Mapping the Galactic Centre with gravitational waves from BHs orbiting Sgr A* : using Pulsar Timing

Alak Ray
Tata Institute of Fundamental Research, Mumbai

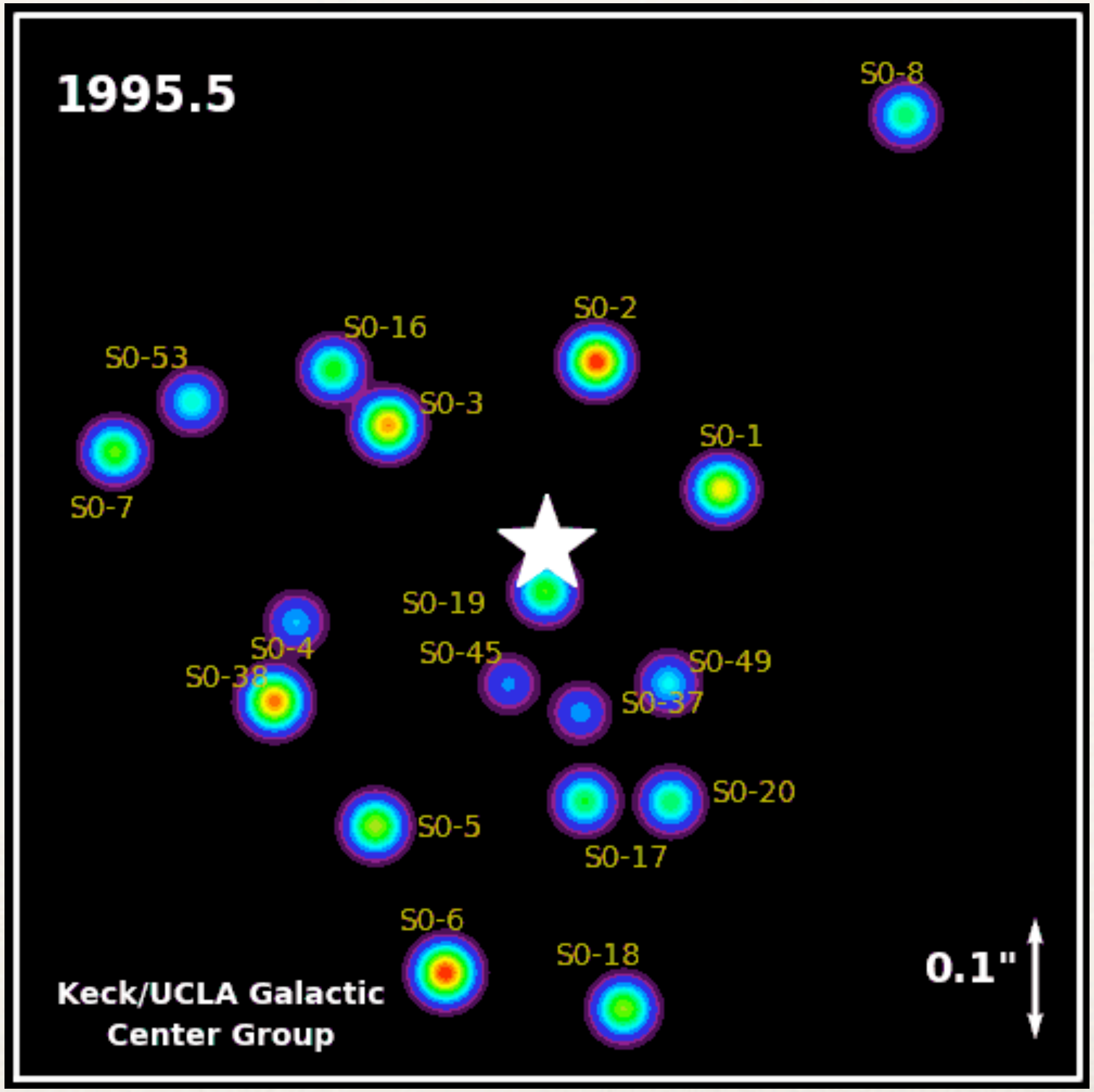
with Bence Kocsis and Simon Portegies-Zwart

ApJ 752, 67, 2012: B. Kocsis et al

October 4, 2013, IAU Symposium 303 Galactic Center, Santa Fe, NM

UCLA Galactic Center Group Animation

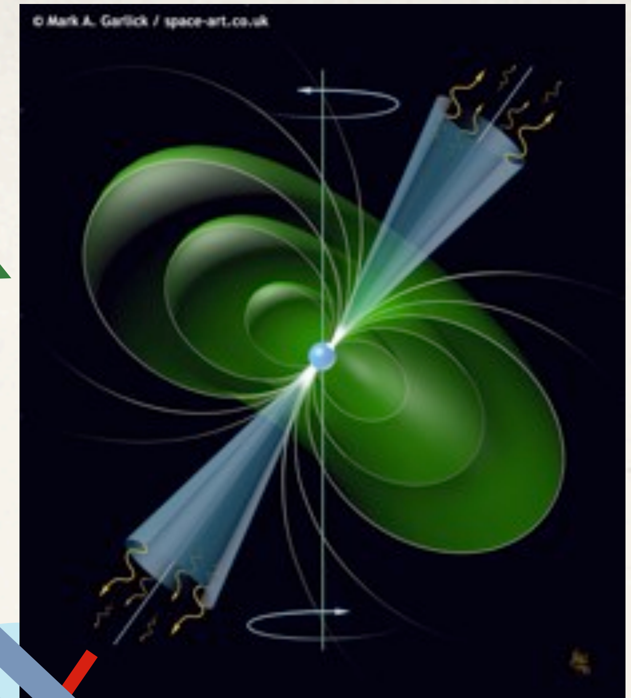
A 2.2 micron animation of the stellar orbits in the central parsec.



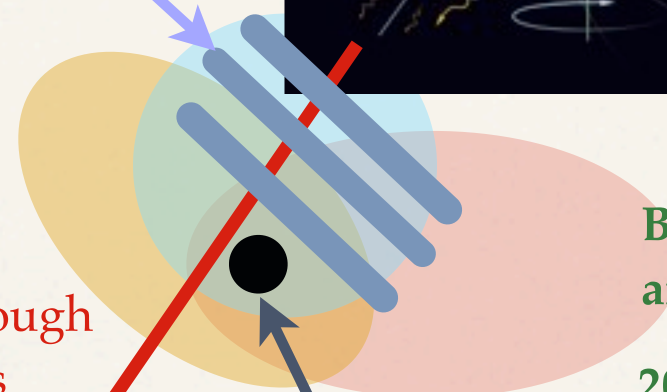
Star S0-2 has orbital period =15.78 years. S0-16 comes a 90 AUs from the central black hole.

Anna Boehle, Poster P38 and other talks at this meeting

Pulsar (clock that
sends out the ticks)



Gravitational Waves
from Galactic centre



Radio waves passing through
region perturbed by GWs

SMBH

BHs, IMBHs in orbit
around SMBH

20,000 stellar mass BHs
(~few to 10 MSun) & a
few IMBHs (10^3 Msun)

Radio Telescope
measures the ticks
of the clock

Direct detection of GW source at GC by a "clock" PSR

- We estimate the ***foreground*** (in contrast to the cosmological background) of Gravitational Waves (GWs) generated by a dense population of compact objects in the GC. Their orbits around the SMBH generate GWs.

- The "probe" or the "clock" -- the radio pulsar sending out regular pulses, show systematic pulse arrival time variations when the spacetime around it gets perturbed by GWs. The probe (PSR) has to be near the source of GWs, and therefore in the ***vicinity*** of the GC.

- Stellar mass BHs and IMBHs are more massive than regular stars populating the GC and they segregate and settle to the core of the GC. Emitted GW signal falls in nHz range probed by pulsar timing arrays.

- If the probe (PSR) is close enough, ***foreground*** signal may exceed GW background locally.

Sources of GWs in the GC

- How many stellar mass (~10 MSun) BHs?

In a relaxed galactic cusp, number density n_* of objects orbiting around a SMBH with semimajor axis a, is:

$$n_\star(a) = \frac{3-\alpha}{4\pi} N_\star \left(\frac{a}{\mathrm{pc}}\right)^{-\alpha} \mathrm{pc}^{-3}$$

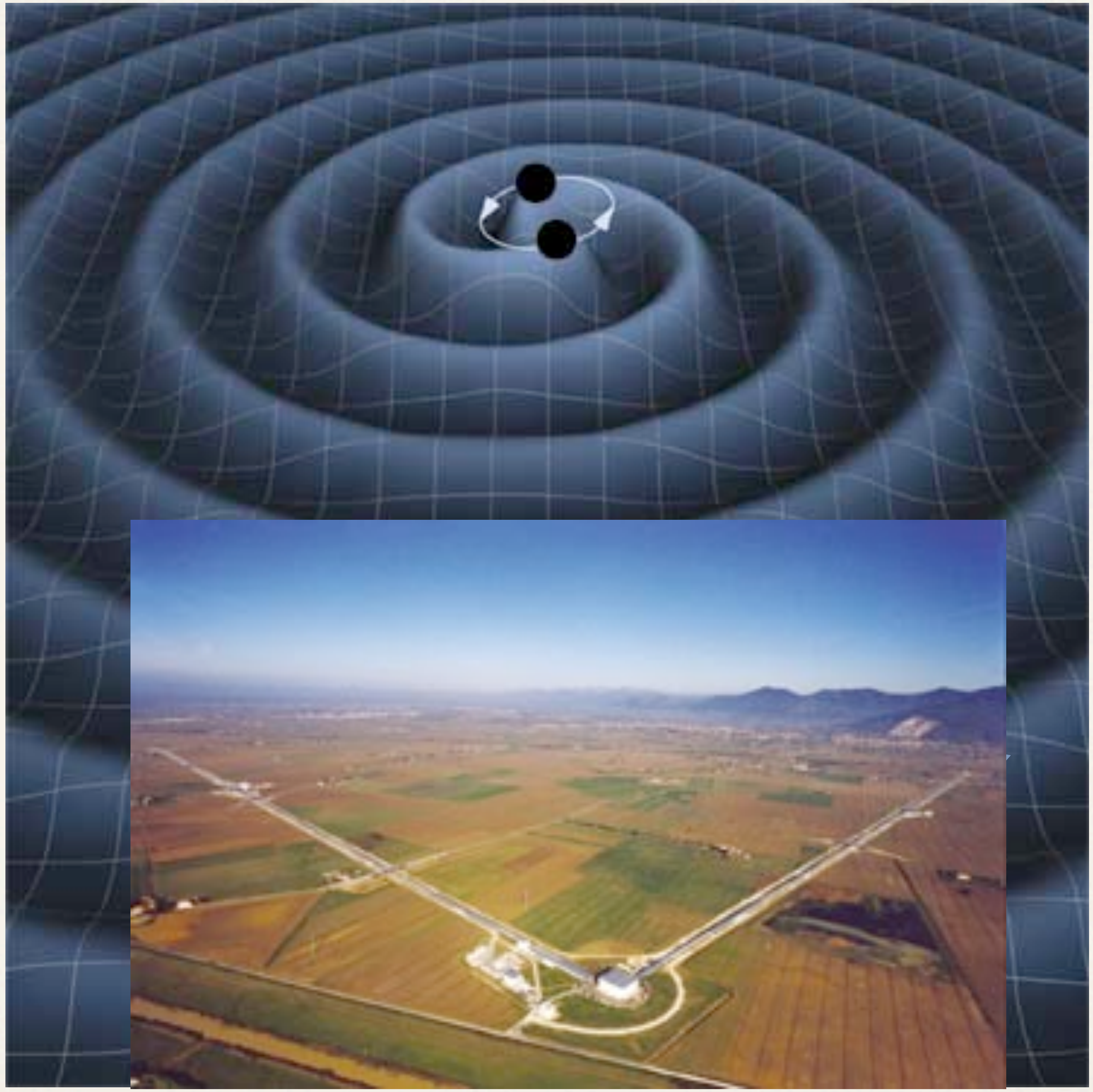
N_* is the total number of objects within 1 pc and $\alpha = 7/4$ for a stationary Bahcall Wolf cusp (Binney & Tremaine 2008).
If the mass function in cusp dominated by heavy objects, the density of light & heavy objects relaxes to: $\alpha \sim 3/2$ and $\alpha \sim 7/4$ (Alexander & Hopman 2009, Keshet et al 2009)

Take $N_* = 20{,}000$ for stellar mass BHs of $m_* = 10$ Msun within r= 1pc (Morris 1993, Miralda Escude & Gould 2000, Freitag et al 2006a) and *assume* $\alpha_{*,\mathrm{BH}} = 2$.

Eccentricity distribution (when used): that for a relaxed thermal distribution of an isotropic cusp, i.e. $dN \propto e\,de$ independent of semi-major axis (Binney & Tremaine 2008)

GW source

Artists's impression of gravitational waves from two orbiting BHs Credits: K. Thorne & T. Carnahan, LISA Gallery



PSR 1

PSR 2

VIRGO detector:
CERN Courier

Courtesy: Andrea Lommen
NANOGrav presentation

Earth

Credits: K. Thorne & T. Carnahan, LISA Gallery

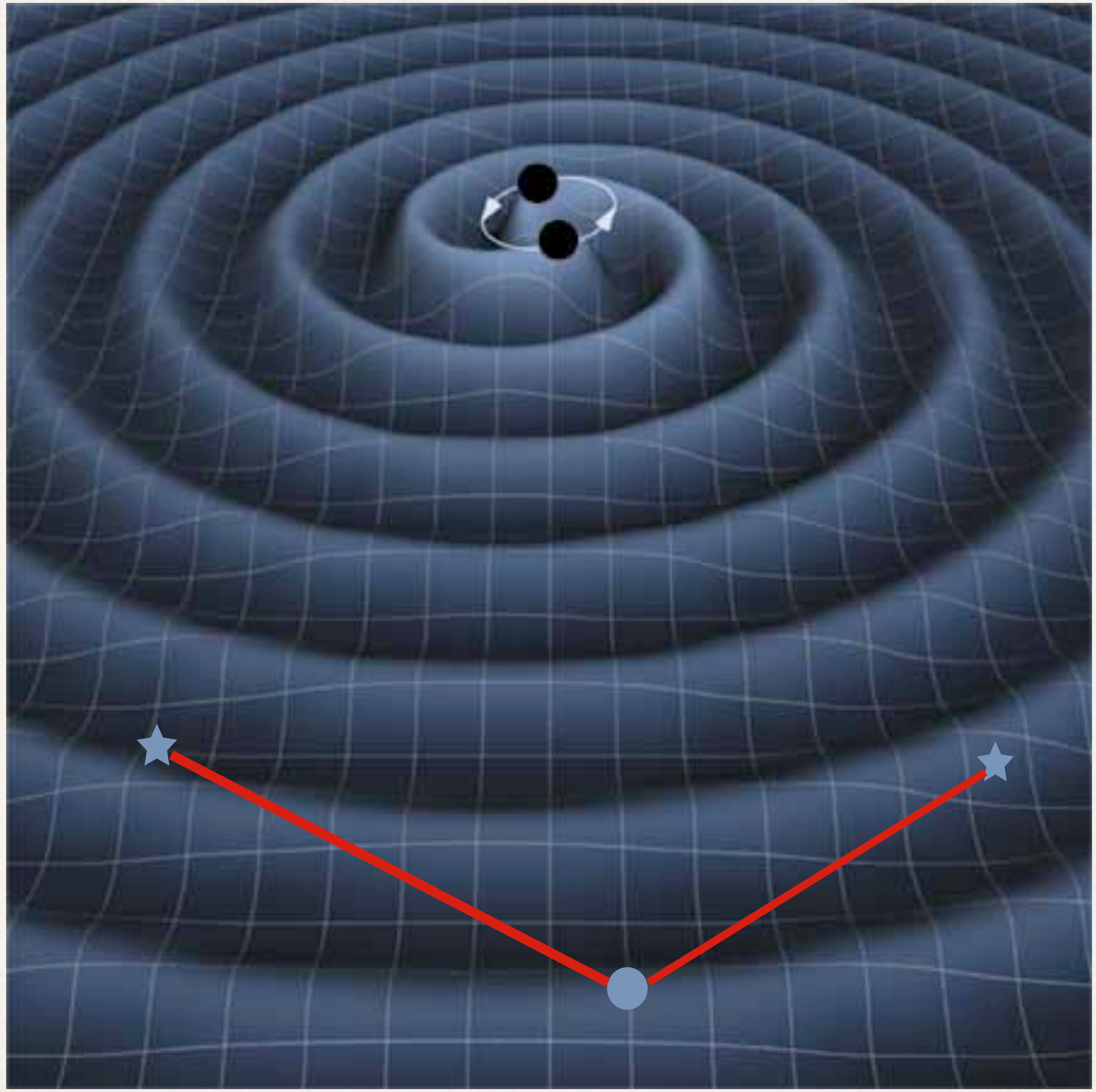
Pulsar Timing Array

Clock Errors: All PSRs have the same TOA variations: monopole signature

Solar-System ephemeris errors: Dipole signature

Gravitational Waves: Quadrupole signature

GW source



PSR 1

PSR 2

Earth

Pulsar-Earth Baselines

Sazhin (1978);
Detweiler (1979)

Hellings & Downs (1983): Pulsar Timing Array

Spectral range and resolution of Pulsar Timing

- If pulsars are observed repeatedly in time intervals Δt for a total time of T, the spectral resolution is $\Delta f = 1/T$
- The observable frequency range is: $1/T < f < 2/\Delta t$
- For T =10 yr and with $\Delta t = 1$ week, this means the range of GW frequencies is: 3×10^{-9} Hz (3 nHz) < f < 3×10^{-6} Hz (3000 nHz)
- Recall that the GW frequency of a circular orbit around the SMBH of mass M_o is twice the orbital frequency, $f = 2\, f_{orb}$ and the corresponding orbital radius is: $r(f) = M_\bullet(\pi M_\bullet f)^{-2/3} = 2.6\, f_8^{-2/3}$ mpc.
- Here, $f_8 = f/\ (10^{-8}$ Hz) and $M_o = 4.3 \times 10^6\, M_{sun}$

Unresolved circular orbit sources of GW

The root mean square strain generated at a measured distance D from the source of GW by a BH of mass m_* orbiting around a SMBH of mass M_o on a circular orbit of radius r in one GW cycle is:

$$h_0(f) = \sqrt{\frac{32}{5}}\,\frac{M_\bullet m_\star}{D\,r(f)} = 8.8\times10^{-15} m_3 D_{\rm pc}^{-1} f_8^{2/3}$$

$m_3 = m_*/10^3$

GW strain of many independent sources with the same frequency but random phase add quadratically. For a signal observed for time T, the spectral resolution is $\Delta f = 1/T$, and the number of sources with overlapping frequencies is:

$$\Delta N = \frac{dN}{dr}\left|\frac{dr}{df}\right|\frac{1}{T}.$$

The integrated GW signal with frequency from a population of sources can be expressed in terms of a strain in a logarithmic frequency bin:

$$h_c^2 = \Delta N\,(fT)h_0^2 = \frac{dN}{dr}\left|\frac{dr}{df}\right| f h_0^2 = \frac{dN}{d\ln f}h_0^2,$$

With $dN/dr = 4\pi r^2 n_*(r)$ we get the characteristic spectral amplitude (squared) as:

$$h_c^2(f) = \frac{8\pi}{3} r^3 n_\star(r)\,h_0^2 = \frac{256\pi}{15}\,\frac{M_\bullet m_\star^2}{D^2}\,(\pi M_\bullet f)^{-2/3}\,n_\star[r(f)]$$

Sensitivity levels of PSR timing

In the far field radiation zone of a circular binary, the metric perturbations induce a change in the observed pulsar frequency ν(t) from ν_0, the unperturbed value (i.e. if GWs were absent), and the Time of Arrivals (TOAs) of the pulses change as: $\delta t = \int z(t) dt,$

$$z(t) \equiv [\nu(t) - \nu_0]/\nu_0 = z_+(t) + z_\times(t),$$

"z"s can be related to the metric perturbations due to GWs following Detweiler (1979, Finn (2009), Cornish (2009). The GWs induce a strain amplitude proportional to $A = M_\bullet m_\star/(D\, r)$

D is the distance between the SMBH and the pulsar while r is the distance between the SMBH and m_*. The PSR is much closer to the SMBH compared to the Earth, so the corresponding Earth term is not included, unlike cosmological GW sources, since that term is negligible. Averaging the redshift factor over an isotropic distribution of the BH-SMBH binary normal and over the corresponding range of vectors pointing from the Earth to that binary and over one GW cycle,

$$z_{\rm GW} \equiv \sqrt{\langle z_+^2 \rangle + \langle z_\times^2 \rangle} = \frac{4}{\sqrt{15}} \frac{M_\bullet m_\star}{D\, r(f)} = \frac{1}{\sqrt{6}} h_0(f),$$

The variations in the TOAs are given by the integrated change $z_{GW}/2\pi f$. For an observation time T, the sensitivity increases with the number of observed cycles as $(fT)^{1/2}$.

$$\delta t_{\rm GW} = \frac{z_{\rm GW}}{2\pi f}(fT)^{1/2} = \frac{1}{\sqrt{6}} \frac{h_c(f)}{2\pi f} = 100\, m_3 D_{\rm pc}^{-1} T_{10}^{1/2} f_8^{1/6} \text{ns}.$$

Assumes contribution from a single source; can generalize to many sources per frequency bin.

Individually resolvable sources

GW foreground generated by a population of BHs is smooth if the average number per frequency bin Δf satisfies $\langle \Delta N \rangle \gg 1$. Spectrum becomes spiky ($\langle \Delta N \rangle \sim 1$) above the frequency (with $\underline{N}_* = N_* / (2 \times 10^4)$):

$$f_{\rm res} = 4.2 \times 10^{-8}\,\mathrm{Hz} \times 10^{9(\alpha-2)/(9-2\alpha)} \left[\frac{(3-\alpha)\bar{N}_\star}{T_{10}} \right]^{3/(9-2\alpha)}$$

$$\begin{aligned} N_{\rm res} &= \int_0^{r_{\rm res}} n_\star(r)\, 4\pi r^2 dr = N_\star \left(\frac{r_{\rm res}}{\rm pc} \right)^{3-\alpha} \\ &= 20 \times 10^{9(\alpha-2)/(9-2\alpha)} \bar{N}_\star \\ &\quad \times \left(\frac{T_{10}}{(3-\alpha)\bar{N}_\star} \right)^{2(3-\alpha)/(9-2\alpha)} . \end{aligned}$$

$$h_{c,1}(f) = (fT)^{1/2} h_0(f) = 1.6 \times 10^{-14} m_3 D_{\rm pc}^{-1} T_{10}^{1/2} f_8^{7/6} .$$

PSR distances for detectability

Using the equation for δt_{GW}, the distance within which a pulsar timing array could measure GWs of an individual source with a fixed timing precision $\delta t = 10\ \delta t_{10}$ ns, is :

$$D_{\delta t} = 14\, m_3 \delta t_{10}^{-1} T_{10}^{1/2} f_8^{1/6}\ \mathrm{pc}.$$

Recall however, that the cosmological GW background constitutes an astrophysical noise for measuring the GWs of objects orbiting SgrA*. At these frequencies, the stochastic background is dominated by SMBH binary inspirals with a characteristic amplitude (Kocsis & Sesana 2011):

$$h_{c,s} = 1.8 \times 10^{-15} f_8^{-2/3}.$$

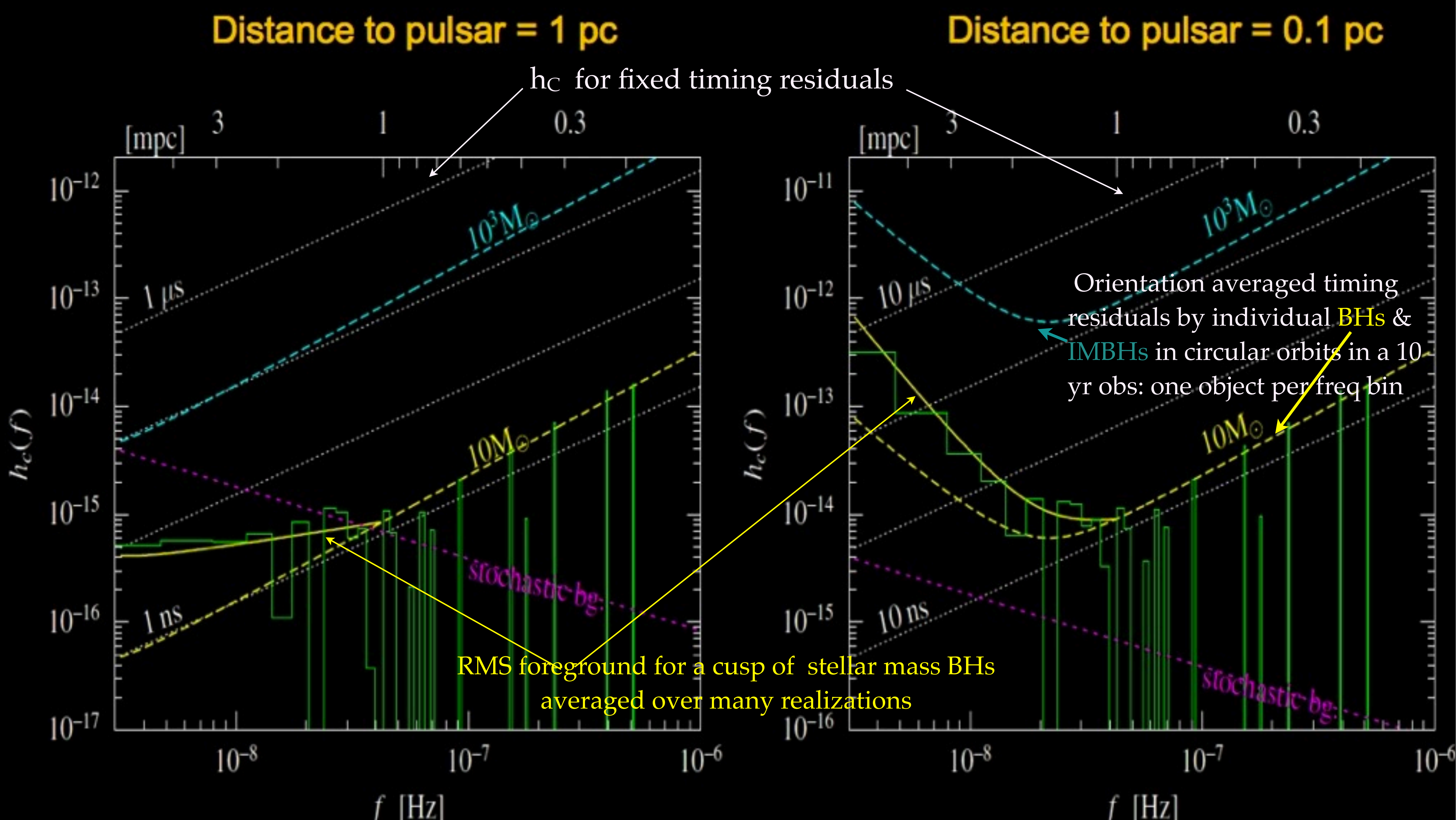
Thus, GWs from an individual BH orbiting the SMBH rises above the stochastic background within:

$$D_{\mathrm{bg}} = 8.7\, m_3 T_{10}^{1/2} f_8^{11/6}\ \mathrm{pc}.$$

Therefore a pulsar within $D_{\delta t}$ and D_{bg} to the GC could be used to detect GWs from individual objects in the GC, provided sufficient timing accuracy can be achieved.

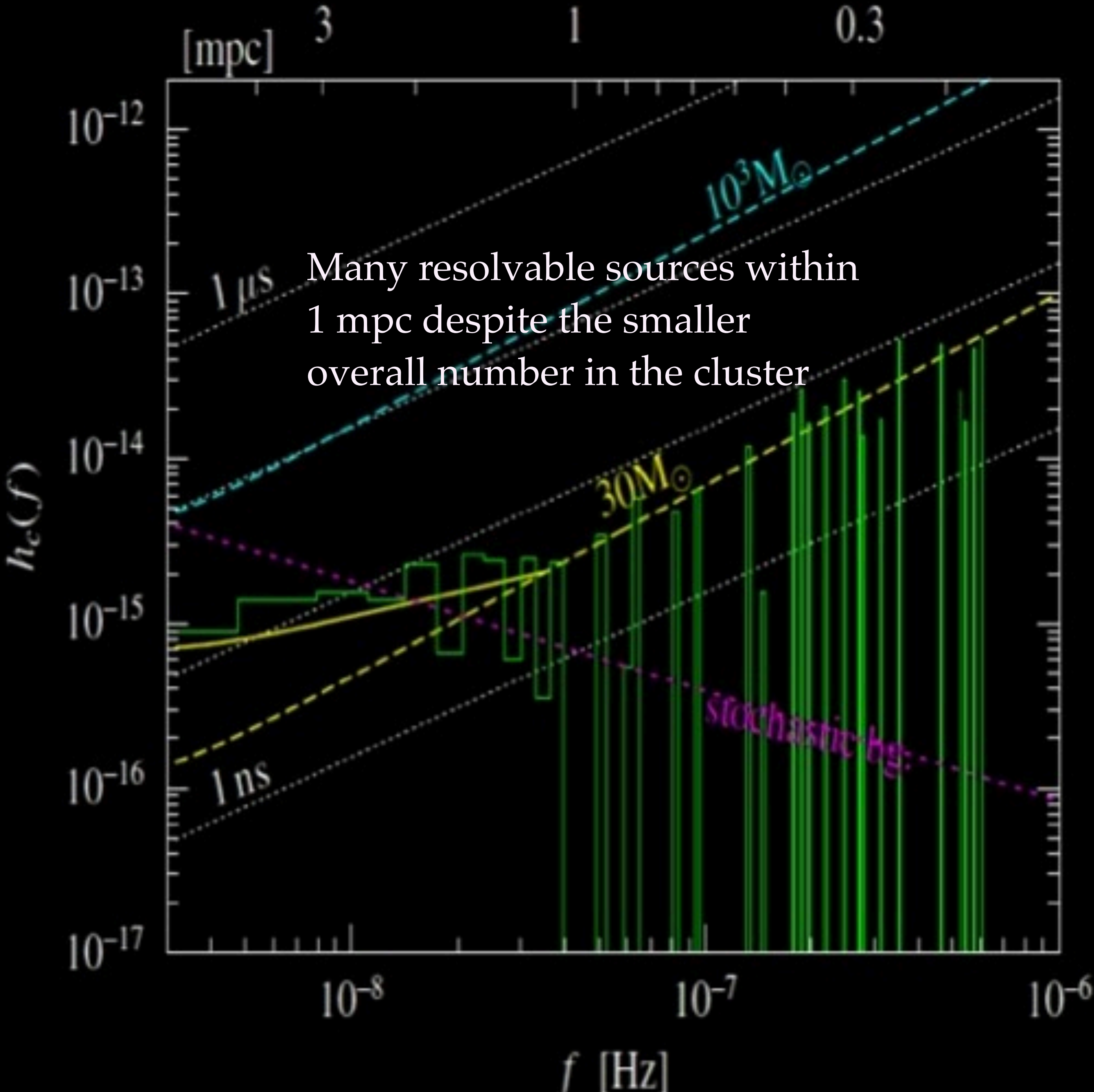
Monte Carlo realization (in green) of BHs in orbit around SMBH in random orientation of circular orbits for a 10 yr observation span

20,000 BHs, 10 M_{Sun}, within 1 pc, $n(r) \sim r^{-2}$

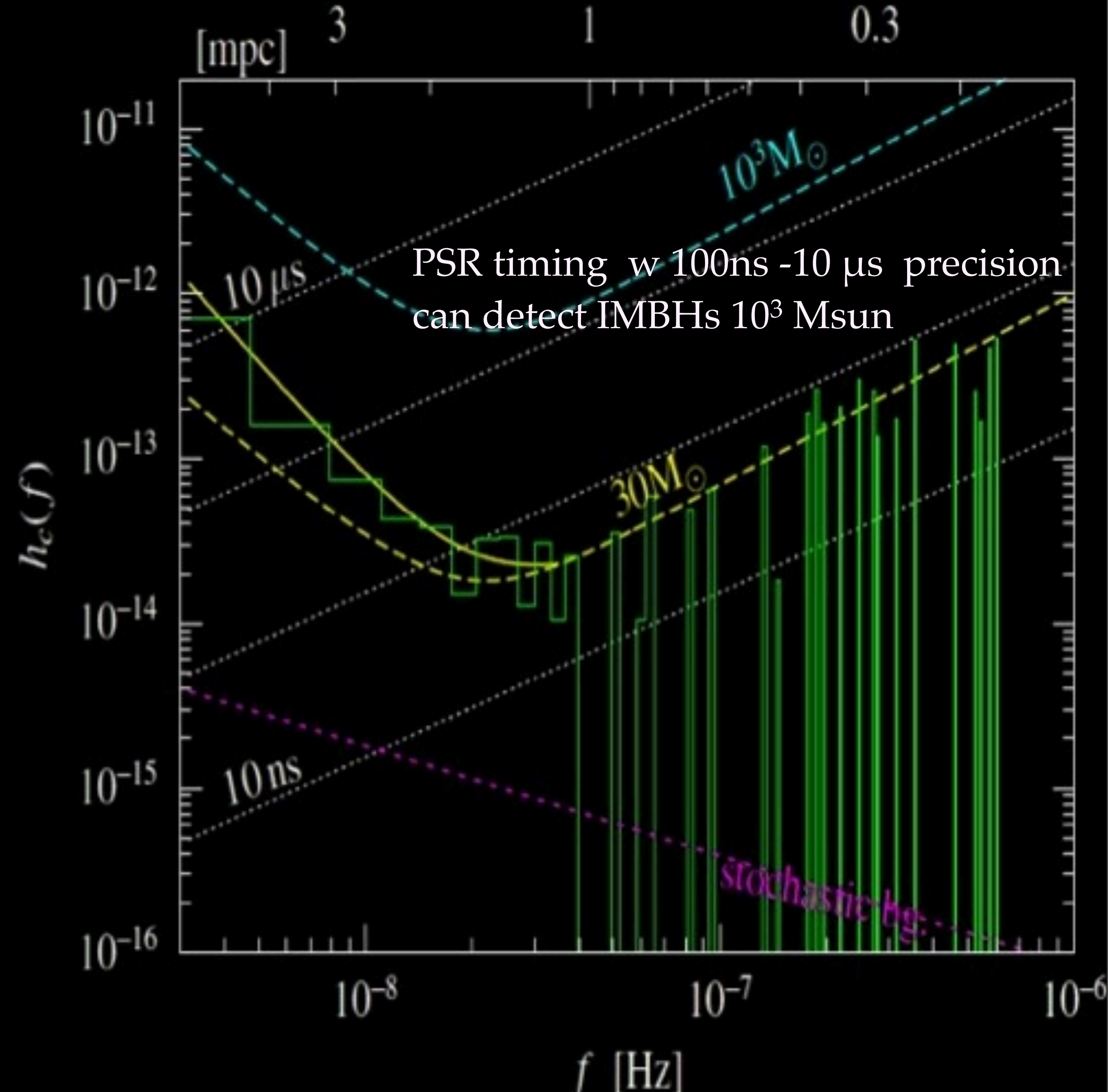


1,000 BHs, 30 M_{Sun}, within 1 pc, $n(r) \sim r^{-2.5}$

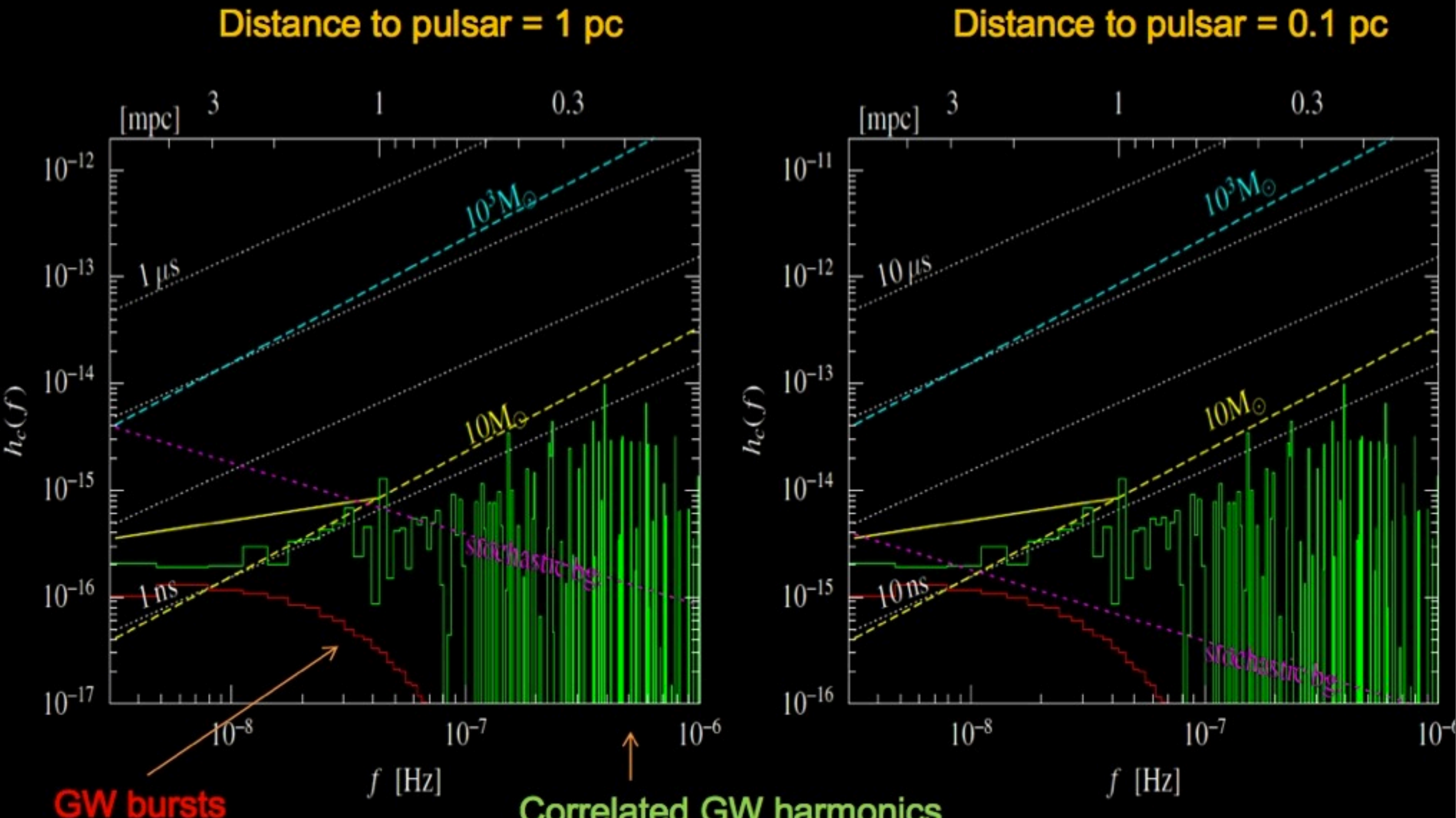
Distance to pulsar = 1 pc



Distance to pulsar = 0.1 pc



Eccentric GW sources



Pulsars near the Galactic Center

- A large population of pulsars may reside inside the GC (Pfahl & Loeb 2004; Lorimer & Kramer 2004). Wharton et al (2012) (also Macquart et al 2010) predict as many as 100 canonical PSRs and ten times larger population of ms PSRs in the central parsec of the GC. Within 100pc of GC at least three PSRs found (Johnston et al 2006; Deneva et al 2009).

- The discovery of GC magnetar SGR J1745-29 in the 1.2- 18.95 GHz radio bands (Eatough et al 2013; Bower et al 2013, Spitler et al 2013) shows: the source angular sizes are consistent with scatter broadened size of SgrA* at each frequency, demonstrating that the two sources, separated by 3" (0.12 pc) are located behind the same hyperstrong scattering medium, placing a "thin" screen at a distance of ~6 kpc from the GC. (Talk by Geoff Bower at this meeting).

- But the measurement of pulse broadening timescale at 1 GHz (Spitler et al 2013) is several orders of magnitude lower than the scattering predicted by NE2001 model (Cordes & Lazio 2002). Implication: the scattering in the GC is much lower than previously thought. The scattering material is patchy at angular scale of 10' arcmin (S. Roy 2013, see Poster P14) and it may be possible to see through such patchiness in the radio for a substantial region around GC.

- Absence of strong temporal broadening in PSRJ 1745-29 coupled with the angular broadening data shows that millisecond PSRs near SgrA* can be detected at frequencies above 10 GHz, while ordinary pulsars can be detected at frequencies of a few GHz.

- Moreover, the radio magnetars are a rare class of PSR (1 in ~500 radio pulsars) and the detection of PSR J1745-29 suggests that a large population of ordinary and ms PSRs may be present. Can utilize the ms PSRs for timing & GW detection.

Square Kilometre Array & PSRs near the Galactic Centre

- Most observed S-stars are within a few mpc of the Sgr A*. The young O/B stars will eventually turn into pulsars in SN explosions. Active pulsars/ ms PSRs may segregate to the outskirts of the GC as heavier objects sink inward.
- Timing accuracy of PSRs in the GC? (Liu et al 2012). Radiometer noise, pulse phase jitter, the interstellar scintillation of ISM towards GC.
- 1 hr timing accuracy of SKA will yield 10-100 μs for regular pulsars. For ms PSRs, it may be 10-100 ns.
- With 10-100 μs accuracy it is possible to individually resolve or rule out the existence of 10^3 Msun IMBHs within 5 mpc of Sgr A*.
- Currently, more than a dozen pulsars are have timing accuracy better than $\delta t \sim 250$ ns, the best ones reaching 20-30 ns (Hobbs et al 2010). SKA may find many more.

Conclusions

- BHs in orbit around SMBH SgrA* generates continuous GW spectrum with $f < 40$ nHz
- Individual BHs within 1 milliparsec to SgrA* stick out in spectrum at higher frequency
- GWs can be resolved by timing PSRs located within this region. A 100 ns- 10 μs timing accuracy with SKA will be sufficient to detect IMBHs (10^3 Msun) in a 3 yr obs if the PSR is 0.1- 1 pc away from SgrA*.
- Unlike EM imaging, can resolve individual objects through GW measurements - improves closer to SgrA*, even when density of objects increases steeply.
- Interstellar scattering: Angular broadening + Pulse broadening of PSR J1745-29 => GC searches can detect ms PSRs ($\nu > 10$ GHz) & ordinary PSRs at even lower frequencies (Bower et al arXiv:1309.4672). ms PSRs for accurate timing.

Thank You

Near field effects

- We assumed so far that the pulsar signal is modulated by the leading order radiative GW terms, and we neglected other post-Newtonian (PN) near-field effects. This is justified, so long as the SMBH-BH binary to the pulsar distance $D_p >> ƛ$, where

$$ƛ = \frac{1}{2\pi f} = 0.16\, f_8^{-1}\,\mathrm{pc}$$

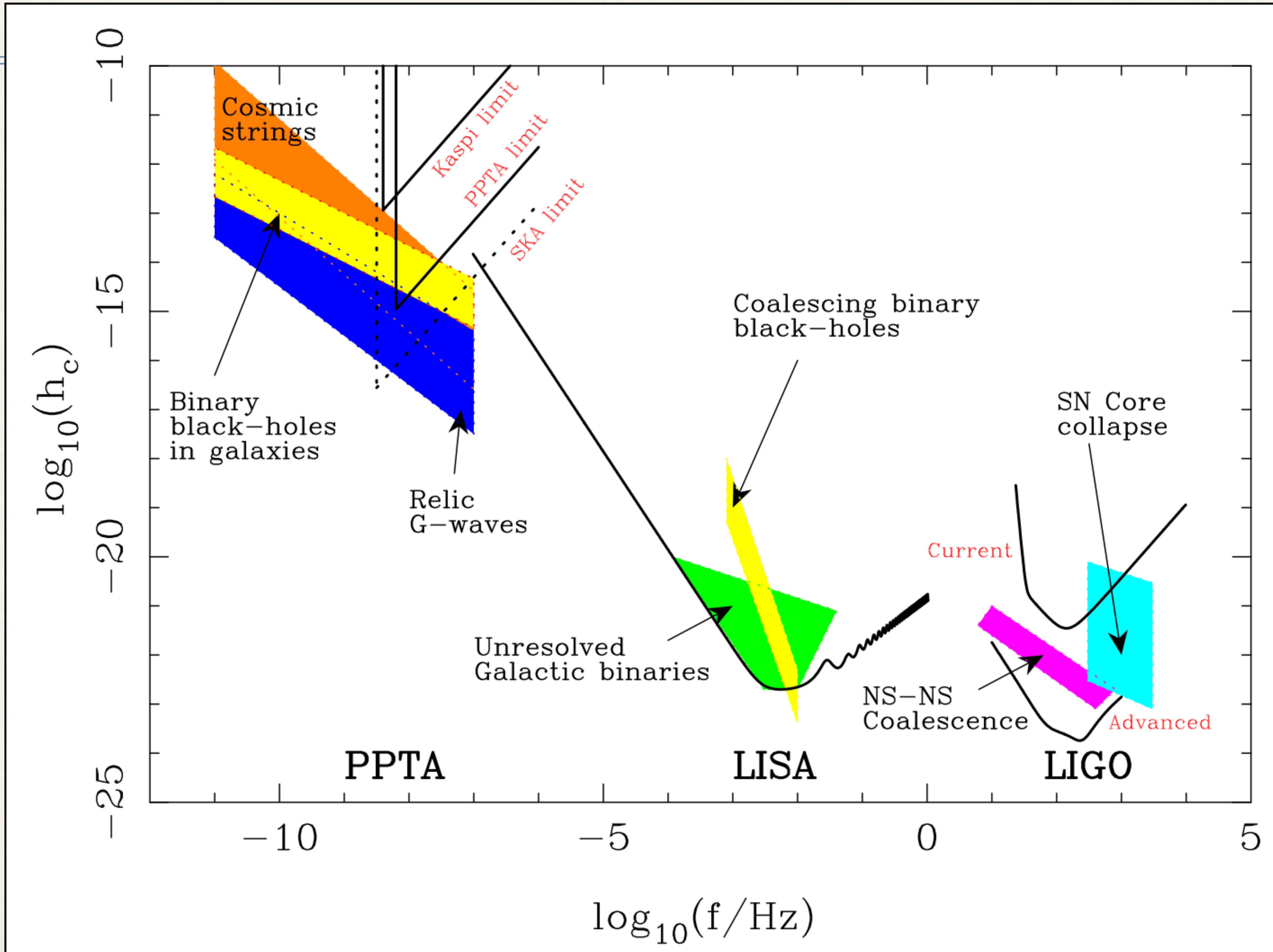
- The near field effects will be negligible for $f_8 > 1$ for $D_p > 1$ pc, but they are significant for pulsars closer to the center. The near zone metric induces a variation in the pulsar position and velocity and modulates the propagation of pulses. These are periodically varying terms.

Gravitational effects on the TOAs due to a nearby binary

Jenet, Creighton, Lommen (2005); Kocsis, Ray, Portegies Zwart (2012)

- Gravitational wave $$\delta t_{\rm GW} \sim \frac{M_\bullet m_\star}{D\, r(f)} \frac{\sqrt{fT}}{2\pi f}$$
- Periodically varying terms in the near-field metric:
  - redshift, Doppler shift, etc.
  - due to current dipole $$\delta t_{\rm cd} \sim \frac{m_\star v r}{D^2} \frac{\sqrt{fT}}{2\pi f}$$
  - and mass quadrupole $$\delta t_{\rm mq} \sim \frac{m_\star r^2}{D^3} \frac{\sqrt{fT}}{2\pi f}$$
- "Indirect" effects analogous to Roemer delay
  - Epicyclic motion of the pulsar induced by the binary
  - Due to current dipole $$\delta t_{\rm cd,R} \sim \frac{m_\star v r}{D^3} \frac{\sqrt{fT}}{(2\pi f)^2}$$
  - and mass quadrupole $$\delta t_{\rm mq,R} \sim \frac{m_\star r^2}{D^4} \frac{\sqrt{fT}}{(2\pi f)^2}$$

Gravitational Wave Spectrum



From Manchester 2006; see also Hobbs arXiv 1006.3969