Imaging Fundamentals



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An interferometer is an indirect imaging device

Young's double slit experiment



2D Fourier transform :



Image = sum of cosine 'fringes'.

Each antenna-pair measures the parameters of one 'fringe'.



Parameters of a Fringe :

Amplitude, Phase

Orientation, Wavelength

Measure the spatial correlation of the E-field incident at each pair of antennas



Parameters of a Fringe :

Amplitude, Phase : $\langle E_i E_j^* \rangle$ is complex. Orientation, Wavelength : \vec{u}, \vec{v} (geometry)

$$\langle E_i E_j^* \rangle \propto V_{ij}(u,v) =$$

$$\iint I^{sky}(l,m) e^{2\pi i (ul+vm)} dldm$$



Aperture Synthesis

Measure many (different) fringes : As much of V(u,v) as possible

 \rightarrow Multiple antenna pairs \rightarrow Multiple times \rightarrow Multiple observing frequencies



Spatial Frequency :

Length and orientation of the vector between two antennas, projected onto the plane perpendicular to the line of sight.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

For each antenna pair, U, V change with time (hour-angle, declination) and observing frequency.

Time and Frequency-resolution of the data samples $\delta \tau$, δv decides δu , δv

Image is real => Visibility function is Hermitian : $V(u, v) = V^*(-u, -v)$

=> One baseline : 2 visibility points



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 5 antennas

477 465 455 455 437 422 40°411 18°59^m45⁶ 35⁶ 30⁶ 25⁸ 20⁶ 15⁸ J2000 Right Ascension



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

4000

3000

2000

1000

-1000

-2000

-3000

-4000

(m) V

Image of the sky using 11 antennas





$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h, \theta) \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas







$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 2 hours 'Earth Rotation Synthesis'







$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 4 hours 'Earth Rotation Synthesis'







$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 4 hours, 2 freqs 'Multi-Frequency Synthesis





$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} R(h,\theta) \\ \delta y \\ \delta z \end{bmatrix}$$

Image of the sky using 27 antennas over 4 hours, 3 freqs 'Multi-Frequency Synthesis







Imaging in practice

Imaging in practice



Image formed by an interferometer : Convolution Equation



You have measured the Convolution of the True Sky with the instrumental PSF.

Recovering True Sky = DE-convolution

--- the impulse-response of the instrument (image of a point-source)

--- the intensity of the diffraction pattern through an array of 'slits' (dishes)

--- a measure of the imaging-properties of the instrument

Imaging in practice

Basic Imaging :

Step 1 : Define image size and cell size

Step 2 : Gridding, data-weighting and FFT

Step 3 : Run iterative deconvolution

$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

Choosing image size, cell-size

- Choosing image 'cell' size : 3-5 pixels within the main lobe of the PSF

PSF beam width :
$$\frac{\lambda}{b_{max}} = \frac{1}{u_{max}}$$
 radians (x $\frac{180}{\pi}$ to convert to degrees)

This is the diffraction-limited angular-resolution of the telescope Ex : Max baseline : 10 km. Freq = 1 GHz. Angular resolution : 6 arcsec

- Choosing image field-of-view (npixels) : As much as desired/practical.

$$\frac{1}{fov_{rad}} = \delta u$$

Field of View (fov) controls the uv-grid-cell size $(\delta u, \delta v)$

- Antenna primary-beam limits the field-of-view ('slits' of finite width)

- Gridding + FFT :

- An interferometer measures irregularly spaced points on the UV-plane.
- Need to place the visibilities onto a regular grid of UV-pixels, and then take an FFT

Basic Imaging :

Step 1 : Define image size and cell size

Step 2 : Gridding, data-weighting and FFT

Step 3 : Run iterative deconvolution

$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

Gridding and Weighting



- Visibility data are interpolated onto a regular grid before taking an i-FFT
- Convolutional Resampling

=> Use a gridding convolution function
 => Use weights per visibility
 (weighted average of all data points per cell)

Image = weighted-average of the data.



PSFs and Observed (dirty) Images



Note the noise-structure. Noise is correlated between pixels by the PSF. Image Units (Jy/beam) ------ All pairs of images satisfy the convolution relation => Need to deconvolve them

An Image is a weighted average of the data.

3000 2000

Weighting-scheme => modify the imaging properties of the instrument => emphasize features/scales of interest => control imaging sensitivity

	Uniform/Robust	Natural/Robust	UV-Taper
-3000 -4000 -3000 -2000 -1000 0 1000 2000 3000 4000 U (m)	All spatial frequencies get equal weight	All data points get equal weight	Low spatial freqs get higher weight than others
Resolution	higher	medium	lower
PSF Sidelobes (VLA)	lower	higher	depends
Point Source Sensitivity	lower	maximum	lower
Extended Source Sensitivity	lower	medium	higher

Basic Imaging :

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$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i (ul+vm)} dl dm$$

Deconvolution – Hogbom CLEAN

Sky Model : List of delta-functions

- (1) Construct the observed (dirty) image and PSF
- (2) Search for the location of peak amplitude.
- (3) Add a delta-function of this peak/location to the model
- (4) Subtract the contribution of this component from the dirty image - a scaled/shifted copy of the PSF

Repeat steps (2), (3), (4) until a stopping criterion is reached.

(5) Restore : Smooth the model with a 'clean beam' and add residuals

The CLEAN algorithm can be formally derived as a model-fitting problem

- model parameters : locations and amplitudes of delta functions
- solution process : χ^2 minimization via an iterative steepest-descent algorithm (method of successive approximation)

Deconvolution – MultiScale (MS)-CLEAN

Multi-Scale Sky Model : Linear combination of 'blobs' of different scale sizes

- Efficient representation of both compact and extended structure (sparse basis)
- A scale-sensitive algorithm
- (1) Choose a set of scale sizes
- (2) Calculate dirty/residual images smoothed to several scales (basis functions)
 - Normalize by the relative sum-of-weights (instrument's sensitivity to each scale)

(3) Find the peak across all scales, update a single multi-scale model as well as all residual images (using information about coupling between scales)

Iterate, similar to Classic CLEAN, and restore at the end.

The MS-CLEAN algorithm can also be formally derived as a model-fitting problem using χ^2 minimization and a basis set consisting of several 'blob' sizes.

Deconvolution – Comparison of Algorithms

CLEAN

MEM

Point source model

Point source model with a smoothness constraint

MS-CLEAN

Multi-Scale model

with a fixed set of

scale sizes

ASP

Multi-Scale model with adaptive bestfit scale per component

 I^m

Deconvolution – Comparison of Algorithms

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Point source model with a smoothness constraint

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 I^m

How can you control the quality of image reconstruction ?

(1) Iterations and stopping criterion

'niter' : maximum number of iterations / components 'threshold' : don't search for flux below this level

- minor cycles can be inaccurate, so periodically trigger major cycles

(2) Using masks

Need masks only if the deconvolution is "hard".

- => Bad PSFs with high sidelobes
- => Leftover bad data causing stripes or ripples
- => Extended emission with sharp edges
- => Extended emission that is seen only by very few baselines
- Draw masks interactively or supply final mask.
- Run Automasking to build up a mask

(3) Self-Calibration

Use your current best estimate of the sky (i.e. the model image) to get new antenna gain solutions. Apply, Image again and repeat.

predictModel (savemodel in tclean)

Image Quality

Noise in the image : Measured from restored or residual images

- With perfect reconstruction, The ideal noise level is : $RMS \propto \frac{0.12 \frac{T_{sys}}{\eta_a}}{\sqrt{N_{ant}(N_{ant}-1) \cdot \delta \tau \cdot \delta \nu \cdot N_{pol}}}$
- In reality, measure the RMS of residual pixel amplitudes

Dynamic Range : Measured from the restored image

- Standard : Ratio of peak brightness to RMS noise in a region devoid of emission.
- More truthful : Ratio of peak brightness to peak error (residual)

Image Fidelity : Correctness of the reconstruction

- remember the infinite possibilities that fit the data perfectly ?
- useful only if a comparison image exists.

Inverse of relative error : _

$$\frac{I^m * I^{beam}}{I^m * I^{beam} - I^{restored}}$$

Model image

Restored image

Residual image

Basic Imaging :

Narrow-frequency range, Small region of the sky

=> The 2D Fourier Transform relations hold
=> Convolution and deconvolution

Basic Imaging :

Narrow-frequency range, Small region of the sky

=> The 2D Fourier Transform relations hold
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Wide-Band Imaging :

=> Sky and instrument change across frequency range

Wide-Field Imaging

=> The 2D Fourier Transform relation breaks

Mosaic Imaging

=> Image an area larger than what each antenna can see.

Wide-band Imaging – Sensitivity and Multi-Frequency Synthesis

Frequency Range :	(1 – 2 GHz)	(4 – 8 GHz)	(8 – 12 GHz)
Bandwidth : $v_{max} - v_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : v_{max} : v_{min}	2:1	2:1	1.5 : 1
Fractional Bandwidth : $(v_{max} - v_{min})/v_m$	_{id} 66%	66%	40%

UV-coverage / imaging properties change with frequency

Sky Brightness can also change with frequency \rightarrow model intensity and spectrum

Spectral Cube (vs) MFS imaging

3 flat-spectrum sources + 1 steep-spectrum source (1-2 GHz VLA observation)

Images made at different frequencies (specmode='cube', deconvolver='hogbom')

35° 30° 25° 20° 15° J2000 Right Ascension

^h59^m45^s 35^s 30^s 25^s 20^s 15ⁱ J2000 Right Ascension

159^m45^s 35^s 30^s 25^s 20^s 15^s J2000 Right Ascension ^h59^m45^s 35^s 30^s 25^s 20^s 15^s J2000 Right Ascension 2 GHz

Add all singlefrequency images (after smoothing to a low resolution)

Use wideband UVcoverage, but ignore spectrum (MFS, nterms=1)

specmode='mfs'

Use wideband UV-coverage + Model and fit for spectra too (MT-MFS, nterms > 1)

Output : Intensity and Spectral-Index

deconvolver='mtmfs' in tclean

Wide-Field Imaging – W-term

$$V^{obs}(u,v) = S(u,v) \iint I(l,m) e^{2\pi i(ul+vm)} dl dm$$

$$V^{obs}(u,v) = S(u,v) \iiint I(l,m) e^{2\pi i (ul+vm+w(n-1))} dl dm dn$$

The 'w' of a baseline can be large, away from the image phase center The 'n' for a source can be large, away from the image phase center

There are algorithms to account for this : Image Faceting, W-Projection.

Wide-Field Imaging – W-term

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The 'w ' of a baseline can be large, away from the image phase center
The 'n ' for a source can be large, away from the image phase center
2D Imaging Facet Imaging W-Projection

Wide-Field Imaging – Primary Beams

Each antenna has a limited field of view => Primary Beam (gain) pattern

=> Sky is (approx) multiplied by PB, before being sampled by the interferometer

 $I^{obs}(l,m) \approx I^{PSF}(l,m) * \left[P^{sky}(l,m) \cdot I^{sky}(l,m)\right]$

The antenna field of view D = antenna diameter λ/D

Compare with angular resolution of the interferometer :

 λ/b_{max}

But, in reality, P changes with time, freq, pol and antenna....

=> Ignoring rotation/scaling limits dynamic range to 10^4 => More-accurate method to account for this : A-Projection Combine data from multiple pointings to form one large image.

One Pointing sees only part of the source

Combine pointings either before or after deconvolution.

Stitched mosaic :

- -- Deconvolve each pointing separately
- -- Divide each image by PB
- -- Combine as a weighted avg

Joint mosaic :

- Combine observed images as a weighted average (or)
 Grid all data onto one UV-grid, and then iFFT
- -- Deconvolve as one large image

Combine data from multiple pointings to form one large image.

Combine pointings either before or after deconvolution.

Stitched mosaic :

- -- Deconvolve each pointing separately
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Joint mosaic :

- Combine observed images as a weighted average (or)
 Grid all data onto one UV-grid, and then iFFT
- -- Deconvolve as one large image

Two Pointings see more.....

Combine data from multiple pointings to form one large image.

Use many pointings to cover the source with approximately uniform sensitivity

Combine pointings either before or after deconvolution.

Stitched mosaic :

- -- Deconvolve each pointing separately
- -- Divide each image by PB
- -- Combine as a weighted avg

Joint mosaic :

- Combine observed images as a weighted average (or)
 Grid all data onto one UV-grid, and then iFFT
- -- Deconvolve as one large image

Frequency dependence of the Primary Beam

Average Primary Beam

45

30'

15'

30'

15'

20^h04^m

02

the main lobe

19^h58^l

J2000 Right Ascension

A very wide shelf of

sensitivity outside

56^m

54ⁿ

40°00'

41°00'

12000 Decl 45'

VLA PBs

2.0 GHz 30' 15' 12000 Declination 45' 30' 30' 15' 40°00' 45' 20^h04ⁿ 54ⁿ 02 00 10^h58^t 56 J2000 Right Ascension

Primary beam scales (or changes) with frequency

Spectral Index of PB

For VLA L-Band (1-2 GHz) - About -0.4 at the PB=0.8 (6 arcmin from the center) - About -1.4 at the HPBW (15 arcmin from the center)

 $I_{wf,wb}^{obs} = \sum_{v} \left[\left(P_{v} \cdot I_{v}^{sky} \right) * PSF_{v} \right]$

Wide Band Primary Beam Correction

5 GHz

41°00

45

000 Declir 45' 30' 15' 40°00'

Cube Imaging

- -- Sky model represents $I(\mathbf{v})P(\mathbf{v})$
- -- Divide the output image at each frequency by P(v)

Multi-Term MFS + Wideband-PBcor

- -- Taylor coefficients represent $I(\mathbf{v})P(\mathbf{v})$

-- Polynomial division by PB Taylor coefficients $\frac{(I_{0,}^{m}I_{1,}^{m}I_{2,}^{m}...)}{(P_{0}P_{1}P_{2}...)} = (I_{0,}^{sky}I_{1,}^{sky}I_{2}^{sky}...)$

Wideband A-Projection

-- Remove $P(\mathbf{v})$ during gridding (before model fitting) $A_{v}^{-1} \approx \frac{A_{v_{c}}^{T}}{A^{T} * A}$ where $P_{v} \cdot P_{v_{c}} \approx P_{v_{mid}}^{2}$

-- Output spectral index image represents only the sky

Choice of gridding convolution function => different instrumental effects

- W-term and Full-beam imaging (gridder = 'wproject', 'mosaic', 'awproject')
- Joint Mosaics (gridder='mosaic', 'awproject') [conjbeam = T/F]

Standard Imaging : Prolate Spheroidal

(gridder = 'standard')

W-Projection : FT of a Fresnel kernel

A-Projection :

Convolutions of Aperture Illumination Funcs + phase gradients for joint mosaics

"A", "W" kernels can change with baseline, time and/or frequency.

Task : tclean

Task : tclean

- Select data (field, spw, etc..) and data weighting scheme
- Make PSF and Observed Image
 - Cube or Continuum imaging
 - Stokes Planes
 - Single field or Multi-field or Multi-Facet or Joint Mosaic
 - Pick a gridder
- Run Deconvolution
 - Hogbom or Multi-scale or Multi-term Wideband
 - Set iteration controls
 - Set up masks
- Restoration
- Save Model Visibilities (for future self-calibration)
- Primary Beam correction

[Parallelization is an option for major cycles]

Image reconstruction is an iterative model-fitting / optimization problem

Measurement Eqn :
$$[A]I^m = V^{obs}$$

Iterative solution : $I^m_{i+1} = I^m_i + g[A^TWA]^+ (A^TW(V^{obs} - AI^m_i))$

Standard gridding, W-Projection, (WB)-A-Projection, Joint Mosaics

Clean (Hogbom, Clark, MultiScale, MultiTerm, etc...)

Standard gridding, W-Projection, (WB)-A-Projection, Joint Mosaics

Cube, MFS, MT-MFS, Faceting, Stokes, Multi-Field Clean (Hogbom, Clark, MultiScale, MultiTerm, etc...)

Imaging Definition : Spectral Cubes

specmode = 'cube'

- N data channels are mapped to M image channels (with binning/interpolation)
- Image coordinates defined by the user : start, width, nchan, outframe (channel, frequency, velocity)
- Image coordinate system is internally stored In LSRK frame, with a conversion layer to allow relabeling to outframe for display/analysis

- All gridding/imaging is done in the LSRK frame with on-the-fly conversions to LSRK (i.e. no cvel needed)

specmode='cubedata' in tclean

- No internal conversion to LSRK.
- Data channels map to image channels (with only binning/interpolation)

specmode = 'cubesrc' in tclean

- Track moving sources via ephemeris tables)

Imaging Definition : Continuum Imaging

specmode = 'mfs', 'cont'

- Data from all channels are gridded onto a single uv-grid, using the appropriate u,v,w coordinates
- Multi-Frequency-Synthesis

- nterms = 1 (flat spectrum assumption
- nterms > 1 (Taylor polynomial spectrum)
 - Major cycle : nterms Taylor-weighted averages of across frequency
 - Minor cycle : Solve for nterms coefficients

Imaging Definition : Correlation and Stokes planes

Users can choose to make images of

R/L => I, Q, U, V, IV, QU, IQUV, R, LL, LR, RL, RLL, RLLR, 'all'

X/Y => I, Q, U, V, IQ, UV, IQUV, XX, YY, XY, YX, XXYY, XYYX, 'all'

Pseudol : Make a stokes I image using data where some correlation pairs are flagged.

Imaging Definition : Multi-field and Facets

Image partitioning : Same data, multiple images

Multiple fields (outlier fields)

- Usually, one large main field and several smaller outlier fields
- To avoid extremely large images
- Outlier file : List of image definition parameters

Multiple Facets :

- Work with smaller field-of-view images To get around the w-term problem (non-coplanarity and sky curvature)
- Grid each facet separately onto a subimage, but do a single joint deconvolution (use PSF from first facet)

Imaging Definition : Stitched and Joint Mosaics

'image mosaic', grid separately but combine before minor cycle)

Joint mosaic

- Grid data from all pointings onto single uv grid, with appropriate phase gradients per pointing
- Joint deconvolution (assumes spatially invariant PSF)

Standard gridding, W-Projection, (WB)-A-Projection, Joint Mosaics

Cube, MFS, MT-MFS, Faceting, Stokes, Multi-Field Clean (Hogbom, Clark, MultiScale, MultiTerm, etc...)