Introduction to Radio Interferometry
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Radio Astronomy

- ALMA
- VLA
- VLBA

Gamma Rays, X-Rays and Ultraviolet Light blocked by the upper atmosphere (best observed from space).

Visible Light observable from Earth, with some atmospheric distortion.

Most of the Infrared spectrum absorbed by atmospheric gasses (best observed from space).

Radio Waves observable from Earth.

Long-wavelength Radio Waves blocked.

NASA/IPAC
Heterodyne Receiver (1901 -)

Observed sky frequencies are converted to lower frequency signals by mixing with a signal artificially created by a Local Oscillator. The output can then be amplified and sampled easily while retaining original phase and amplitude information.
Mapping the Sky in Radio Astronomy

- In astronomy, we wish to know the angular distribution of electromagnetic emission.
- ‘Angular Distribution’ means we are interested in the **brightness** of the emission, not just the **flux**.
- Measuring the brightness means making a map.
- Our targets are far away, the emission is extremely weak, and of very small angular size.
- Early radio surveys employed single dishes.

- **Nowadays, most (but not all!) observations are done with interferometers.**
Why Interferometry?

• It’s all about **Diffraction** - a consequence of the wave nature of light.

• Radio telescopes coherently sum electric fields over an aperture of size $D$. For this, diffraction theory applies – the angular resolution is: $\theta_{\text{rad}} \approx \frac{\lambda}{D}$

• To obtain 1 arcsecond resolution at a wavelength of 21 cm, we require an aperture of ~42 km!

• The (currently) largest single, fully-steerable aperture are the 100-m antennas in Effelsberg and Green Bank. Nowhere big enough.

• **Solution: Synthesize an aperture of that size with pairs of antennas!**
Interferometry - Basic Concept

- A parabolic dish coherently sums EM fields at the focus.
- The same result by adding in a network voltages from Individual elements.
- This is the basic concept of interferometry.
- Aperture Synthesis is an extension of this concept.
Subtitle: It’s important to note that while the effective size of the dog can be arbitrarily large, it’s not any more of a good dog than the two original dogs.
Stationary, Monochromatic Interferometer

- A small (but finite) frequency width, and no motion.
- Consider radiation from a small solid angle, from direction $\mathbf{s}$.

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$

$$V = V_1 \cos[\omega(t - \tau_g)]$$

Multiply

Average

$$R_c = \frac{V_1 V_2 \cos(\omega \tau_g)}{2} = \left[ V_1 V_2 \cos \left( 2\pi v \frac{\mathbf{b} \cdot \mathbf{s}}{c} \right) \right]$$
Examples of the Signal Multiplications

The two input signals are shown in red and blue. The desired coherence is the average of the product (black trace)

**In Phase:** \( \tau_g = n\lambda/c \)

**Quadrature Phase:**
\[
\tau_g = (2n+1)\lambda/4c
\]

**Anti-Phase:**
\[
\tau_g = (2n + 1)\lambda/2c
\]
Schematic primary beam vs. synthesized beam

Antenna Radiation Pattern

Synthesized Beam Pattern

1st sidelobe

Main Lobe
Complex Visibility

• Combine the cosine and sine responses (two independent correlator outputs) to form the complex visibility:
  \[ V = R_c - iR_s = Ae^{-i\phi} \]

• where the amplitude is: \( A = \sqrt{R_c^2 + R_s^2} \)

• where the phase is: \( \phi = \tan^{-1}(R_s/R_c) \)

• This also gives the relationship between the source brightness, and the response of an interferometer:
  \[ V(b) = R_c - iR_s = \int \int I_v(s)e^{-2\pi ivbs/c} \]
Visualizing the Visibility

The intensity, $I_\nu$, is in black, the ‘fringes’ in red. The visibility is the net dark green area.
Fourier Transform

Relate measured interference pattern (fringes) to the radio intensity on the sky.

- A radio interferometer measures the interference pattern (fringes) produced by pairs of apertures.

- The fringe pattern is directly related to the source brightness, where for small field-of-view: the complex visibility, $V(u,v)$ is the 2D Fourier transform of the brightness on the sky.

(Van Cittert-Zernike theorem)
Real Fringes

The fringe separation is given by baseline length in wavelengths
The orientation is given by the orientation of the baseline

East-West baseline makes vertical fringes
North-South baseline makes horizontal fringes
Rotated baseline makes rotated fringes

Fringe angular spacing given by baseline length in wavelengths:

\[ \Delta \theta = \lambda / b \]
Longer Baselines => Smaller Fringes

With longer baselines (in wavelengths!) come finer fringes:

- 250 meter baseline
  - 120 arcsecond fringe

- 1000 meter baseline
  - 30 arcsecond fringe

- 5000 meter baseline
  - 6 arcsecond fringe

What the interferometer measures is the integral (sum) of the product of this pattern with the actual brightness.
Cyg A Fringe example

- Cygnus A is a powerful, nearby radio galaxy.
- The left panel shows the actual brightness.
- The other two panels show how the 5km-baseline interferometer ‘sees’ it

Zero-Spacing Image
Sum = 999 Jy

5 km EW spacing
Sum = 61 Jy

5 km NS spacing
Sum = -16 Jy
Visibility to Image Space

- Fourier Space \((u,v,w)\) – right handed coordinate system:

\[
V(u, v, w) = \iiint I(x, y, z)e^{2\pi i(ux+vy+wz)} \, dx \, dy \, dz
\]

- Image Space \((x,y,z)\)

\[
I(x, y, z) = \iiint V(u, v, w)e^{-2\pi i(ux+vy+wz)} \, du \, dv \, dw
\]

Can simplify and drop \(w\) (z) term.

(a lot more information can be found in, e.g. Thompson, Moran & Swenson)
Visibility and Intensity in 1D

Input: Even brightness distributions.

Output: Real, even visibility functions.
Filling the Fourier Plane in 2D
Earth Rotation Synthesis

Snapshot (u,v)-coverage

12 hour track (u,v)-coverage
Making Images (FT)

\textit{sky + interferometer response}

Snapshot dirty image

12 hour track dirty image

Uniform weighting scheme of intensities.
Making Images - Deconvolution

Limited sampling of the (u,v)-plane causes defects in the FT images.

“The ‘CLEAN’ algorithm is most commonly used in radio interferometry to solve the convolution equation by representing a radio source by a number of point sources in an otherwise empty field of view. The final ‘deconvolved’ image, usually referred to as ‘CLEAN’ image, is the sum of these point components convolved with a ‘CLEAN’, usually Gaussian, beam to de-emphasize the higher spatial frequencies.” (Cornwell & Braun 1989)
# Visibility Weighting Schemes

<table>
<thead>
<tr>
<th></th>
<th>Robust/Uniform</th>
<th>Natural</th>
<th>Taper</th>
</tr>
</thead>
<tbody>
<tr>
<td>resolution</td>
<td>higher</td>
<td>medium</td>
<td>lower</td>
</tr>
<tr>
<td>sidelobes</td>
<td>lower</td>
<td>higher</td>
<td>Depends on # of baselines</td>
</tr>
<tr>
<td>Point source sensitivity</td>
<td>lower</td>
<td>maximum</td>
<td>lower</td>
</tr>
<tr>
<td>Extended source sensitivity</td>
<td>lower</td>
<td>medium</td>
<td>higher</td>
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*Image Credit: D. Wilner (SIW 2018)*
Interferometry Basic Parameters

- **Sensitivity**: Is given by the number of antennas times their effective collecting area.

- **Resolution**: Is given by the largest distance between antennas (longest baseline length). This determines the smallest scale of the synthesized beam.

- **Largest Angular Scale**: This is determined by the shortest distance between antennas.

- **Field of View**: Is given by the size of the beam of a single antenna (corresponding to the resolution of a single dish).
General Observing/Calibration Concepts

Delay

For example, VLA digital data transport system (DTS) introduces small delay offsets between antennas that are not accounted for during correlation. These are small and we can determine those by searching for the maximum fringe amplitude on a bright astronomical target applying a range of delay offsets.
General Observing/Calibration Concepts

Bandpass

Observe a bright calibrator (S/N per channel of 5-10) to obtain per antenna bandpass shapes.

VLA example:
General Observing/Calibration Concepts
Bandpass/Flux Density Scale

Observe a bright calibrator (S/N per channel of 5-10) to obtain per antenna bandpass shapes. And scale gains to known flux density.

VLA example:
General Observing/Calibration Concepts

Complex Gain Calibration

Tracking time variable changes, like atmosphere or changing gain in the instrument.

Short time solution

Long time solution
Standard Continuum Observing Procedure

• Flux Density Scale/Bandpass/Delay Calibrator
• Phase Calibrator
• Target
• Phase Calibrator
• Target
• ...

More detailed information:
https://science.nrao.edu/facilities/vla/docs/manuals/obsguide/calibration
Concluding Remarks

• Interferometry samples Fourier components of sky brightness.
• Using the Fourier transform we can reconstruct the sky brightness distribution.
• Deconvolution attempts to reconstruct the sky brightness given imperfect sampling.
• Some care has to be taken when setting up observations in order to be able to calibrate and achieve the set science goals.

• Radio Interferometry is challenging. However instrumentation is reasonably well understood to make the lives of astronomers easier.
Further Reading


• … there are many more references on radio interferometry with each focusing on different aspects …