NAASC

A Crash Course in Radio Astronomy and Interferometry: 1. Basic Radio/mm Astronomy

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(thanks to S. Dougherty, C. Chandler, D. Wilner & C. Brogan)

Intensity & Flux Density

EM power in bandwidth δν from solid angle δΩ intercepted by surface δA is:

$$\delta W = I_\nu \delta\Omega \delta A \delta\nu$$

Defines surface brightness $\boldsymbol{I_\nu}$ (W m^{-2} Hz^{-1} sr^{-1} ; aka specific intensity)

Flux density $\boldsymbol{S_\nu}$ (W m^{-2} Hz^{-1}) – integrate brightness over solid angle of source

$$S_\nu = \int_{\Omega_s} I_\nu d\Omega$$

Convenient unit – the **Jansky** ➔ 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Note: $S_\nu = L_\nu / 4\pi d^2$ ie. distance dependent

$\Omega \propto 1/d^2 \Rightarrow I_\nu \propto S_\nu / \Omega$ ie. distance independent

Surface Brightness

In general surface brightness is position dependent, ie. $I_\nu = I_\nu(\theta,\phi)$

$$I_\nu(\theta,\varphi) = \frac{2k\nu^2 T(\theta,\varphi)}{c^2}$$

(if I_ν described by a blackbody in the Rayleigh-Jeans limit; $h\nu/kT << 1$)

Back to flux:

$$S_\nu = \int_{\Omega_s} I_\nu(\theta,\varphi)d\Omega \quad = \frac{2k\nu^2}{c^2}\int T(\theta,\varphi)d\Omega$$

In general, a radio telescope maps the *temperature distribution of the sky*

Brightness Temperature

Many astronomical sources DO NOT emit as blackbodies!

However….

Brightness temperature (T_B) of a source is defined as the temperature of a blackbody with the same surface brightness at a given frequency:

$$I_\nu = \frac{2k\nu^2 T_B}{c^2}$$

This implies that the flux density $S_\nu = \int_{\Omega_s} I_\nu d\Omega = \frac{2k\nu^2}{c^2} \int T_B d\Omega$

What does a Radio Telescope Detect?

Recall : $$\delta W = I_\nu \delta\Omega \delta A \delta \nu$$

Telescope of effective area A_e receives power P_{rec} per unit frequency from an unpolarized source but is only sensitive to one mode of polarization:

$$P_{rec} = \frac{1}{2} I_\nu A_e \delta\Omega$$

Telescope is sensitive to radiation from more than one direction with *relative* sensitivity given by the normalized antenna pattern $P_N(\theta,\varphi)$:

$$P_{rec} = \frac{1}{2} A_e \int_{4\pi} I_\nu(\theta,\varphi) P_N(\theta,\varphi) d\Omega$$

Antenna Temperature

Johnson-Nyquist theorem (1928): $P = kT$

Power received by the antenna: $P_{rec} = kT_A$

$$P_{rec} = \frac{A_e}{2} \int_{4\pi} I_\nu(\theta,\varphi) P_N(\theta,\varphi)\, d\Omega$$

$$\therefore T_A = \frac{A_e}{2k} \int_{4\pi} I_\nu(\theta,\varphi) P_N(\theta,\varphi)\, d\Omega$$

Antenna temperature is what is observed by the radio telescope.

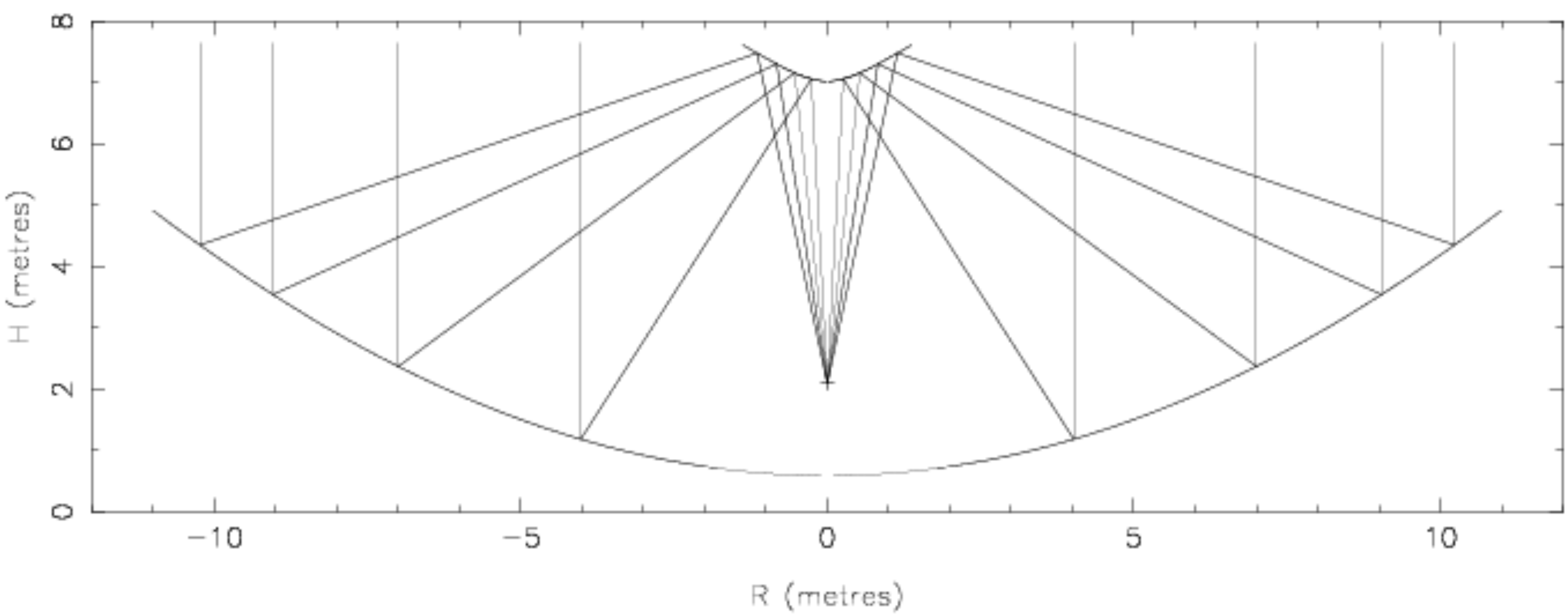
A "convolution" of sky brightness with the beam pattern

It is an inversion problem to determine the source temperature distribution.

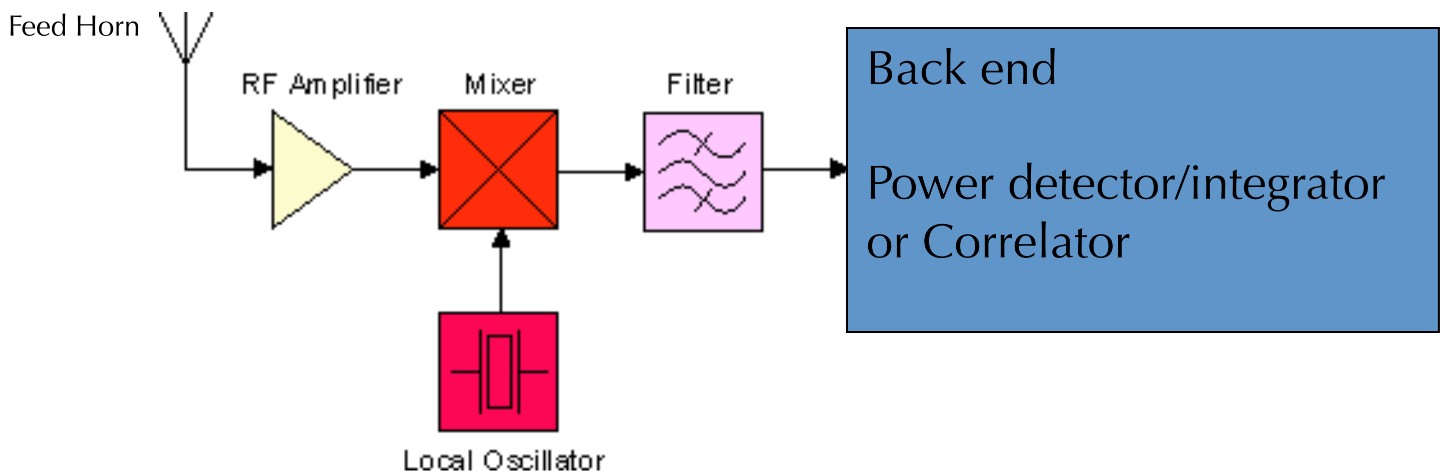
Radio Telescopes

The *antenna* collects the E-field over the aperture at the focus

The *feed horn* at the focus adds the fields together, guides signal to the *front end*



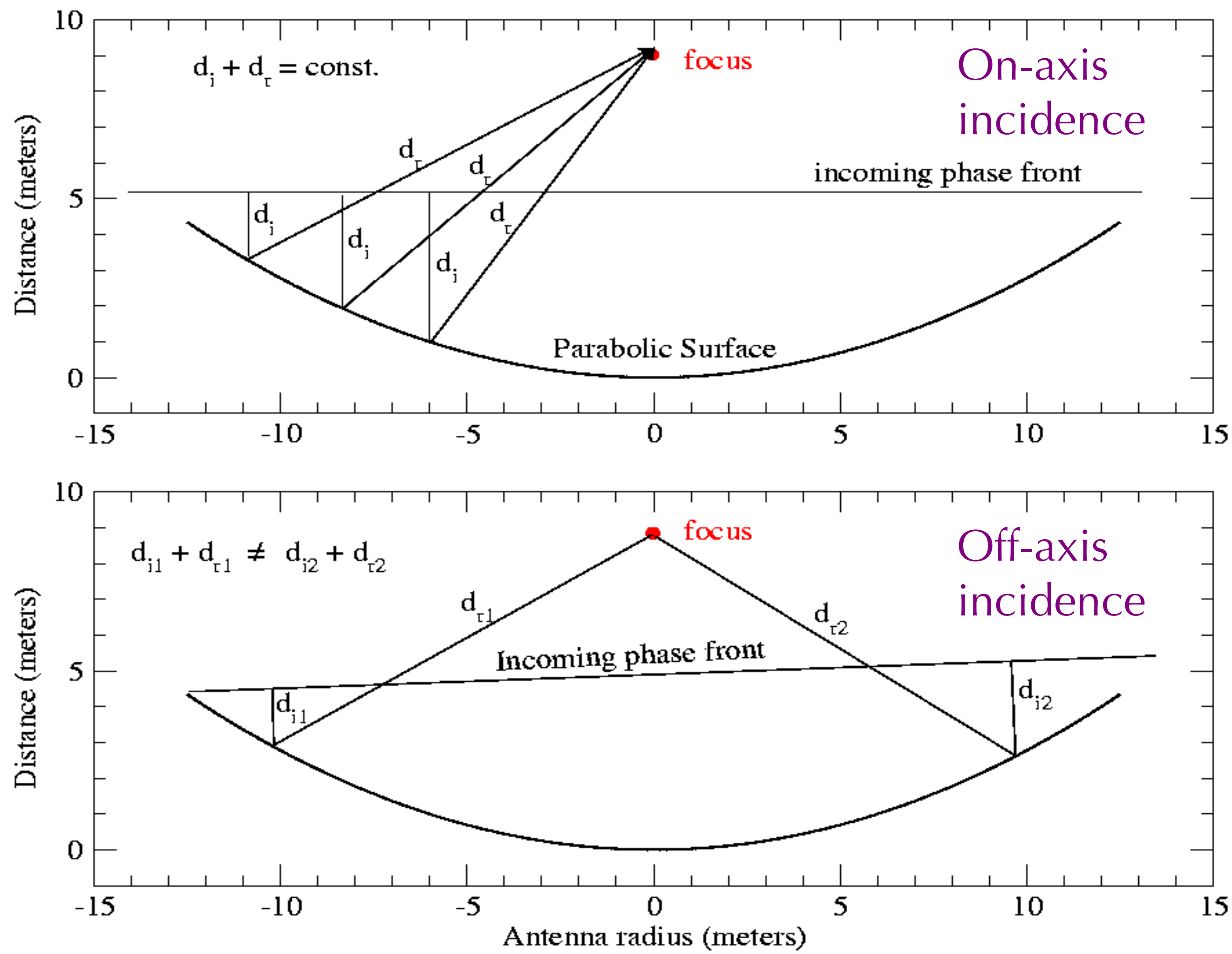
Components of a Heterodyne System



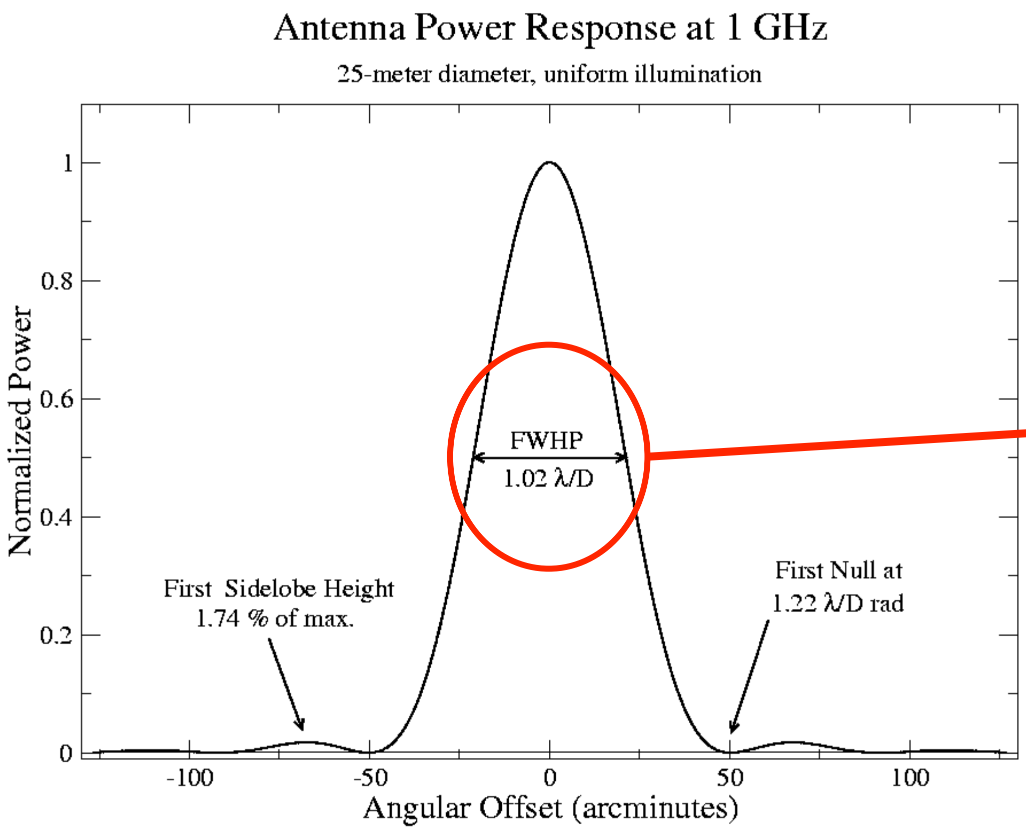
- *Amplifier*
  - amplifies a very weak radio frequency (RF) signal, is stable & low noise
- *Mixer*
  - produces a stable lower, intermediate frequency (IF) signal by mixing the RF signal with a stable local oscillator (LO) signal, is tunable
- *Filter* – selects a narrow signal band out of the IF
- *Backend* – either total power detector or more typically today, a correlator

Origin of the Beam Pattern

- Antenna response is a coherent phase summation of the E-field at the focus
- First null occurs at the angle where one extra wavelength of path is added across the full aperture width, i.e., $\theta \sim \lambda/D$



Antenna Power Pattern



Defines telescope resolution

- The voltage response pattern is the FT of the aperture distribution
- The power response pattern, $P(\theta) \propto V^2(\theta)$, is the FT of the autocorrelation function of the aperture
- for a uniform circle, $V(\theta)$ is $J_1(x)/x$ *and* $P(\theta)$ is the Airy pattern, $(J_1(x)/x)^2$

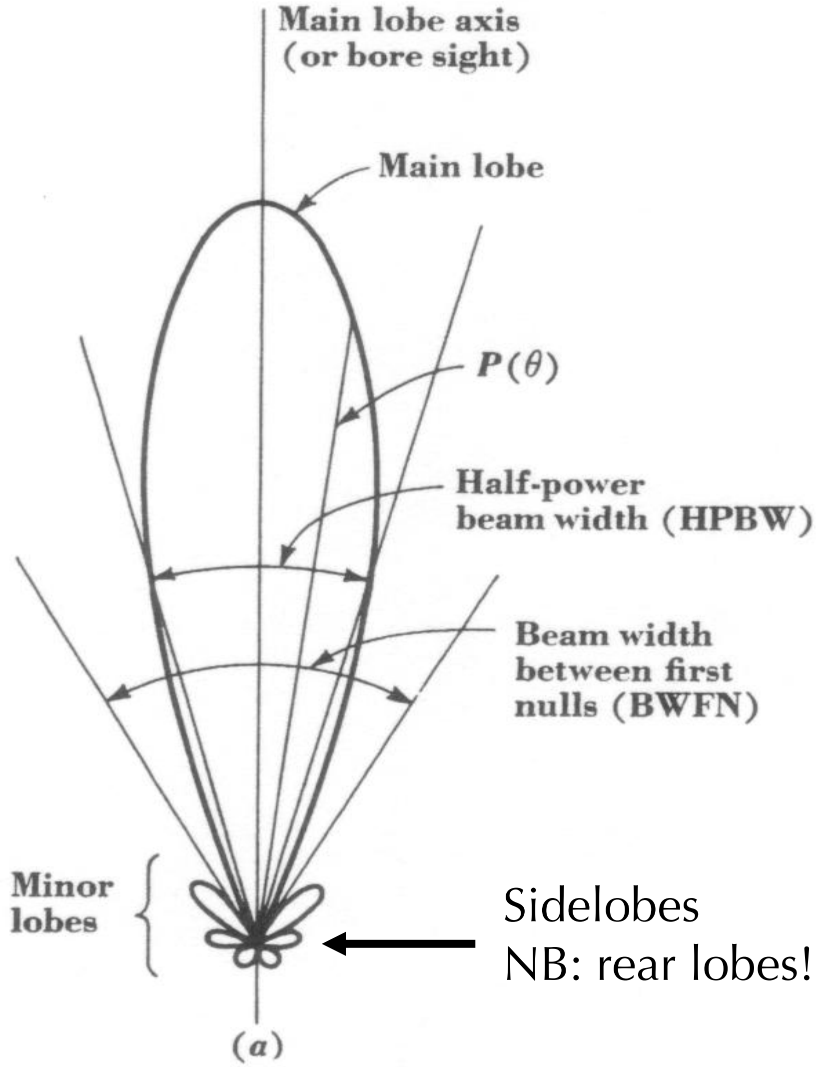
The Beam

The antenna "beam" solid angle on the sky is:

$$\Omega_A = \int_{4\pi} P(\theta,\phi)\, d\Omega$$

Telescope beams @ 345 GHz

	D (m)	*θ (")*
AST/RO	1.7	103
JCMT	15	15
LMT	50	4.5
SMA	508	0.35
ALMA	15 000	0.012



Sidelobes
NB: rear lobes!

Sensitivity (Noise)

Unfortunately, the telescope system itself contributes noise to the the signal detected by the telescope, i.e.,

$$P_{out} = P_A + P_{sys} \rightarrow T_{out} = T_A + T_{sys}$$

The *system temperature*, T_{sys}, represents noise added by the system:

$$T_{sys} = T_{bg} + T_{sky} + T_{spill} + T_{loss} + T_{cal} + T_{rx}$$

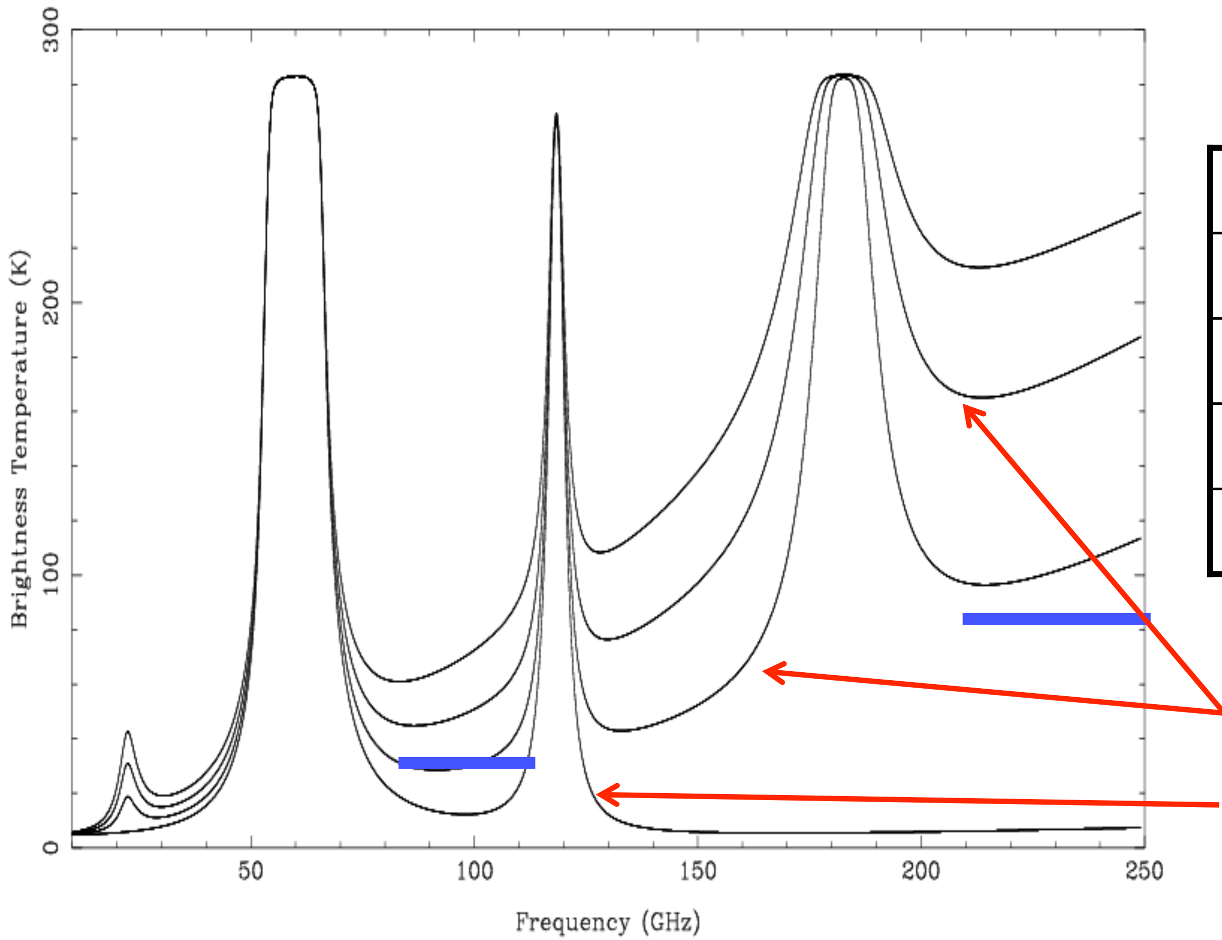
T_{bg} = microwave and galactic background (3K, except below 1GHz)
T_{sky} = atmospheric emission (increases with frequency--dominant in mm)
T_{spill} = ground radiation (via sidelobes) (telescope design)
T_{loss} = losses in the feed and signal transmission system (design)
T_{cal} = injected calibrator signal (usually small)
T_{rx} = receiver system (often dominates at cm — a design challenge)

Note that T_{bg}, T_{sky}, and T_{spill} vary with sky position and T_{sky} is time variable

Sensitivity (Noise)

In the mm/submm regime, T_{sky} is the challenge (especially at low elevations)

In general, T_{rx} is essentially at the quantum limit, and $T_{rx} < T_{sky}$



"Wet" component: H_2O

"Dry" component: O_2

ALMA T_{rx}

	GHz	T_{rx} (K)
Band 3	84-116	< 37
Band 6	211-275	< 83
Band 7	275-370	< 83
Band 9	602-720	< 175

Sensitivity (Noise)

Q: How can you detect T_A (signal) in the presence of T_{sys} (noise)?
A: The signal is correlated from one sample to the next but the noise is not

For bandwidth $\Delta\nu$, samples taken less than $\Delta\tau = 1/\Delta\nu$ are not independent
(Nyquist sampling theorem!)

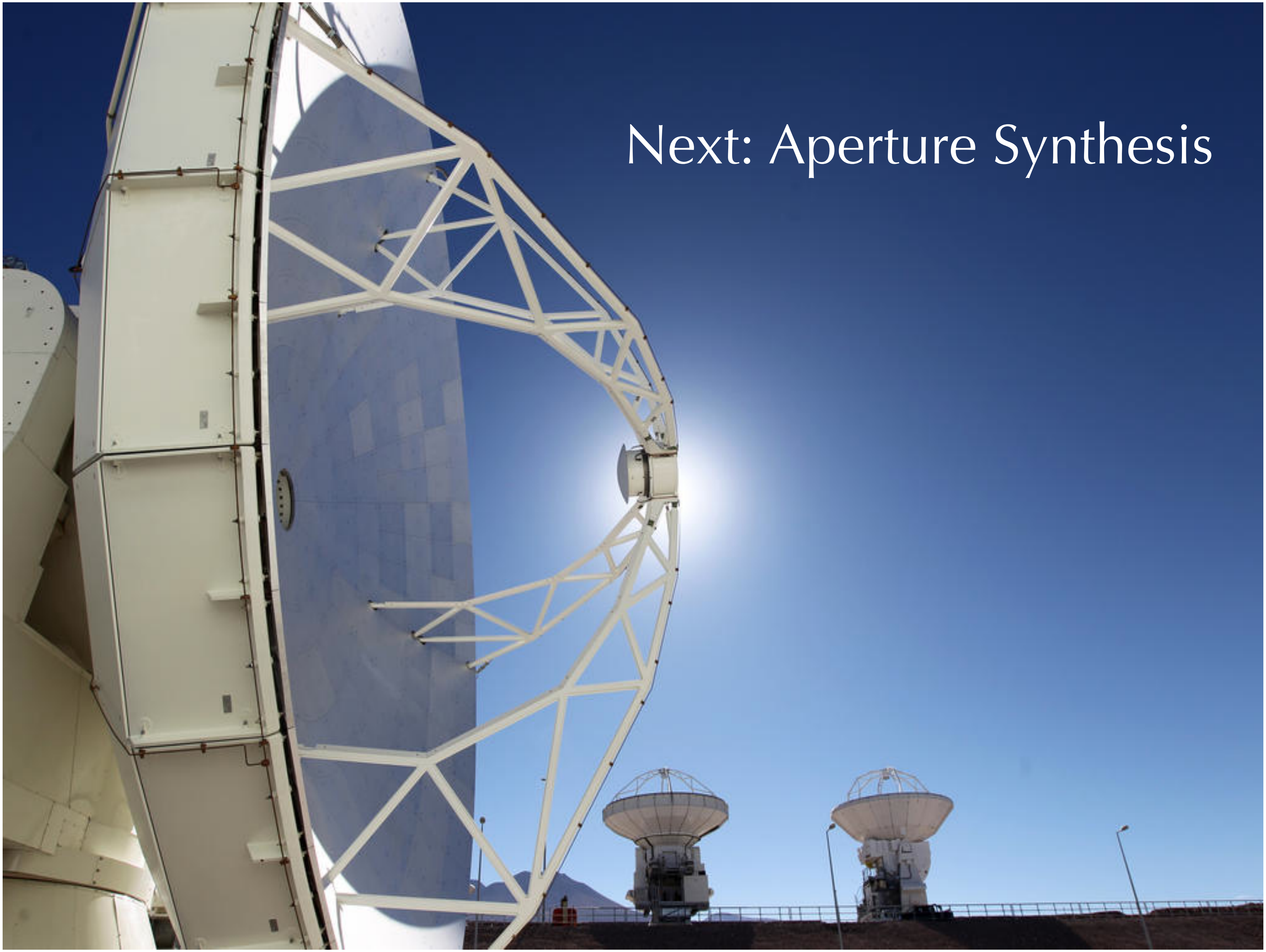
Time τ contains $N = \tau / \Delta\tau = \tau\,\Delta\nu$ independent samples

For Gaussian noise, total error for *N* samples is $1/\sqrt{N}$ that of single sample

$$\therefore \quad \frac{\Delta T_A}{T_{sys}} = \frac{1}{\sqrt{\tau\,\Delta\nu}}$$

Radiometer equation

$$SNR = \frac{T_A}{\Delta T_A} = \frac{T_A}{T_{sys}}\sqrt{\tau\,\Delta\nu}$$


Next: Aperture Synthesis