

SKA Reflector Design

Robert Lehmensiek
EMSS Antennas

Dirk de Villiers
Stellenbosch University



science and technology

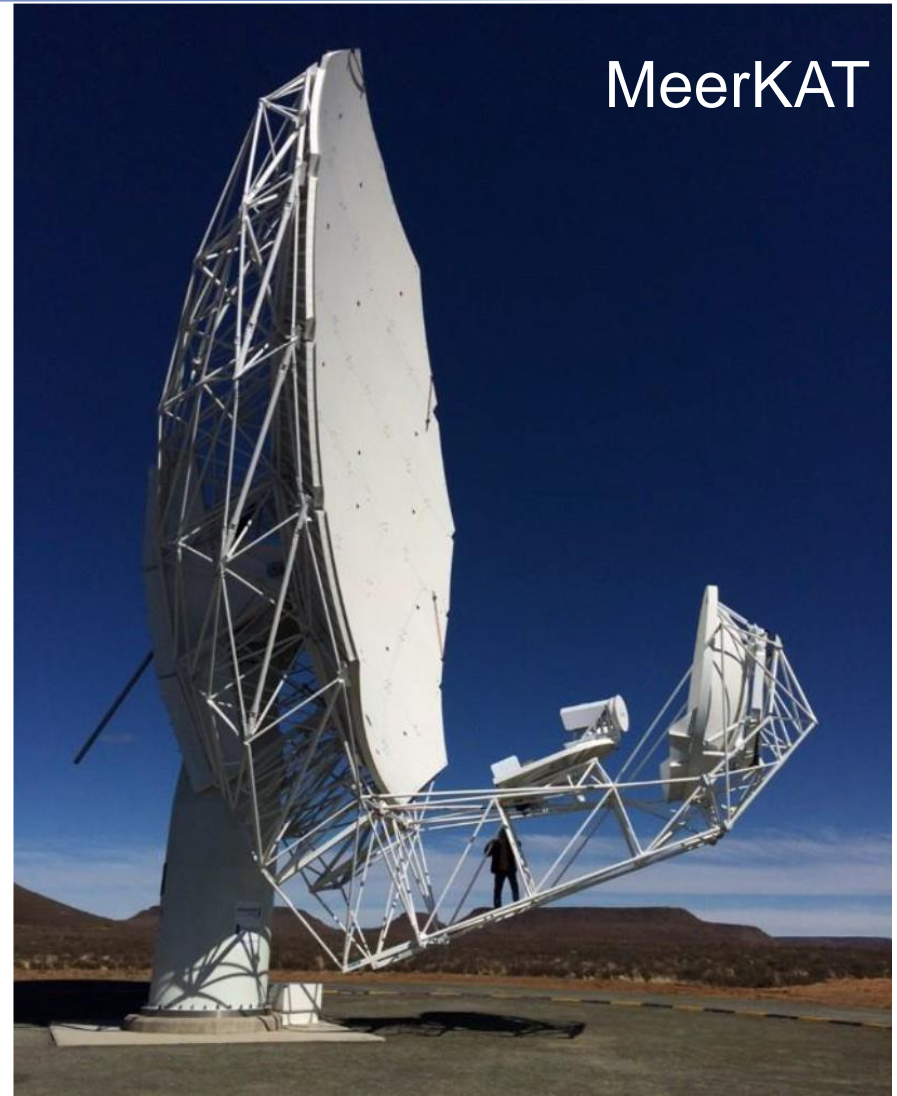
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Background



XDM: 2005 - 2007

- 15 m prime focus cluster feed
- Single antenna, Haartebeeshoek
- Subtended half-angle: $\sim 53^\circ$
- 1414 – 1670 MHz



R. Lehmsiek and I. P. Theron, "On the design of the feed horns for the Karoo Array Telescope dish antennas," in *Proc. IEEE Africon Conf.*, Windhoek, Namibia, Sep. 2007, pp. 1-5.

KAT-7: 2008 - 2012

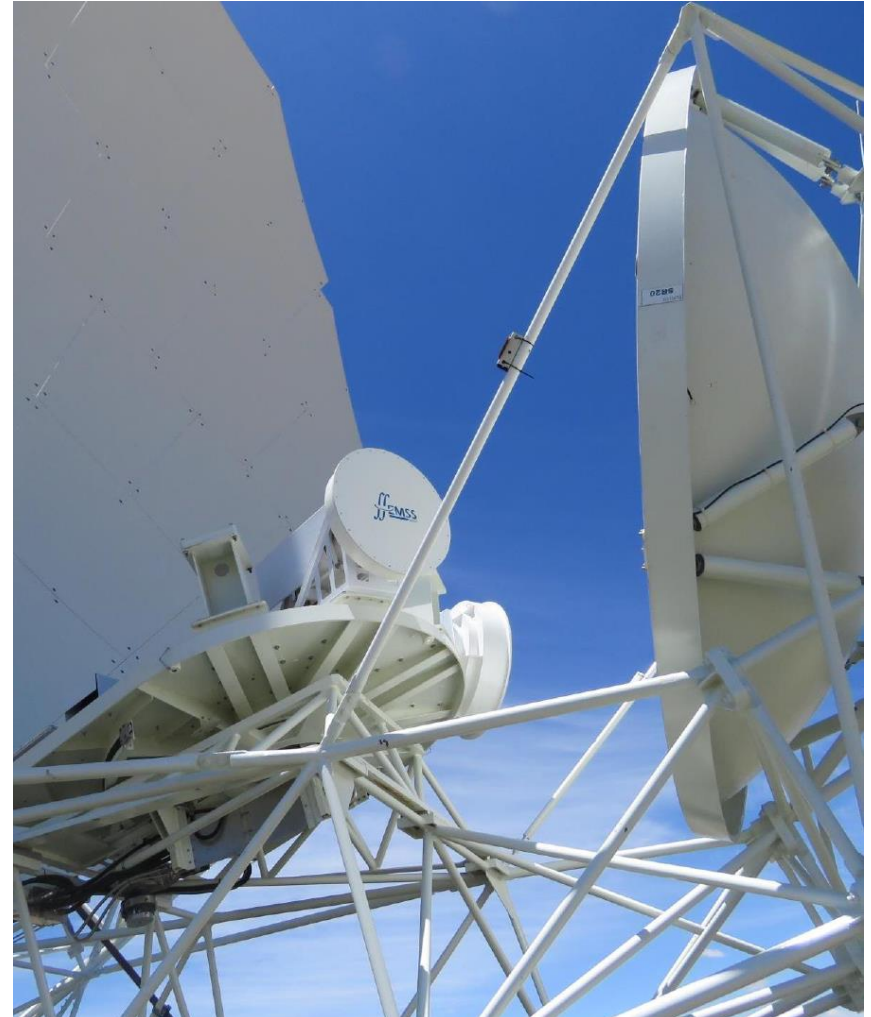
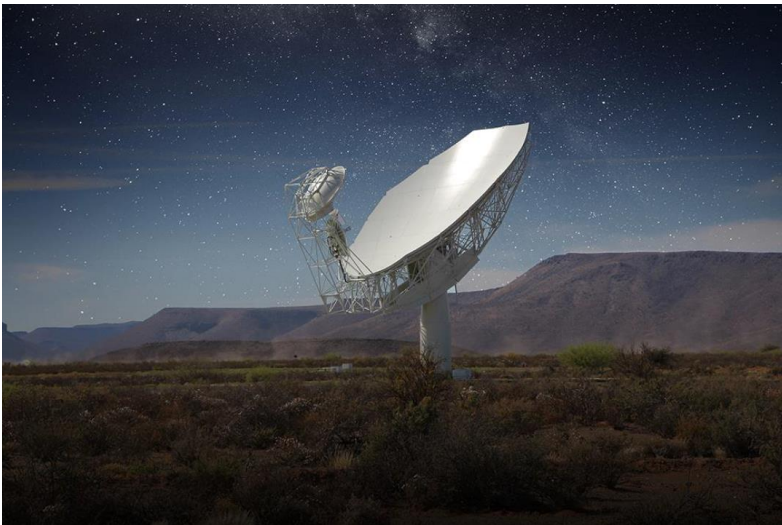
- 12 m prime focus
- 7 antennas,
SKA core site
- Subtended half-angle: $\sim 67^\circ$
- 1200 – 1950 MHz



A. R. Foley, et al., “Engineering and science highlights of the KAT-7 radio telescope,” *Mon. Not. R. Astron. Soc.*, vol. 460, no. 2, pp. 1664-1679, Aug. 2016.

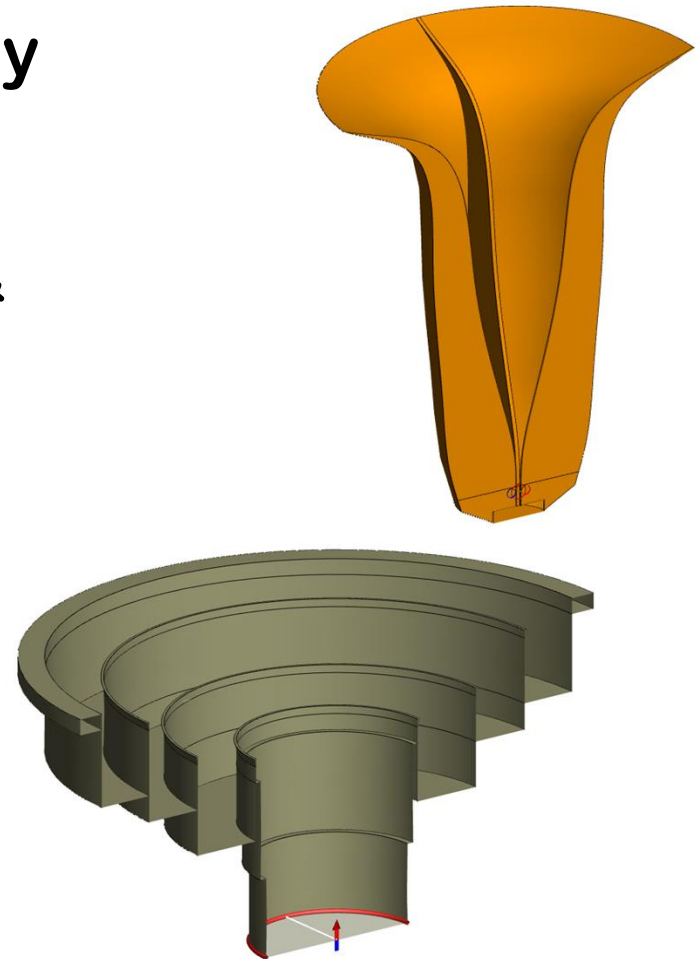
MeerKAT: 2011 - 2018

- 13.8 m offset Gregorian
- 64 antennas, SKA core site
- Subtended half-angle: $\sim 49^\circ$
- UHF: 580 – 1050 MHz
L-band: 900 – 1670 MHz
S-band: 1750 – 3440 MHz
X-band: planned



F. Camilo, *et al.*, “Revival of the magnetar PSR J1622-4950: Observations with MeerKAT, Parkes, XMM-Newton, Swift, Chandra, and NuSTAR,” *The Astrophysical Journal*, vol. 856, no. 2, article id. 180, pp. 1-11, Apr. 2018.

- **SKA – Square Kilometre Array**
 - 1 km² receiving aperture
 - **SKA1-mid (~10% of full SKA)**
 - 64 x 13.5 m MeerKAT (precursor) & 133 x 15 m SKA1 shaped dishes
 - **Six single pixel feeds:**
 - Band 1: 350 – 1050 MHz (3:1)
 - Band 2: 950 – 1760 MHz (1.85:1)
 - Band 3: 1.65 – 3.05 GHz (1.85:1)
 - Band 4: 2.8 – 5.18 GHz (1.85:1)
 - Band 5a: 4.6 – 8.5 GHz (1.85:1)
 - Band 5b: 8.3 – 15.4 GHz (1.86:1)
 - **SKA1-low (Log-Periodic)**
 - **Advanced program on PAFs**

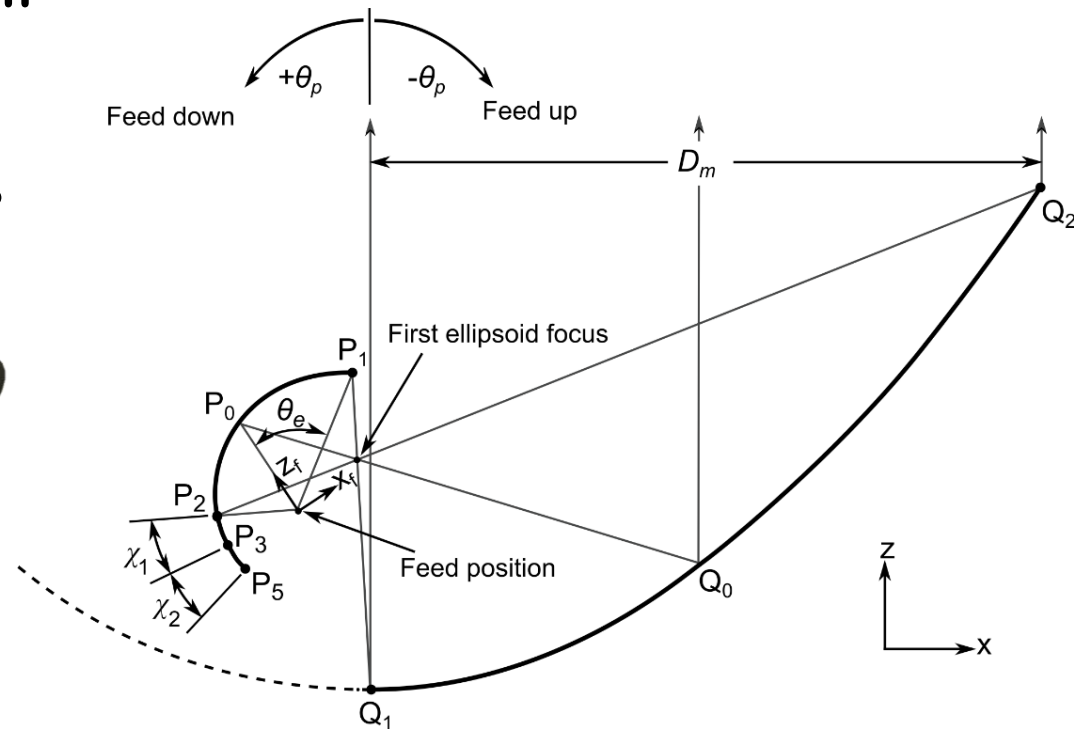


G. H. Tan, R. Lehmensiek, B. Billade, K. Caputa, S. Gauffre, I. P. Theron, M. Pantaleev, Z. Ljusic, B. Quartier and A. Peens-Hough, “An innovative, highly sensitive receiver system for the Square Kilometre Array Mid Radio Telescope,” in *Proc. SPIE Astronomical Telescopes and Instrumentation*, Edinburgh, United Kingdom, Jun. 2016, vol. 9906.

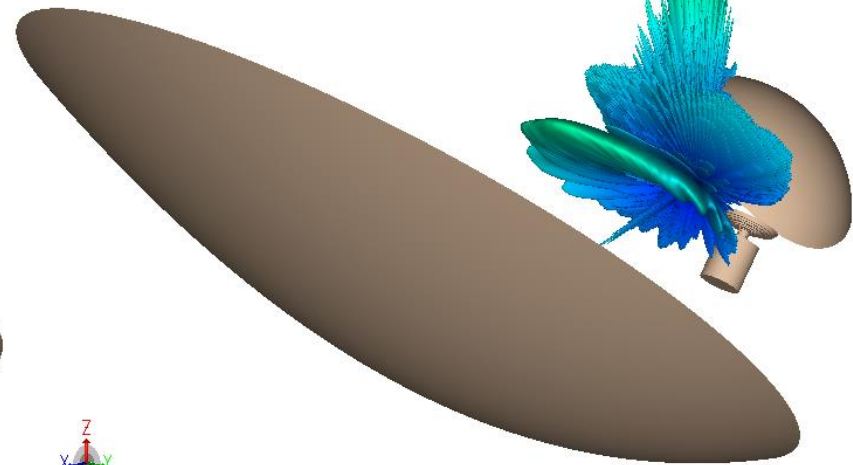
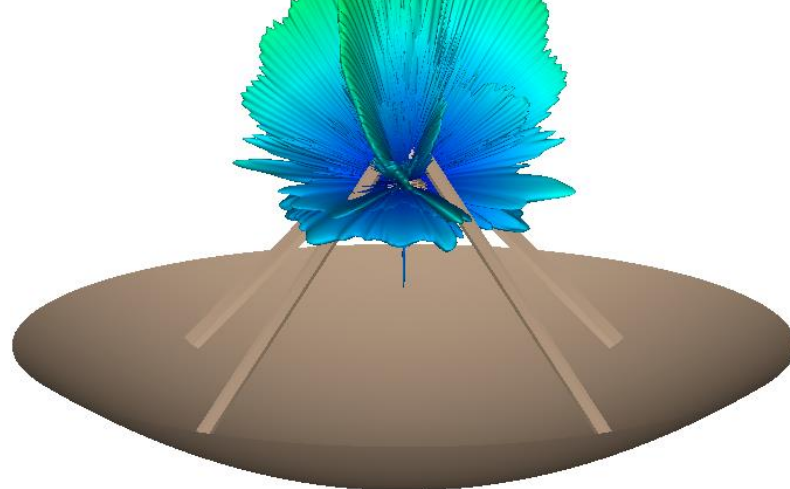
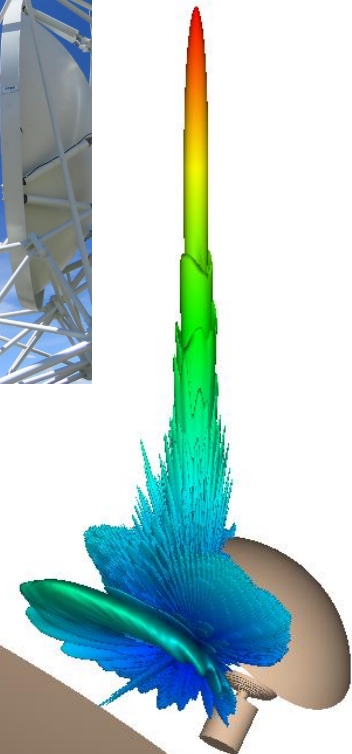
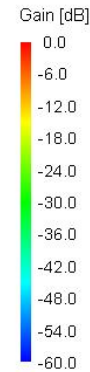
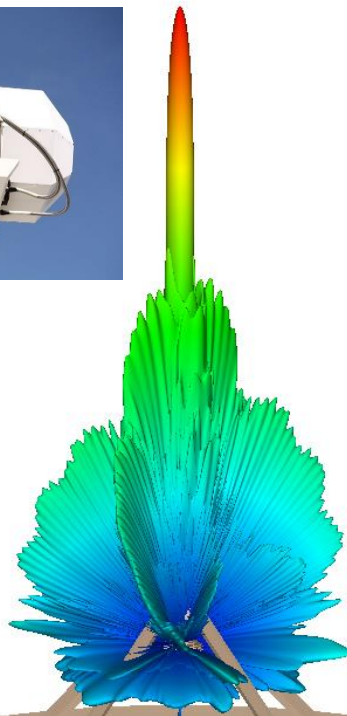
Reflector system

- **Offset Gregorian reflector system**

- **Main:** ➤ $D_m = 15$ m
 - $\|Q_1Q_2\| \approx 18.2$ m
- **Sub:** ➤ $\|x_{P_1}x_{Q_1}\| \approx 0.5$ m
 - $\|P_1P_3\| = 5$ m
 - $\theta_e = 58^\circ$
- **Extension:** ➤ $\chi_{1,2} = 40^\circ$
- **Tipping:** Feed down

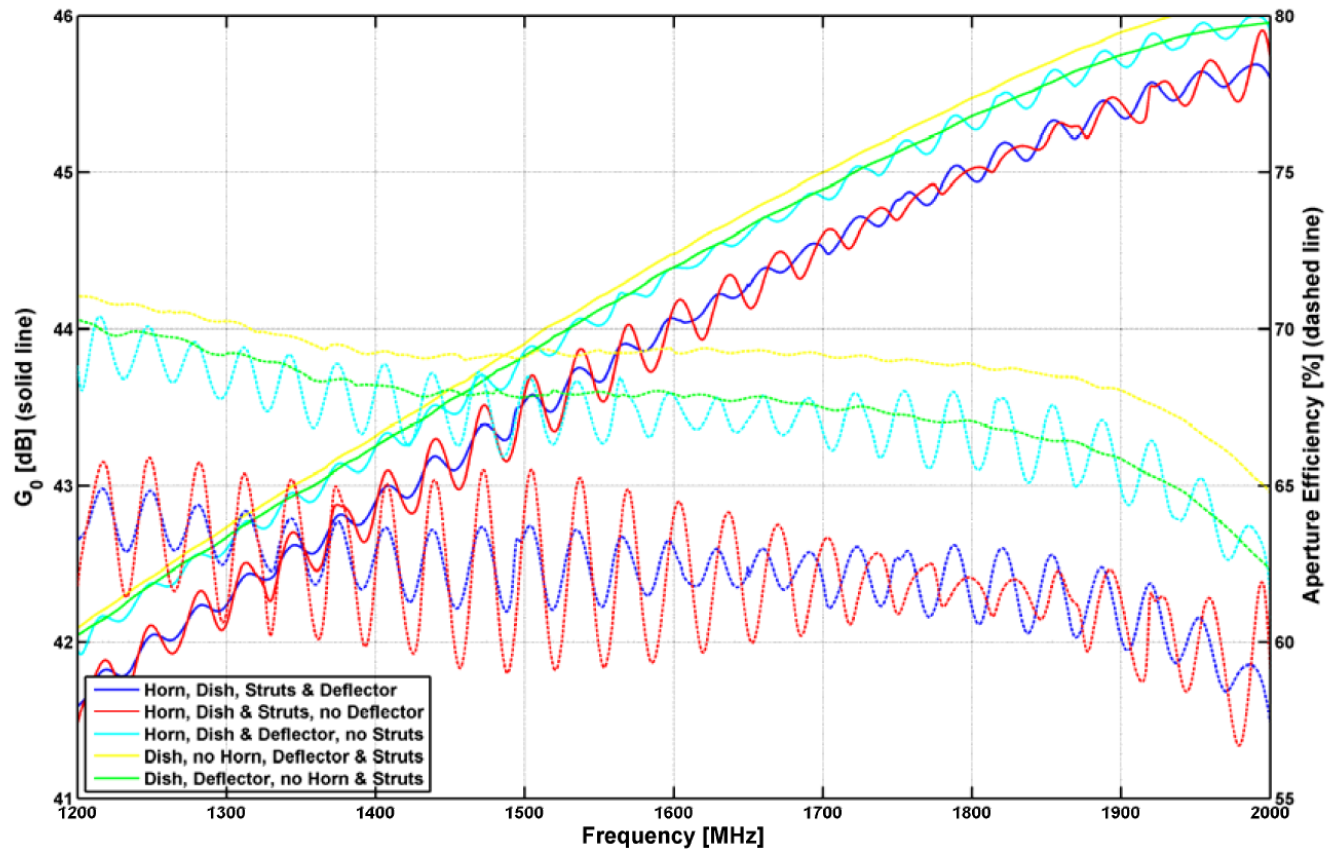


Prime Focus vs Offset Gregorian



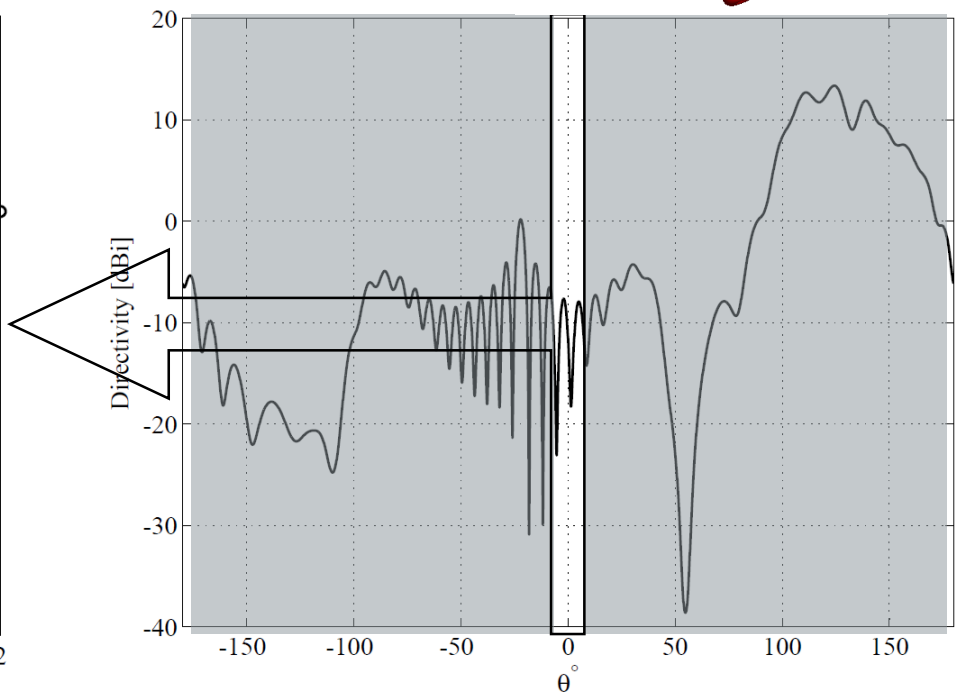
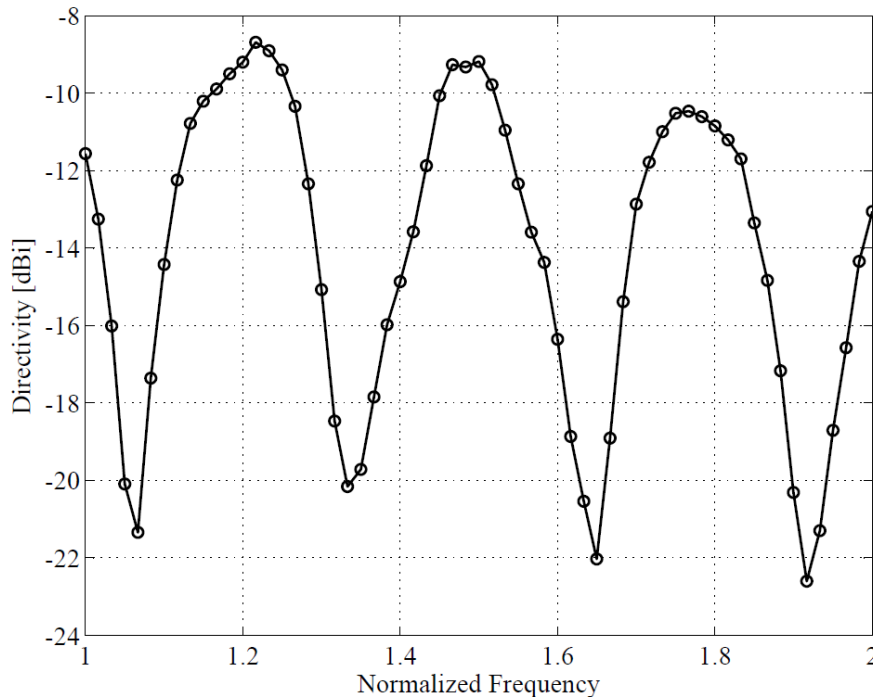
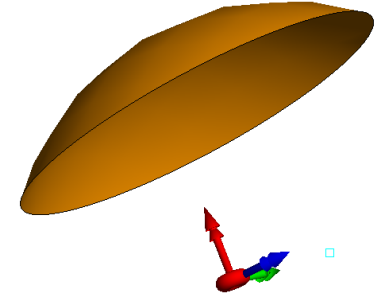
I. P. Theron, R. Lehmensiek and D. I. L. de Villiers, "The design of the MeerKAT dish optics," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Cape Town, South Africa, Sep. 2012, pp. 539-542.

Frequency ripple: KAT-7



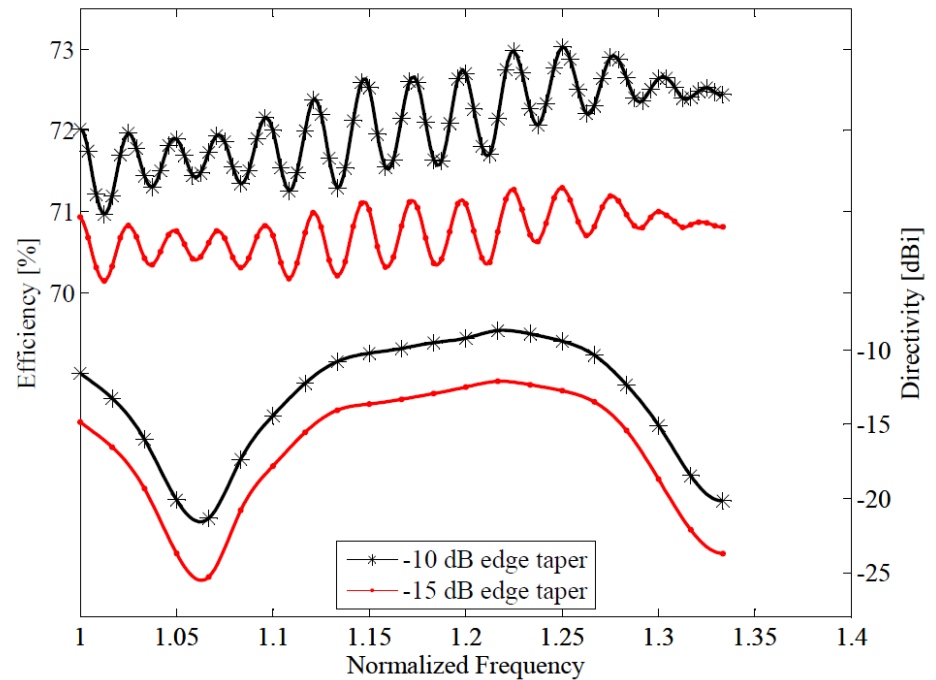
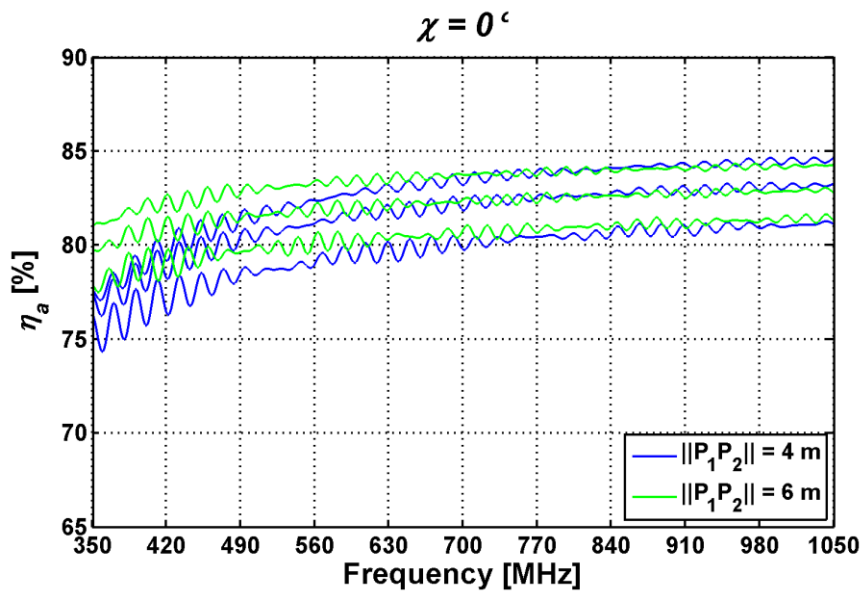
Frequency ripple: Offset Gregorian

- Diffraction from the sub-reflector can be significant in small to medium sized systems (tens of wavelengths SR size – result below for 10λ case)



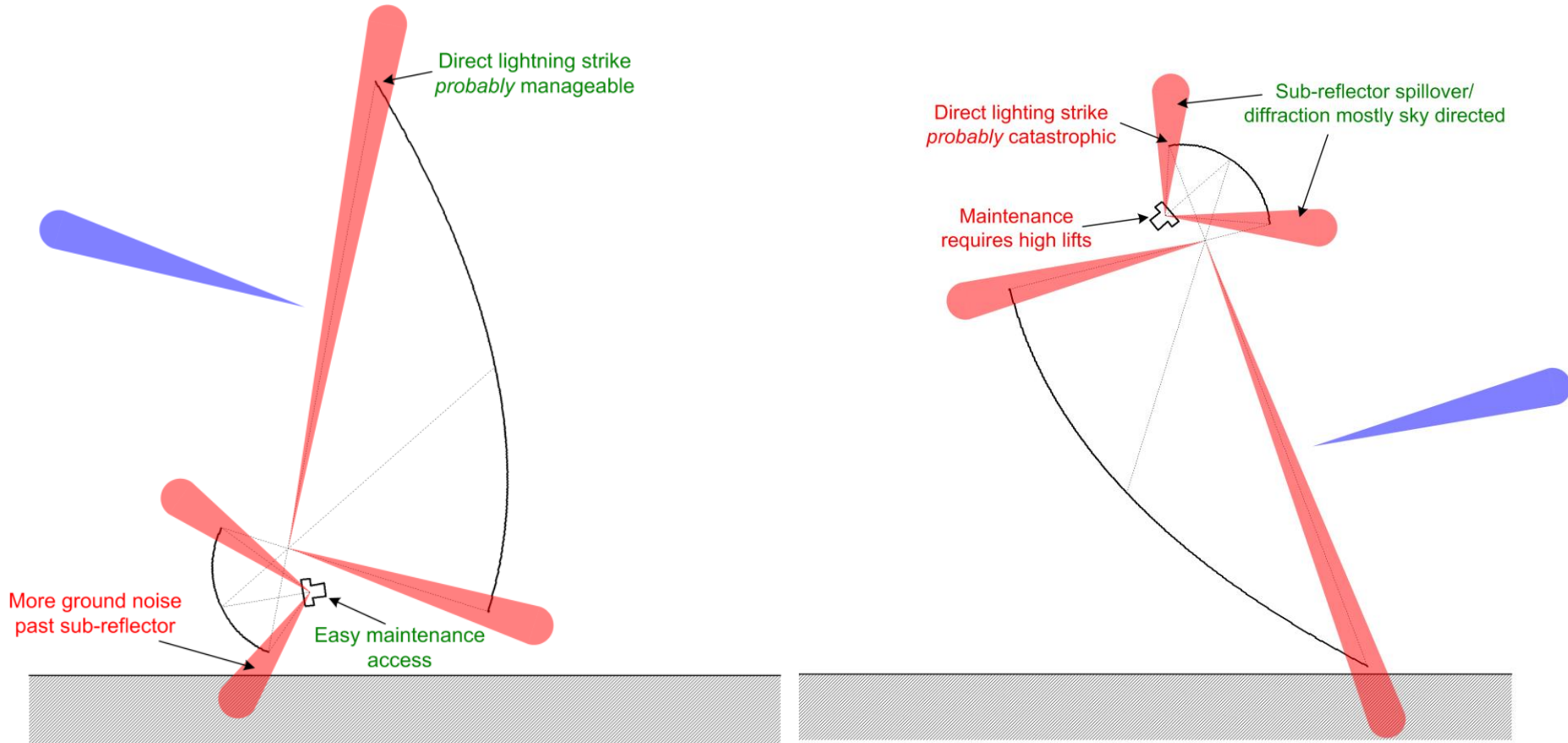
D.I.L. de Villiers, "Prediction of aperture efficiency ripple in clear aperture offset Gregorian antennas", *IEEE Trans. Antennas Propag.*, vol. 61, no. 5, pp. 2457 – 2465, May, 2013.

Frequency ripple: Offset Gregorian

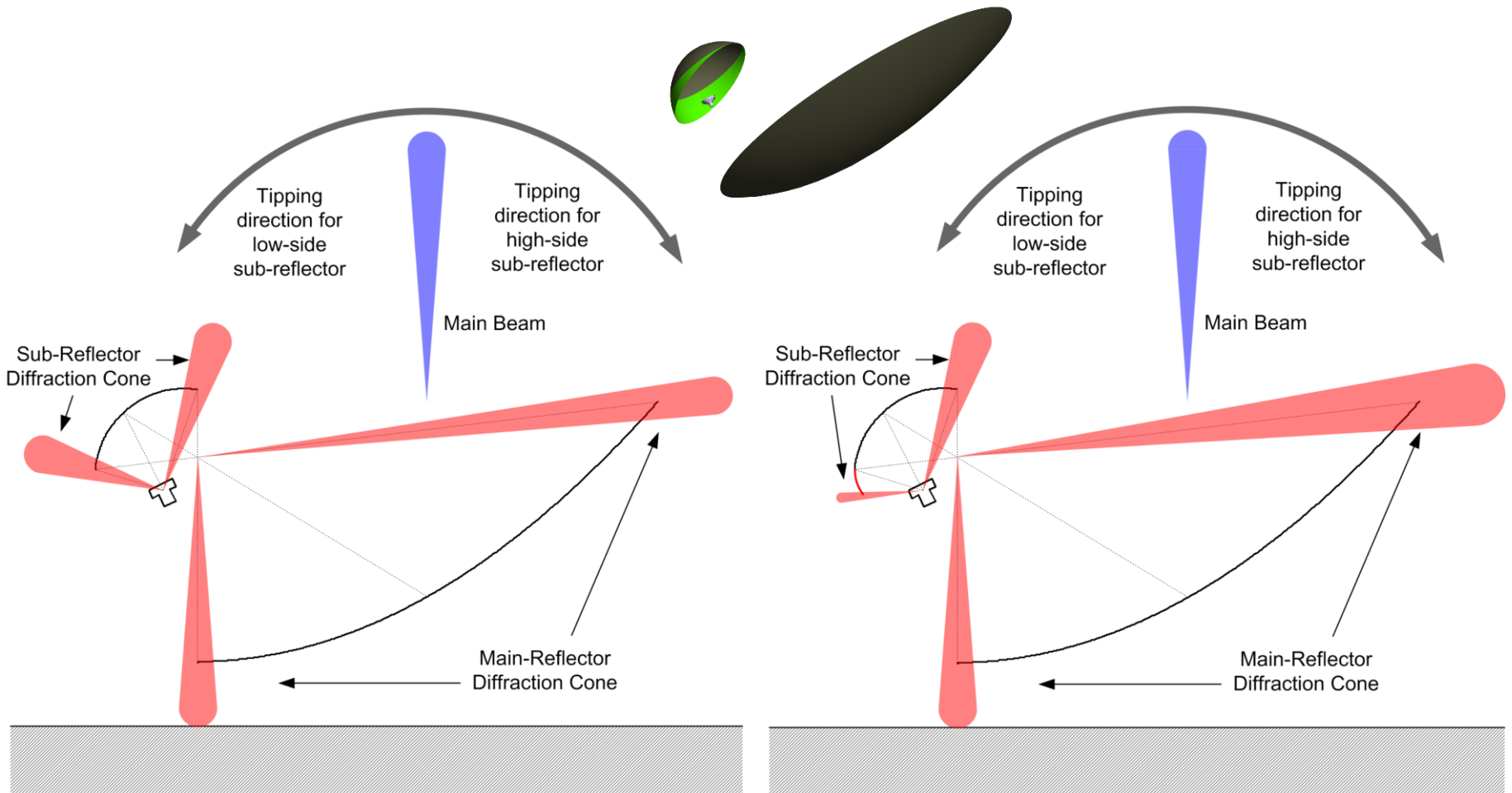


D. I. L. de Villiers, R. Lehmsiek and M. V. Ivashina, "Effects of diffraction and feed pattern variation in shaped offset Gregorian reflectors," *IEICE Trans. Commun.*, vol. E101-B, no. 2, pp. 316-323, Feb. 2018.

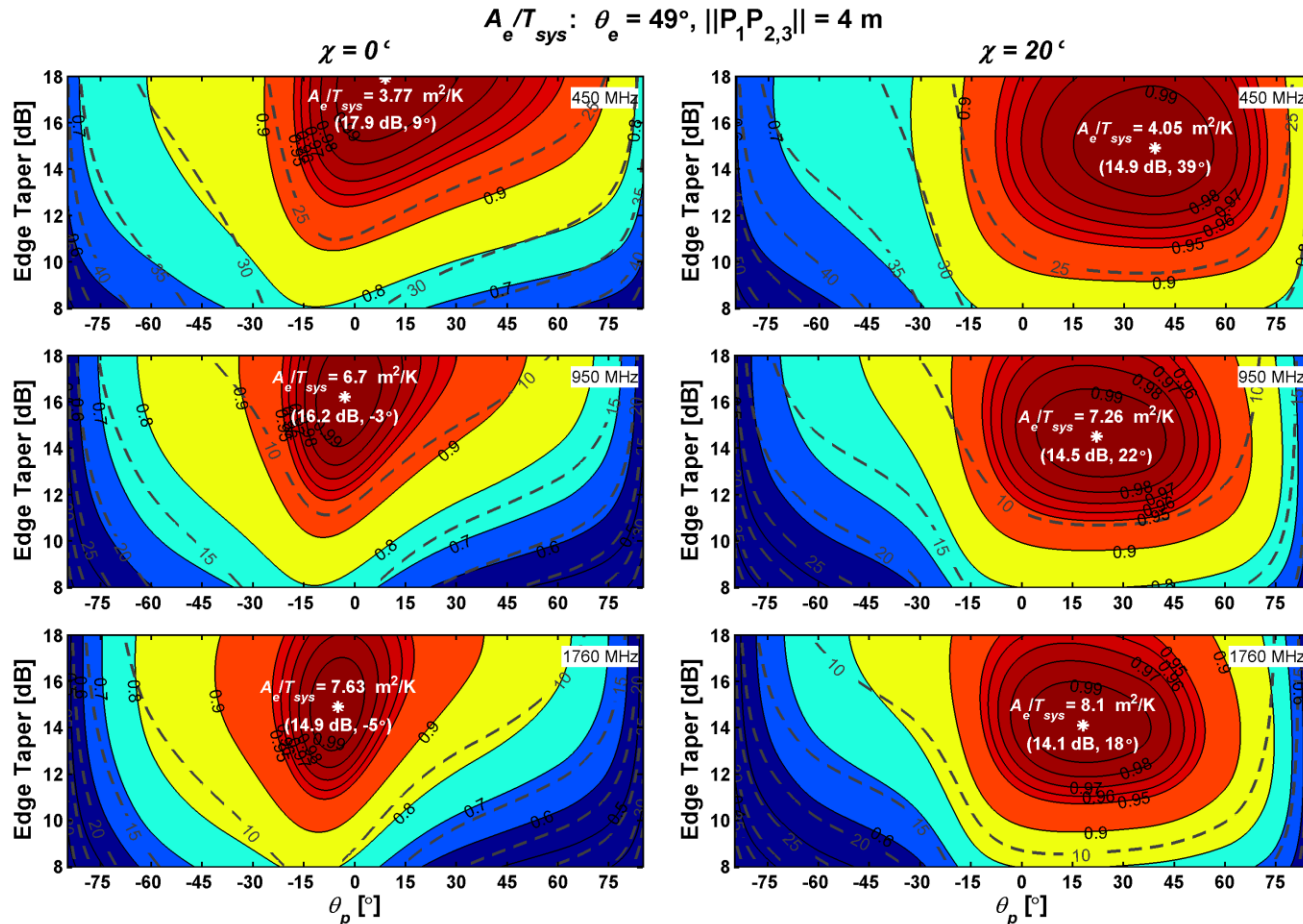
Feed up vs Feed down



Feed up vs Feed down



Feed up vs Feed down



- D.I.L. de Villiers and R. Lehmensiek, “Sub-Reflector Extensions in Shaped Dual Reflector Systems”, *IEEE AP-S International Symposium on Antennas and Propagation*, Memphis, TN, USA, pp. 1481-1482, July, 2014.
- D.I.L. de Villiers and R. Lehmensiek, ‘Sub-reflector extension shapes for reduced far-out sidelobes in offset Gregorian antennas’, *International Symposium on Antennas, Propagation and Electromagnetic Theory (ISAPE)*, Xi’An, China, pp. 55 – 58, October, 2012.

- **Dish and Feed Design Methodology**
 - **Parametric study of unshaped dishes with Gaussian feeds**
 - Reflector dimensions, θ_e
 - **Designed feeds for a subset of unshaped dish**
 - **Shaped dish given the optimised (for unshaped) feeds**
 - Exhaustive search of a parameterised mapping function, $\rho(\theta_f)$
 - **Re-optimised the feeds on the optimum shaped dish**
- **Full-wave analyses to determine system performance**
 - Take into account diffraction effects

I. P. Theron, R. Lehmensiek and D. I. L de Villiers, "Towards an optics design for the SKA," in *Proc. IEEE Africon/URSI Conf.*, Mauritius, Sep. 2013, pp. 1313-1317.

Metric

- **Receiving sensitivity, A_e / T_{sys} (m^2/K)**

$$T_{sys} = T_A + T_{rec}$$

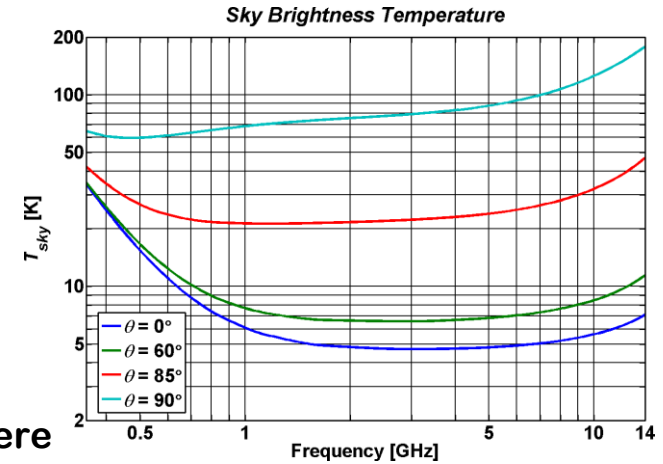
$$T_A(f | \hat{\mathbf{r}}_0) = \frac{\iint_{4\pi} T_b(f, \theta, \phi) P_n(f, \theta, \phi) \sin \theta d\theta d\phi}{\iint_{4\pi} P_n(f, \theta, \phi) \sin \theta d\theta d\phi}$$

$P_n(f, \theta, \phi)$ - antenna radiation pattern

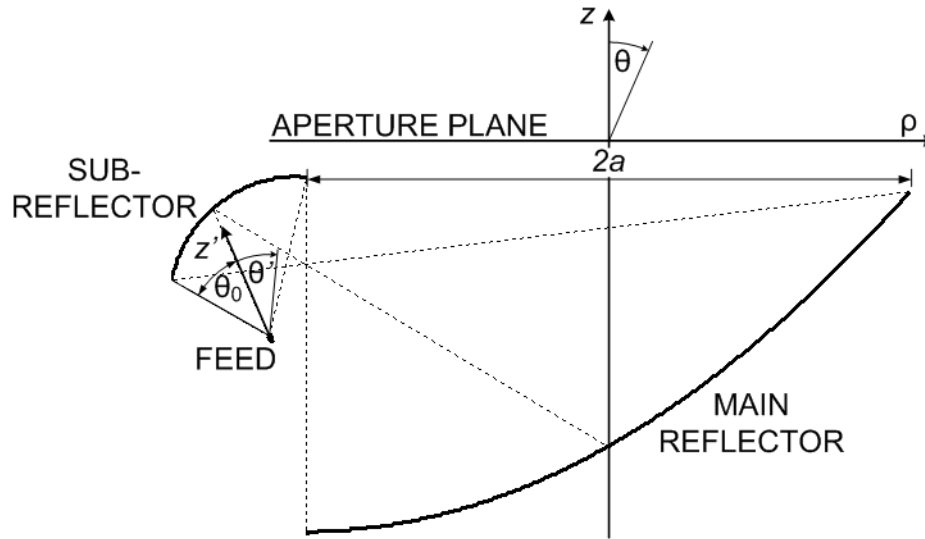
$T_b(f, \theta, \phi)$ - scene brightness temperature

- emission from water vapour and oxygen in atmosphere
- cosmic microwave background and galactic emission (average parameter values) as observed through the atmosphere
- ground emission includes the sky radiation reflected from the ground and the ground emission

- **Sidelobe levels, 2nd and higher**
 - SLL₁ – imaged as part of the main beam
 - SLL₂ – typically highest
- **Cross-polarisation levels (IXR), -1dB and -3dB contour**
- **For optimisation purposes**
 - Average over frequency and tipping angles (0° - 70°)



Shaping



$$\rho(\theta_f)\rho'(\theta_f) = \frac{|G(\theta_f)|^2 \sin \theta_f}{V_c |E(\rho)|^2}$$

Shaped offset Gregorian system:

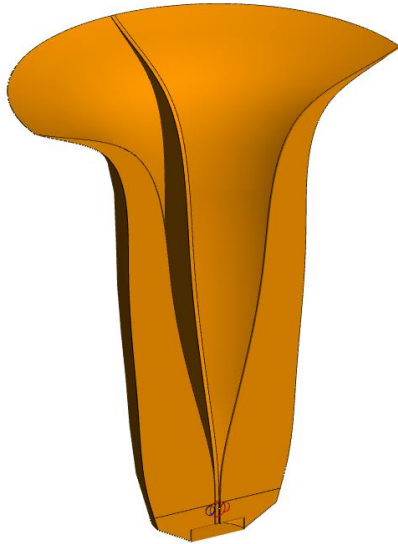
1. Conservation of energy
2. Equal path length
3. Snell's law reflection
4. Specified θ - ρ mapping

- Maps feed pattern to a desired aperture field distribution
- Assumes:
 - Axially symmetric feed pattern
 - Frequency invariance
 - No diffraction effects

Band 1

350 – 1050 MHz

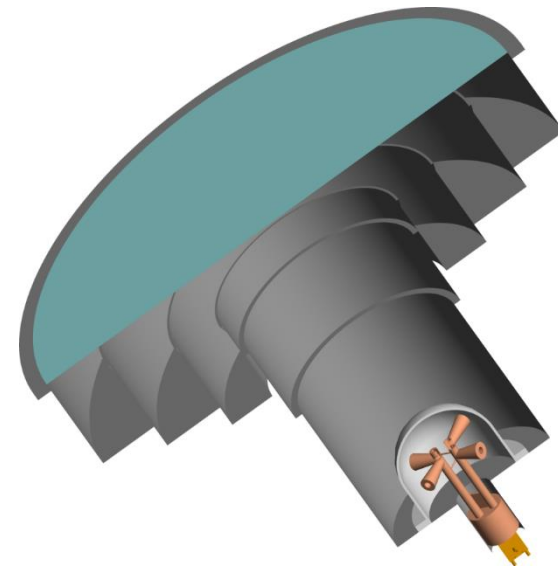
Quad-ridge flared horn



Band 2

950 – 1760 MHz

Corrugated conical horn
(OMT & Coupler)



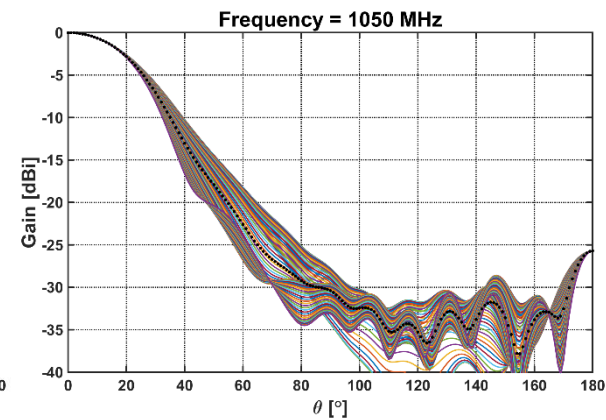
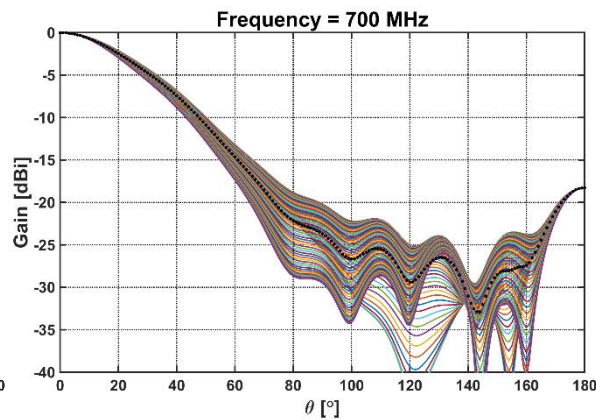
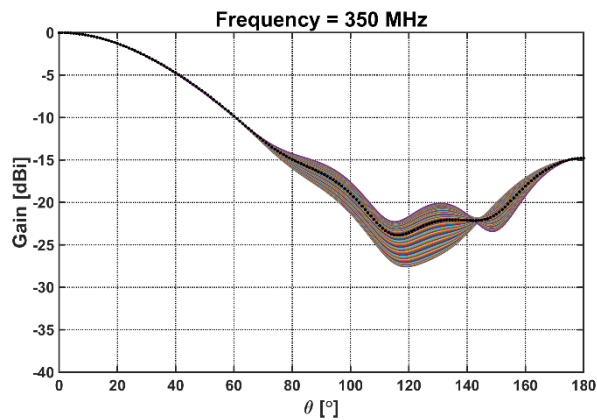
R. Lehmensiek and I. P. Theron, "The design of the MeerKAT UHF band feed," in *Proc. 8th Eur. Conf. Antennas and Propag. (EuCAP)*, The Hague, Netherlands, Apr. 2014, pp. 880-884.

R. Lehmensiek, "A design methodology of the wideband orthogonal mode transducer for the SKA band 2 feed," in *Proc. 10th Eur. Conf. Antennas and Propag. (EuCAP)*, Davos, Switzerland, Apr. 2016, pp. 1-4.

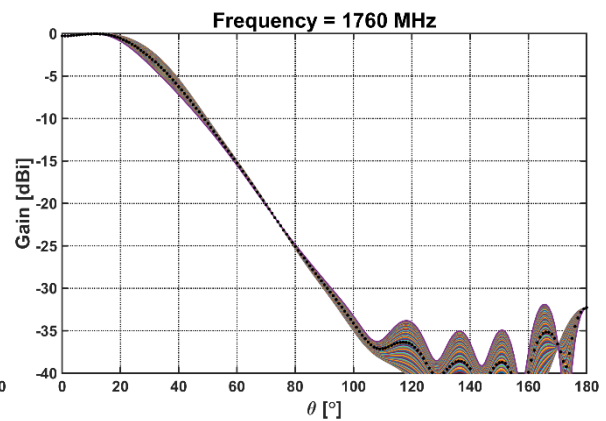
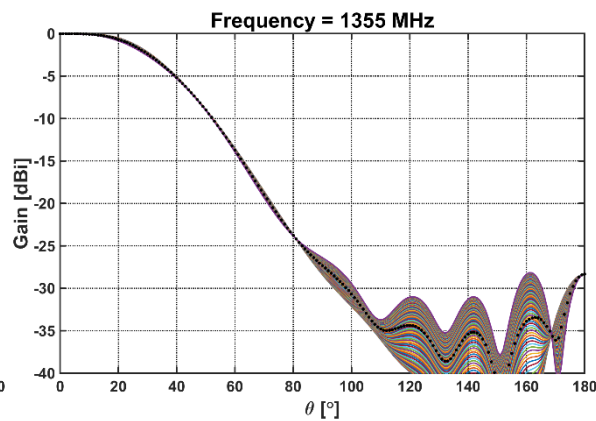
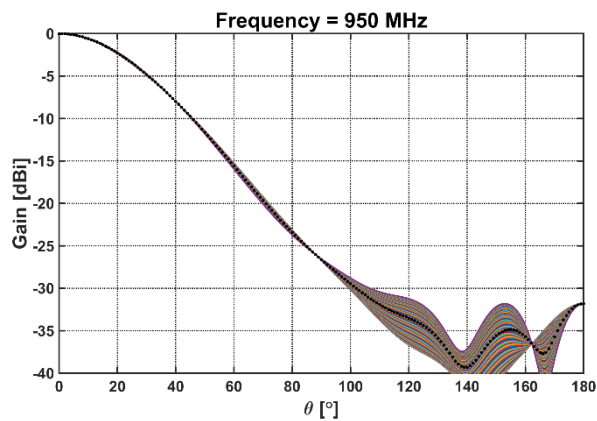
R. Lehmensiek and I. P. Theron, "The design of the MeerKAT L-band feed," in *Proc. Int. Conf. Electromagn. Adv. Appl. (ICEAA)*, Cape Town, South Africa, Sep. 2012, pp. 321-324.

Feeds

Band 1

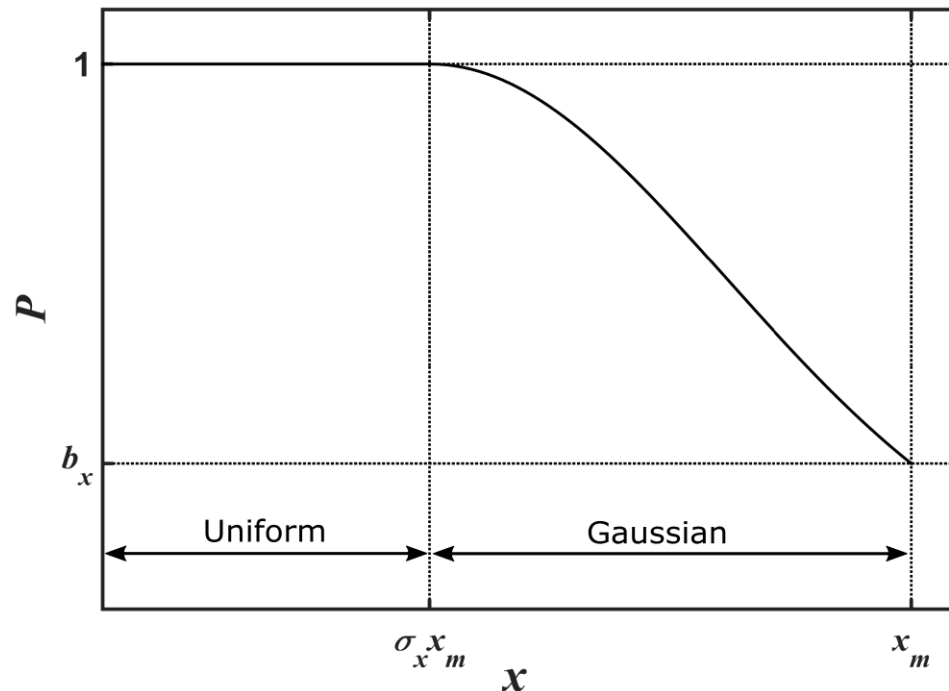


Band 2



- hybrid uniform/Gaussian function:

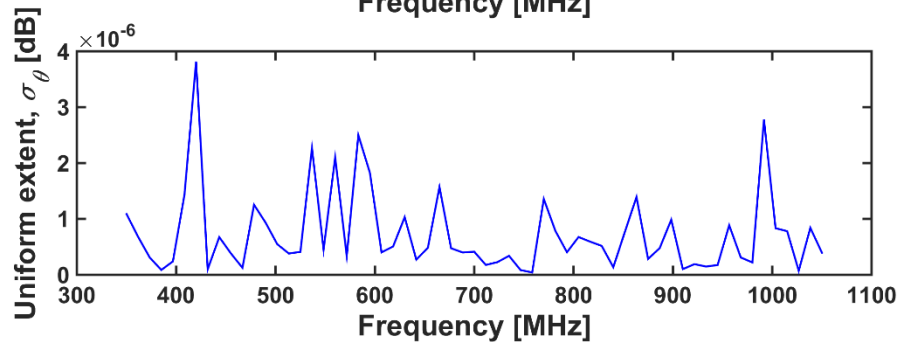
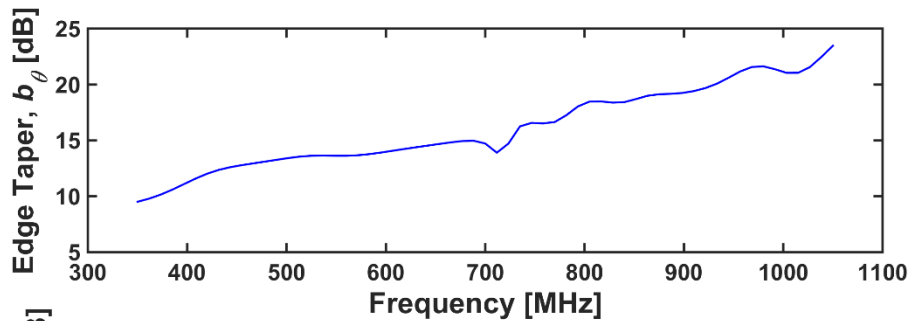
$$P = \begin{cases} 1, & 0 \leq x \leq \sigma_x x_m \\ \exp\left[-b_x \left(\frac{x - \sigma_x x_m}{x_m(1 - \sigma_x)}\right)^2\right], & \sigma_x x_m \leq x \leq x_m \end{cases}$$



Feeds

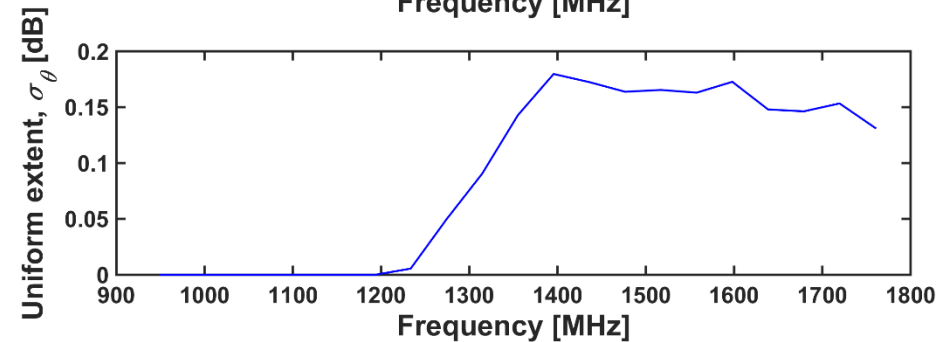
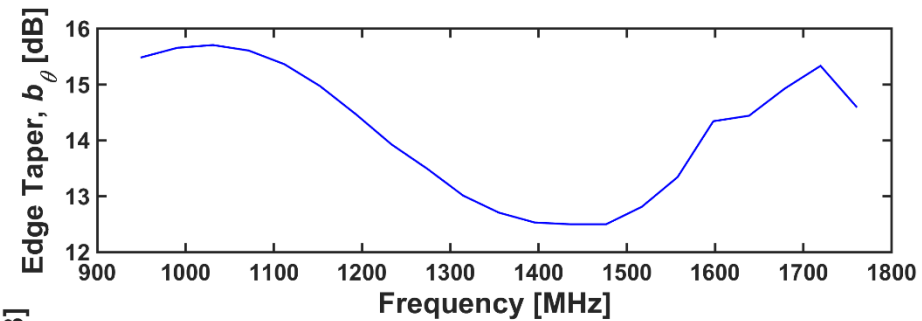
Band 1

Quad-ridge flared horn



Band 2

Corrugated conical horn



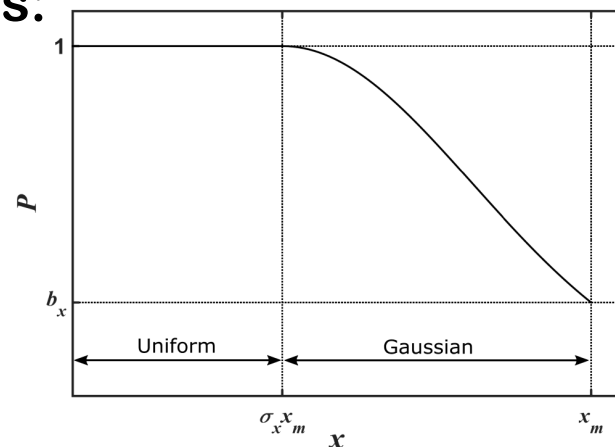
Mapping function

- Variety of feeds with pattern variation (phi, frequency, band)
- Analyses on dish leads to significant variation in performance parameters (diffraction)
- Choosing a mean feed pattern and shaping for a desired aperture distribution will not necessarily lead to an optimal mapping function
- Determine mapping function by optimisation
- Parameterized mapping function
- Use a parameterized feed pattern and aperture field distribution
- Model by hybrid uniform/Gaussian functions:

$$P = \begin{cases} 1, & 0 \leq x \leq \sigma_x x_m \\ \exp\left[-b_x \left(\frac{x - \sigma_x x_m}{x_m(1 - \sigma_x)}\right)^2\right], & \sigma_x x_m \leq x \leq x_m \end{cases}$$

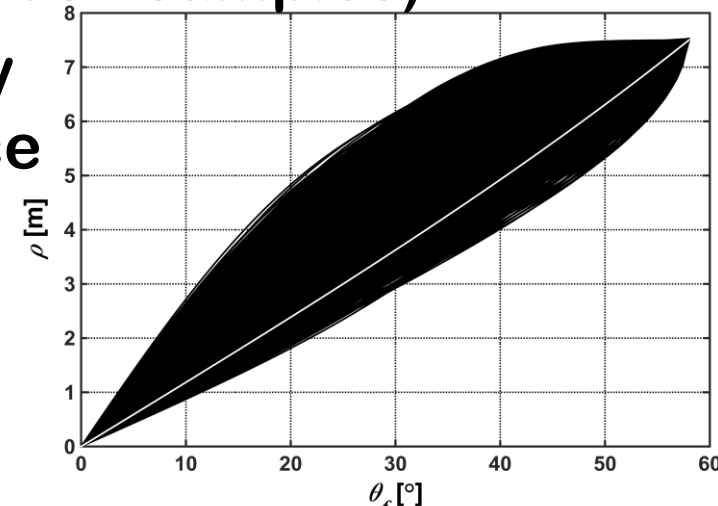
- $x \rightarrow \theta_f$ or ρ

- $P \rightarrow |G(\theta_f)|^2$ or $|E(\rho)|^2$



Optimization methodology

- **Optimization parameter space:**
 - **Feed:** $\sigma_\theta = [0, 0.5]$, $b_\theta = [-17, -5]$ dB
 - **Aperture:** $\sigma_\rho = [0, 0.7]$, $b_\rho = [-17, -5]$ dB
- **Grid search optimization**
 - **Not computationally effective (2352 samples)**
 - **Together with interpolation fully determines the parameter space**
 - **Exhaustive set of mapping functions**



R. Lehmensiek, I. P. Theron and D. I. L. de Villiers, "Deriving an optimum mapping function for the SKA shaped offset Gregorian reflectors," *IEEE Trans. Antennas Propag.*, vol. 63, no. 11, pp. 4658-4666, Nov. 2015.

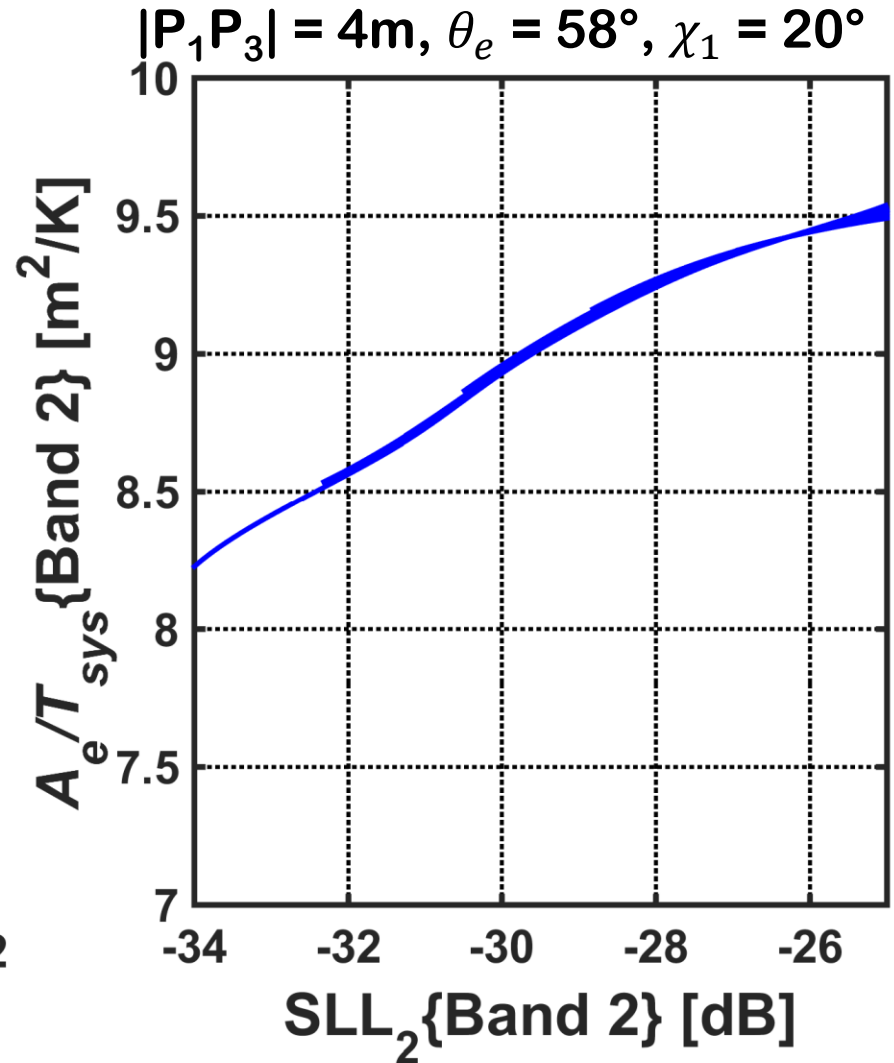
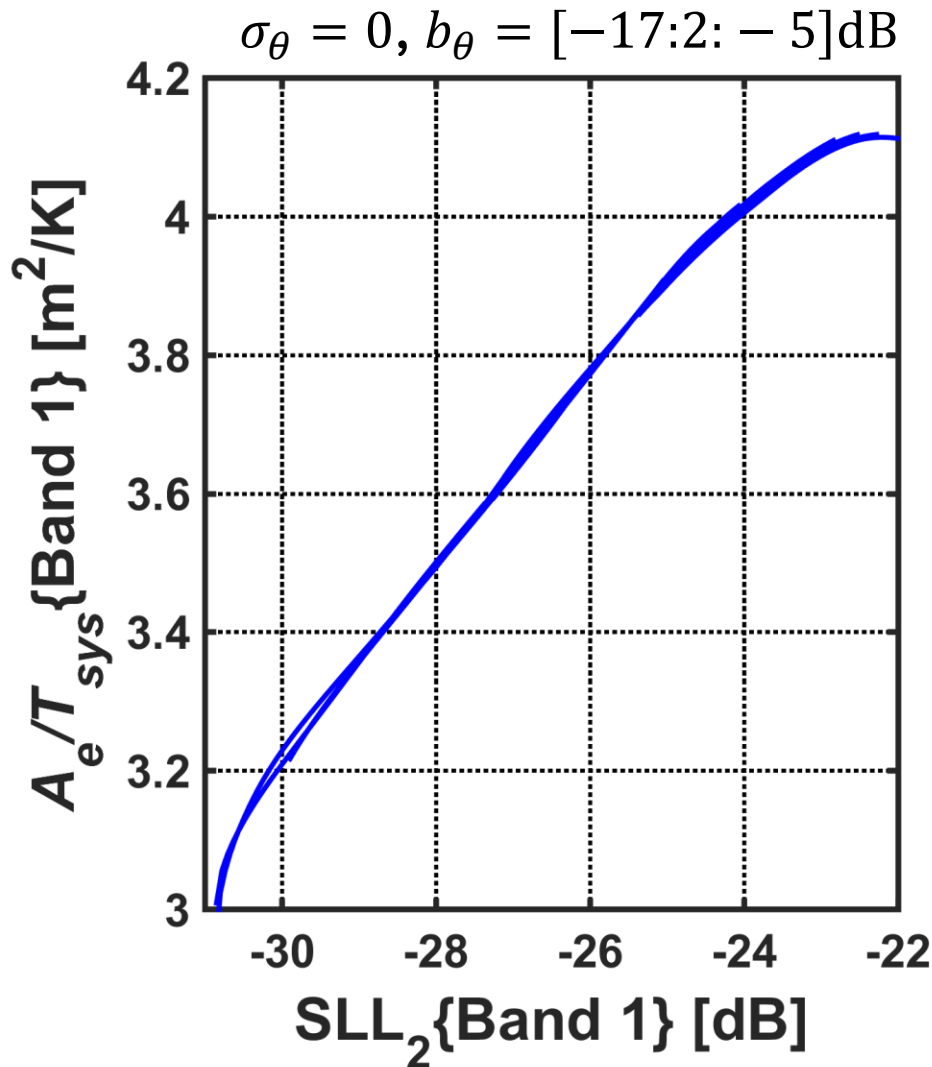
Computations

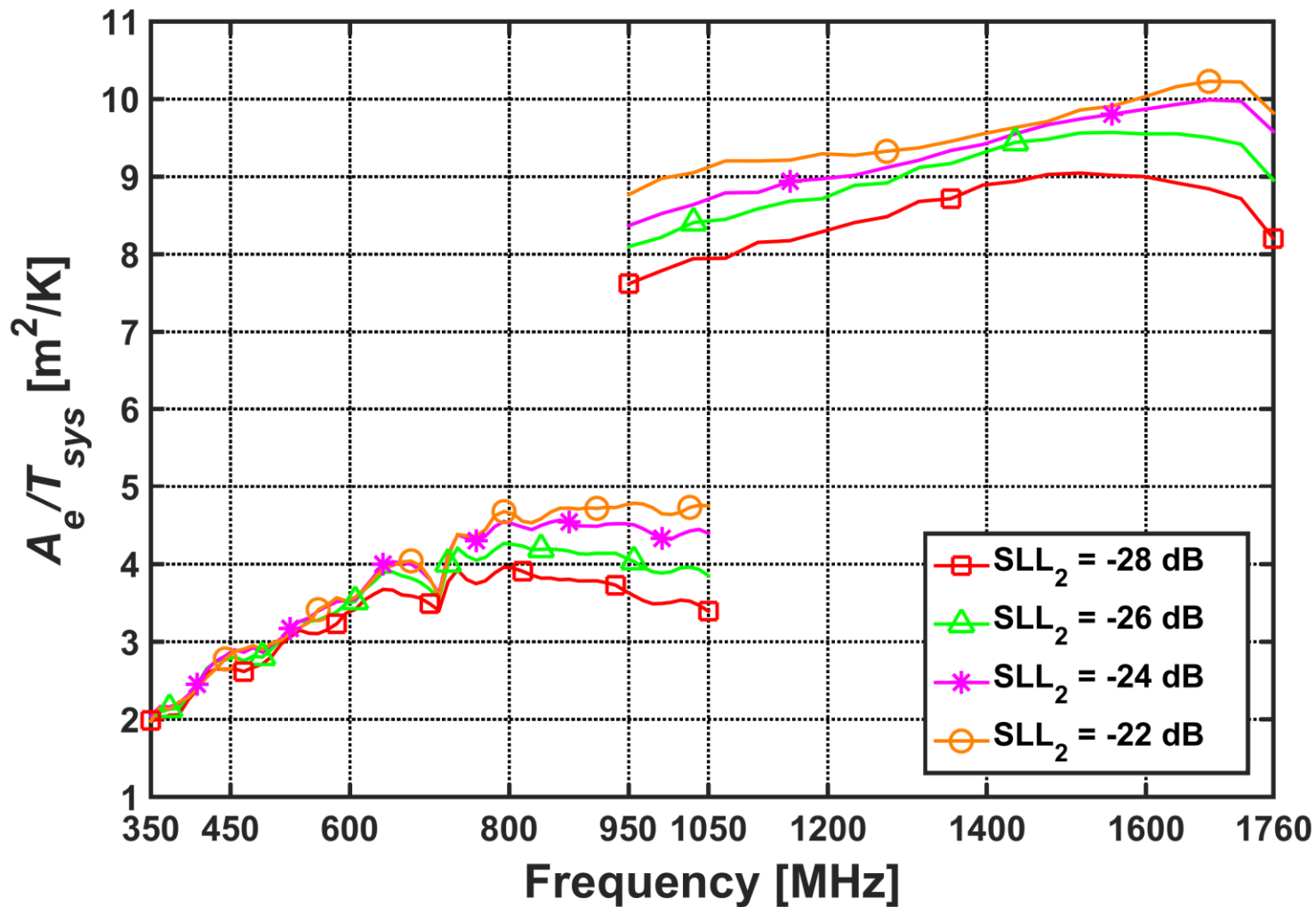
- Feed pattern → dish → secondary pattern → performance parameters
- How to make the computations tractable?
- Approximations:
 - Simplistic dish model, no support structures, deformations, blockage
 - GRASP: PO/PTD, with spherical mode source
 - Linear vertical polarization
 - Number of frequency points: 61 (Band 1), 21 (Band 2)
 - Antenna noise temperature, diffraction compensated main reflector masking technique, $\theta_p = [0^\circ: 10^\circ: 70^\circ]$

D. I. L. de Villiers and R. Lehmensiek, “Rapid calculation of antenna noise temperature in offset Gregorian reflector systems,” *IEEE Trans. Antennas Propag.*, vol. 63, no. 4, pp. 1564-1571, Apr. 2015.

R. Lehmensiek and D. I. L. de Villiers, “Noise temperature approximations for offset Gregorian reflector systems,” *IEICE Trans. Commun.*, vol. E101-B, no. 2, pp. 332-339, Feb. 2018.

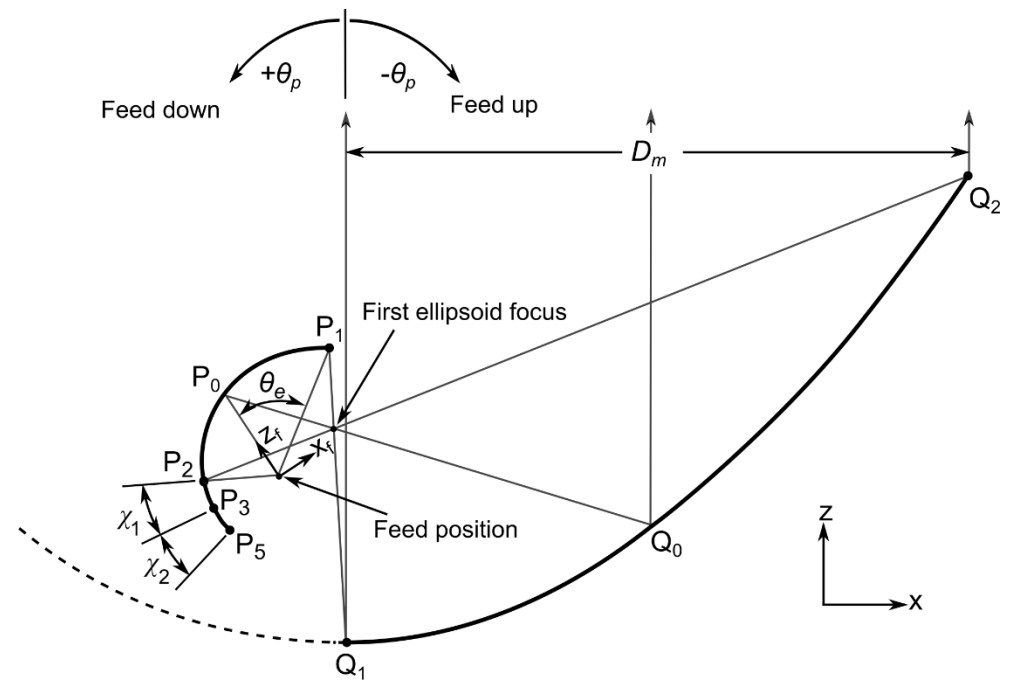
Sensitivity versus SLL_2



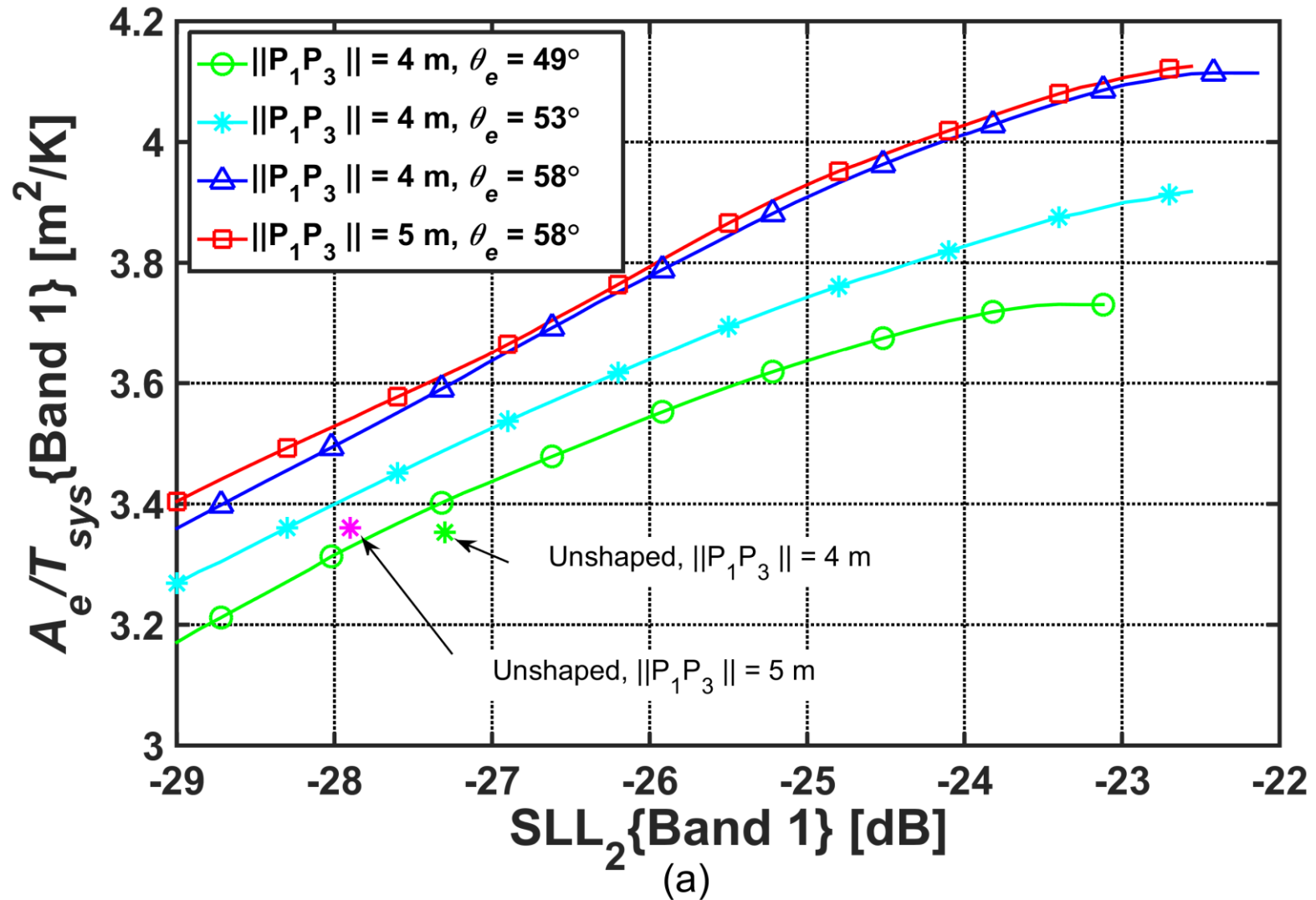


Degrees of shaping

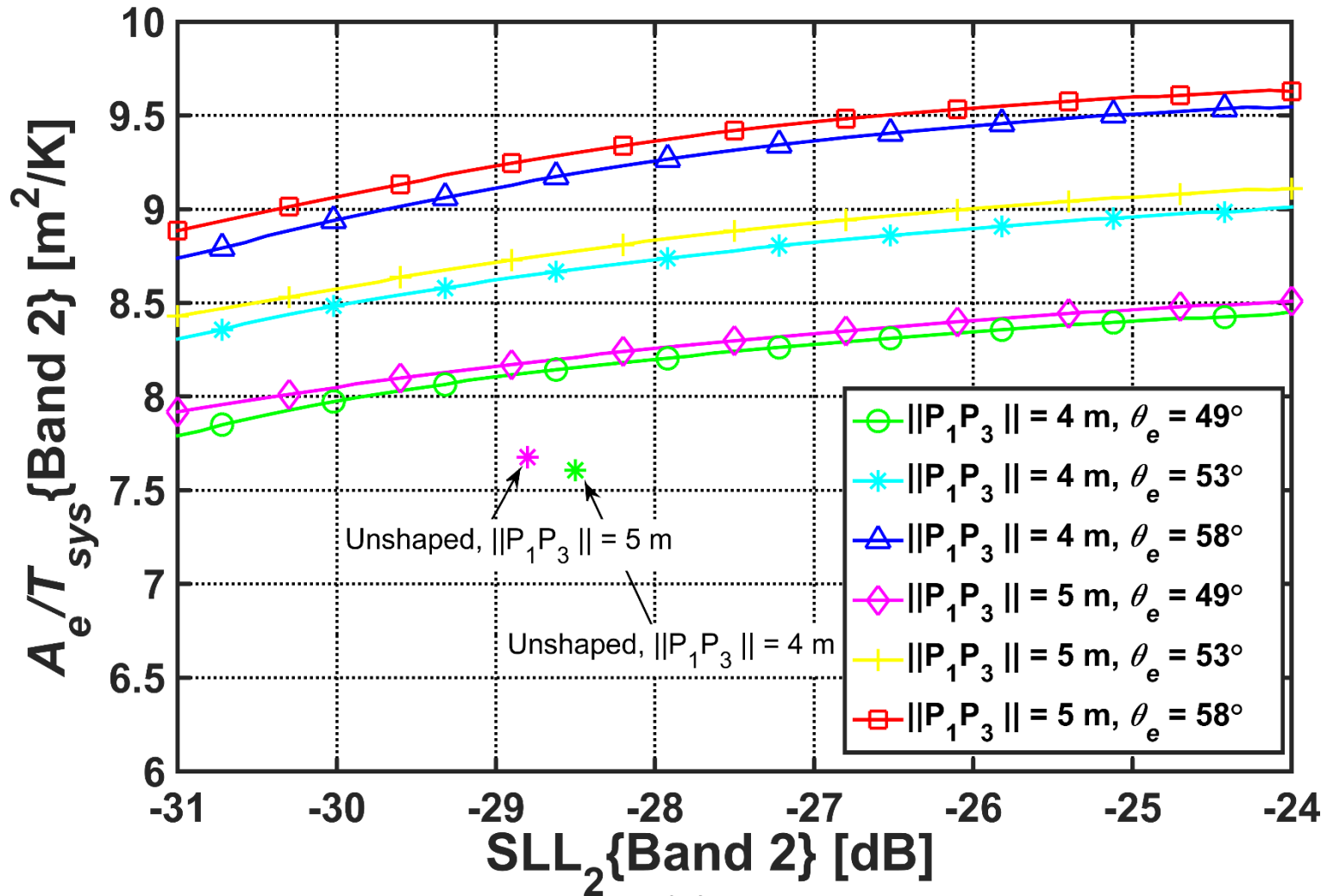
- Results for: $|P_1P_3| = 4\text{m}$, $\theta_e = 58^\circ$, $\chi_1 = 20^\circ$
- Effects of changing parameters
 - $|P_1P_3| = 4\text{m}$ and 5m
 - $\theta_e = 49^\circ$, 53° and 58°
 - $\chi_2 = 0^\circ$, 10° , 20° , 30°



Degrees of shaping

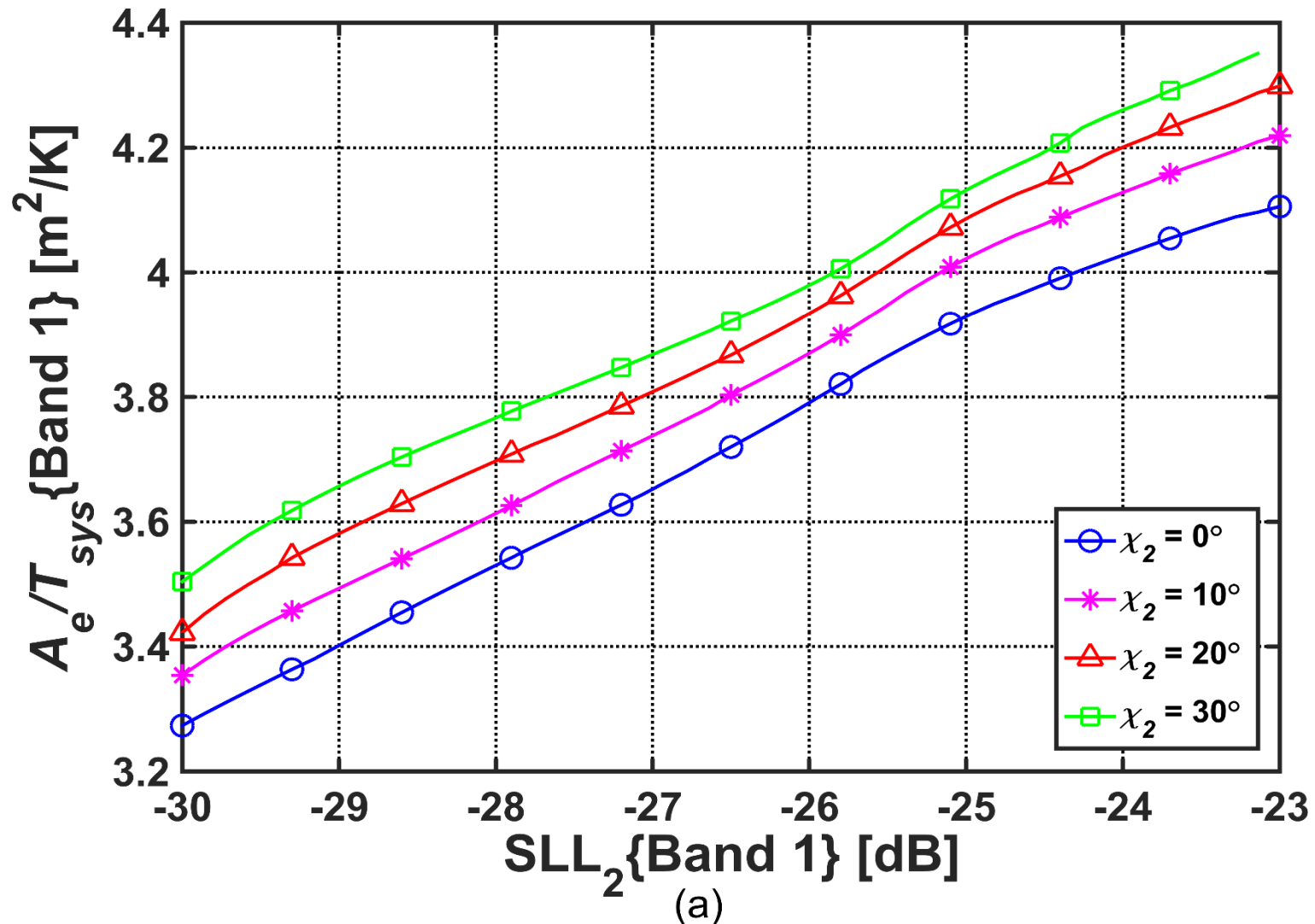


Degrees of shaping

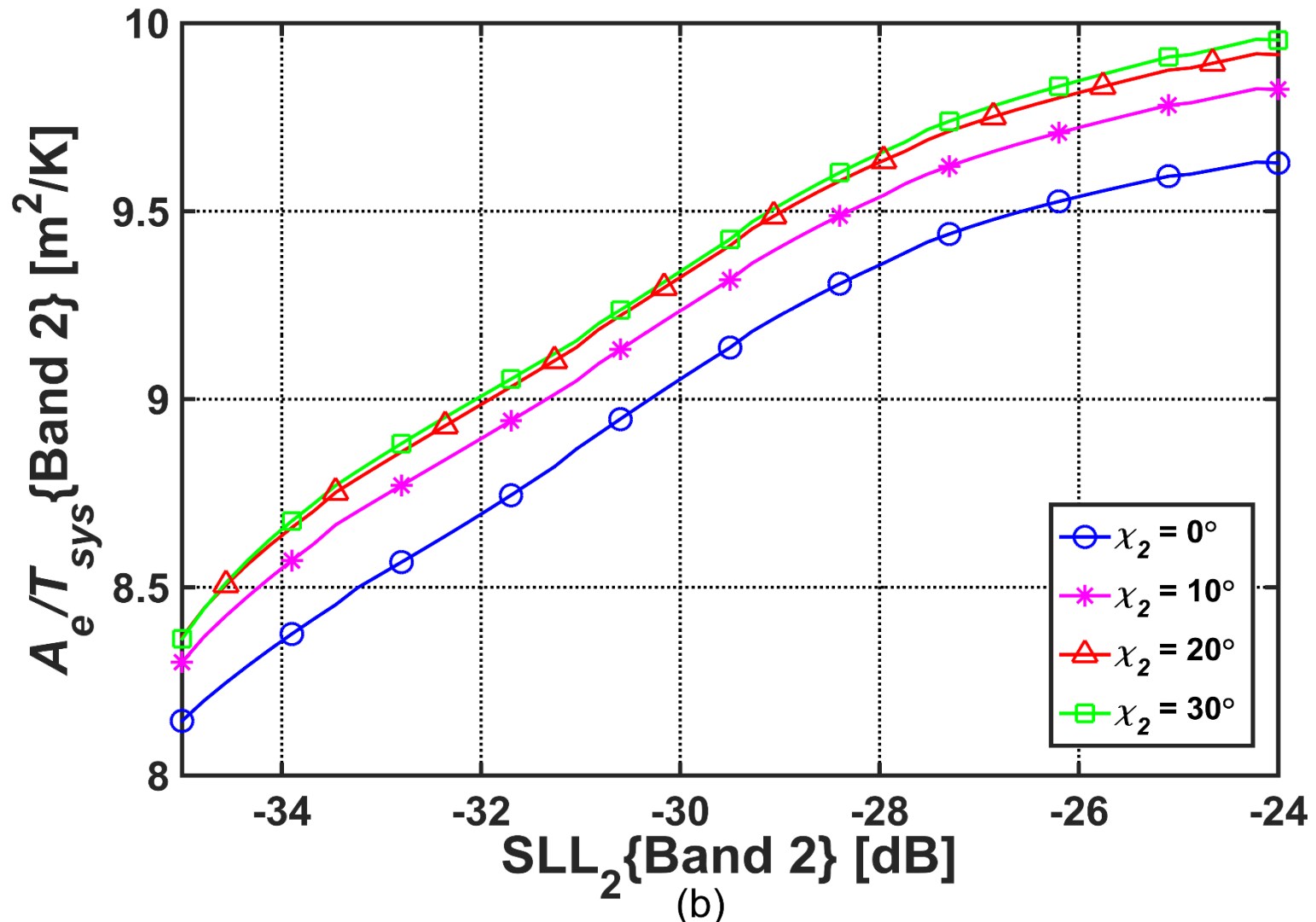


(b)

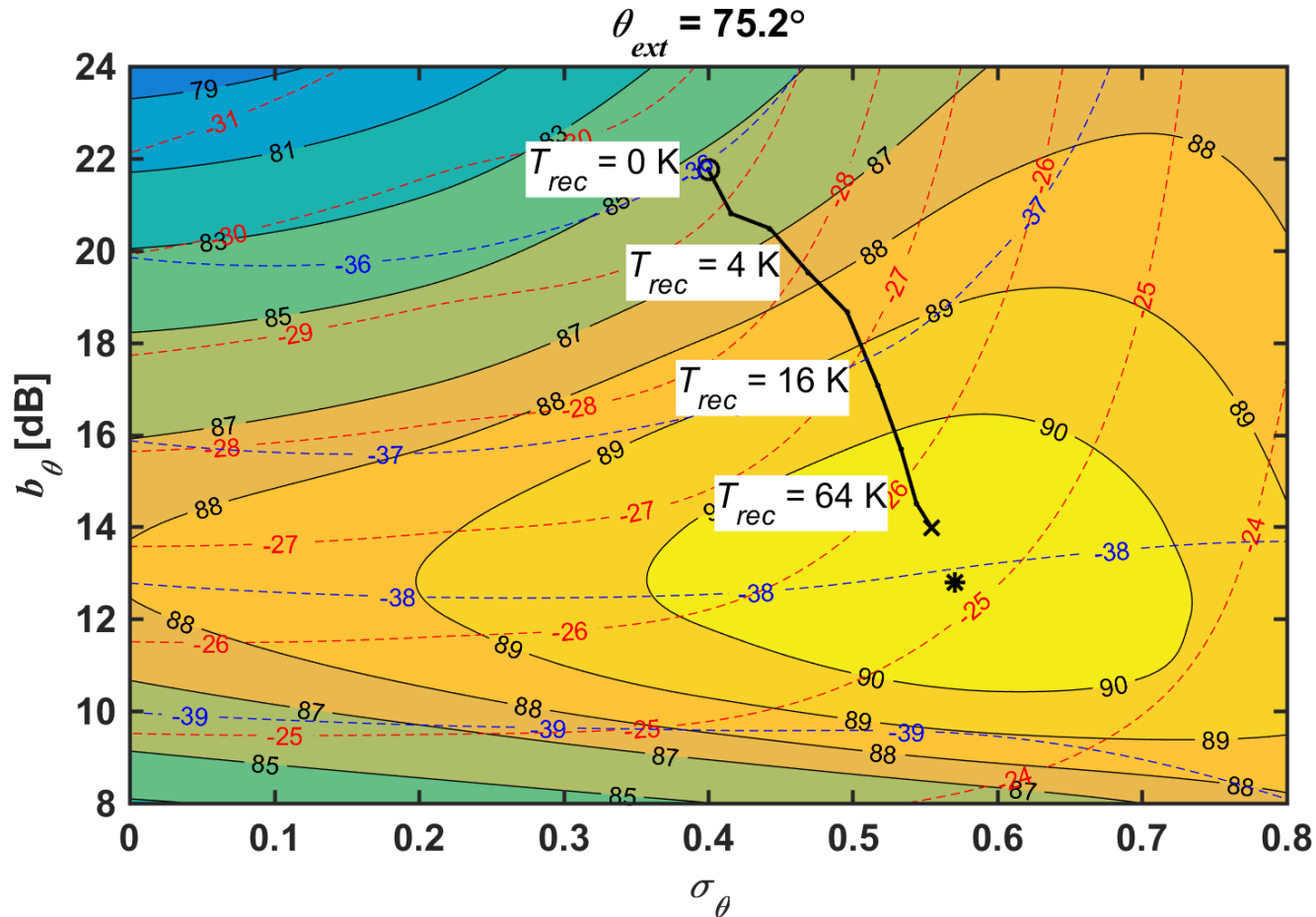
Sub-reflector extensions



Sub-reflector extensions



Ideal feed performance limits

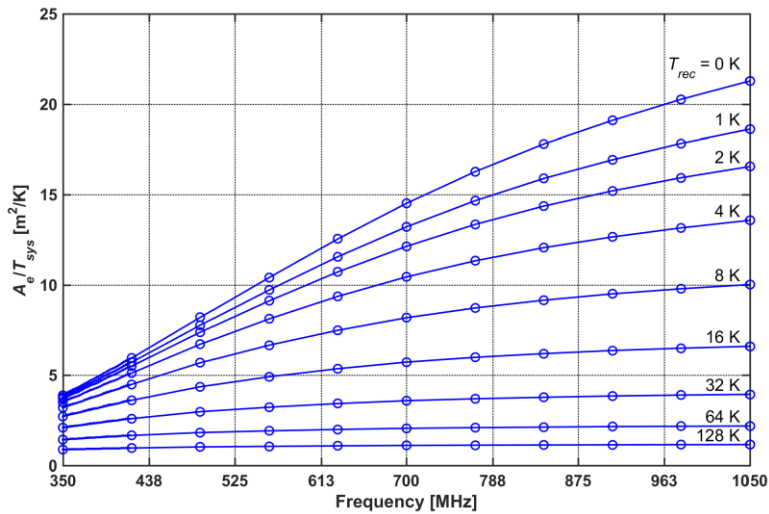


R. Lehmensiek and D. I. L. de Villiers, "Aperture efficiency performance limits of the SKA reflector system," in *Proc. IEEE Int. Symp. AP & USNC/URSI Nat. Radio Sci. Meet.*, San Diego, CA, Jul. 2017, pp. 989-990.

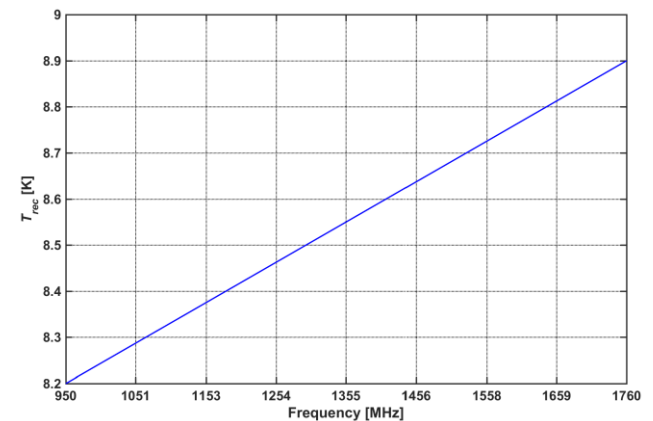
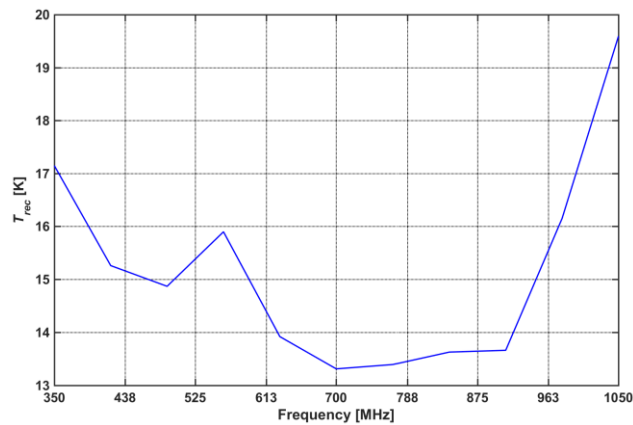
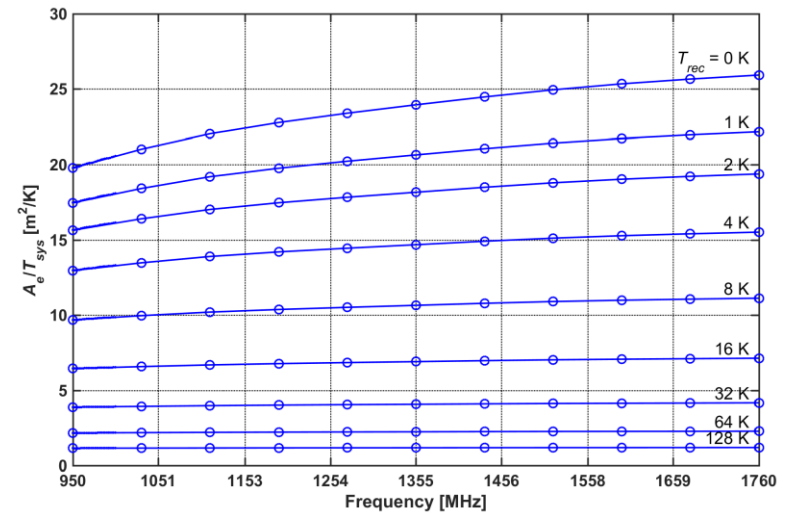
R. Lehmensiek and D. I. L. de Villiers, "On the performance limits of the SKA1-mid reflector system," in *32nd URSI General Assembly and Scientific Symposium (GASS)*, Montreal, QC, Canada, Aug. 2017.

Performance limits

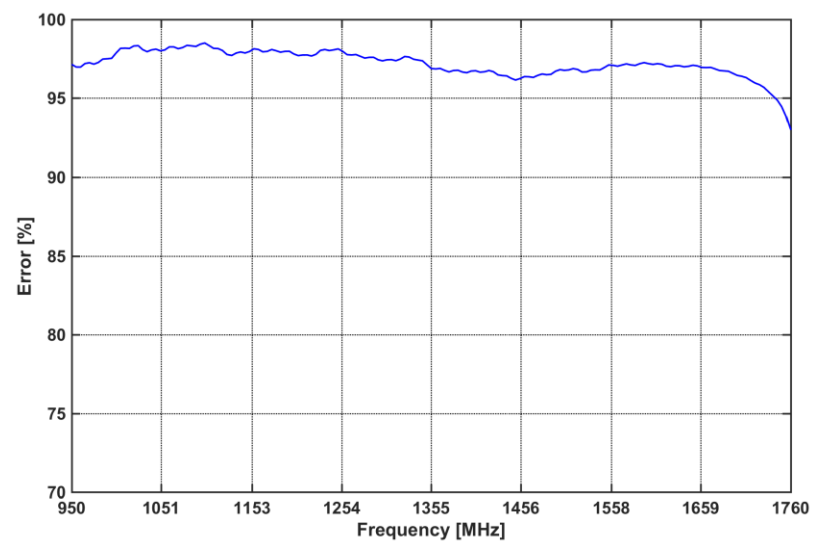
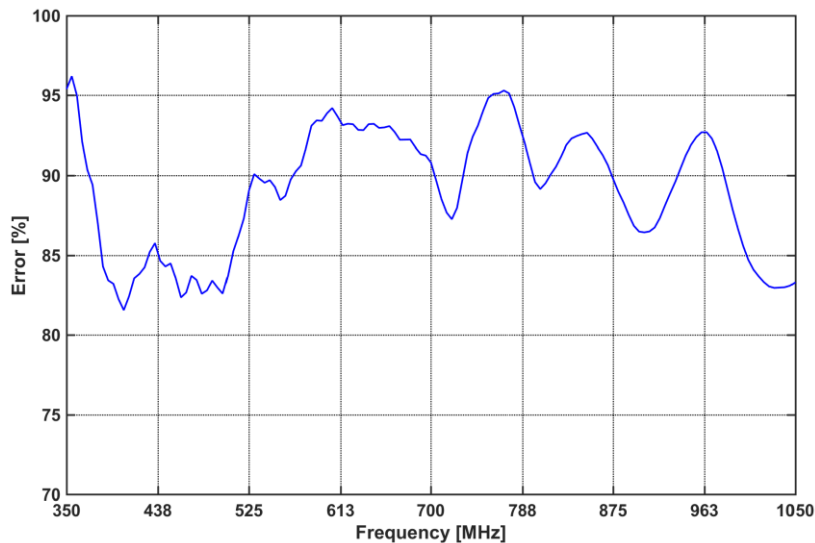
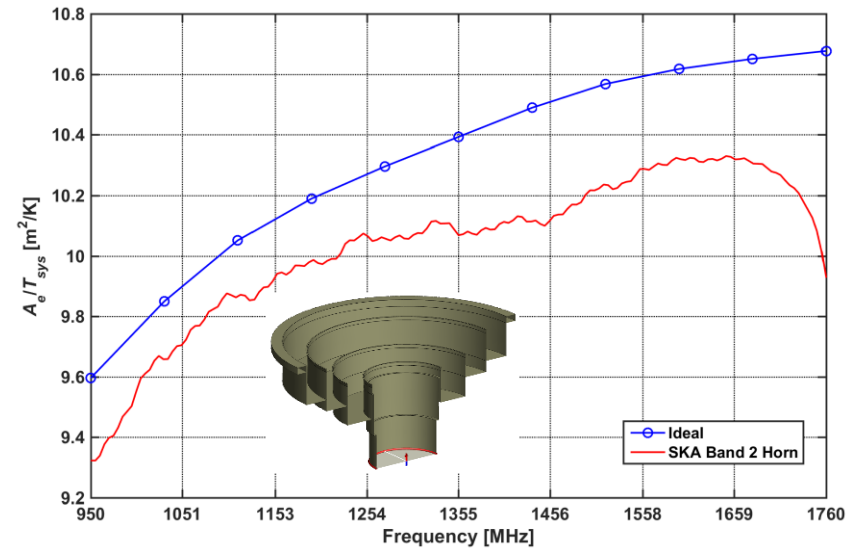
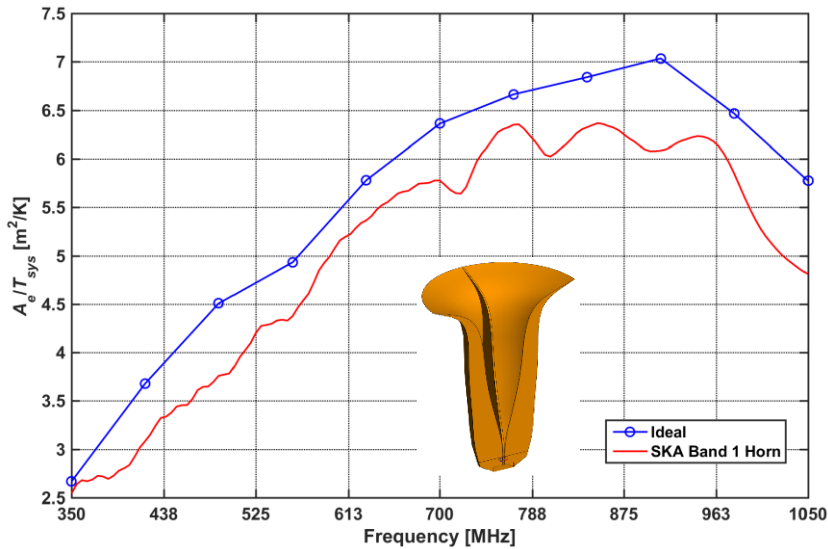
Band 1



Band 2



Performance limits

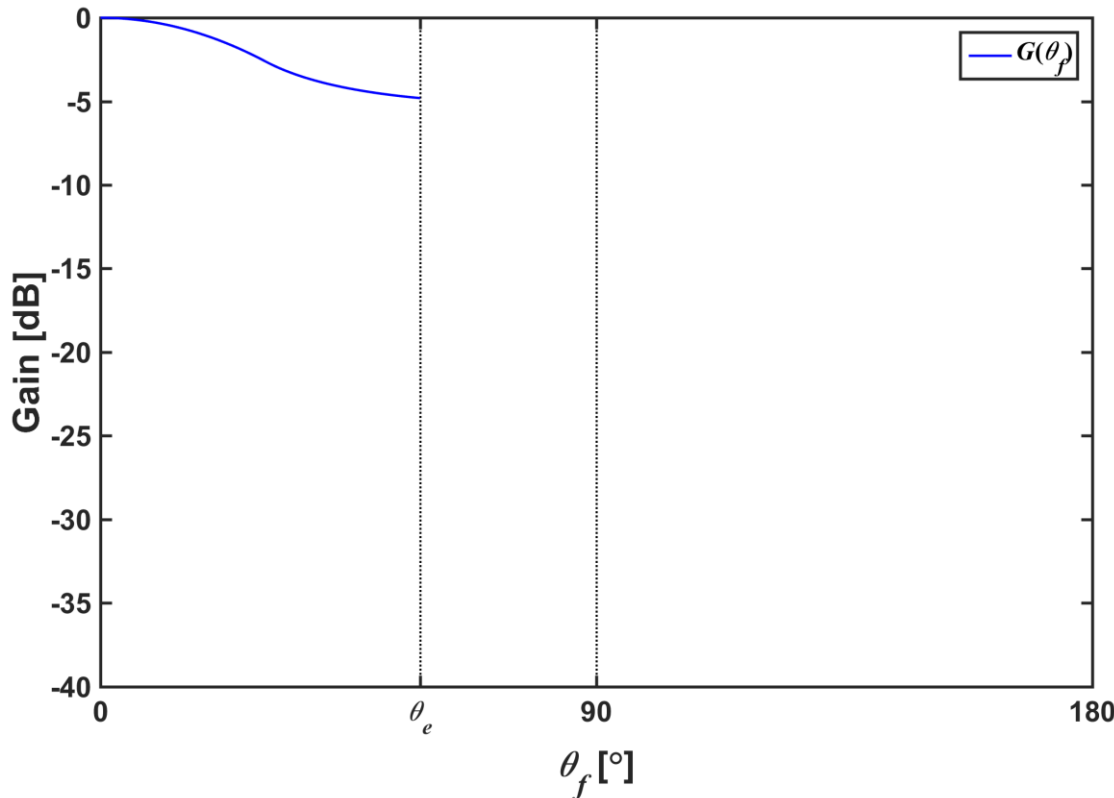


Conclusion

- Realistic frequency-dependent feed patterns
- Realistic receiver temperatures
- Accurate numerical EM analyses
- Exhaustive mapping function space
- Performance parameters: A_e/T_{sys} versus SLL_2

- Sub-reflector size only marginally improves performance
- Minimized edge illumination has best performance
- Optimizing for Band 1 does not significantly compromise the performance of Band 2
- Choice for SKA:
 - -24dB SLL_2 in Band 1 (-26 dB in Band 2)
 - $\theta_e = 58^\circ$, $|P_1 P_3| = 5$ m (PAF driven)
 - $\chi_1 = 20^\circ$, $\chi_2 = 20^\circ$

The parameterised Feed



- Determine parameterised feed model

- $\rho(\theta_f)$ from optics design

- Maximise η_a , $|E(\rho)|^2 = 1$

$$|G(\theta_f)|^2 = \frac{V_c |E(\rho)|^2 \rho(\theta_f) \rho'(\theta_f)}{\sin \theta_f} \quad \theta_f \leq \theta_e$$

- Maximise η_{spill}

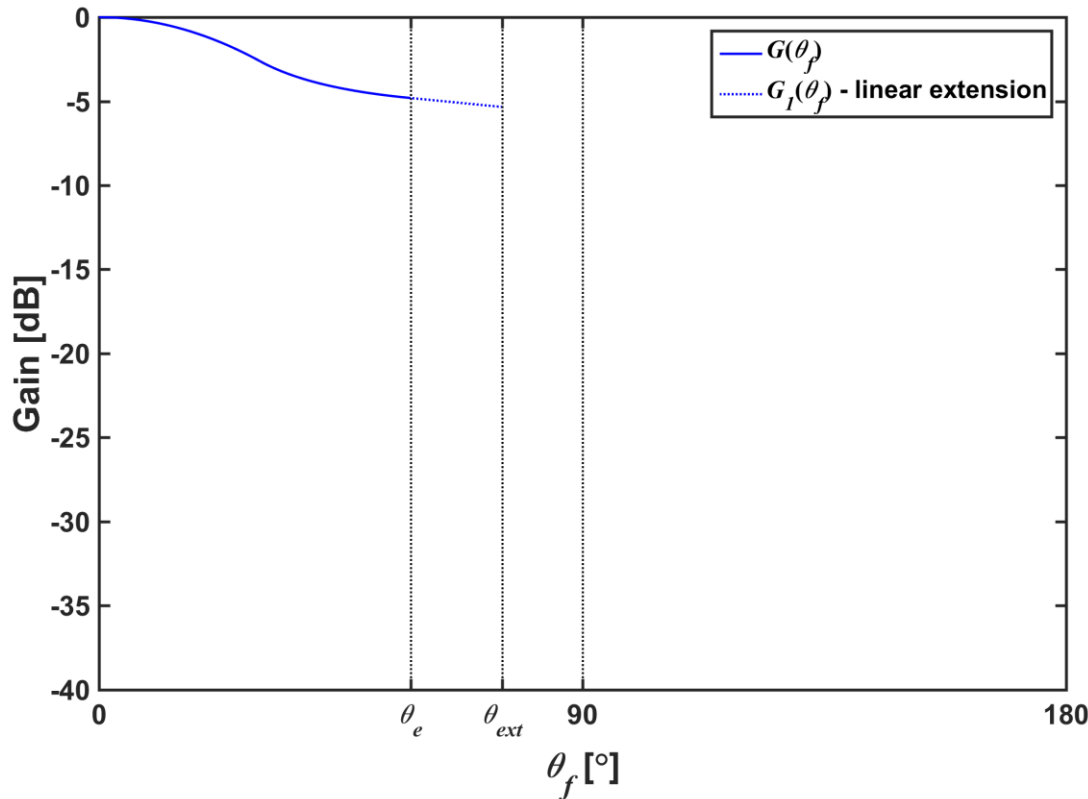
- Minimise T_{spill}

$$|G(\theta_f)|^2 = -100 \text{ dB} \quad \theta_f > \theta_e$$

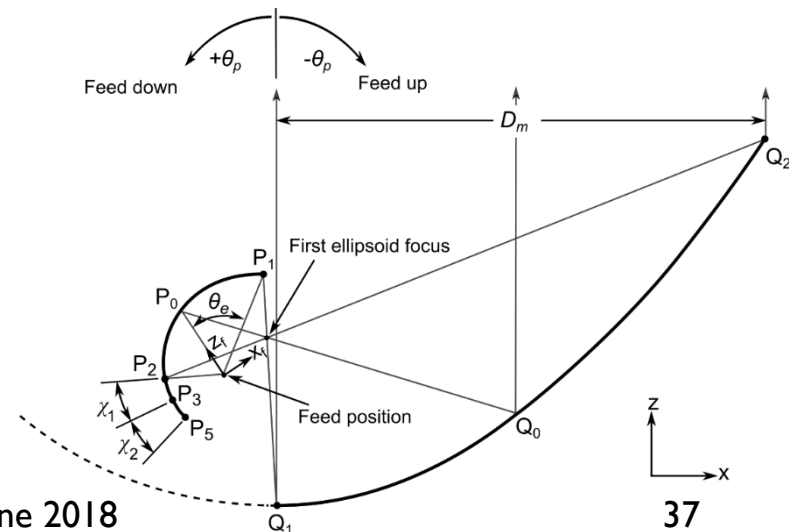
- GO limit

- Ignores edge diffraction

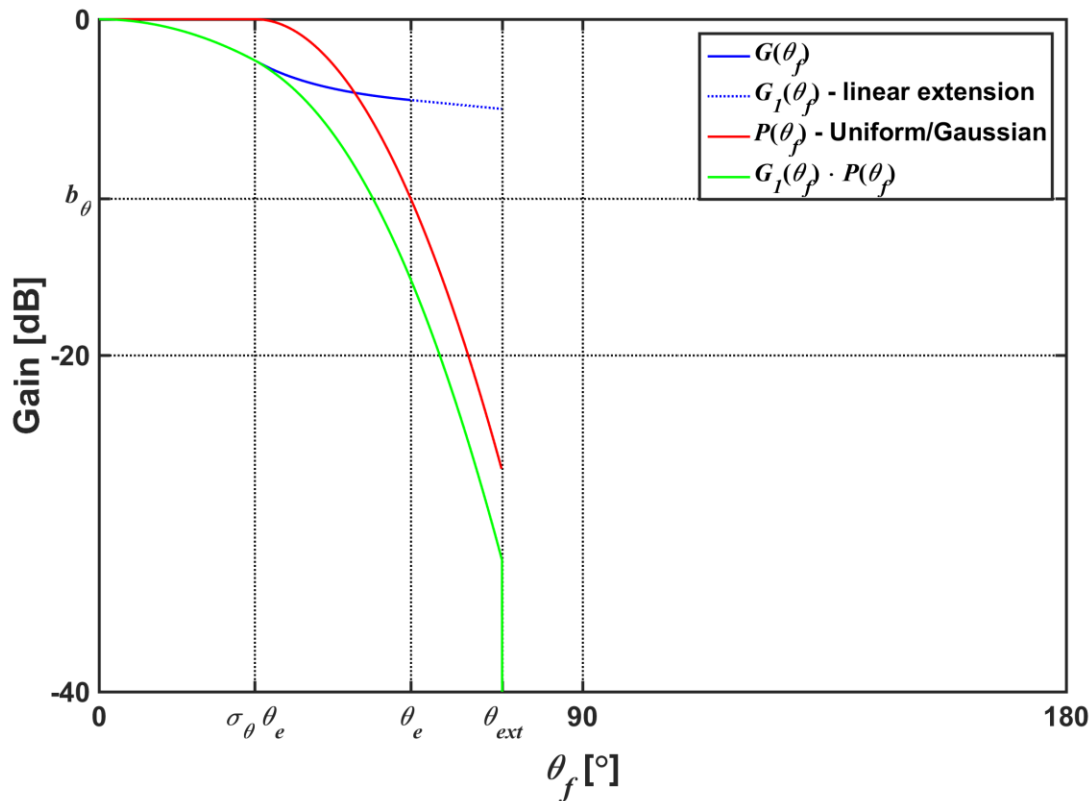
The Feed



- SR extension - unsymmetric
- Trade-off between η_{ill} and η_{spill} (T_{spill})
- Slightly over illuminate the SR (extension)
- Linear interpolation
 - Parameter 1: θ_{ext}



The Feed



- **Weighting function $P(\theta_f)$: Uniform/Gaussian**

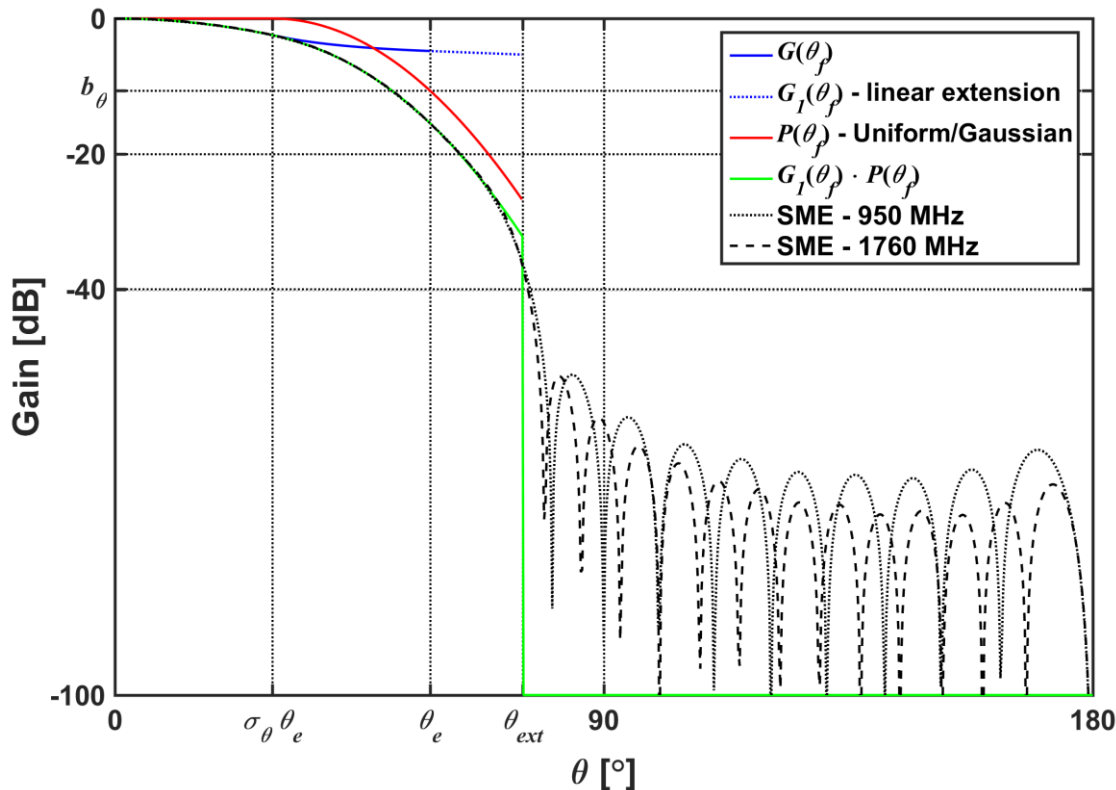
- Parameter 2: b_θ
- Parameter 3: σ_θ

- **Parameterised Feed Pattern**

- Central part – identical to ideal case
- Edge - stronger taper

- **Discontinuous pattern**

The Feed



- Limit feed size:
 - Band 1: $2r_0 = 1.3$ m
 - Band 2: $2r_0 = 0.8$ m

- Spatial filter
- Spherical Mode expansion
- Limit number of modes:
 - In ϕ : $\text{BOR}_1, M = 1$
 - In θ :

$$N = kr_0 + \max(3.6^3 \sqrt{kr_0}, 10)$$

F. Jensen and A. Frandsen, "On the number of modes in spherical wave expansions," in *Proc. 26th Antenna Meas. Tech. Association (AMTA)*, Stone Mountain Park, GA, Oct. 2004, pp. 489-494.