SKA Reflector Design

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[Logos and emblems of various institutions]
Background

XDM

KAT-7

MeerKAT
XDM: 2005 - 2007

• 15 m prime focus cluster feed
• Single antenna, Haartebeeshoek
• Subtended half-angle: ~ 53°
• 1414 – 1670 MHz

KAT-7: 2008 - 2012

- 12 m prime focus
- 7 antennas, SKA core site
- Subtended half-angle: ~ 67°
- 1200 – 1950 MHz

• 13.8 m offset Gregorian
• 64 antennas, SKA core site
• Subtended half-angle: ~ 49°
• UHF: 580 – 1050 MHz
  L-band: 900 – 1670 MHz
  S-band: 1750 – 3440 MHz
  X-band: planned

SKA

- **SKA – Square Kilometre Array**
  - 1 km² receiving aperture
  - **SKA1-mid (~10% of full SKA)**
    - 64 x 13.5 m MeerKAT (precursor) & 133 x 15 m SKA1 shaped dishes
  - Six single pixel feeds:
    - Band 1: 350 – 1050 MHz (3:1)
    - Band 2: 950 – 1760 MHz (1.85:1)
    - Band 3: 1.65 – 3.05 GHz (1.85:1)
    - Band 4: 2.8 – 5.18 GHz (1.85:1)
    - Band 5a: 4.6 – 8.5 GHz (1.85:1)
    - Band 5b: 8.3 – 15.4 GHz (1.86:1)

- **SKA1-low (Log-Periodic)**

- **Advanced program on PAFs**

**Reflector system**

- **Offset Gregorian reflector system**
  - **Main:**
    - $D_m = 15 \text{ m}$
    - $\|Q_1Q_2\| \approx 18.2 \text{ m}$
  - **Sub:**
    - $\|x_{P1}x_{Q1}\| \approx 0.5 \text{ m}$
    - $\|P_1P_3\| = 5\text{ m}$
    - $\theta_e = 58^\circ$
  - **Extension:**
    - $\chi_{1,2} = 40^\circ$
  - **Tipping:** Feed down

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Prime Focus vs Offset Gregorian


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Frequency ripple: KAT-7
Frequency ripple: Offset Gregorian

- Diffraction from the sub-reflector can be significant in small to medium sized systems (tens of wavelengths SR size – result below for $10 \lambda$ case).

Feed up vs Feed down

- Direct lightning strike: probably manageable
- More ground noise past sub-reflector
- Easy maintenance access

- Direct lightning strike: probably catastrophic
- Maintenance requires high lifts
- Sub-reflector spillover/diffraction mostly sky directed

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Feed up vs Feed down

Tipping direction for low-side sub-reflector

Tipping direction for high-side sub-reflector

Sub-Reflector Diffraction Cone

Main-Reflector Diffraction Cone

Main Beam

Tipping direction for low-side sub-reflector

Tipping direction for high-side sub-reflector

Sub-Reflector Diffraction Cone

Main-Reflector Diffraction Cone

Main Beam

Dish optics design

• **Dish and Feed Design Methodology**
  • Parametric study of unshaped dishes with Gaussian feeds
    ➢ Reflector dimensions, $\theta_e$
  • Designed feeds for a subset of unshaped dish
  • Shaped dish given the optimised (for unshaped) feeds
    ➢ Exhaustive search of a parameterised mapping function, $\rho(\theta_f)$
  • Re-optimised the feeds on the optimum shaped dish

• **Full-wave analyses to determine system performance**
  ➢ Take into account diffraction effects

Metric

- **Receiving sensitivity, \( A_e / T_{sys} \) (m²/K)**

\[
T_{sys} = T_A + T_{rec}
\]

\[
T_A(f | \hat{\mathbf{r}}_0) = \frac{\iiint_{4\pi} T_b(f, \theta, \phi) P_n(f, \theta, \phi) \sin \theta \, d\theta \, d\phi}{\iiint_{4\pi} P_n(f, \theta, \phi) \sin \theta \, d\theta \, d\phi}
\]

- \( P_n(f, \theta, \phi) \) - antenna radiation pattern
- \( T_b(f, \theta, \phi) \) - scene brightness temperature
  - emission from water vapour and oxygen in atmosphere
  - cosmic microwave background and galactic emission (average parameter values) as observed through the atmosphere
  - ground emission includes the sky radiation reflected from the ground and the ground emission

- **Sidelobe levels, 2\(^{nd}\) and higher**
  - SLL\(_1\) – imaged as part of the main beam
  - SLL\(_2\) – typically highest

- **Cross-polarisation levels (IXR), -1dB and -3dB contour**

- **For optimisation purposes**
  - Average over frequency and tipping angles (0° - 70°)
Shaping

- Maps feed pattern to a desired aperture field distribution
- Assumes:
  - Axially symmetric feed pattern
  - Frequency invariance
  - No diffraction effects

Shaped offset Gregorian system:
1. Conservation of energy
2. Equal path length
3. Snell’s law reflection
4. Specified \( \theta-\rho \) mapping

\[
\rho(\theta_f) \rho'(\theta_f) = \frac{|G(\theta_f)|^2 \sin \theta_f}{V_c |E(\rho)|^2}
\]
Feeds

**Band 1**
350 – 1050 MHz
Quad-ridge flared horn

**Band 2**
950 – 1760 MHz
Corrugated conical horn (OMT & Coupler)


Feeds

Band 1

Frequency = 350 MHz

Frequency = 700 MHz

Frequency = 1050 MHz

Band 2

Frequency = 950 MHz

Frequency = 1355 MHz

Frequency = 1760 MHz
• hybrid uniform/Gaussian function:

\[
P = \left\{ \begin{array}{ll}
1, & 0 \leq x \leq \sigma_x x_m \\
\exp\left[-b_x \left( \frac{x - \sigma_x x_m}{x_m (1 - \sigma_x)} \right)^2 \right], & \sigma_x x_m \leq x \leq x_m
\end{array} \right.
\]
Feeds

Band 1
Quad-ridge flared horn

Band 2
Corrugated conical horn
Mapping function

- Variety of feeds with pattern variation (phi, frequency, band)
- Analyses on dish leads to significant variation in performance parameters (diffraction)
- Choosing a mean feed pattern and shaping for a desired aperture distribution will not necessarily lead to an optimal mapping function
- Determine mapping function by optimisation
- Parameterized mapping function
- Use a parameterized feed pattern and aperture field distribution
- Model by hybrid uniform/Gaussian functions:

\[
P = \begin{cases} 
1, & 0 \leq x \leq \sigma_x x_m \\
\exp\left[-b_x \left(\frac{x - \sigma_x x_m}{x_m (1 - \sigma_x)}\right)^2\right], & \sigma_x x_m \leq x \leq x_m 
\end{cases}
\]

- \(x \rightarrow \theta_f \) or \(\rho\)
- \(P \rightarrow |G(\theta_f)|^2 \) or \(|E(\rho)|^2\)
Optimization methodology

- **Optimization parameter space:**
  - **Feed:** $\sigma_\theta = [0, 0.5], b_\theta = [-17, -5]$ dB
  - **Aperture:** $\sigma_\rho = [0, 0.7], b_\rho = [-17, -5]$ dB

- **Grid search optimization**
  - Not computationally effective (2352 samples)
  - Together with interpolation fully determines the parameter space
  - Exhaustive set of mapping functions

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Computations

• Feed pattern $\rightarrow$ dish $\rightarrow$ secondary pattern $\rightarrow$ performance parameters

• How to make the computations tractable?

• Approximations:
  • Simplistic dish model, no support structures, deformations, blockage
  • GRASP: PO/PTD, with spherical mode source
  • Linear vertical polarization
  • Number of frequency points: 61 (Band 1), 21 (Band 2)
  • Antenna noise temperature, diffraction compensated main reflector masking technique, $\theta_p = [0^\circ: 10^\circ: 70^\circ]$

Sensitivity versus SLL$_2$

$\sigma_\theta = 0, b_\theta = [-17:2: -5]$dB

$|P_1P_3| = 4$m, $\theta_e = 58^\circ$, $\chi_1 = 20^\circ$
Degrees of shaping

- Results for: $|P_1P_3| = 4m$, $\theta_e = 58^\circ$, $\chi_1 = 20^\circ$
- Effects of changing parameters
  - $|P_1P_3| = 4m$ and $5m$
  - $\theta_e = 49^\circ$, $53^\circ$ and $58^\circ$
  - $\chi_2 = 0^\circ$, $10^\circ$, $20^\circ$, $30^\circ$
Degrees of shaping

![Graph showing the relationship between SLL and A/T sys for different degrees of shaping.](graph.png)
Degrees of shaping

![Graph showing degrees of shaping](image-url)

- Unshaped, $\|P_1 P_3\| = 5$ m
- Unshaped, $\|P_1 P_3\| = 4$ m

**Equations and Parameters**
- $A_e / T_{sys\{Band 2}\} [m^2/K]$ vs. $SLL_{2\{Band 2\}} [dB]$
- $\|P_1 P_3\| = 4$ m, $\theta_e = 49^\circ$
- $\|P_1 P_3\| = 4$ m, $\theta_e = 53^\circ$
- $\|P_1 P_3\| = 4$ m, $\theta_e = 58^\circ$
- $\|P_1 P_3\| = 5$ m, $\theta_e = 49^\circ$
- $\|P_1 P_3\| = 5$ m, $\theta_e = 53^\circ$
- $\|P_1 P_3\| = 5$ m, $\theta_e = 58^\circ$
Sub-reflector extensions

![Graph showing the relationship between $A_e/T_{sys,\{Band 1\}}$ [m$^2$/K] and $SLL_{2,\{Band 1\}}$ [dB]. The graph includes lines for different values of $\chi_2$: $0^\circ$, $10^\circ$, $20^\circ$, and $30^\circ$.](image-url)
Sub-reflector extensions

![Graph showing the relationship between A_e/T_{sys} and SLL_{2,Band 2} for different values of \(\chi_2\).]
Ideal feed performance limits


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Performance limits

Band 1

Band 2
Performance limits
Conclusion

- Realistic frequency-dependent feed patterns
- Realistic receiver temperatures
- Accurate numerical EM analyses
- Exhaustive mapping function space
- Performance parameters: $A_e/T_{sys}$ versus $SLL_2$

- Sub-reflector size only marginally improves performance
- Minimized edge illumination has best performance
- Optimizing for Band 1 does not significantly compromise the performance of Band 2
- Choice for SKA:
  - -24dB $SLL_2$ in Band 1 (-26 dB in Band 2)
  - $\theta_e = 58^\circ$, $|P_1P_3| = 5$ m (PAF driven)
  - $\chi_1 = 20^\circ$, $\chi_2 = 20^\circ$
The parameterised Feed

- Determine parameterised feed model
- $\rho(\theta_f)$ from optics design
- Maximise $\eta_a$, $|E(\rho)|^2 = 1$

$$|G(\theta_f)|^2 = \frac{V_c|E(\rho)|^2 \rho(\theta_f)\rho'(\theta_f)}{\sin \theta_f} \quad \theta_f \leq \theta_e$$

- Maximise $\eta_{\text{spill}}$
- Minimise $T_{\text{spill}}$

$$|G(\theta_f)|^2 = -100 \text{dB} \quad \theta_f > \theta_e$$

- GO limit
- Ignores edge diffraction
The Feed

- SR extension - unsymmetric
- Trade-off between $\eta_{ill}$ and $\eta_{spill} (T_{spill})$
- Slightly over illuminate the SR (extension)
- Linear interpolation
  - Parameter 1: $\theta_{ext}$

![Graph showing gain vs. angle $\theta_f$ with linear interpolation and extensions](image)
The Feed

- Weighting function $P(\theta_f)$: Uniform/Gaussian
  - Parameter 2: $b_\theta$
  - Parameter 3: $\sigma_\theta$

- Parameterised Feed Pattern
  - Central part – identical to ideal case
  - Edge - stronger taper

- Discontinuous pattern
The Feed

- **Limit feed size:**
  - Band 1: $2r_0 = 1.3$ m
  - Band 2: $2r_0 = 0.8$ m

- **Spatial filter**
- **Spherical Mode expansion**

- **Limit number of modes:**
  - $\ln \phi : \text{BOR}_1, M = 1$
  - $\ln \theta$:
    \[ N = kr_0 + \max(3.63^3 \sqrt{kr_0}, 10) \]