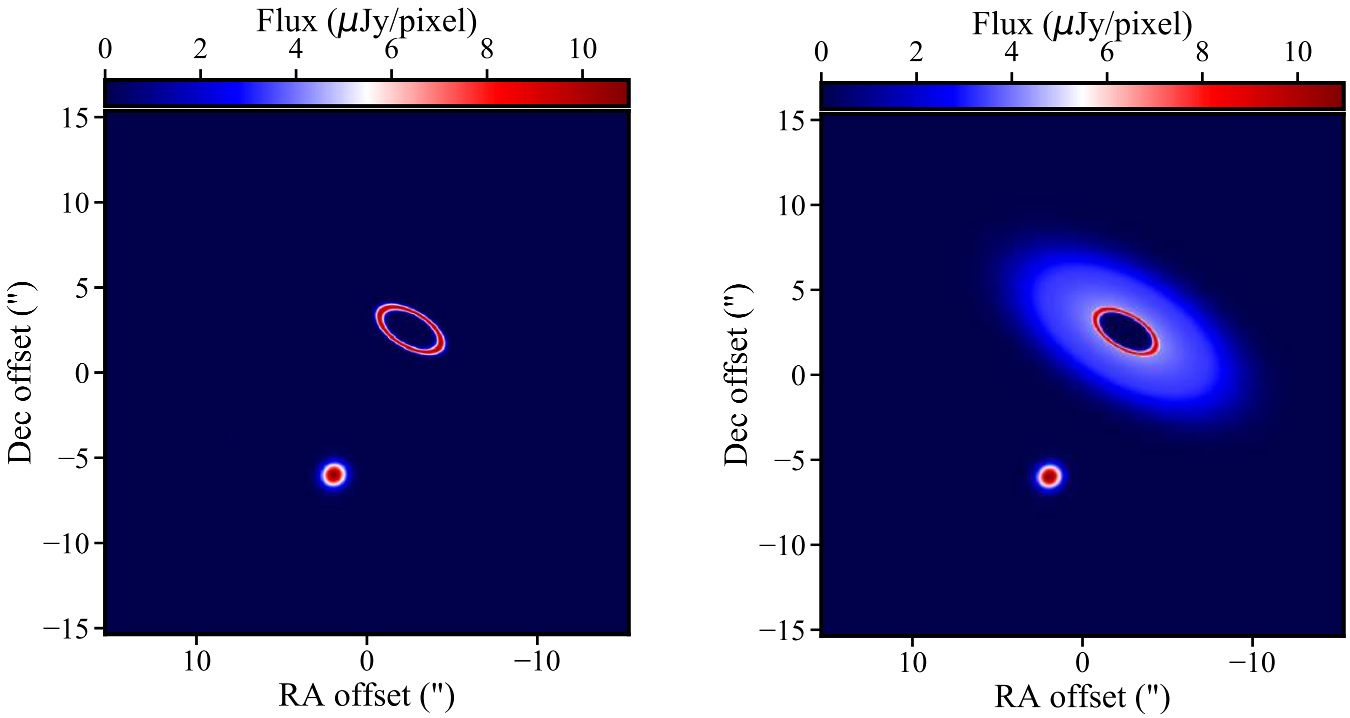
ACA

12 x 7m: covers 7 to12m+

4 x 12m single dishes: 0 to 7m

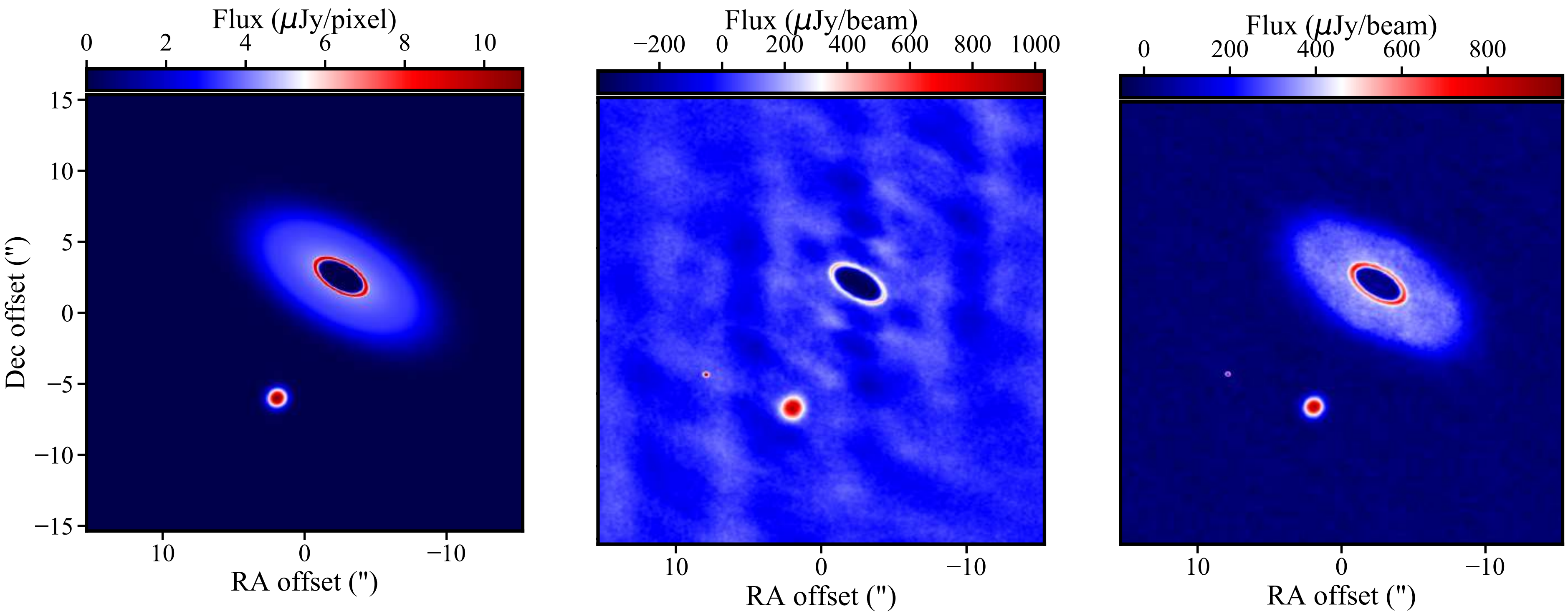
# Missing Short Baselines: Demonstration

- Do the visibilities observed in our example discriminate between these two models of sky brightness *T(l,m)*?



# Missing Short Baselines: Demonstration

*T(l,m)* | *ALMA 12m array* | *ALMA 12m array + ACA*




*n.b. clean does not reach theoretical rms due to poorly sampled extended structure*

*much improved*

# Example: Missing Short Baselines

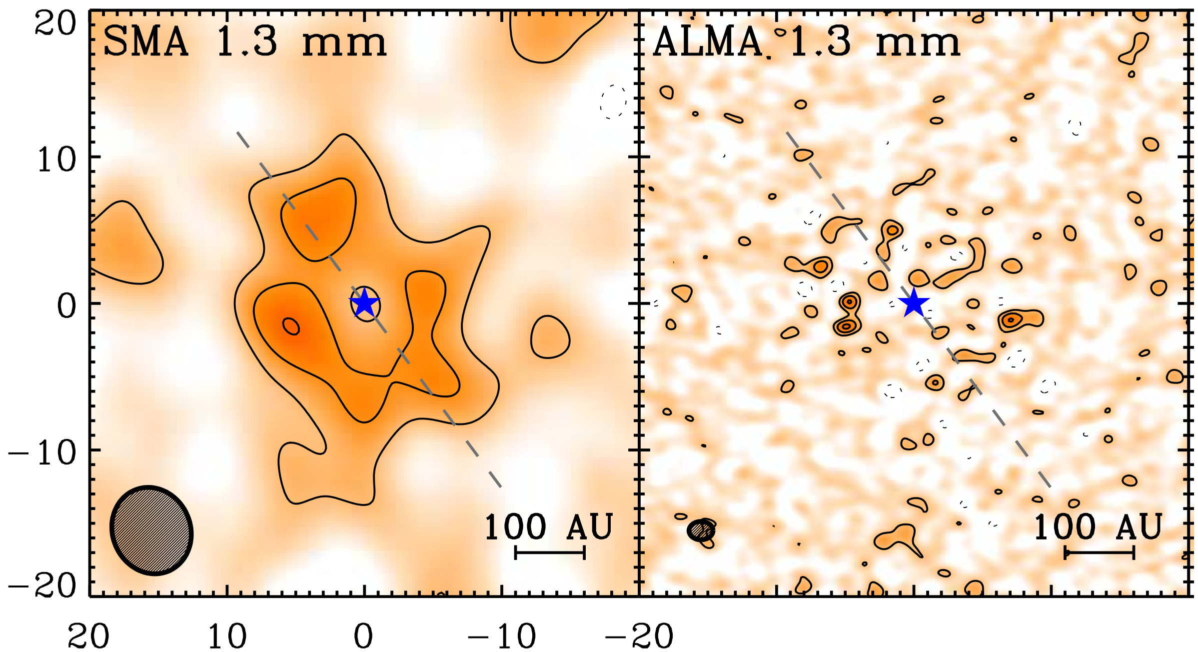
The Astrophysical Journal, 855:56 (10pp), 2018 March 1
https://doi.org/10.3847/1538-4357/aaacd7


CrossMark

## Resolved Millimeter Observations of the HR 8799 Debris Disk

David J. Wilner[1], Meredith A. MacGregor[1,2,6], Sean M. Andrews[1], A. Meredith Hughes[3], Brenda Matthews[4], and Kate Su[5]

SMA 8x6m




Booth et al. 2016
ALMA 38x12m

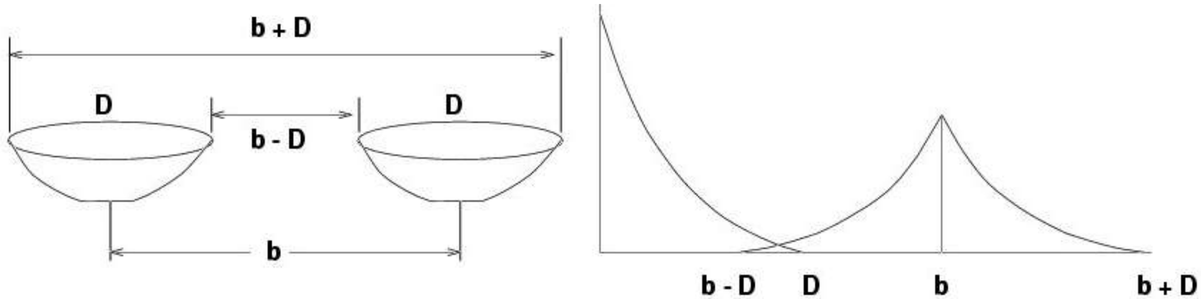
beam 6.1x5.6 arcsec
rms 180 μJy/beam

beam 1.7x1.2 arcsec
rms 16 μJy/beam

# Techniques to Obtain Short Spacings (III)

mosaic with a homogeneous array

- recover a range of spatial frequencies around the nominal baseline b using knowledge of A(l,m), shortest spacings from single dishes (Ekers & Rots 1979)




- *V(u,v)* is a linear combination of baselines from b-D to b+D
- depends on pointing direction *($l_0$,$m_0$)* as well as on *(u,v)*

$$V(u,v;l_0,m_0) = \int\int T(l,m)A(l-l_0,m-m_0)e^{i2\pi(ul+vm)}dldm$$

- Fourier transform with respect to pointing direction *($l_0$,$m_0$)*

$$V(u-u_0,v-v_0) = \left(\int\int V(u,v;l_0,m_0)e^{i2\pi(u_0l_0+v_0m_0)}dl_0dm_0\right)/a(u_0,v_0)$$

# Self Calibration

- *a priori* calibration from external calibrators must be interpolated from different time and sky direction from source, which leaves errors
- self calibration corrects for antenna based phase and amplitude errors *together with imaging* to create an improved source model
- why should this work?
  - at each time, measure N complex gains and N(N-1)/2 visibilities
  - source structure can be represented by small number of parameters
  - a highly overconstrained problem if N large and source simple
- in practice, an iterative, non-linear relaxation process
  - assume source model $\rightarrow$ solve for time dependent gains $\rightarrow$ form new source model from corrected data using clean $\rightarrow$ solve for new gains
  - requires sufficient signal-to-noise at each solution interval

- loses absolute phase from calibrators and therefore position information
- dangerous with small N arrays, complex sources, low signal-to-noise

# Concluding Remarks

- interferometry samples Fourier components of sky brightness
- make an image by Fourier transforming sampled visibilities
- deconvolution attempts to correct for incomplete sampling

- remember
  - there are an infinite number of images compatible with the visibilities
  - missing (or corrrupted) visibilities affect the entire image
  - astronomers must use judgement in imaging and deconvolution
- it's fun and worth the trouble $\rightarrow$ high resolution images!

*many, many issues not covered in this talk, see references*

# END